Part A - Comments on the papers of Burovski *et al*.

Part B - On Superfluid Properties of Asymmetric Dilute Fermi Systems

Part A

Comments on papers of

E. Burovski, N. Prokof'ev, B. Svistunov and M. Troyer

- Phys. Rev. Lett. 96, 160402 (2006)
- cond-mat/0605350 version 2, New Journal of Physics, in e-press, August (2006)

by A. Bulgac, J.E. Drut and P. Magierski

Determinant Diagrammatic Monte Carlo

The partition function is expanded in a power series in the interaction

$$Z = \sum_{n=0}^{\infty} (-U)^n \sum_{\mathbf{x}_1 \ldots \mathbf{x}_n} \int_{0<\tau_1<\tau_2<\ldots<\beta} \prod_{j=1}^{n} d\tau_j \, \mathrm{Tr}\left[e^{-\beta H_0} \prod_{j=1}^{n} c_\uparrow^\dagger(\mathbf{x}_j\tau_j) c_\uparrow(\mathbf{x}_j\tau_j) c_\downarrow^\dagger(\mathbf{x}_j\tau_j) c_\downarrow(\mathbf{x}_j\tau_j) \right]$$

It is notoriously known that the pairing gap is a non-analytical function of the interaction strength and that no power expansion of pairing gap exists. It is completely unclear why an expansion of this type should describe correctly the pairing properties of a Fermi gas at unitarity.

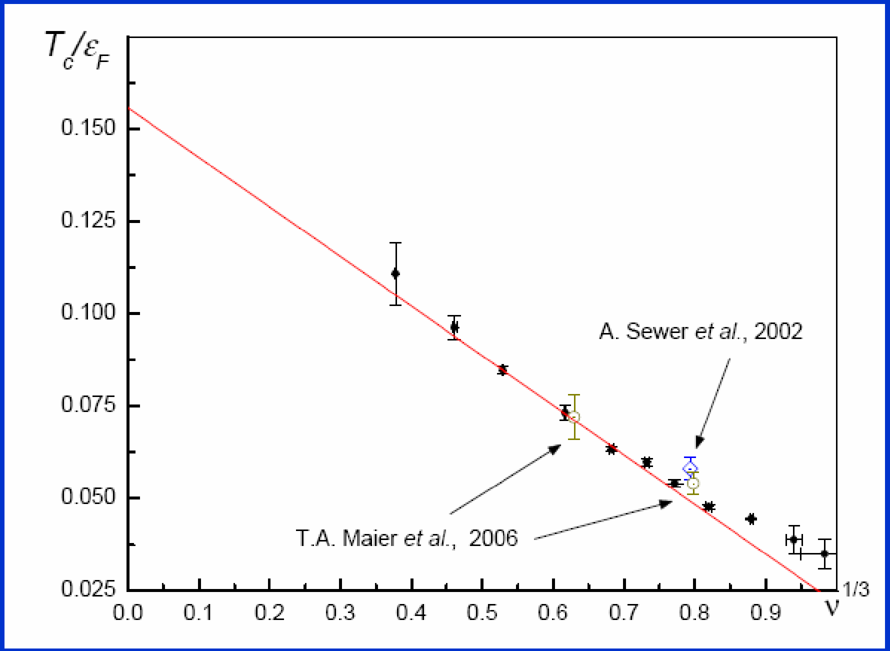
Extrapolation prescription used by Burovski *et al.*

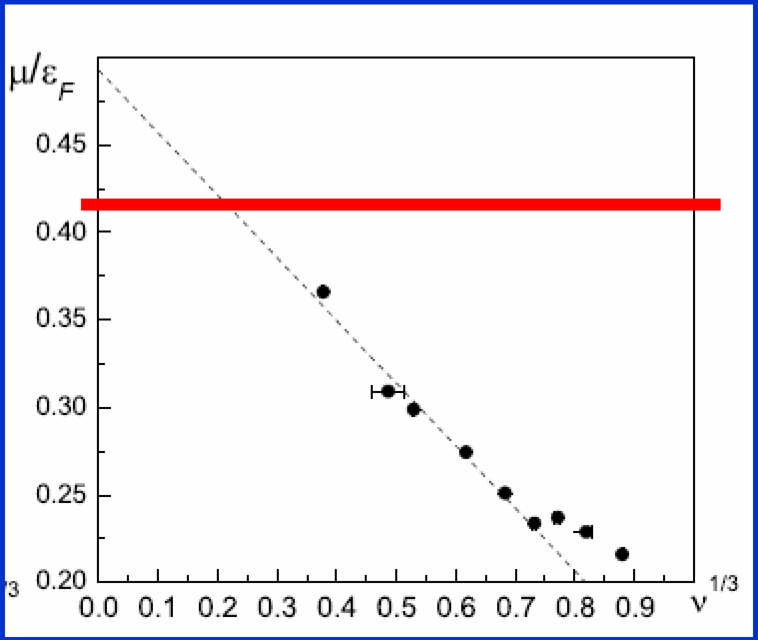
Argument based on comparing the continuum and lattice T-matrix at unitarity.

$$\frac{\Gamma(\xi,\mathrm{p}) - \Gamma_{\mathrm{cont}}(\xi,\mathrm{p})}{\Gamma(\xi,\mathrm{p})} \sim \nu^{1/3}$$

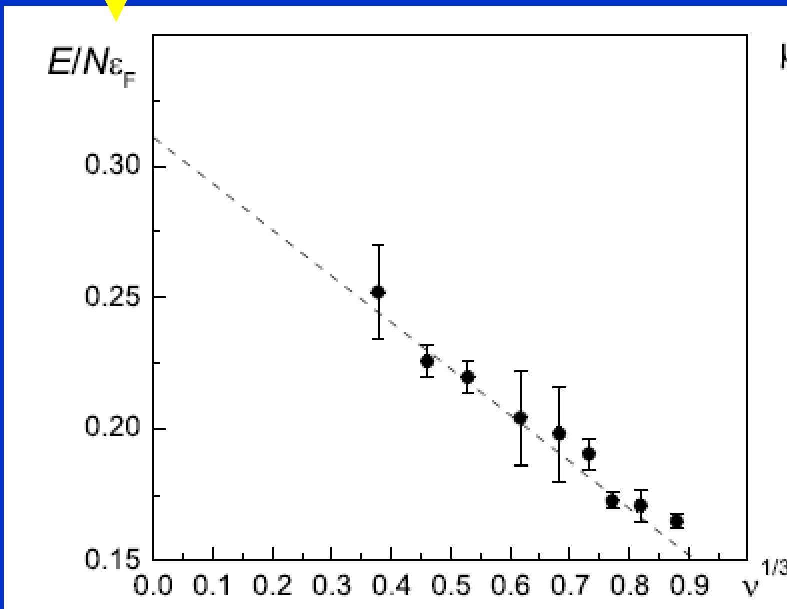
However:

- **T-matrix is governed by the scattering length, which is infinite at unitarity**
- **many Fermion system is governed by Fermi wave length, which is finite at unitarity**
- **large error bars and clear non-linear dependence**

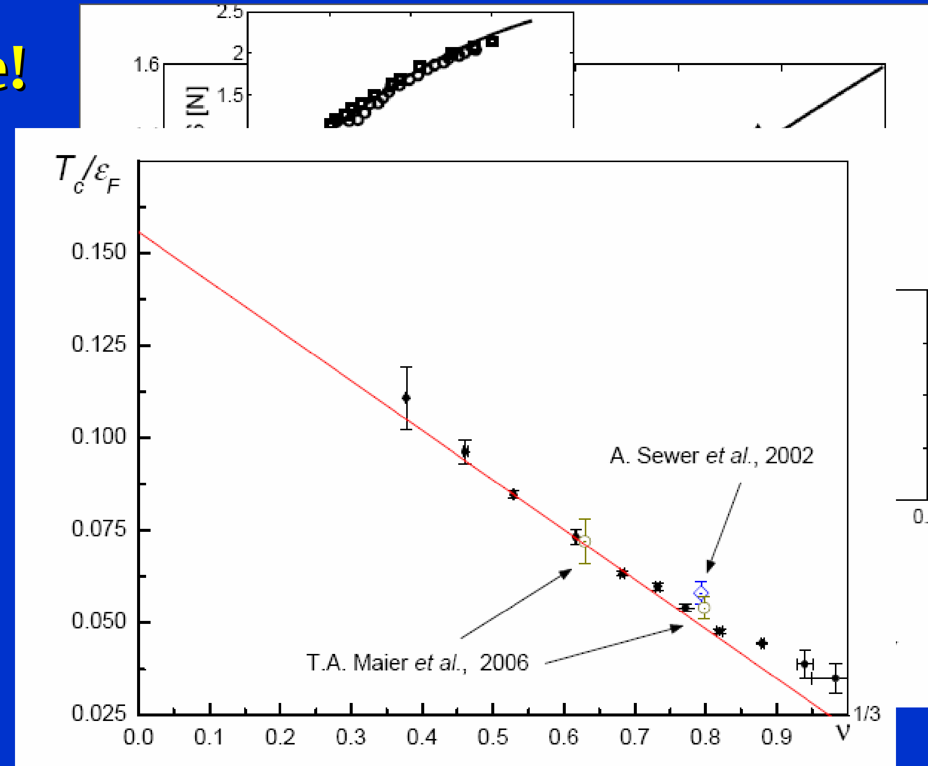




Energy at critical temperature
Notice the strong size dependence!



Burovski *et al.*



Bulgac *et al.*

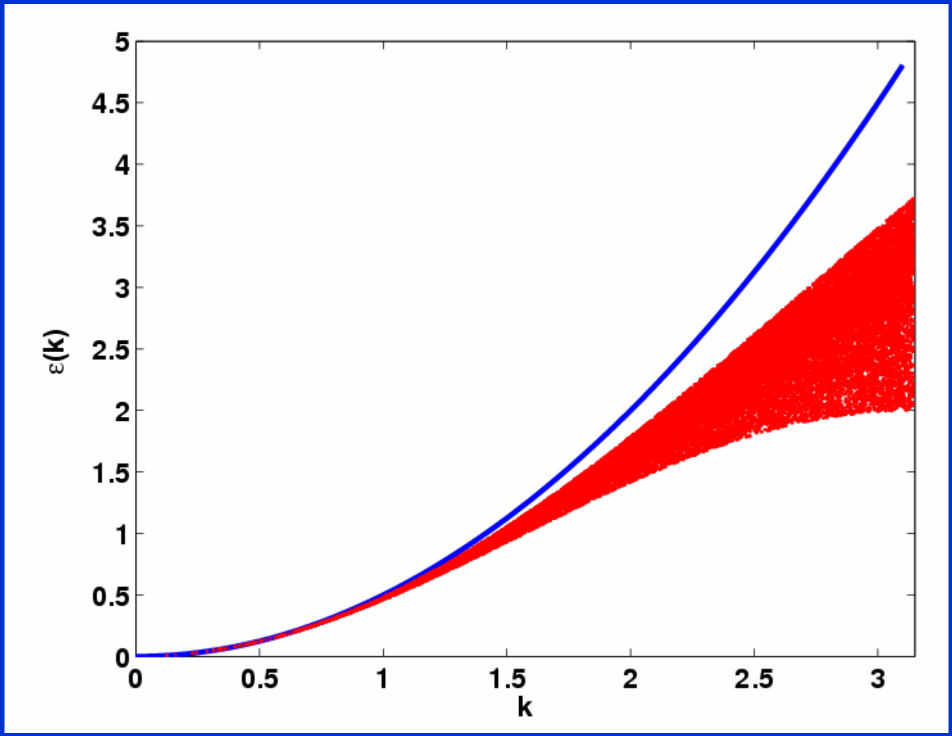
extrapolation. The overall shape of the caloric curve seem to be little affected by the finite volume of the system. This is hardly surprising since even in the thermodynamic limit $E(T)$ and its derivative $\mathrm{d}E/\mathrm{d}T$ are continuous at the transition point. These

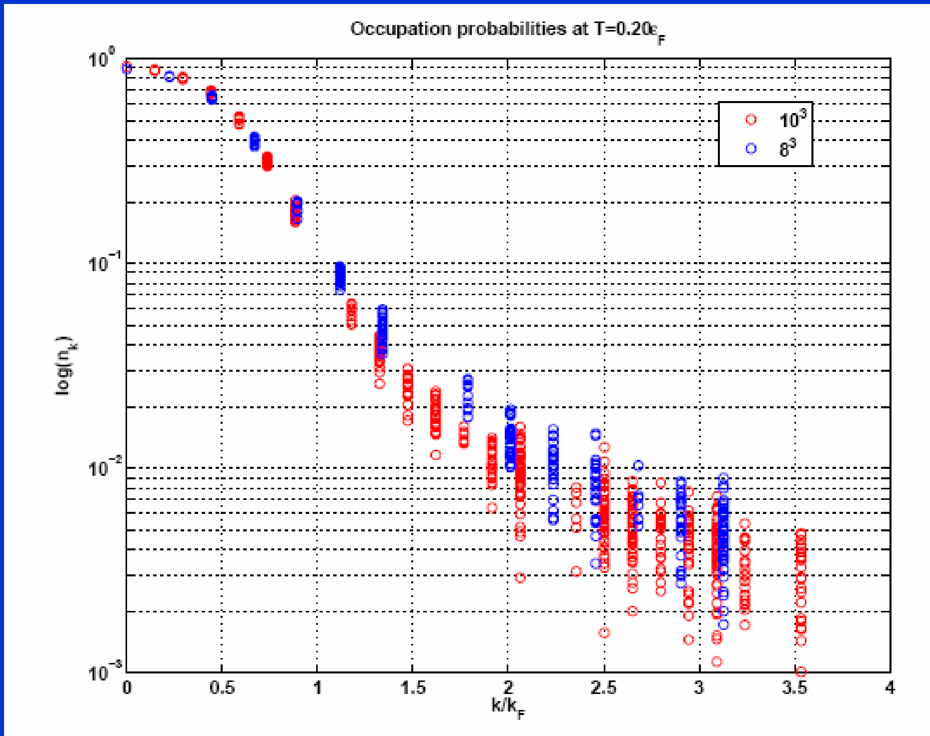
It is clearly seen that the presence of the lattice suppresses the critical temperature considerably, nearly by a factor of 4, depending on the filling factor. Strong dependence of T_c on ν is in apparent contradiction with Ref. [24], which claims weak or no ν-dependence. This disagreement might be due to the difference in the single-particle spectra $\varepsilon_{\mathbf{k}}$ used: Ref. [24] employs the parabolic spectrum with spherically symmetric

Burovski *et al.*

Single particle kinetic energy and occupation probabilities

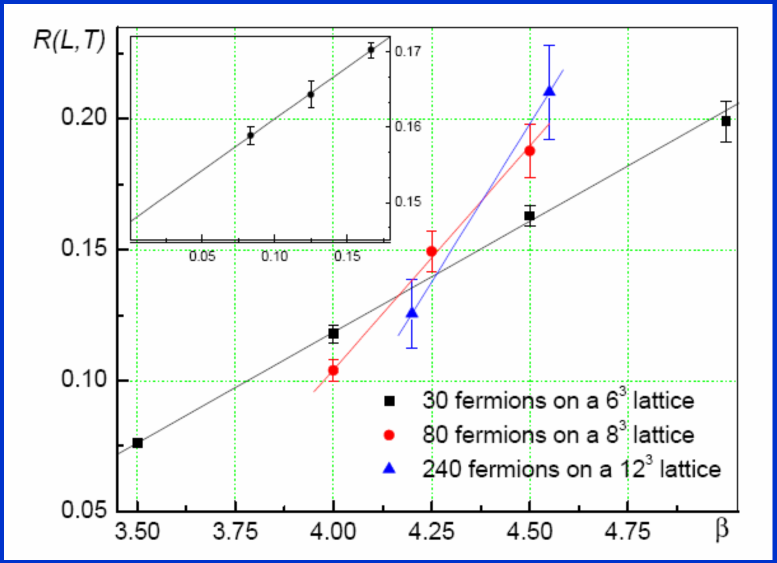
- We have found that in order to have a reasonable accuracy the highest momentum states should have an occupation probability of less than 0.01!
-Notice the large difference, and the spread of values, between the kinetic energy of the free particle and the kinetic energy in the Hubbard model, even at half the maximum momentum (one quarter of the maximum energy)





Occupation probabilities are from our results, were we treat the kinetic energy exactly. Burovski *et al.* have not considered them this quantity. Since both groups have similar filling factors, we expect large deviations from continuum limit.

Finite size scaling

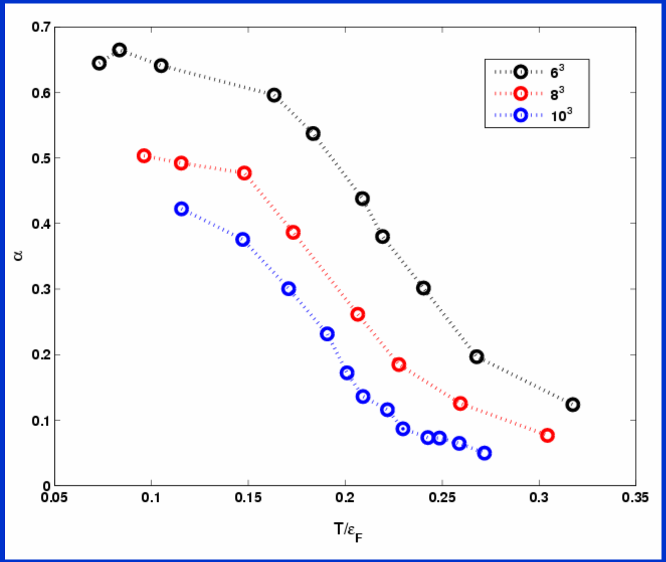


Burovski *et al.*

$$T_c = 0.157(7)\ \varepsilon_F$$

These authors however never displayed the order parameter as function of T and we have to assume that the phase transition really exists in their simulation.

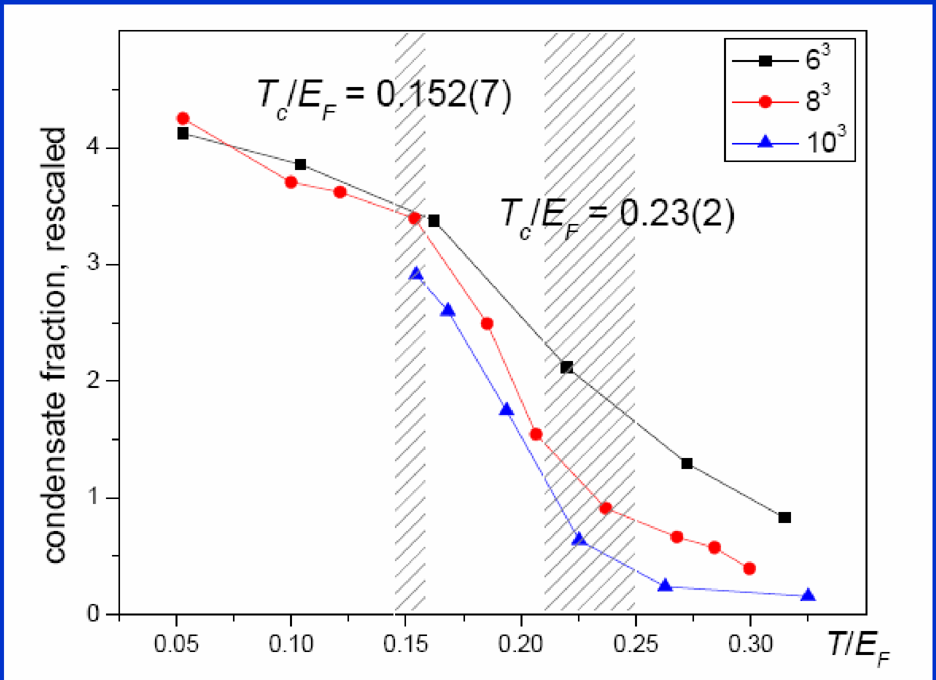
Condensate fraction

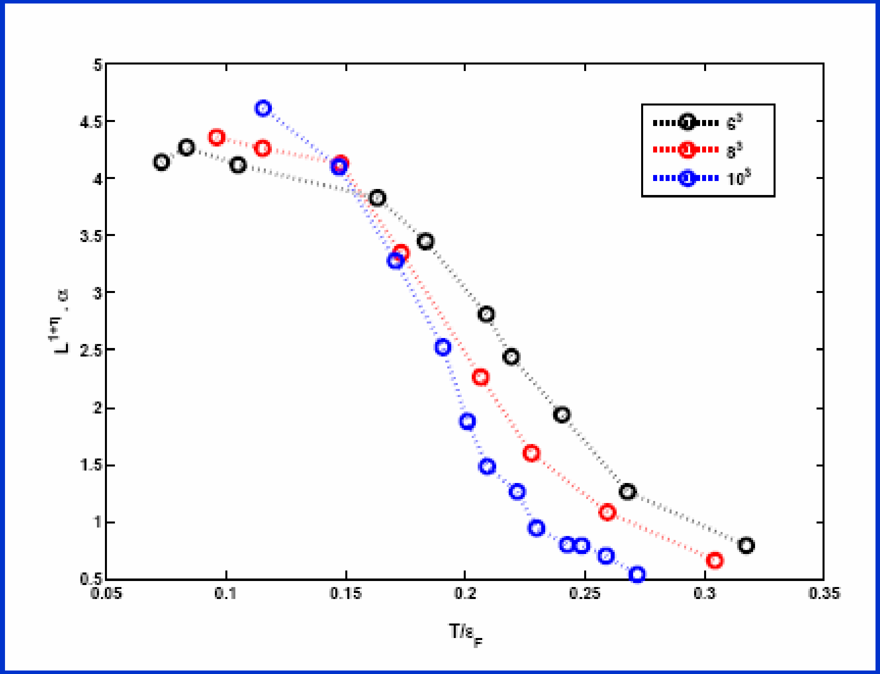


Bulgac *et al.*
(new, preliminary results)

$$T_c \approx 0.23(3)\ \varepsilon_{\mathrm{F}}$$

Value consistent with behavior of other thermodynamic quantities





Preliminary new data!

Finite size scaling consistent with our previously determined value

$$T_c \approx 0.23\ \varepsilon_F$$

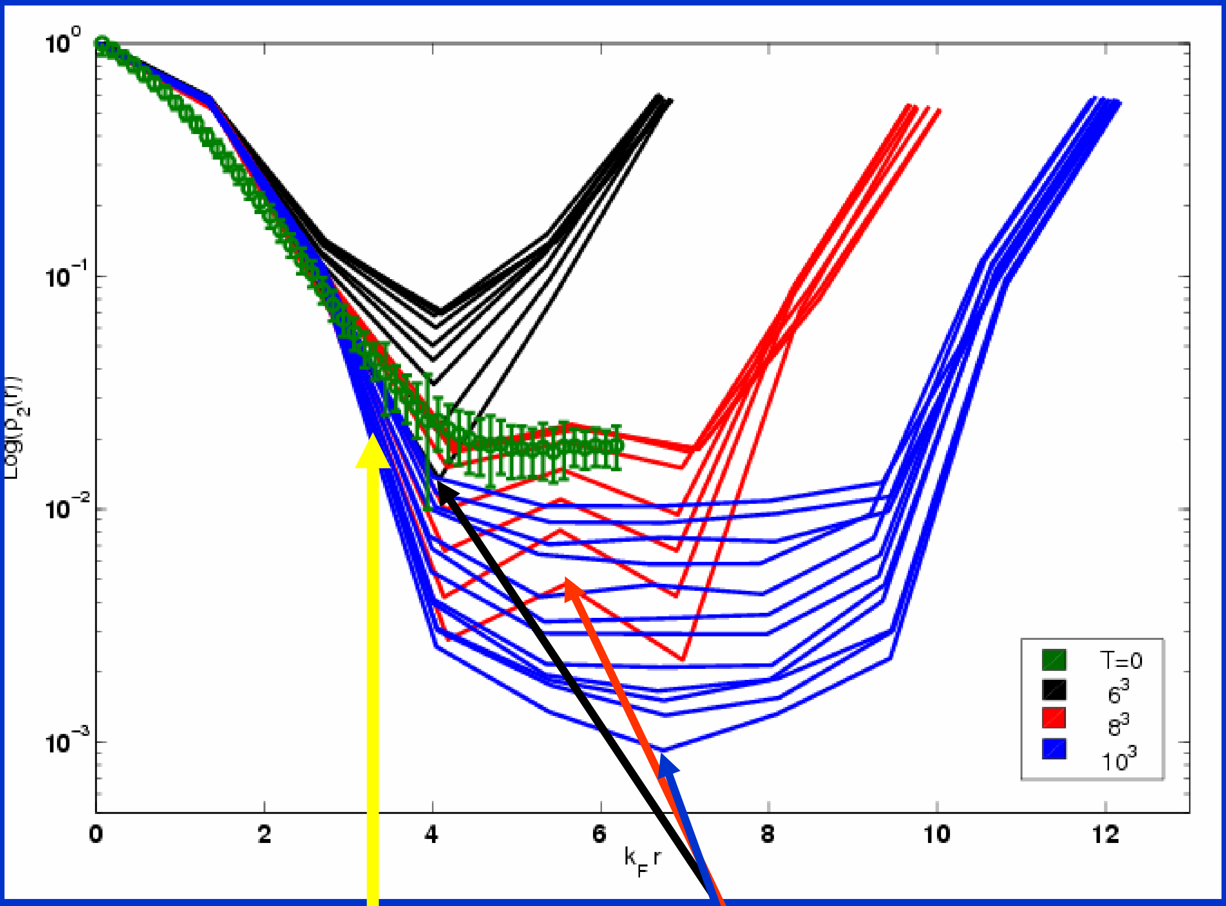
temperature. The intersection of scaled curves turns out to be inconsistent with the estimate for T_c derived from the caloric curve inspection.

?

Burovski *et al.*

However, see next slide

Two-body correlation function, condensate fraction



System dependent scale
Here the Fermi wavelength

L/2

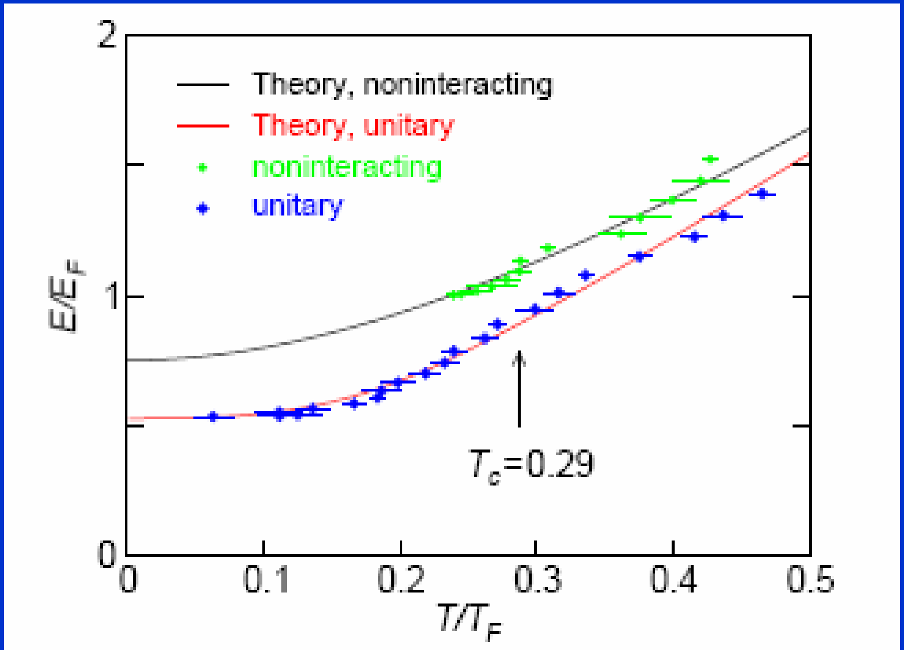
Green symbols, T=0 results of Astrakharchick et al, PRL 95, 230405 (2005)

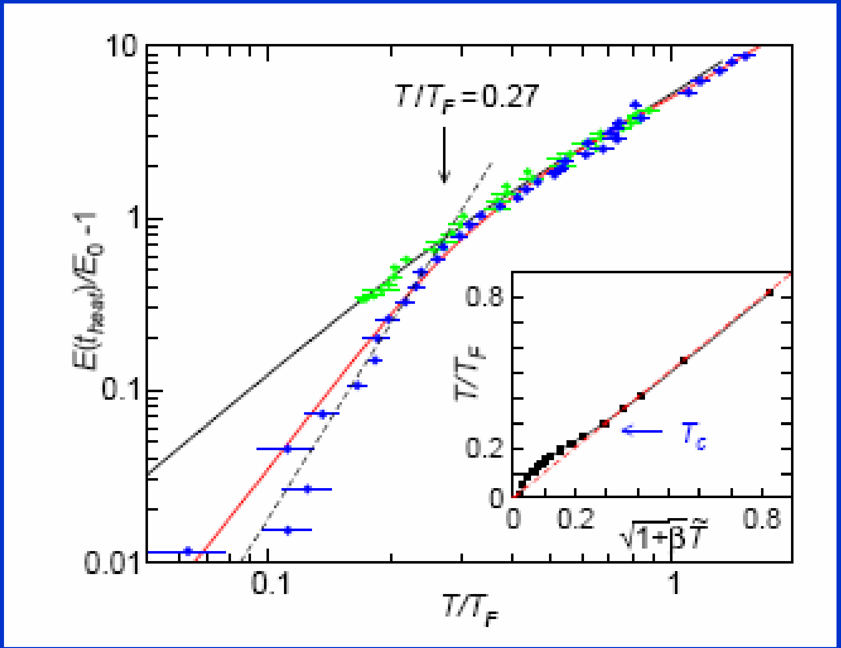
Power law critical scaling expected between the Fermi wavelength and L/2 !!! <u>Clearly L is in all cases too small!</u>

The energy of a Fermi gas at unitarity in a trap at the critical temperature is determined experimentally, even though T is not.

$$\frac{E(T_c)}{E(0)} - 1 = \begin{cases} 0.8 & \text{Kinast et al, Science 307, 1296(2005)} \\ 0.2 & \text{Burovski et al. cond-mat/0605350} \\ 0.6 & \text{Bulgac et al. PRL 96, 090404 (2006)} \end{cases}$$

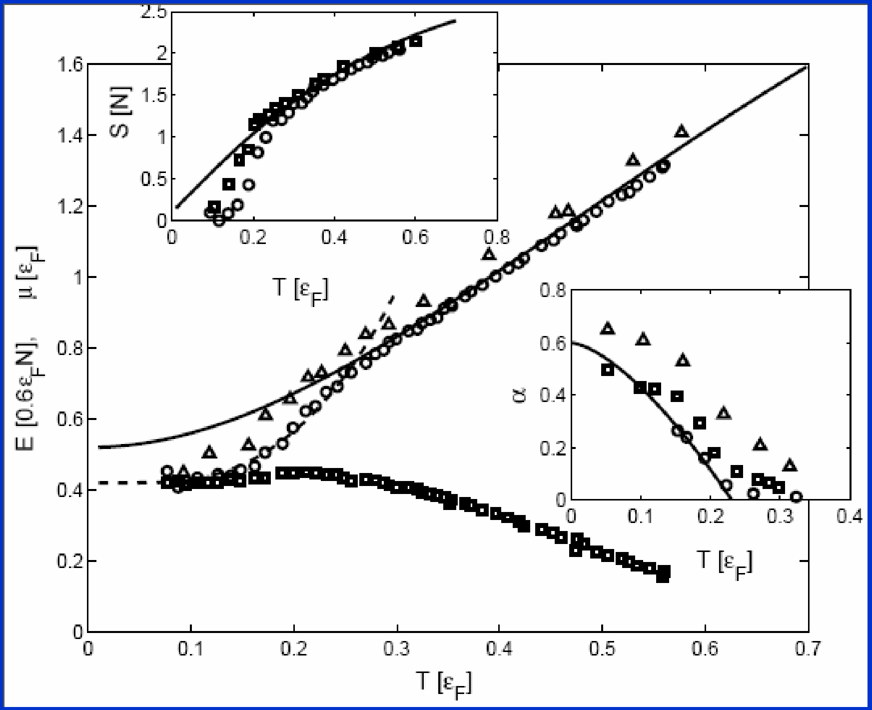
estimated





Kinast et al. Science, 307, 1296 (2005)

Our conclusions based on our results



Below the transition temperature the system behaves as a free condensed Bose gas (!), which is superfluid at the same time! No thermodynamic hint of Fermionic degrees of freedom!

Above the critical temperature one observes the thermodynamic behavior of a free Fermi gas! No thermodynamic trace of bosonic degrees of freedom!

New type of fermionic superfluid.

Part B

On Superfluid Properties of Asymmetric Dilute Fermi Systems

Aurel Bulgac, Michael McNeil Forbes and Achim Schwenk
Department of Physics, University of Washington

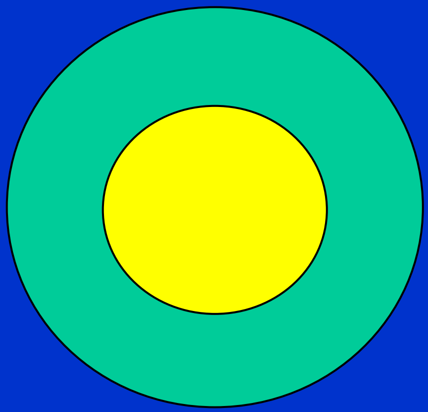
Outline:

- **Induced *p*-wave superfluidity in asymmetric Fermi gases**

  Bulgac, Forbes, and Schwenk, cond-mat/0602274, Phys. Rev. Lett. 97, 020402 (2006)

- **T=0 thermodynamics in asymmetric Fermi gases at unitarity**

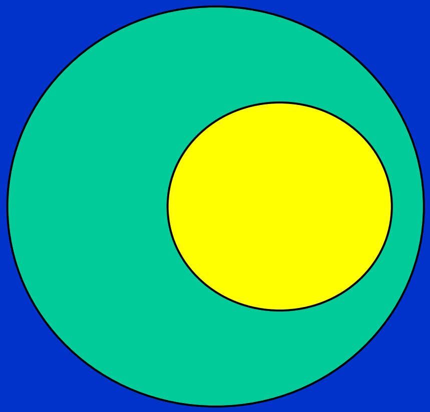
  Bulgac and Forbes, cond-mat/0606043



Green – spin up

Yellow – spin down

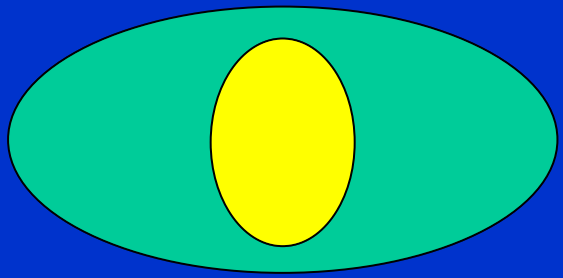
If $\left|\mu_\uparrow - \mu_\downarrow\right| < \frac{\Delta}{\sqrt{2}}$ the same solution as for $\mu_\uparrow = \mu_\downarrow$



LOFF (1964) solution

Pairing gap becomes a spatially varying function

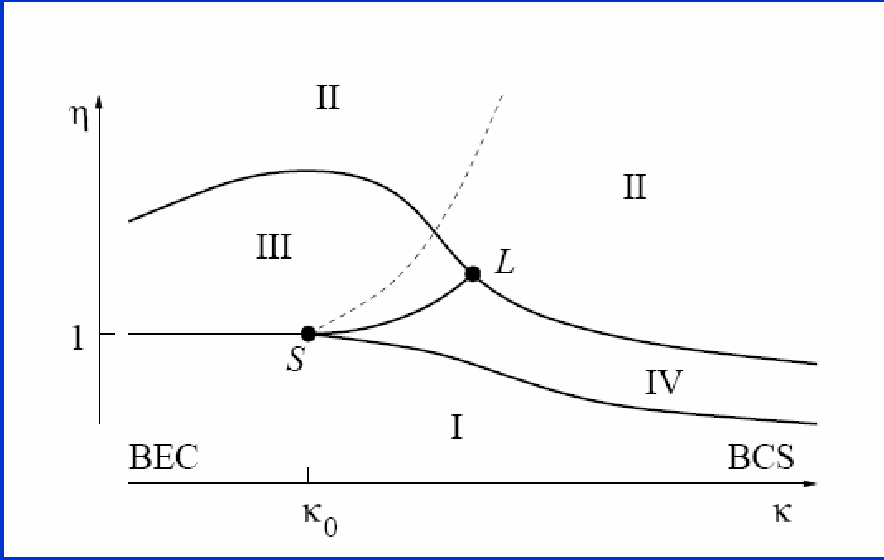
Translational invariance broken



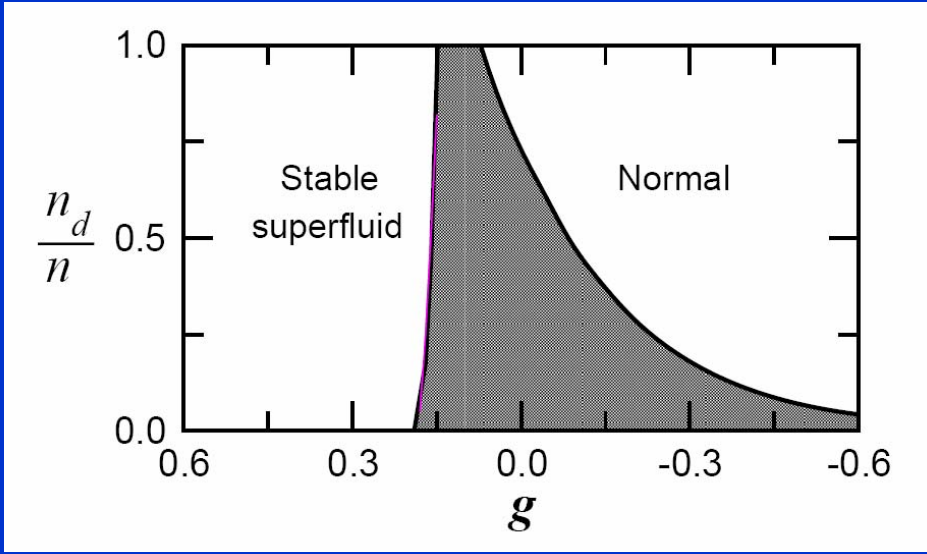
Muether and Sedrakian (2002)

Translational invariant solution

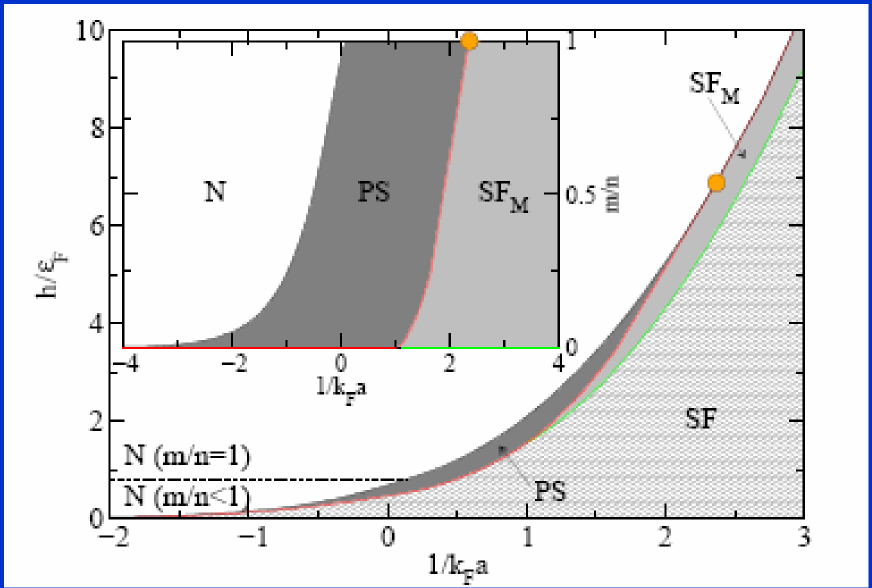
Rotational invariance broken



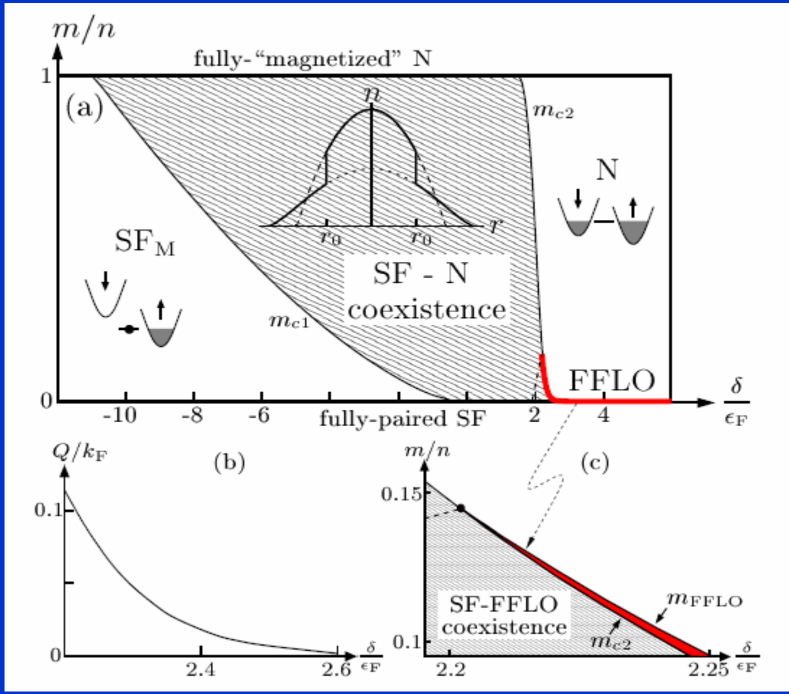
Son and Stephanov, cond-mat/0507586



Pao, Wu, and Yip, PR B 73, 132506 (2006)



Parish, Marchetti, Lamacraft, Simons cond-mat/0605744



Sheeny and Radzihovsky, PRL 96, 060401(2006)

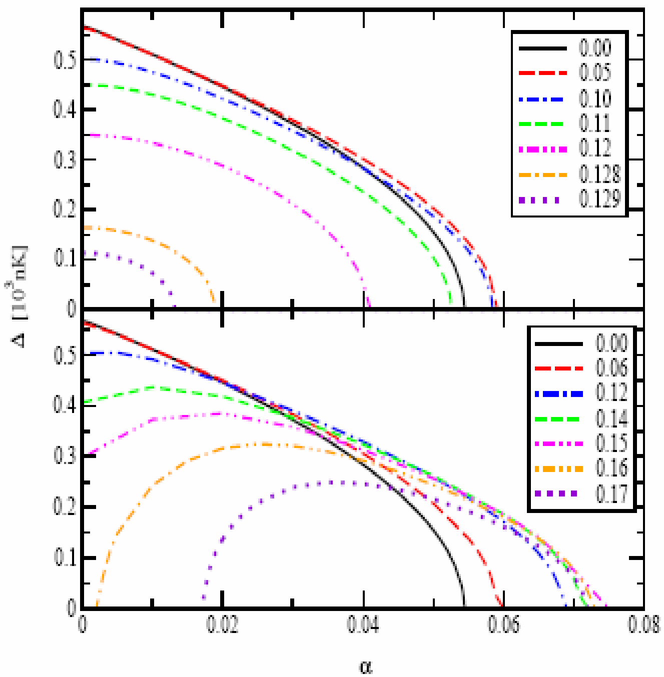


FIG. 1: (Color online) The dependence of the pairing gaps in the LOFF phase (upper panel) and the DFS phase (lower panel) on the asymmetry parameter for several values of the the total momentum P/k_F and deformation parameter $\delta\epsilon$ which are indicated in the panels.

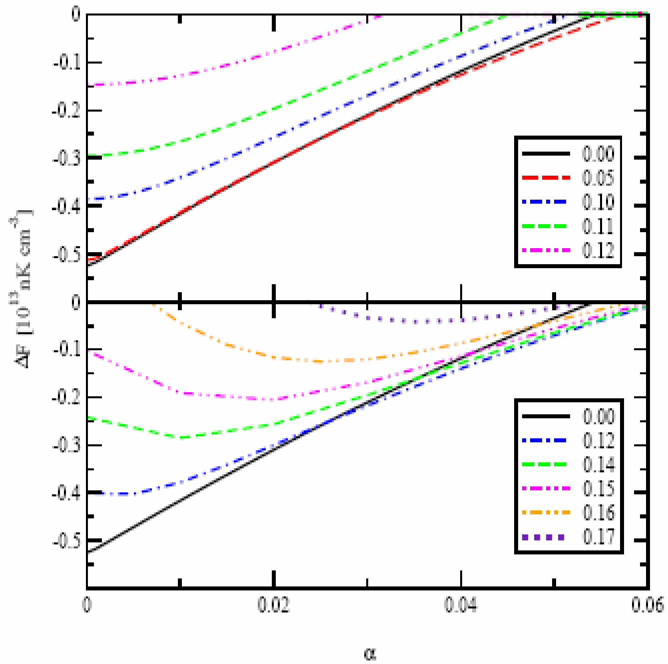


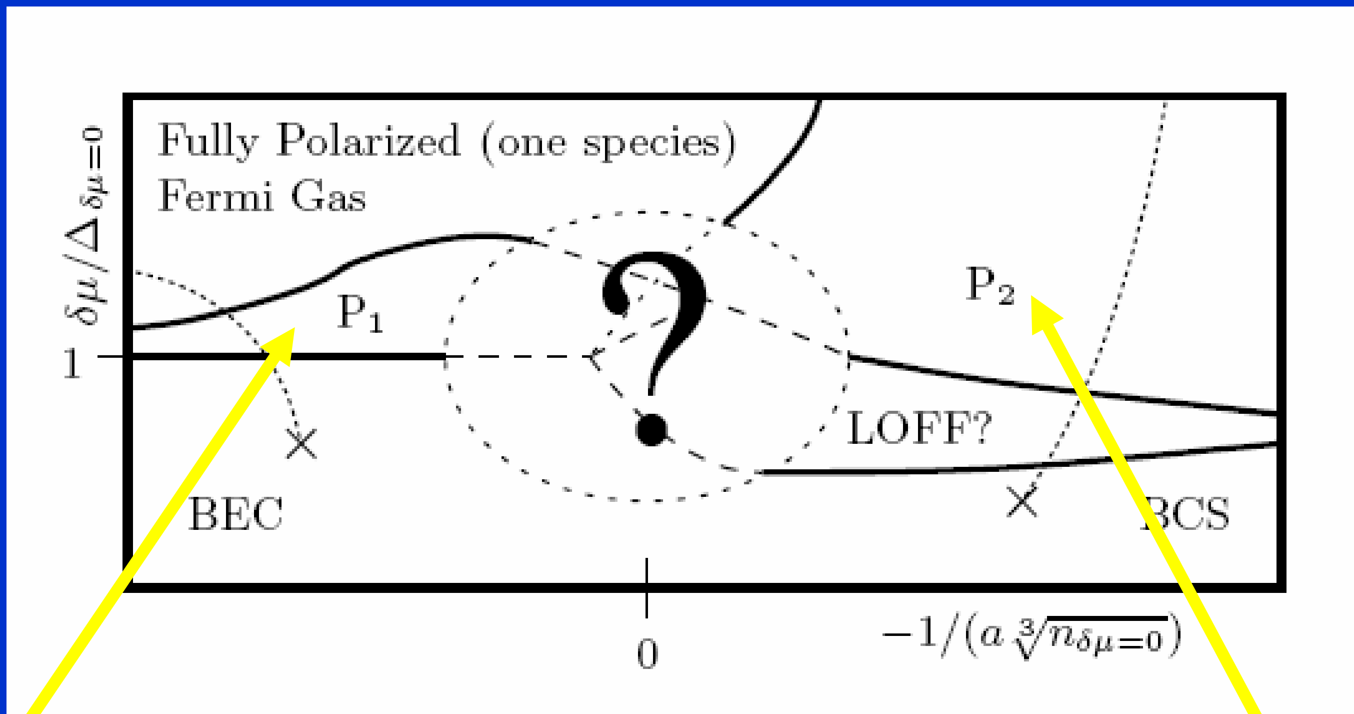
FIG. 4: (Color online) The dependence of the free energy of the LOFF (upper panel) and the plane-wave DFS phase (lower panel) on the asymmetry parameter for several values of the deformation parameter $\delta\epsilon$ and the total momentum P/k_F which are indicated in the panels.

Sedrakian, Mur-Petit, Polls, Muether
Phys. Rev. A 72, 013613 (2005)

What we predict?

Induced *p*-wave superfluidity in asymmetric Fermi gases

Two new superfluid phases where before they were not expected



Bulgac, Forbes, Schwenk

One Bose superfluid coexisting with one P-wave Fermi superfluid

Two coexisting P-wave Fermi superfluids

BEC regime

- **all minority (spin-down) fermions form dimers and the dimers organize themselves in a Bose superfluid**

- **the leftover/un-paired majority (spin-up) fermions will form a Fermi sea**

- **the leftover spin-up fermions and the dimers coexist and, similarly to the electrons in a solid, the leftover spin-up fermions will experience an attraction due to exchange of Bogoliubov phonons of the Bose superfluid**

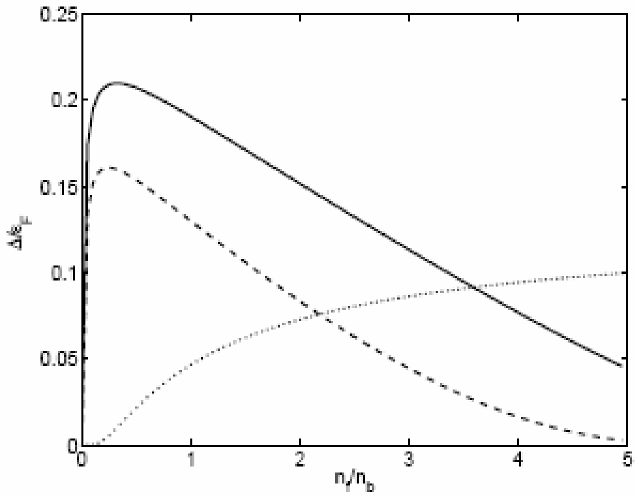


FIG. 1: The ratio Δ/ε_F ($\varepsilon_F = \hbar^2 k_F^2/2m$) as a function of n_f/n_b, for a fixed boson number density $n_b = 10^{13}\ cm^{-3}$ and $n_b a^3 = 0.064$ (solid line) and $n_b a^3 = 0.037$ (dashed line) respectively. The dots show the value of the gap in the case of p-wave paring for $n_b a^3 = 0.064$.

p-wave gap

Bulgac, Bedaque, Fonseca, cond-mat/030602

$$\Delta_p \sim \varepsilon_F \exp\left(-0.44\frac{n_b}{n_f}\right), \qquad \text{if} \qquad \frac{n_f}{n_b} \ll k_F a \ll 1$$

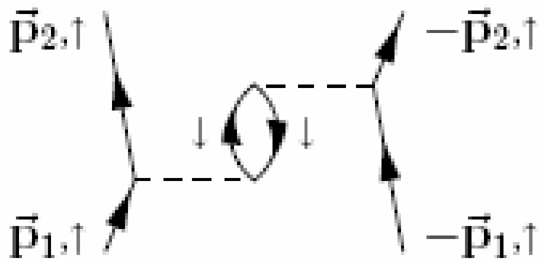
$$\Delta_p \sim \varepsilon_F \exp\left(-\frac{6\pi^2}{\alpha_{fb}^2 (k_F a)^2 \ln\left(x^2\right)}\frac{n_b}{n_f}\right), \qquad \text{if} \qquad \frac{n_f}{n_b} \gg k_F a, \qquad x^2 = \left(\frac{\hbar k_F}{m_b c}\right)^2$$

$$\Delta_p\Big|_{\max} \sim \varepsilon_F \exp\left(-\frac{5.6}{k_F a}\right), \qquad \text{if} \qquad \frac{n_f}{n_b} \approx 0.44 k_F a \ll 1$$

!!!

BCS regime:

$$U_{\mathrm{ind}}^{\uparrow\uparrow}(0, \vec{\mathrm{p}}_1 - \vec{\mathrm{p}}_2) =$$



$$= -N_{\mathrm{F}}^{\downarrow}\left(\frac{4\pi a\hbar^2}{m}\right)^2 L\left(|\vec{\mathrm{p}}_1 - \vec{\mathrm{p}}_2|/(2\hbar k_{\mathrm{F}}^{\downarrow})\right).$$

The same mechanism works for the minority/spin-down component

$$\Delta_p^{\uparrow} \sim \varepsilon_F^{\uparrow} \exp\left(\frac{1}{N_F^{\uparrow} U_p^{\uparrow\uparrow}}\right) = \varepsilon_F^{\uparrow} \exp\left(-\frac{\pi^2}{4 k_F^{\uparrow} k_F^{\downarrow} a^2 L_p\left(\dfrac{k_F^{\uparrow}}{k_F^{\downarrow}}\right)}\right)$$

$$L_p(z) = \frac{5z^2 - 2}{15z^4} \ln\left|1 - z^2\right| - \frac{z^2 + 5}{30z} \ln\left|\frac{1 - z}{1 + z}\right| - \frac{z^2 + 2}{15z^2}$$

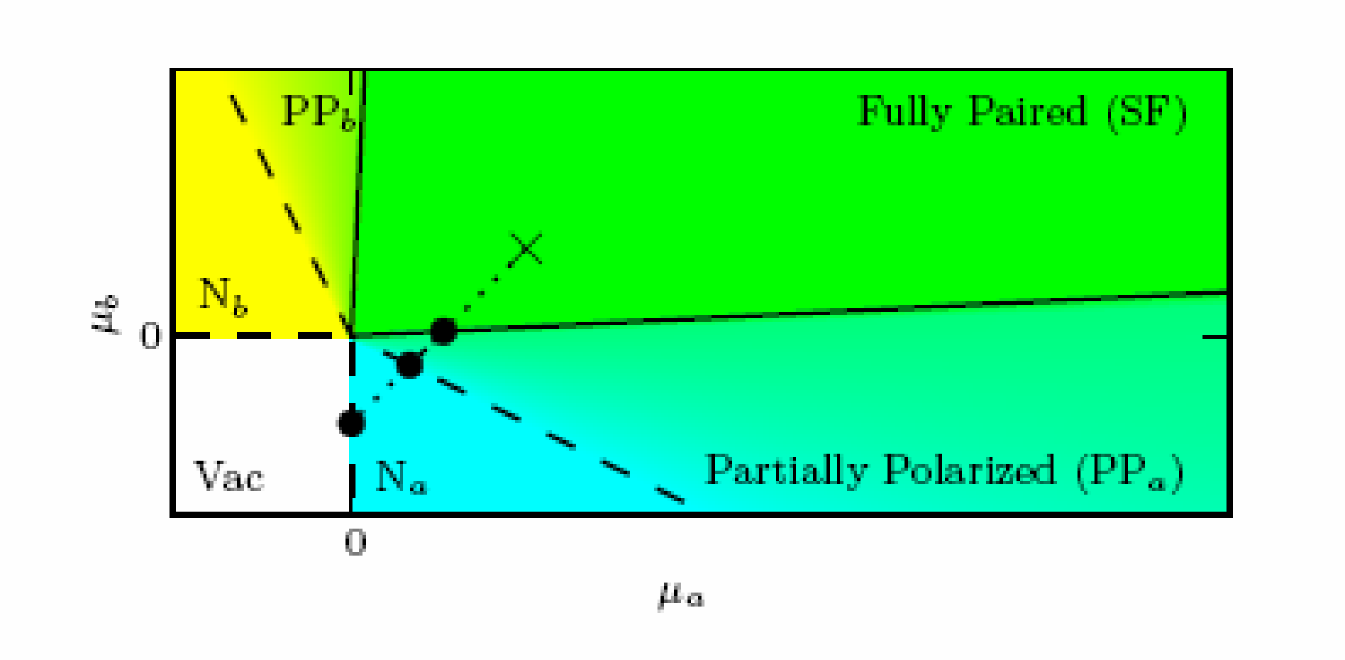
$$\left.\Delta_p^{\uparrow}\right|_{\max} \sim \varepsilon_F^{\uparrow} \exp\left(-\frac{\pi^2}{0.11\left(2k_F^{\uparrow} a\right)^2}\right), \qquad \text{for } k_F^{\downarrow} \approx 0.77 k_F^{\uparrow} \text{ and fixed } k_F^{\downarrow}$$

$$\left.\begin{aligned} \Delta_p^{\uparrow} &\sim \varepsilon_F^{\uparrow} \exp\left(-\frac{3\pi^2}{2\left(2k_F^{\uparrow} a\right)^2 \ln\left(\dfrac{k_F^{\uparrow}}{k_F^{\downarrow}}\right)} \frac{k_F^{\uparrow}}{k_F^{\downarrow}}\right) \\ \Delta_p^{\downarrow} &\sim \varepsilon_F^{\downarrow} \exp\left(-\frac{18\pi^2}{\left(2k_F^{\downarrow} a\right)^2} \frac{k_F^{\uparrow}}{k_F^{\downarrow}}\right) \end{aligned}\right\} \quad \text{for } k_F^{\uparrow} \gg k_F^{\downarrow}$$

➢T=0 thermodynamics in asymmetric Fermi gases at unitarity

What we think is going on:

At unitarity the equation of state of a two-component fermion system is subject to rather tight theoretical constraints, which lead to well defined predictions for the spatial density profiles in traps and the grand canonical phase diagram is:



In the grand canonical ensemble there are only two dimensionfull quantities

We use both micro-canonical and grand canonical ensembles

$$x = \frac{n_b}{n_a} \le 1, \qquad y = \frac{\mu_b}{\mu_a} \le 1$$

$$E(n_a, n_b) = \frac{3}{5}\alpha\left[n_a g(x)\right]^{5/3}$$

$$P(\mu_a, \mu_b) = \frac{2}{5}\beta\left[\mu_a h(y)\right]^{5/2} = \mu_a n_a + \mu_b n_b - E(n_a, n_b) = \frac{2}{3}E(n_a, n_b)$$

$$y = \frac{g'(x)}{g(x) - xg'(x)}, \qquad h(y) = \frac{1}{g(x) - xg'(x)}$$

$$x = \frac{h'(y)}{h(y) - yh'(y)}, \qquad g(x) = \frac{1}{h(y) - yh'(y)}$$

The functions g(x) and h(y) determine fully the thermodynamic properties and only a few details are relevant

Both g(x) and h(y) are convex functions of their argument.

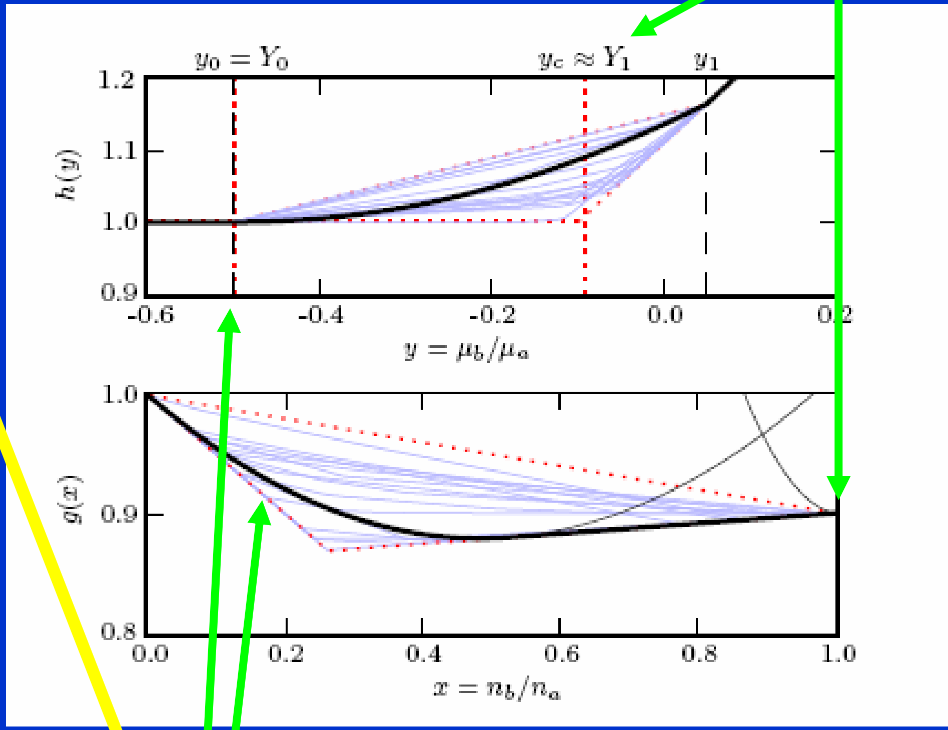
Bounds given by GFMC

Non-trivial regions exist!

$$h(y) = \begin{cases} 1 & \text{if} \quad y \le y_0 \\ \dfrac{1+y}{(2\xi)^{3/5}} & \text{if} \quad y \in [y_1, 1] \end{cases}$$

$$h''(y) \ge 0$$

$$y_0 \le Y_0 < y_c < Y_1 \le 1, \qquad y_c = (2\xi)^{3/5} - 1$$



$$g(0) = 1, \qquad g(x) = (2\xi)^{3/5}$$

$$g''(x) \ge 0$$

$$g'(0) \le Y_0 \quad \text{and} \quad g'(1) \in \left[\frac{g(1)}{1+Y_1^{-1}}, \frac{g(1)}{2}\right]$$

Bounds from the energy required to add a single spin-down particle to a fully polarized Fermi sea of spin-up particles

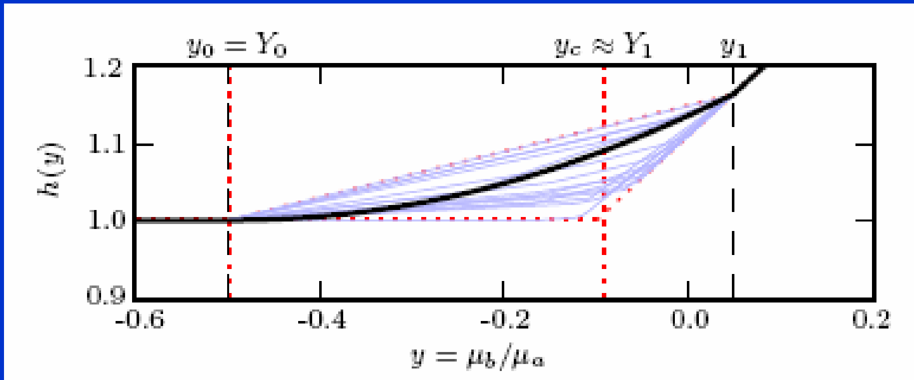
Now put the system in a trap

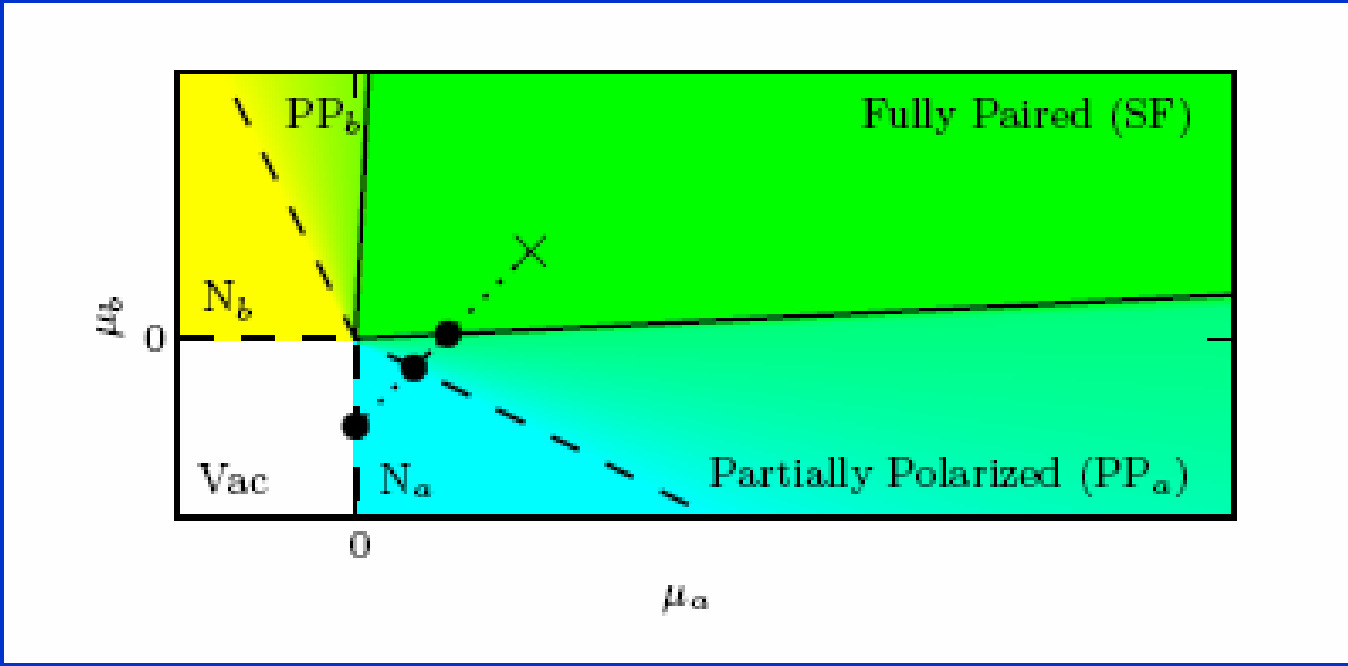
$$\mu_{a,b}(\vec{r}) = \lambda_{a,b} - V(\vec{r}), \qquad y(\vec{r}) = \frac{\mu_b(\vec{r})}{\mu_a(\vec{r})}$$

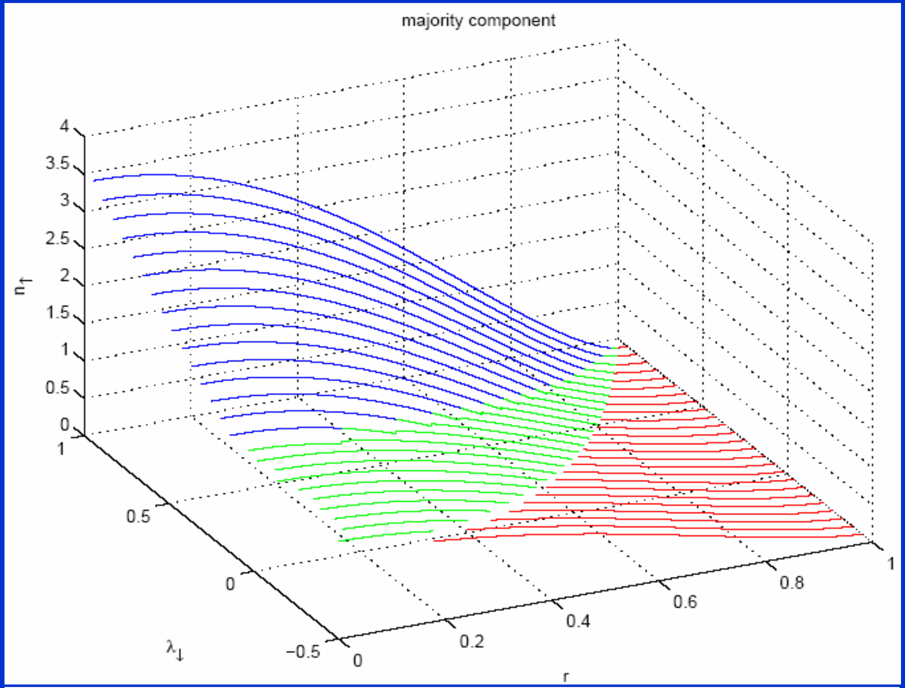
$$2\mu_- = \lambda_a - \lambda_b$$

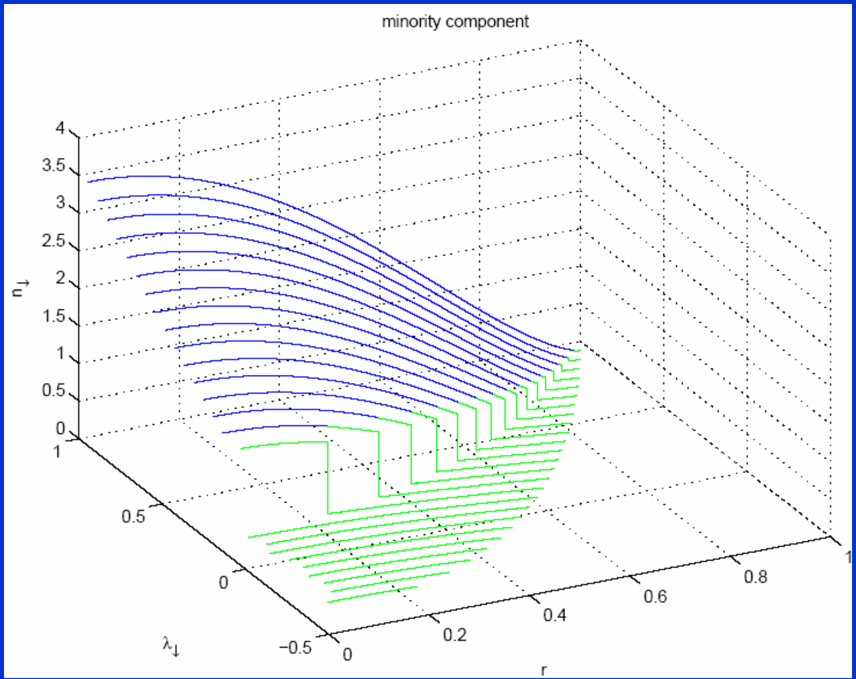
$$n_a(\vec{r}) = \beta\left[\mu_a(\vec{r})h(y(\vec{r}))\right]^{3/2}\left[h(y(\vec{r})) - y(\vec{r})h'(y(\vec{r}))\right]$$

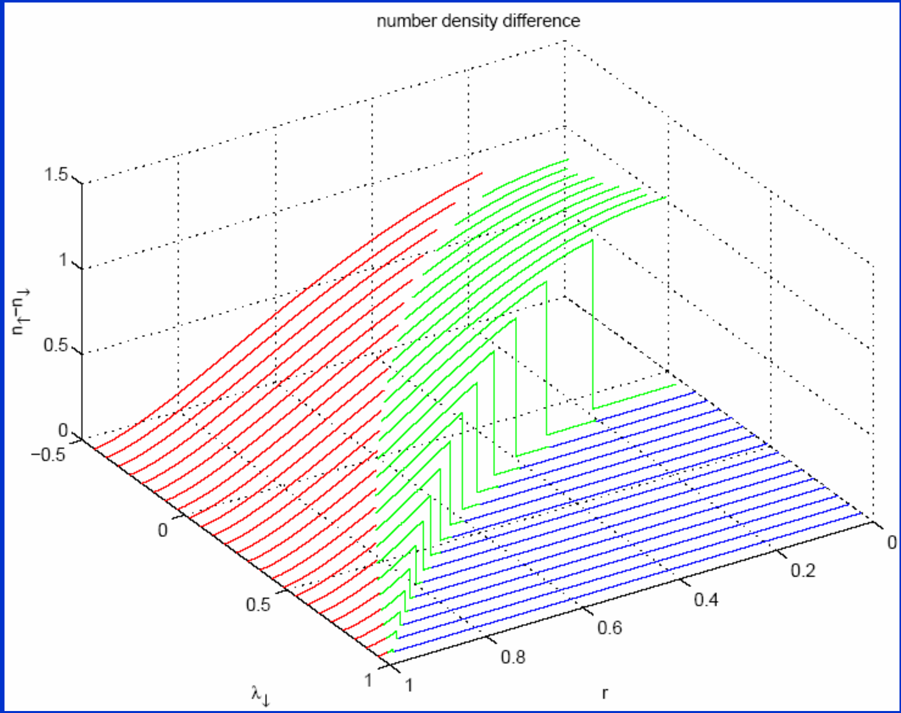
$$n_b(\vec{r}) = \beta\left[\mu_a(\vec{r})h(y(\vec{r}))\right]^{3/2} h'(y(\vec{r}))$$









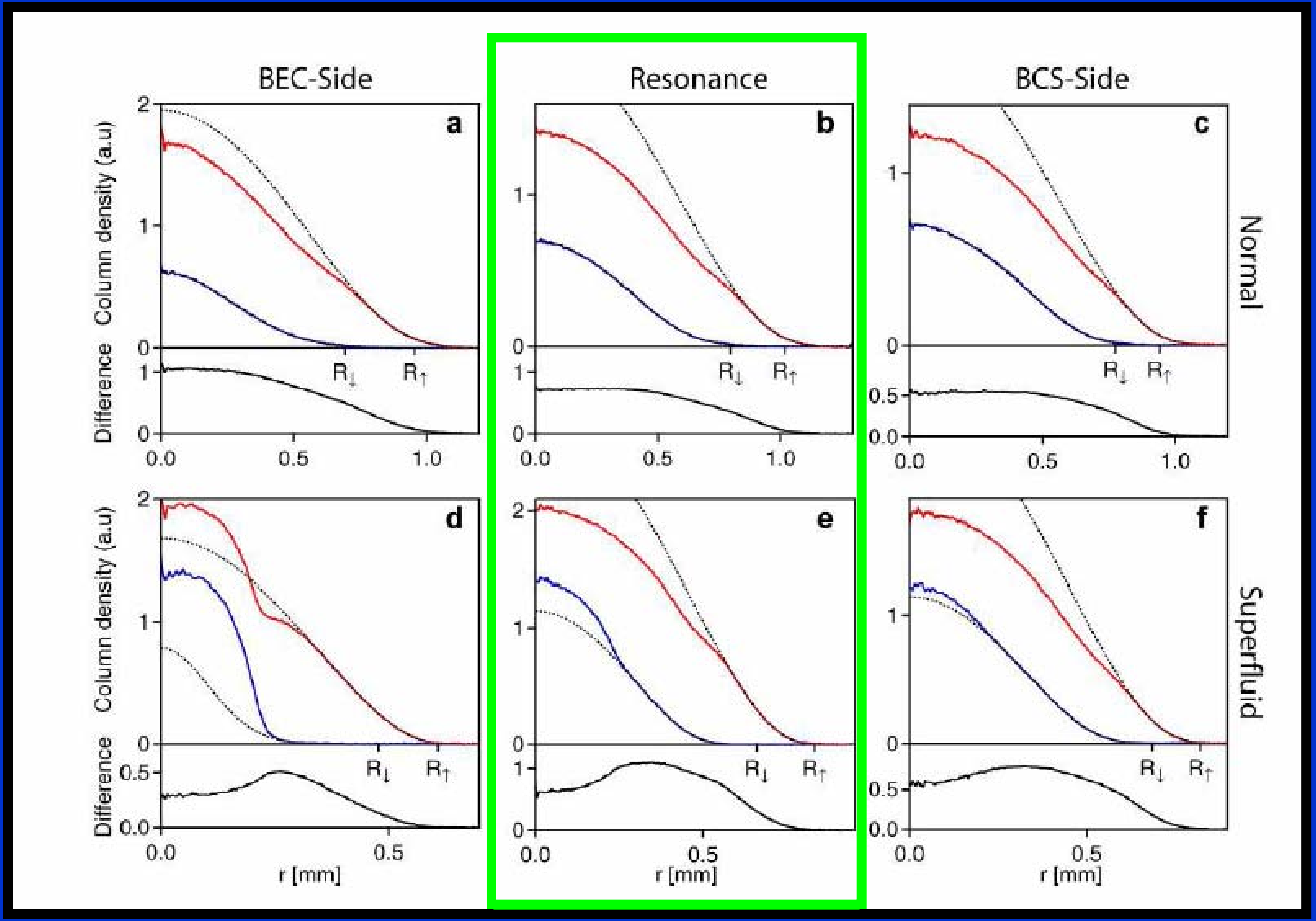


- **blue - P = 0 region**
- **green - 0 < P < 1 region**
- **red - P= 1 region**

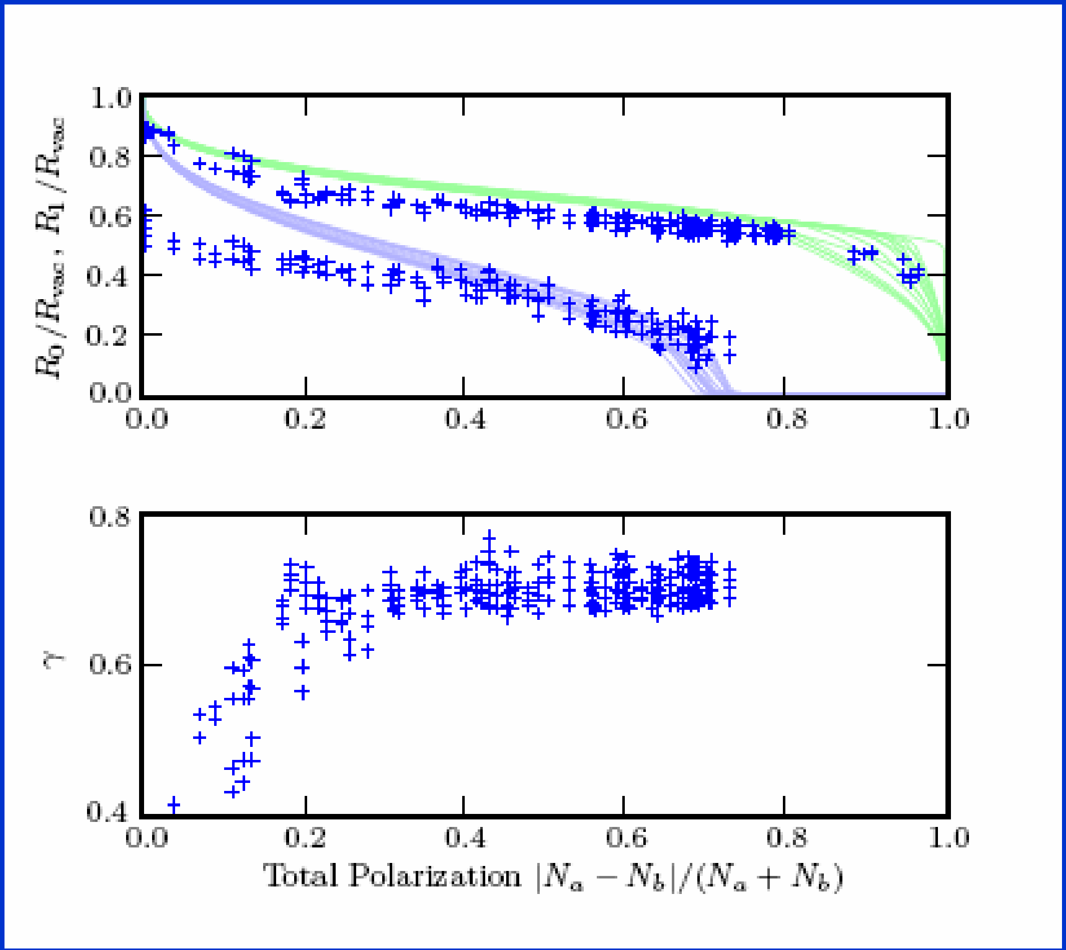
Column densities (experiment)

Normal

Superfluid



Zweirlein *et al.* cond-mat/0605258

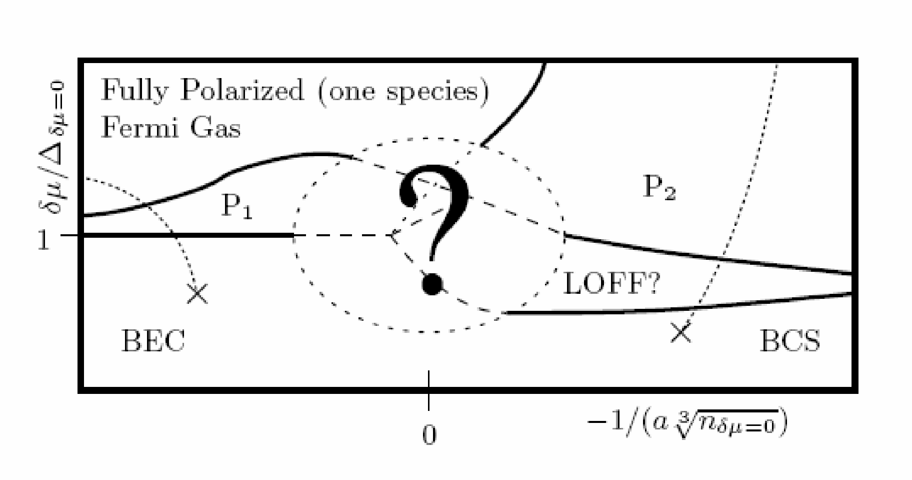


$$\gamma = \frac{y_1 - 1}{y_0 - 1} = \frac{R_0^2 - R_{vac}^2}{R_1^2 - R_{vac}^2} \approx 0.70(5)$$

Experimental data from Zwierlein *et al.* cond-mat/0605258

Main conclusions:

- At T=0 a two component fermion system is always superfluid, irrespective of the imbalance and a number of unusual phases should exists.



- At T=0 and unitarity an asymmetric Fermi gas has non-trivial partially polarized phases