What do we know about the state of cold fermions in the unitary regime?

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University of Washington, Seattle, WA

Also in Warsaw

Now in Lund

Outline

- What is the unitary regime?
- The two-body problem, how one can manipulate the two-body interaction?
- What many/some theorists know and suspect that is going on?
- What experimentalists have managed to put in evidence so far and how that agrees with theory?

➢ What is the unitary regime?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

The system is very dilute, but strongly interacting!

$$\boxed{n\, r_0^3 \ll 1} \qquad \boxed{n\, |a|^3 \gg 1}$$

n - number density

$$\boxed{r_0 \ll n^{-1/3} \approx \lambda_F/2 \ll |a|}$$

r_0 - range of interaction

a - scattering length

What is the *Holy Grail* of this field?

Fermionic superfluidity!

Superconductivity and superfluidity in Fermi systems

20 orders of magnitude over a century of (low temperature) physics

- Dilute atomic Fermi gases — $T_c \approx 10^{-12} - 10^{-9}$ eV
- ✓ Liquid ^{3}He — $T_c \approx 10^{-7}$ eV
- ✓ Metals, composite materials — $T_c \approx 10^{-3} - 10^{-2}$ eV
- ✓ Nuclei, neutron stars — $T_c \approx 10^{5} - 10^{6}$ eV
- QCD color superconductivity — $T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV $\approx$ 10^4 K)

One of my favorite times in the academic year occurs in early spring when I give my class of extremely bright graduate students, who have mastered quantum mechanics but are otherwise unsuspecting and innocent, a take-home exam in which they are asked to deduce superfluidity from first principles. There is no doubt a special place in hell being reserved for me at this very moment for this mean trick, for the task is impossible. Superfluidity, like the fractional quantum Hall effect, is an emergent phenomenon – a low-energy collective effect of huge numbers of particles that cannot be deduced from the microscopic equations of motion in a rigorous way and that disappears completely when the system is taken apart[A)]. There are prototypes for superfluids, of course, and students who memorize them have taken the first step down the long road to understanding the phenomenon, but these are all approximate and in the end not deductive at all, but fits to experiment. The students feel betrayed and hurt by this experience because they have been trained to think in reductionist terms and thus to believe that everything not amenable to such thinking is unimportant. But nature is much more heartless than I am, and those students who stay in physics long enough to seriously confront the experimental record eventually come to understand that the reductionist idea is wrong a great deal of the time, and perhaps always.

Robert B. Laughlin, Nobel Lecture, December 8, 1998

Bertsch Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

Why? Besides pure theoretical curiosity, this problem is relevant to neutron stars!

In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not! A number of people argued that under such conditions fermionic matter is unstable.

- ***systems of bosons are unstable (Efimov effect)***
- ***systems of three or more fermion species are unstable (Efimov effect)***

- **Baker (winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)**

- **Carlson et al (2003) Fixed-Node Green Function Monte Carlo and Astrakharchik et al. (2004) FN-DMC provided the best theoretical estimates for the ground state energy of such systems.**

- **Thomas' Duke group (2002) demonstrated experimentally that such systems are (meta)stable.**

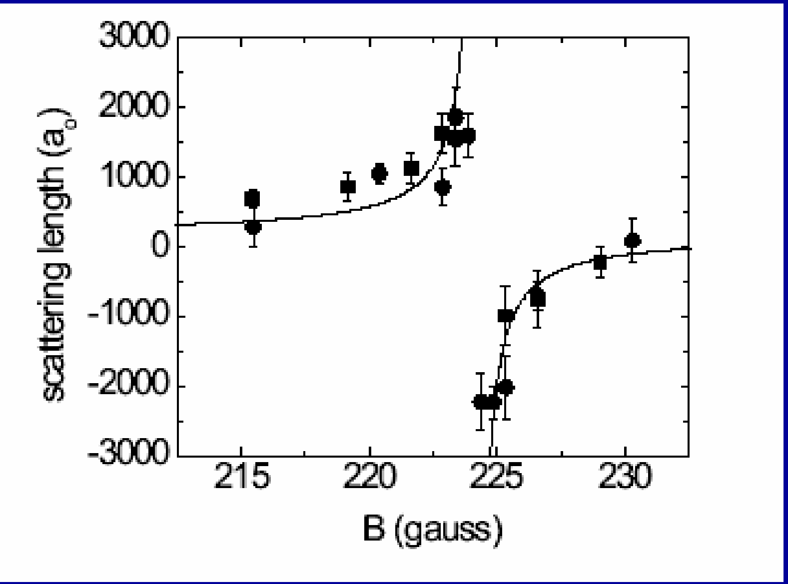
Feshbach resonance

$$H = \frac{\vec{p}^{\,2}}{2\mu_r} + \sum_{i=1}^{2}(V_i^{hf} + V_i^{Z}) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + \cancel{V^d}$$

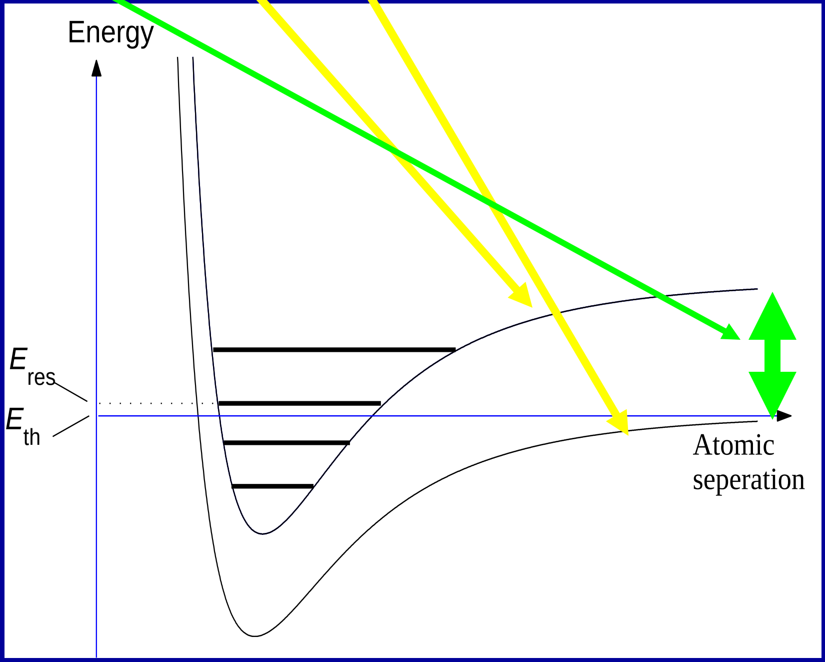
Channel coupling

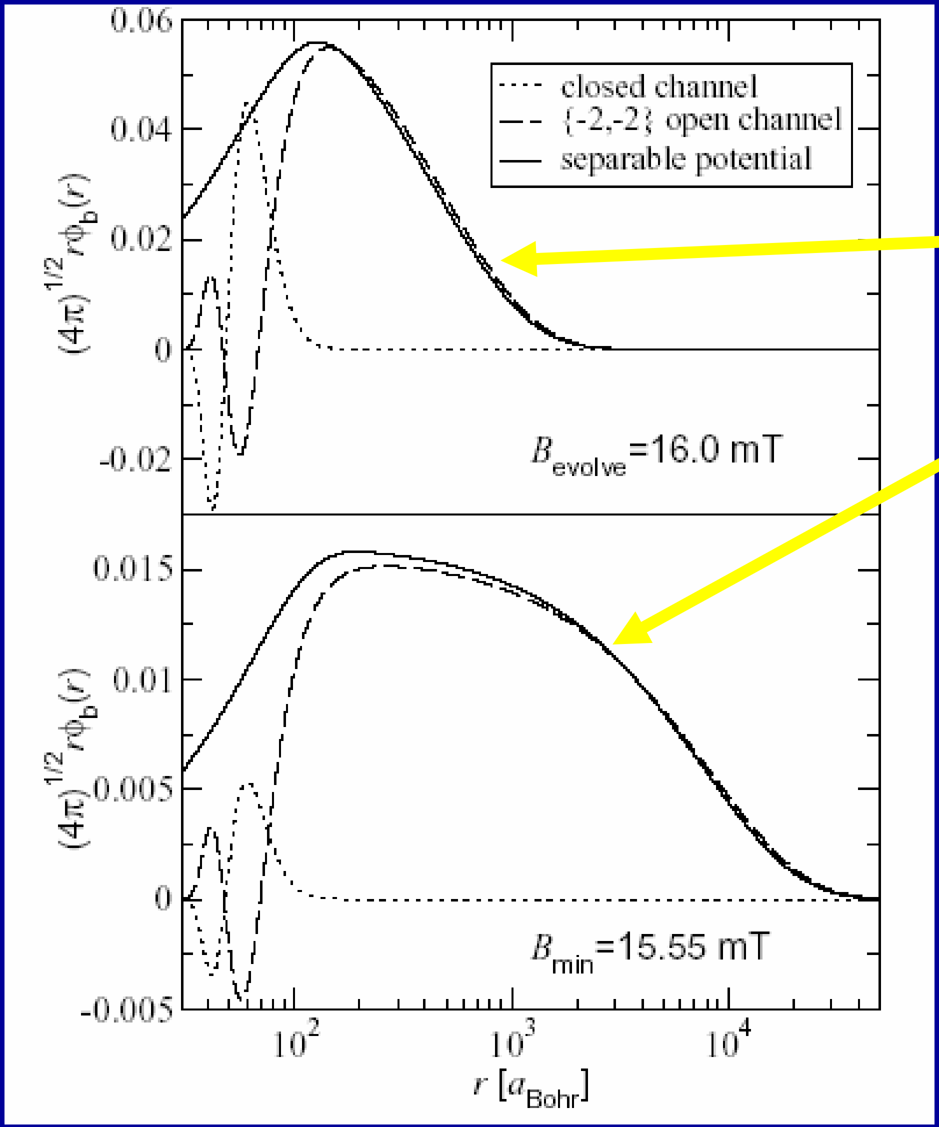
$$V^{hf} = \frac{a_{hf}}{\hbar^2}\vec{S}^{e}\cdot\vec{S}^{n},\quad V^{Z} = (\gamma_e S_z^e - \gamma_n S_z^n)B$$

Tiesinga, Verhaar, Stoof
Phys. Rev. A**47**, 4114 (1993)



Regal and Jin
Phys. Rev. Lett. **90**, 230404 (2003)



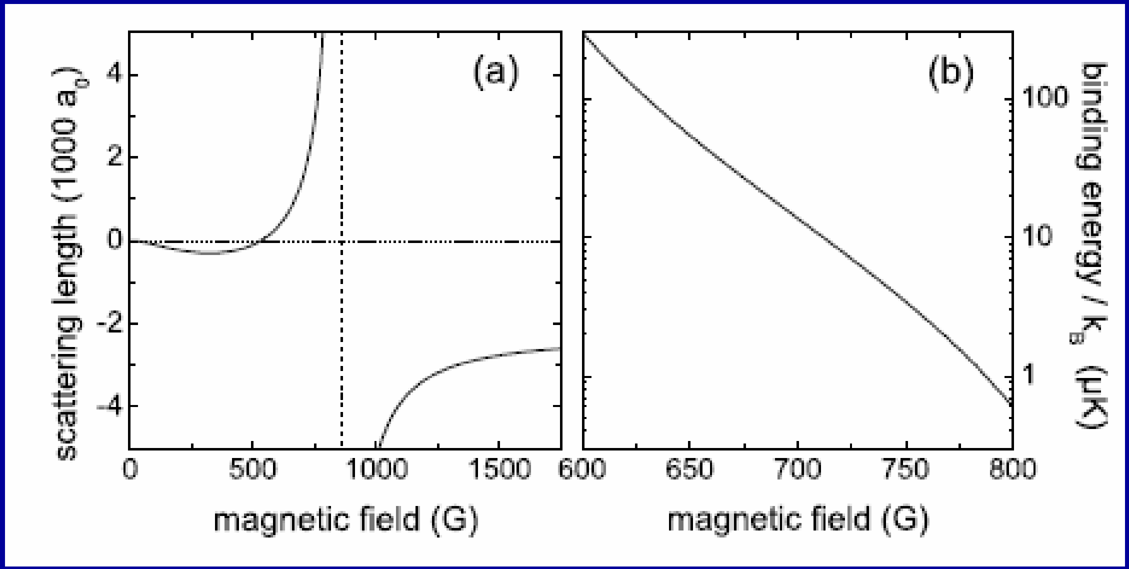


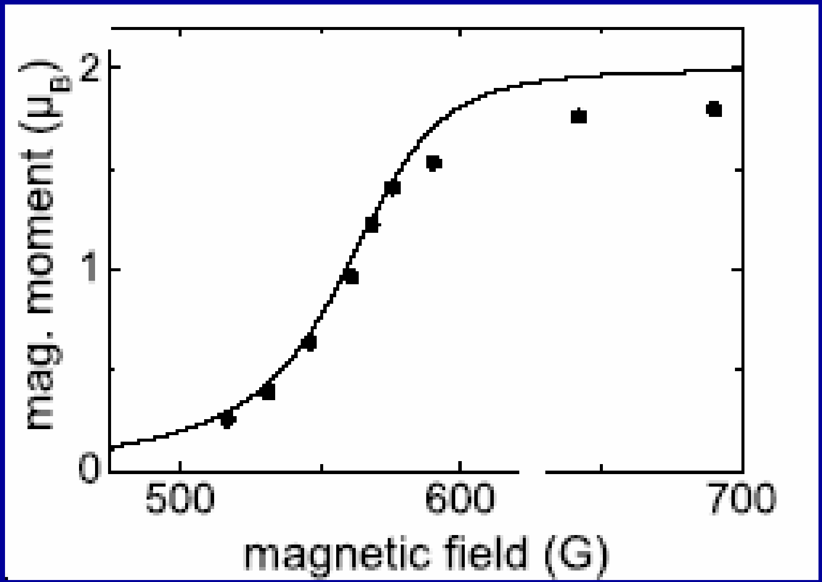
Halo dimer
(open channel)

$$\frac{P(r > r_0)}{P(r < r_0)} \propto \frac{a}{r_0} \quad >> \; 1$$

Most of the time two atoms are at distances greatly exceeding the range of the interaction!

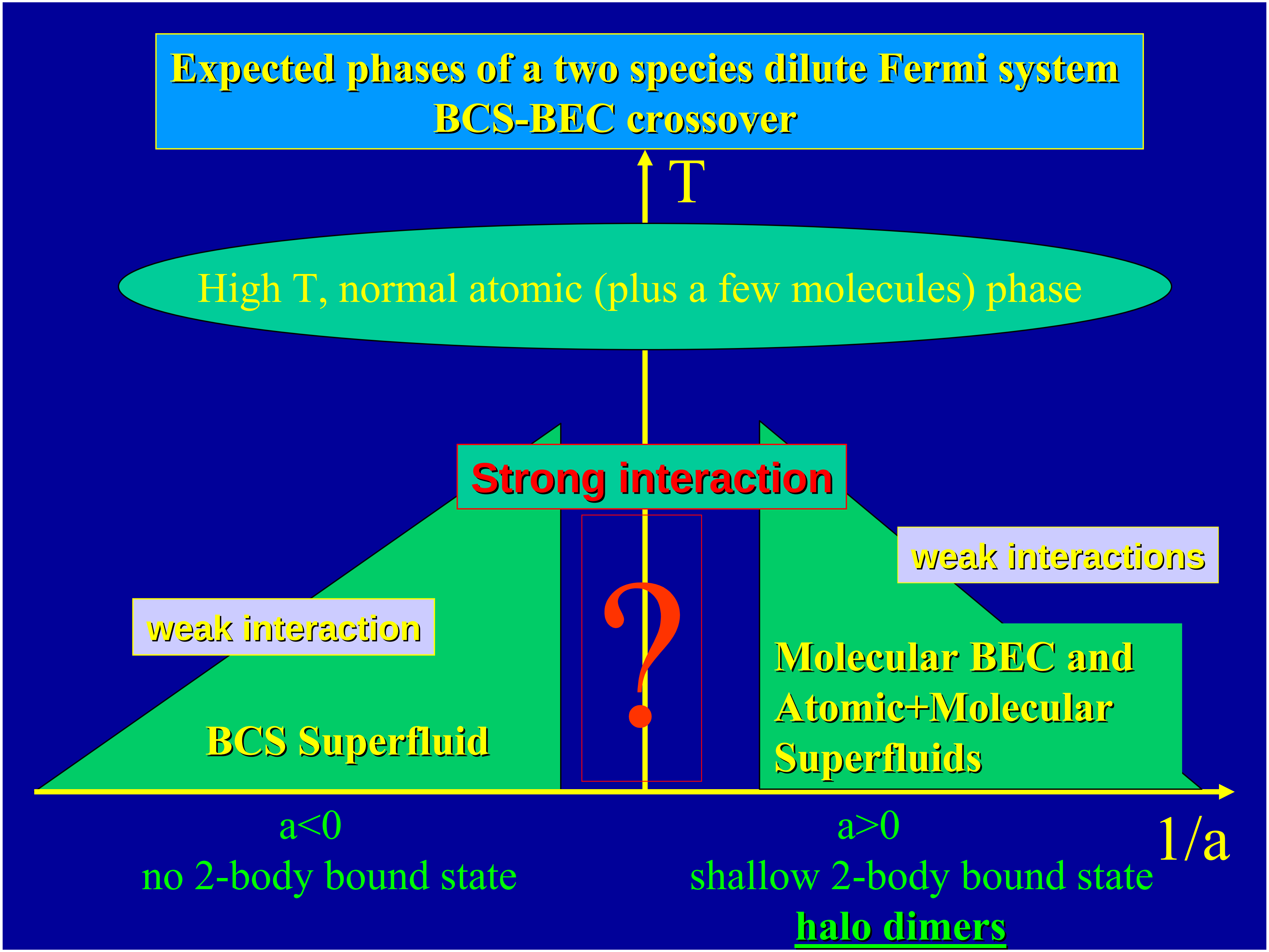
Köhler *et al.* Phys. Rev. Lett. 91, 230401 (2003), inspired by Braaten *et al.* cond-mat/0301489





When the system is in the unitary regime the atom pairs are basically pure triplets and thus predominantly in the open channel, where they form spatially large pairs **halo dimers** *(if a>0)*

Jochim et al. Phys. Rev. Lett. 91, 240402 (2003)


Expected phases of a two species dilute Fermi system
BCS-BEC crossover
T
High T, normal atomic (plus a few molecules) phase
Strong interaction
weak interaction
weak interactions
?
BCS Superfluid
Molecular BEC and
Atomic+Molecular
Superfluids
a<0
no 2-body bound state
a>0
shallow 2-body bound state
halo dimers
1/a

Early theoretical approach

Eagles (1969), Leggett (1980) …

$$|gs\rangle = \prod_k \left(\mathrm{u_k} + \mathrm{v_k} a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow}\right)|vacuum\rangle$$ BCS wave function

$$\frac{m}{4\pi\hbar^2 a} = \sum_k \left(\frac{1}{2\varepsilon_k} - \frac{1}{2\mathrm{E}_k}\right)$$ gap equation

$$n = 2\sum_k \left(1 - \frac{\varepsilon_k - \mu}{\mathrm{E}_k}\right)$$ number density equation

$$\Delta \approx \frac{8}{\mathrm{e}^2}\varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$ pairing gap

$$\mathrm{E}_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta^2}$$ quasi-particle energy

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}$$

$$\mathrm{u_k^2} + \mathrm{v_k^2} = 1, \quad \mathrm{v_k^2} = \frac{1}{2}\left(1 - \frac{\varepsilon_k - \mu}{\mathrm{E}_k}\right)$$

Consequences:

- Usual BCS solution for small and negative scattering lengths, with exponentially small pairing gap

- For small and positive scattering lengths this equations describe a gas a weakly repelling (weakly bound/shallow) molecules, essentially all at rest (almost pure BEC state)

$$\Psi\left(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4,\ldots\right) \approx \mathcal{A}\left[\varphi(\vec{r}_{12})\varphi(\vec{r}_{34})\ldots\right]$$

In BCS limit the particle projected many-body wave function has the same structure (BEC of spatially overlapping Cooper pairs)

- For both large positive and negative values of the scattering length these equations predict a smooth crossover from BCS to BEC, from a gas of spatially large Cooper pairs to a gas of small molecules

What is wrong with this approach:

• The BCS gap is overestimated, thus critical temperature and condensation energy are overestimated as well.

• In BEC limit (small positive scattering length) the molecule repulsion is overestimated

• The approach neglects of the role of the "meanfield (HF) interaction," which is the bulk of the interaction energy in both BCS and unitary regime

• All pairs have zero center of mass momentum, which is reasonable in BCS and BEC limits, but incorrect in the unitary regime, where the interaction between pairs is strong !!! (similar to superfluid ^{4}He)

Fraction of non-condensed pairs (perturbative result)!?!

$$\frac{n_{ex}}{n_0} = \frac{8}{3\sqrt{\pi}}\sqrt{n_m a_{mm}^3}, \qquad n_m = \frac{n}{2}, \qquad a_{mm} \approx 0.6a$$

What people use a lot ?

(Basically this is Eagles' and Leggett's model, somewhat improved.)

VOLUME 83, NUMBER 14 PHYSICAL REVIEW LETTERS 4 OCTOBER 1999

Rarified Liquid Properties of Hybrid Atomic-Molecular Bose-Einstein Condensates

Eddy Timmermans,[1] Paolo Tommasini,[2] Robin Côté,[2,*] Mahir Hussein,[2,3] and Arthur Kerman[4]

$$\hat{H} = \int d^3r\, \hat{\psi}_a^\dagger \left[-\frac{\hbar^2\nabla^2}{2m} + \frac{\lambda_a}{2}\hat{\psi}_a^\dagger\hat{\psi}_a + \lambda\hat{\psi}_m^\dagger\hat{\psi}_m \right]\hat{\psi}_a + \int d^3r\, \hat{\psi}_m^\dagger \left[-\frac{\hbar^2\nabla^2}{4m} + \frac{\lambda_m}{2}\hat{\psi}_m^\dagger\hat{\psi}_m + \epsilon \right]\hat{\psi}_m + \frac{\alpha}{\sqrt{2}}\int d^3r\, \{\hat{\psi}_m^\dagger\hat{\psi}_a\hat{\psi}_a + \hat{\psi}_m\hat{\psi}_a^\dagger\hat{\psi}_a^\dagger\}, \tag{2}$$

VOLUME 88, NUMBER 9 PHYSICAL REVIEW LETTERS 4 MARCH 2002

Signatures of Resonance Superfluidity in a Quantum Fermi Gas

M. L. Chiofalo,* S. J. J. M. F. Kokkelmans, J. N. Milstein, and M. J. Holland

$$H = \sum_{k\sigma}\epsilon_k a_{k\sigma}^\dagger a_{k\sigma} + \nu\sum_k b_k^\dagger b_k + U\sum_{qkk'} a_{q/2+k\uparrow}^\dagger a_{q/2-k\downarrow}^\dagger a_{q/2-k'\downarrow} a_{q/2+k'\uparrow} + \left(g\sum_{kq} b_q^\dagger a_{q/2-k\downarrow} a_{q/2+k\uparrow} + \text{H.c.}\right),$$

Why?

Everyone likes doing simple meanfield (and sometimes add fluctuations on top) calculations!

Timmermans *et al*. realized that a contact interaction proportional to either a very large or infinite scattering length makes no sense in meanfield approximation.

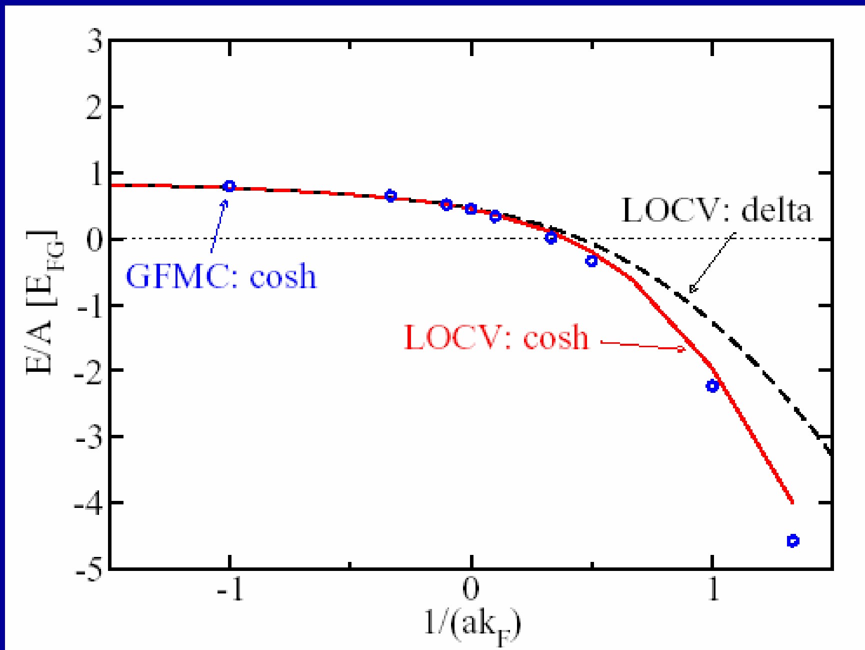
$$U(\vec{r}_1 - \vec{r}_2) = \frac{4\pi\hbar^2 a}{m}\delta(\vec{r}_1 - \vec{r}_2)$$

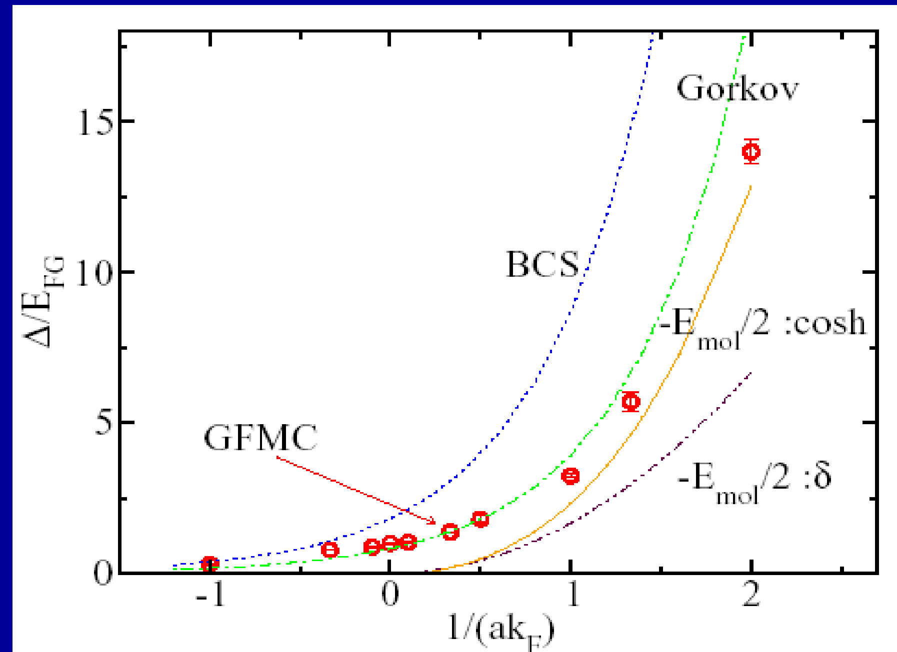
The two-channel approach, which they introduced initially for bosons, does not seem, superficially at least, to share this difficulty. However, one can show that corrections to such a meanfield approach will be governed by the parameter **na^3** anyway, so, the problem has not been really solved.

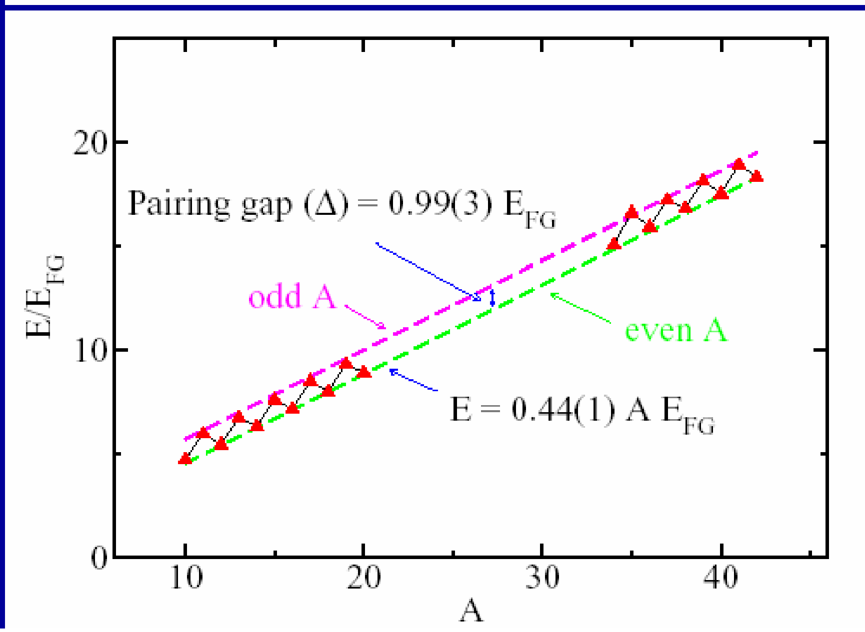
Is there a better approach?

Full blown many body calculations!

Fixed-Node Green Function Monte Carlo approach at T=0





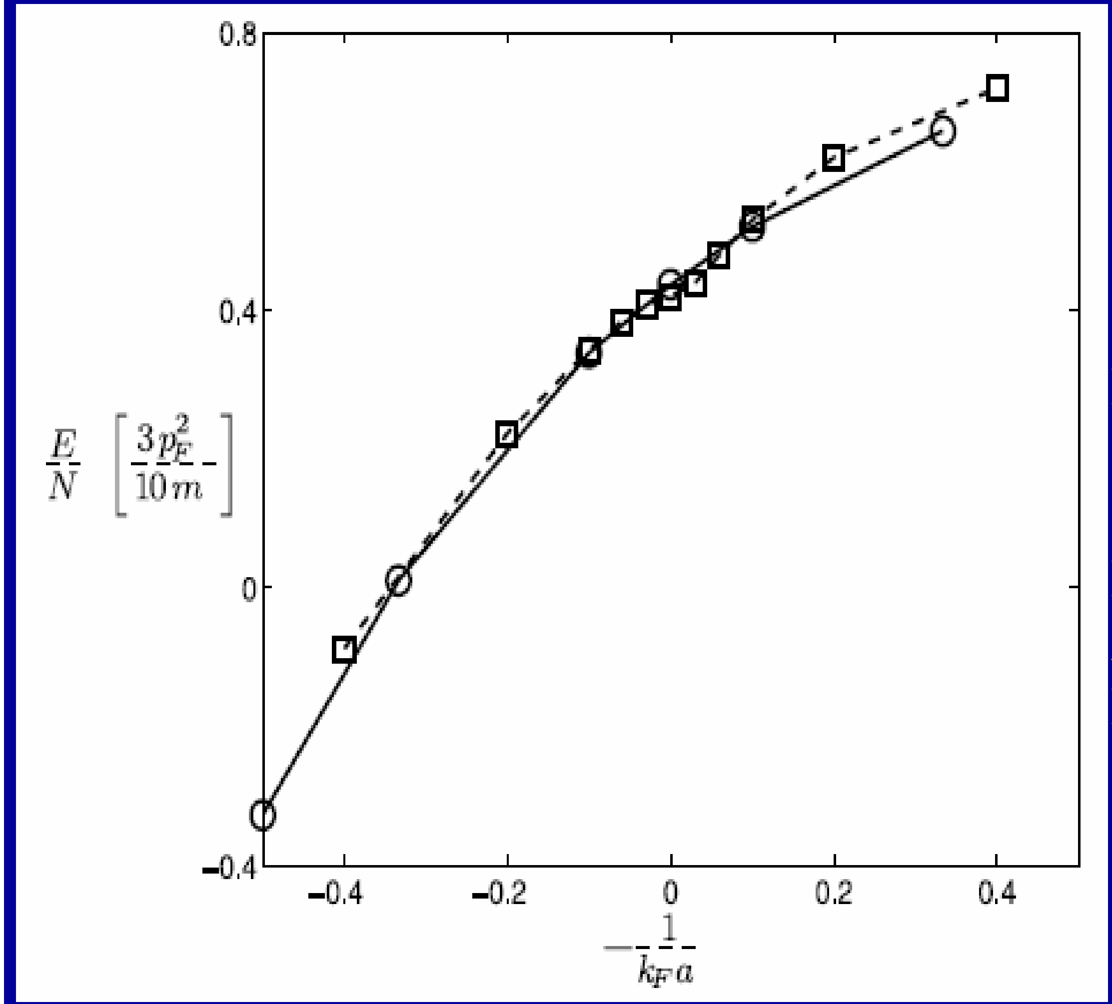


$$\Delta_{\mathrm{BCS}} \approx \frac{8}{\mathrm{e}^2} \varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

$$\Delta_{\mathrm{Gorkov}} \approx \left(\frac{2}{e}\right)^{7/3} \varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

Carlson *et al.* PRL <u>91</u>, 050401 (2003)
Chang *et al.* PRA <u>70</u>, 043602 (2004)

Energy per particle near the Feshbach resonance from Fixed Node Green Function/Diffusion Monte Carlo calculations



Solid line with circles
Chang *et al.*
Phys. Rev. A **70**, 043602 (2004)
(both even and odd particle numbers)

Dashed line with squares
Astrakharchik *et al.*
Phys. Rev. Lett. **93**, 200404 (2004)
(only even particle numbers)

$$r_0 \ll \quad \frac{1}{n^{1/3}} \approx \frac{\lambda_F}{2} \quad \ll |a|$$

$$\left.\frac{E}{N}\right|_{GFMC} = \varepsilon[n] \approx \frac{3}{5}\varepsilon_F\left[\xi - \frac{\varsigma}{k_F a} - \frac{5\iota}{3\left(k_F a\right)^2}\right], \qquad \xi \approx 0.44, \qquad \varsigma \approx 1, \qquad \iota \approx 1$$

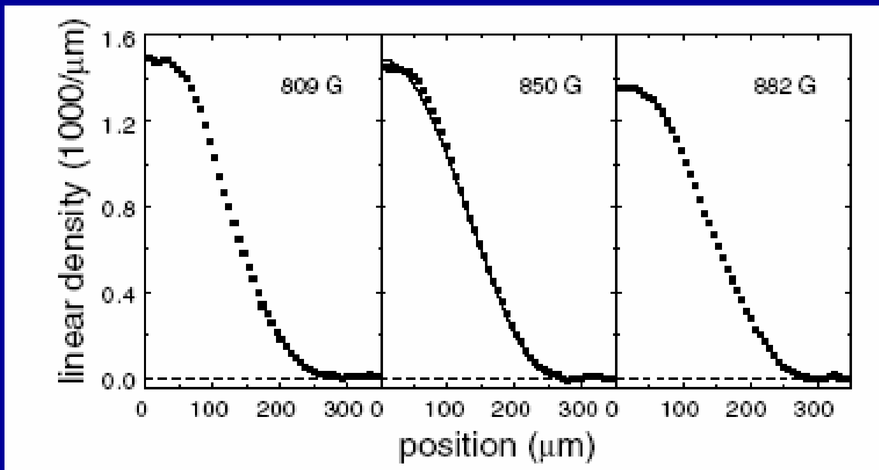
$$\Delta_{GFMC} \approx \varepsilon_F \left(\frac{2}{e}\right)^{7/3} \exp\left(\frac{\pi}{2k_F a}\right), \qquad n = \frac{k_F^3}{3\pi^2}, \quad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}, \quad x = \frac{1}{k_F a}$$

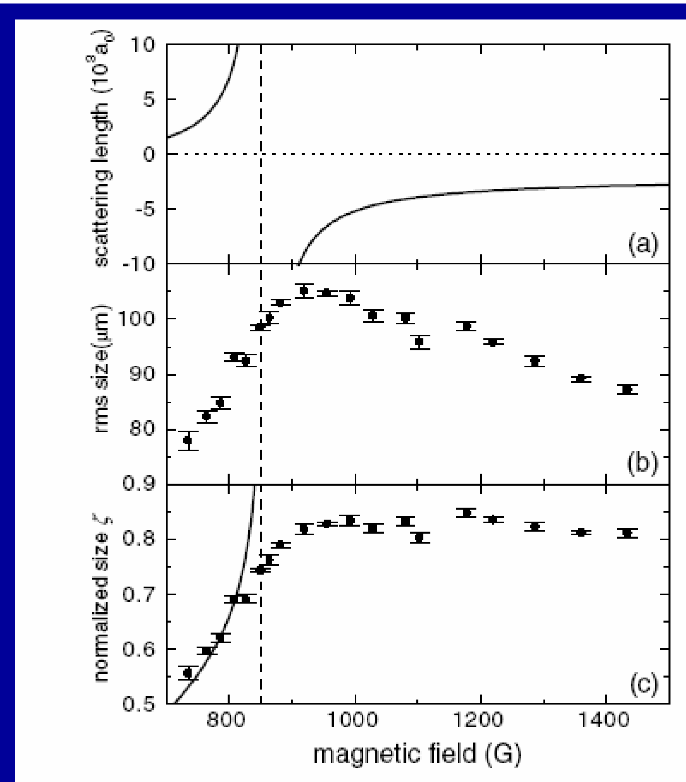
$$\varepsilon_{SLDA}[n]n = \varepsilon_{kin} n + \frac{\hbar^2}{m}\beta[x]\, n^{5/3} + \frac{\hbar^2}{m}\gamma[x]\, \frac{|\nu|^2}{n^{1/3}} \qquad + \quad \text{Renormalization}$$

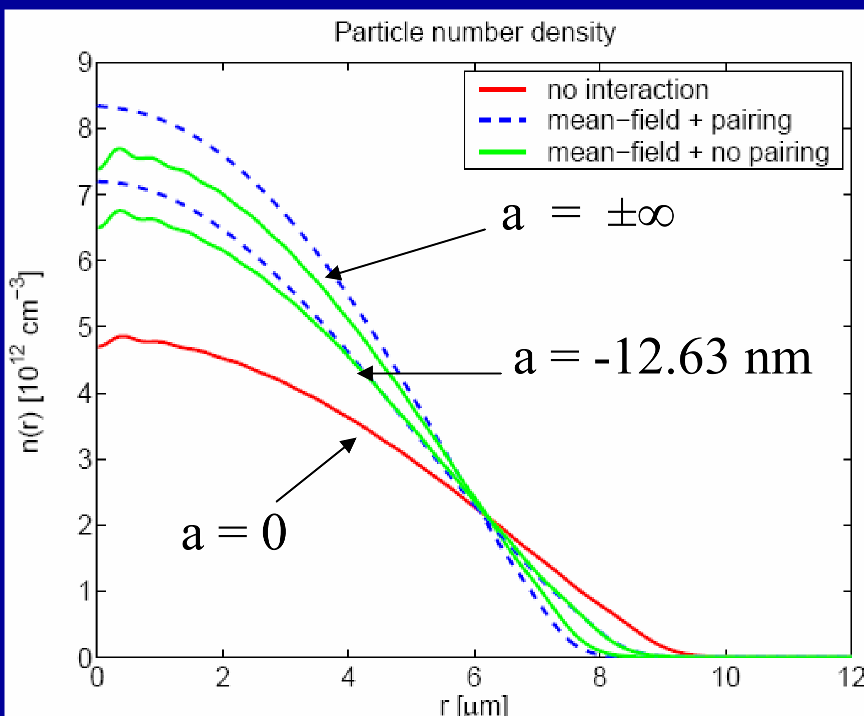
Dimensionless coupling constants

Superfluid LDA (SLDA) is the generalization of Kohn-Sham to superfluid fermionic systems

Jochim et al. Phys.Rev.Lett. 91, 240402 (2003)







Y. Yu, July, 2003, unpublished

5200 ^{40}K atoms in a spherical trap
ħω=0.568 x 10^{-12} eV

SLDA calculation using
GFMC equation of state of
Carlson *et al.* PRL 91, 050401 (2003)

Sound in infinite fermionic matter

$$\omega = \mathrm{v}_s k$$

	Local shape of Fermi surface	Sound velocity	
Collisional Regime - <u>high T!</u> Compressional mode	Spherical	$\mathrm{v}_s \approx \frac{\mathrm{v}_F}{\sqrt{3}}$	First sound
Superfluid collisionless- <u>low T!</u> Compressional mode	Spherical	$\mathrm{v}_s \approx \frac{\mathrm{v}_F}{\sqrt{3}}$	Bogoliubov-Anderson sound
Normal Fermi fluid collisionless - <u>low T!</u> (In)compressional mode	Elongated along propagation direction	$\mathrm{v}_s = s\mathrm{v}_F$ $s > 1$	Landau's zero sound Need repulsion !!!

$$\varepsilon(n) = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m}\left[\xi - \frac{\varsigma}{k_F a} - \frac{5\iota}{(k_F a)^2} + O\left(\frac{1}{(k_F a)^3}\right)\right]$$

$$\xi \approx 0.44, \qquad \varsigma \approx 1, \qquad \iota \approx 1$$

$$U = \frac{m\omega_0^2\left(x^2 + y^2 + \lambda^2 z^2\right)}{2}$$

$$\frac{\delta\omega^2}{\omega^2} = \frac{\varsigma}{\xi}\frac{1}{k_F(0)a}K$$

Adiabatic regime
Spherical Fermi surface

Bogoliubov-Anderson modes in a trap

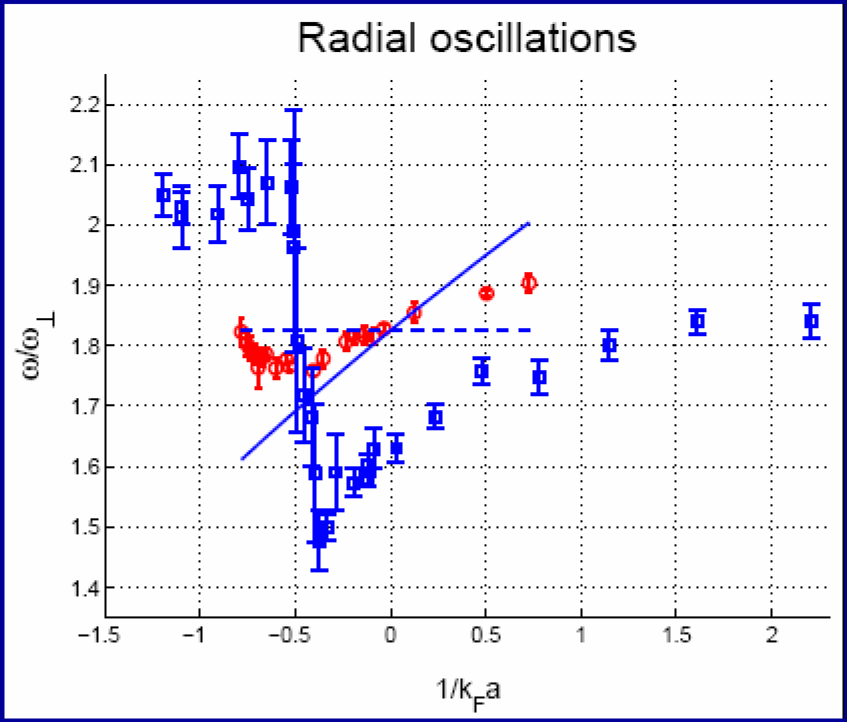
Perturbation theory result using GFMC equation of state in a trap

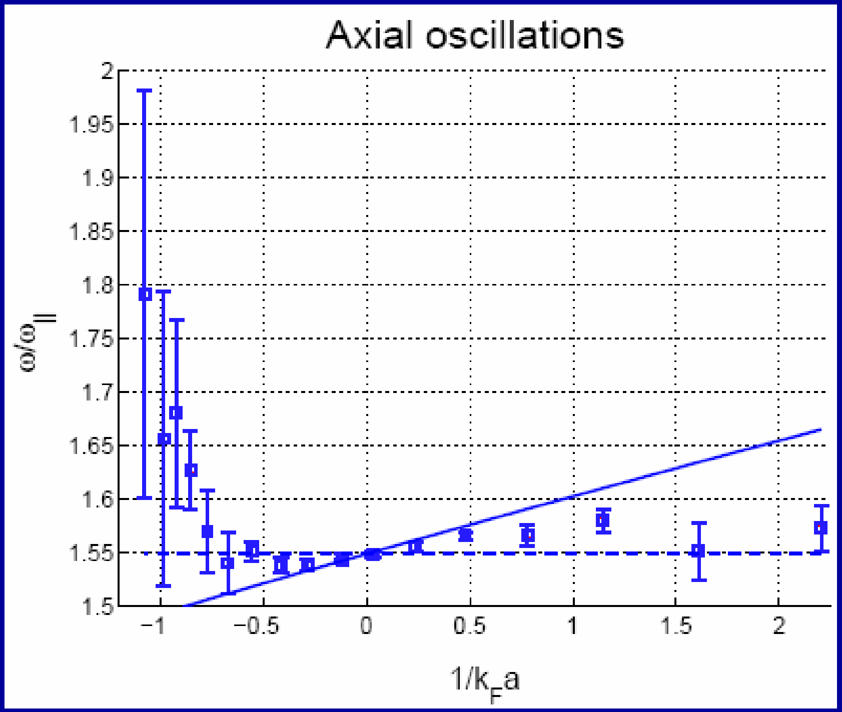
TABLE II: Results for K.

trap type	mode	f_1	ω	K
spherical	dipole	z	ω_0	0
$\lambda = 1$	monopole	$1 - 2r^2$	$2\omega_0$	$\frac{256}{525\pi}$
	quadrupole	xy	$\sqrt{2}\omega_0$	0
axial	$M = \pm 2$	$xy,\ x^2 - y^2$	$\sqrt{2}\omega_0$	0
$\lambda \ll 1$	$M = \pm 1$	$xz,\ yz$	ω_0	0
	radial	$x^2 + y^2 + \frac{2}{5}\lambda^2 z^2 - \frac{2}{5}$	$\sqrt{\frac{10}{3}}\omega_0$	$\frac{1024}{2625\pi}$
	axial	$1 - 6\lambda^2 z^2$	$\sqrt{\frac{12}{5}}\lambda\omega_0$	$\frac{256}{2625\pi}$

Only compressional modes are sensitive to the equation of state and experience a shift!

Innsbruck's results - blue symbols
Duke's results - red symbols



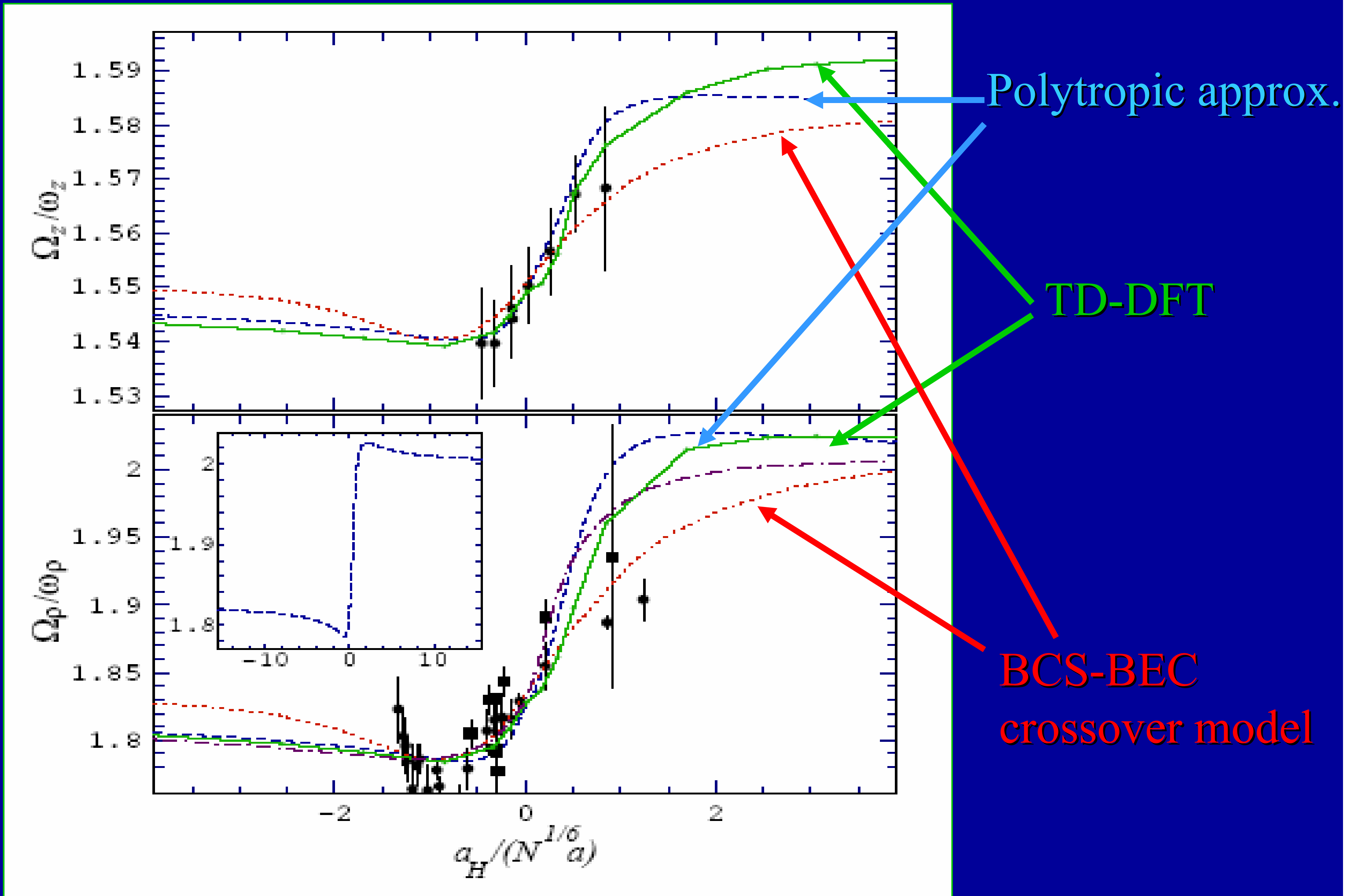


First order perturbation theory prediction (blue solid line)

Unperturbed frequency in unitary limit (blue dashed line)
Identical to the case of non-interacting fermions

If the matter at the Feshbach resonance would have a bosonic character then the collective modes will have significantly higher frequencies!

Innsbruck's results



Duke's result

Manini and Salasnich, cond-mat/0407039

How should one describe a fermionic system in the unitary regime at finite T?

Grand Canonical Path-Integral Monte Carlo calculations on 4D-lattice

$$H = T + V = \int d^3x \left[\psi_\uparrow^\dagger(\vec{x})\left(-\frac{\hbar^2\Delta}{2m}\right)\psi_\uparrow(\vec{x}) + \psi_\downarrow^\dagger(\vec{x})\left(-\frac{\hbar^2\Delta}{2m}\right)\psi_\downarrow(\vec{x})\right] - g\int d^3x\, n_\uparrow(\vec{x}) n_\downarrow(\vec{x})$$

$$N = \int d^3x \left[n_\uparrow(\vec{x}) + n_\downarrow(\vec{x})\right]$$

Trotter expansion (*trotterization* of the propagator)

$$Z(\beta) = \mathrm{Tr}\, \exp\left[-\beta\left(H - \mu N\right)\right] = \mathrm{Tr}\, \left\{\exp\left[-\tau\left(H - \mu N\right)\right]\right\}^{N_\tau}, \qquad \beta = \frac{1}{T} = N_\tau \tau$$

Recast the propagator at each time slice and use FFT

$$\exp\left[-\tau\left(H - \mu N\right)\right] \approx \exp\left[-\tau\left(T - \mu N\right)/2\right] \exp(-\tau V) \exp\left[-\tau\left(T - \mu N\right)/2\right] + O(\tau^3)$$

Discrete Hubbard-Stratonovich transformation

$$\exp(-\tau V) = \prod_{\vec{x}} \sum_{\sigma_\pm(\vec{x}) = \pm 1} \left\{1 + \sigma_\pm(\vec{x}) A\left[n_\uparrow(\vec{x}) + n_\downarrow(\vec{x})\right]\right\}, \qquad A = \sqrt{\exp(\tau g) - 1}$$

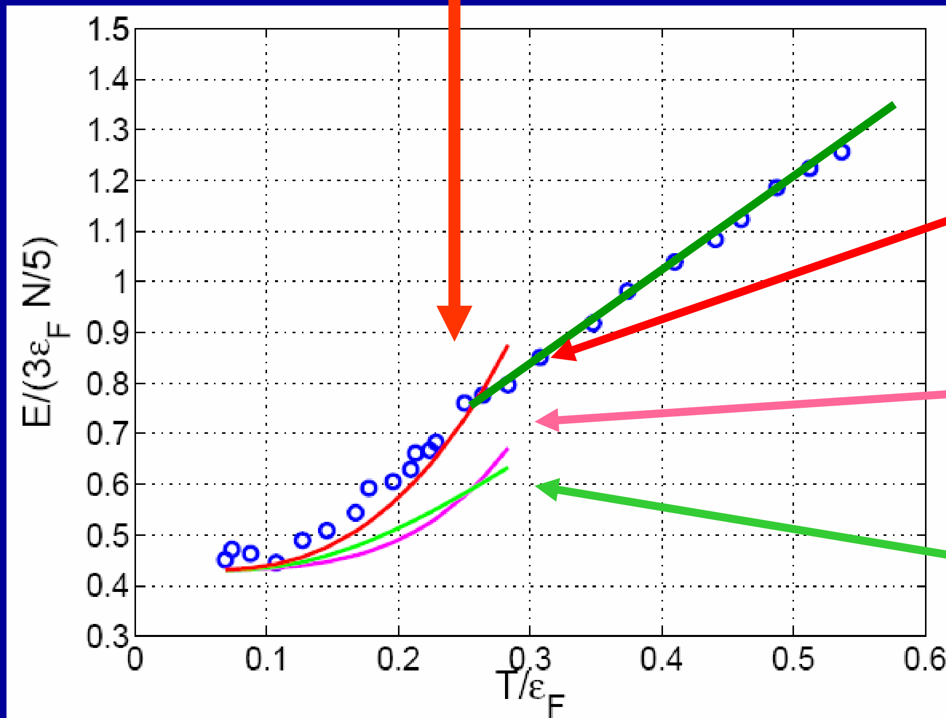
σ-fields fluctuate both in space and imaginary time

$$\frac{m}{4\pi\hbar^2 a} = -\frac{1}{g} + \frac{m k_{cut-off}}{2\pi^2\hbar^2}$$

Running coupling constant g defined by lattice

A. Bulgac, J.E. Drut and P.Magierski

Superfluid to Normal Fermi Liquid Transition



Bogoliubov-Anderson phonons and quasiparticle contribution (red line)

Bogoliubov-Anderson phonons contribution only (magenta line)
People never consider this ???

Quasi-particles contribution only (green line)

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.44$$

$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}}\exp\left(-\frac{\Delta}{T}\right)$$

$$\Delta = \left(\frac{2}{e}\right)^{7/3}\varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

- **Lattice size:**
  from 6^3 x 112 at low T
  to 6^3 x 30 at high T

- **Number of samples:**
  Several 10^5's for T
- **Also calculations for 4^3 lattices**
- **Limited results for 8^3 lattices**

Specific heat of a fermionic cloud in a trap

- **Typical traps have a cigar/banana shape and one distinguish several regimes because of geometry only!**

  - Specific heat exponentially damped if $T \ll \hbar\omega_{\parallel}$
  - If $\hbar\omega_{\parallel} \ll T \ll \hbar\omega_{\perp}$ then $E(T) \approx E_{gs} + \frac{\sqrt{3}\pi^2}{6}\frac{T^2}{\hbar\omega_{\parallel}}$
  - If $T \gg \hbar\omega_{\perp}$ then surface modes dominate

$$E(T) = E_{gs} + \sum_{nl} \frac{(2l+1)\hbar\Omega_{nl}}{\exp(\beta\hbar\Omega_{nl})-1} \approx E_{gs} + 142\frac{T^5}{\hbar^4\omega^4}$$

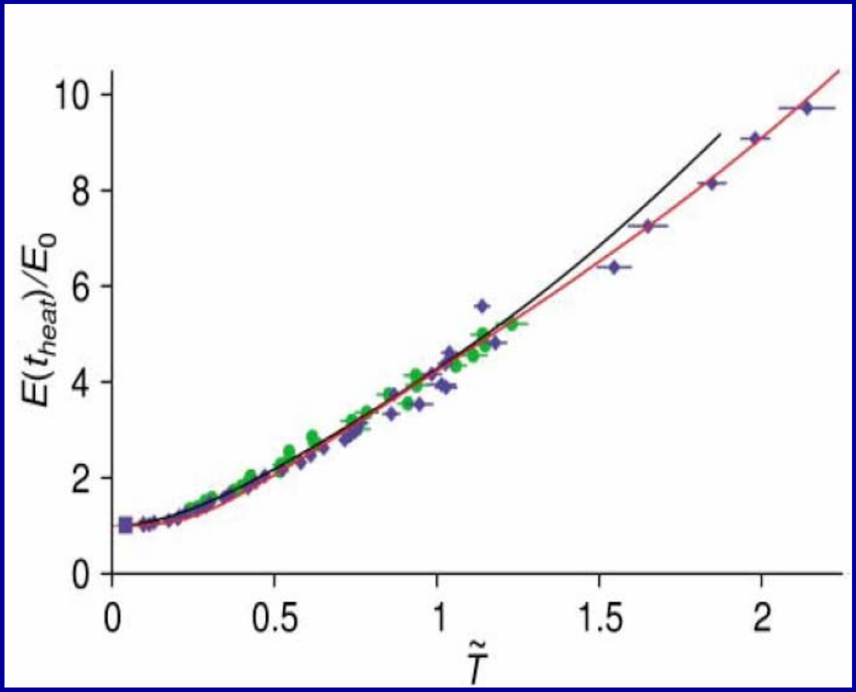
Unexpected!

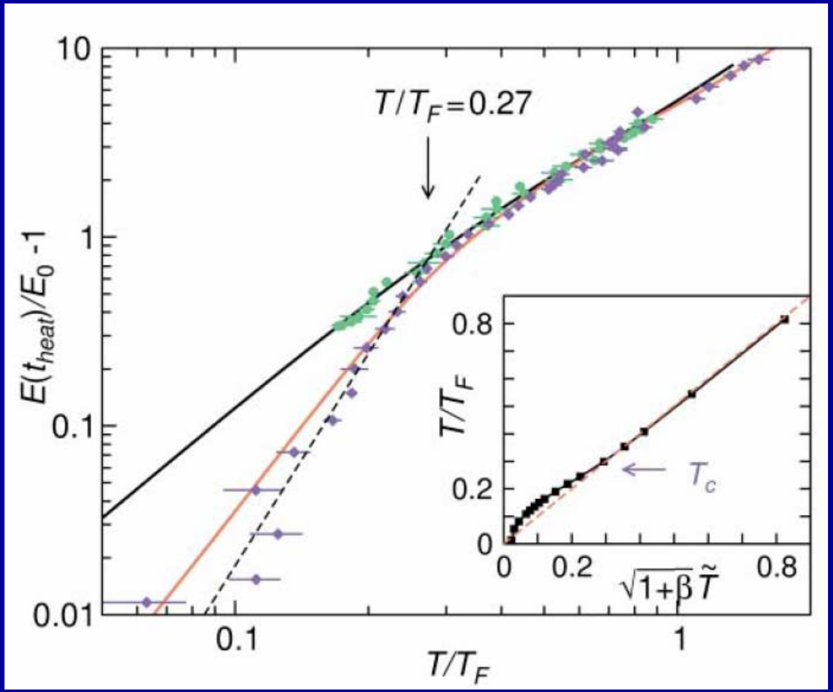
$$\hbar\Omega_{nl} = \hbar\omega\sqrt{\frac{4}{3}n\left(n+l+2\right)+l} \approx \hbar\omega\sqrt{l}$$

$$E(T) = E_{gs} + \frac{3^{1/2}\pi^4}{10}\frac{T^4}{\hbar^3\omega^3}$$

Expected bulk behavior (if no surface modes)

What experiment (with some theoretical input) tells us?

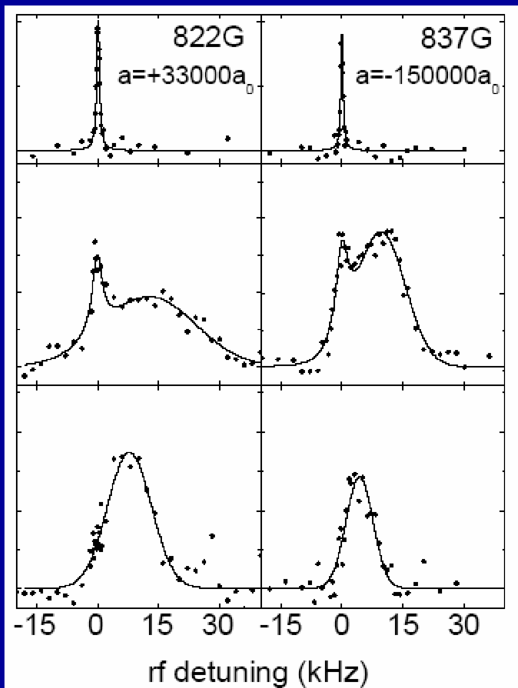




Specific Heat of a Fermi Superfluid in the Unitary Regime

Kinast et al. Science <u>307</u>, 1296 (2005)
Blue symbols – Fermi Gas in the Unitary regime
Green symbols – Non-interacting Fermi Gas

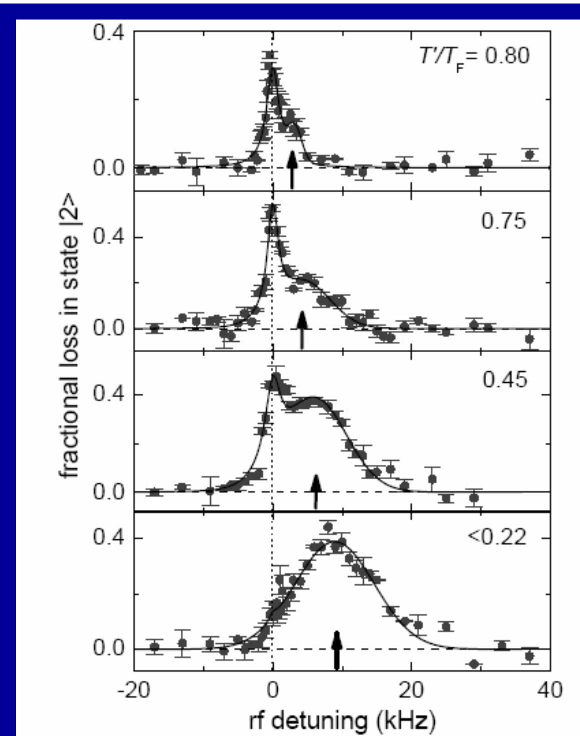
How about the gap?

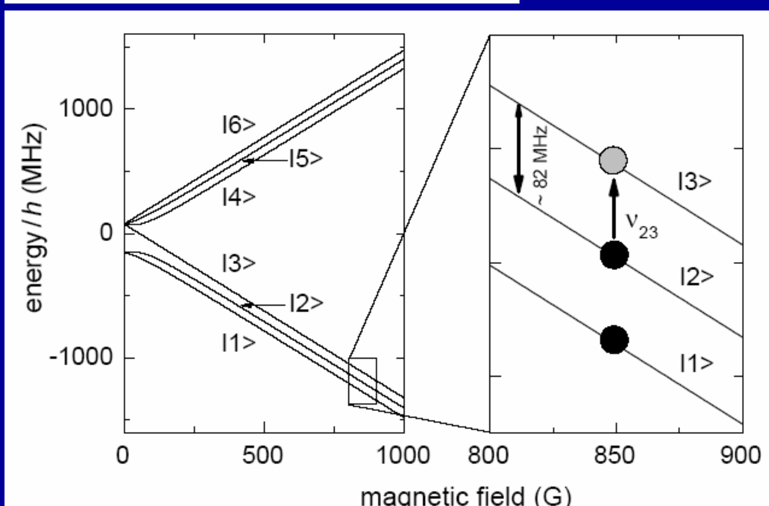


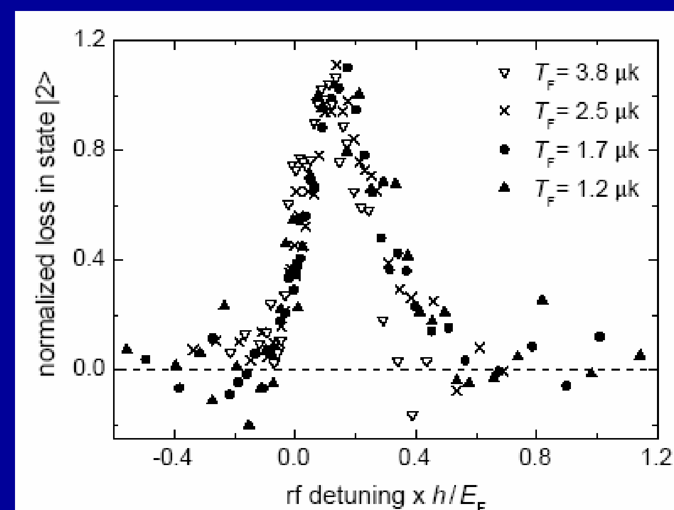
$$\Delta_{\text{exp}} \approx 0.2\varepsilon_F$$

$$\Delta_{\text{theor}} \approx 0.5\varepsilon_F$$

$$T_c \approx ?$$







Chin et al. Science **305**, 1128 (2004)

This shows scaling expected in unitary regime

Key experiments seem to confirm to some degree what theorists have expected. <u>However!</u>

✓ The collective frequencies in the two experiments show significant and unexplained differences.

✓ The critical temperature, allegedly determined in the two independent experiments, does not seem to be the same.

✓ The value of the pairing gap also does not seem to have been pinpointed down in experiments yet!

A liberal quote from a talk of Michael Turner of University of Chicago and NSF

No experimental result is definite until confirmed by theory!

Physics aims at understanding and is not merely a collection of facts.

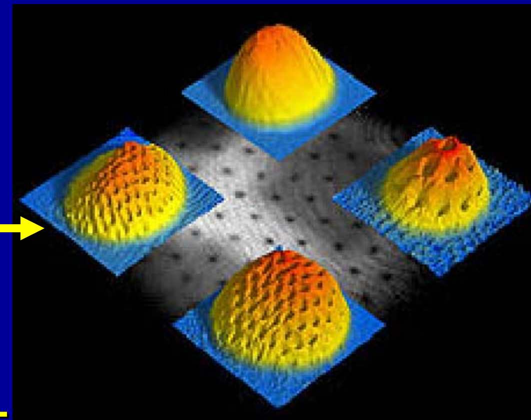
Ernest Rutherford said basically the same thing in a somewhat different form.

If we set our goal to prove that these systems become superfluid, there is no other way but to show it!

Is there a way to put directly in evidence the superflow?

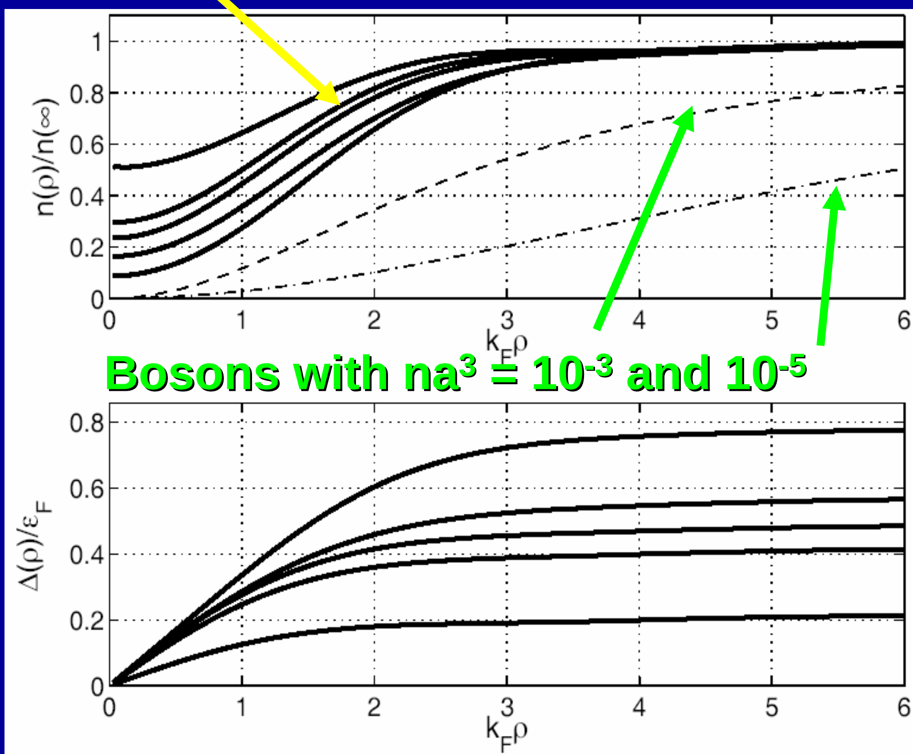
Vortices!

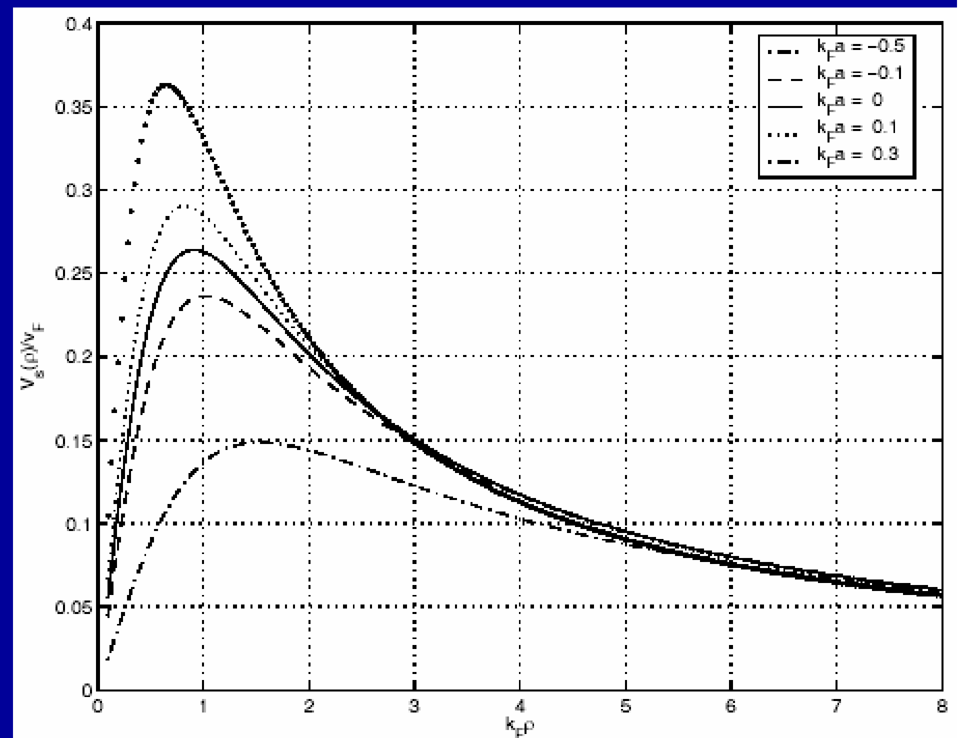
The depletion along the vortex core is reminiscent of the corresponding density depletion in the case of a vortex in a Bose superfluid, when the density vanishes exactly along the axis for 100% BEC.



From Ketterle's group

Fermions with $1/k_F a$ = 0.3, 0.1, 0, -0.1, -0.5





Extremely fast quantum vortical motion!

Number density and pairing field profiles

Local vortical speed as fraction of Fermi speed

Main conclusions

TABLE I: Character of the condensate as a function of the inverse scattering length a^{-1} in various in intervals, the approximate boundaries of these intervals being shown in the second row. The total electron spin and spin projection (S, S_Z) along the magnetic field for various pairs are shown in the last row.

$a^{-1} > 0$			$a^{-1} < 0$	
$+\infty$	r_0^{-1} k_F	0	0 k_F	$-\infty$
molecules	halo dimers (+ atoms ?[15])	?	BCS strong coupling	BCS weak coupling
(0,0)	(1,-1)	(1,-1)	(1,-1)	(1,-1)

Theory: **easy** **easy** **hard** **hard** **easy**

✓ **Fermion superfluidity, more specificaly <u>superflow,</u> has not yet been demonstrated unambiguously experimentally.**

✓ **There is lots of circumstantial evidence and facts in qualitative agreement with theoretical models assuming its existence.**

✓ **<u>Vortices!</u>**