What do we know about the state of cold fermions in the unitary regime?

Aurel Bulgac, George F. Bertsch, Joaquin E. Drut, Piotr Magierski, Yongle Yu
University of Washington, Seattle, WA

Also in Warsaw

Now in Lund

Outline

- Lots of others people results (experiment mainly and theory) throughout the entire presentation

- What is the unitary regime?

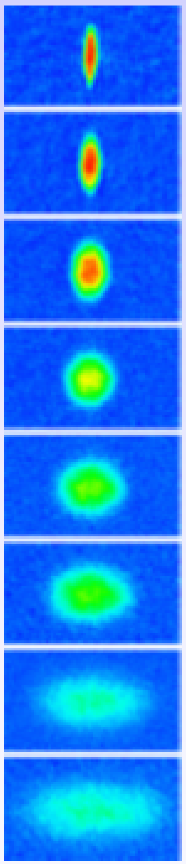
- The two-body problem, how one can manipulate the two-body interaction?

- What many/some theorists know and suspect that is going on?

- What experimentalists have managed to put in evidence so far and how that agrees with theory?

Why Study Fermi Gases ?

Duke Physics
Atom Cooling and Trapping



- Fermions are the building blocks of matter
- Strongly-interacting Fermi gases are stable
- Link to other interacting Fermi systems:
  - High-T_C superconductors – Neutron stars
  - Lattice field theory
  - Quark-gluon plasma of Big Bang
  - String theory!

O'Hara et al., Science 2002

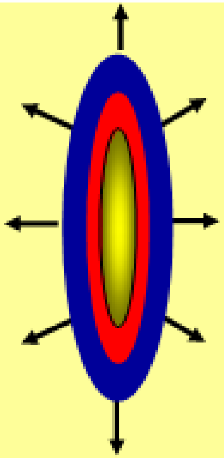
From a talk of J.E. Thomas (Duke)

Finite Temperature Hydrodynamics

Duke Physics

Atom Cooling and Trapping

2) Finite Temperature Hydrodynamics: Breathing Mode or Expansion



Scale factor

$\sigma_x(t) = \sigma_x b_x(t)$

$\mathbf{u}(\mathbf{x},t) =$ velocity field

$$\nabla P_0(\tilde{\mathbf{x}}) + n_0(\tilde{\mathbf{x}})\nabla U(\tilde{\mathbf{x}}) = 0$$

If isentropic: $\mathbf{u}_n = \mathbf{u}_s = \mathbf{u}$

$$m\frac{\partial \mathbf{u}}{\partial t} = -\nabla\left(\tfrac{m}{2}\mathbf{u}^2 + U(\mathbf{x})\right) - \frac{\nabla P(\mathbf{x})}{n(\mathbf{x},\mathrm{t})}$$

$$x = \tilde{x}\, b_x(t) \qquad n(\mathbf{x},\mathrm{t}) = \frac{n_0(\tilde{\mathbf{x}})}{\Gamma}$$

$$u_x = \tilde{x}\, \dot{b}_x(t)$$

$$P(n,T) = \tfrac{2}{3} n \varepsilon_F(n) f_E\left[\frac{T}{T_F(n)}\right]$$

If isentropic expansion: $\Rightarrow$ $\boxed{P(\mathbf{x}) = \dfrac{P_0(\tilde{\mathbf{x}})}{\Gamma^{5/3}}}$

$$m\frac{\partial \mathbf{u}}{\partial t} = -\nabla\left(\tfrac{m}{2}\mathbf{u}^2 + U(\mathbf{x}) - \frac{U(\tilde{\mathbf{x}})}{\Gamma^{2/3}}\right)$$

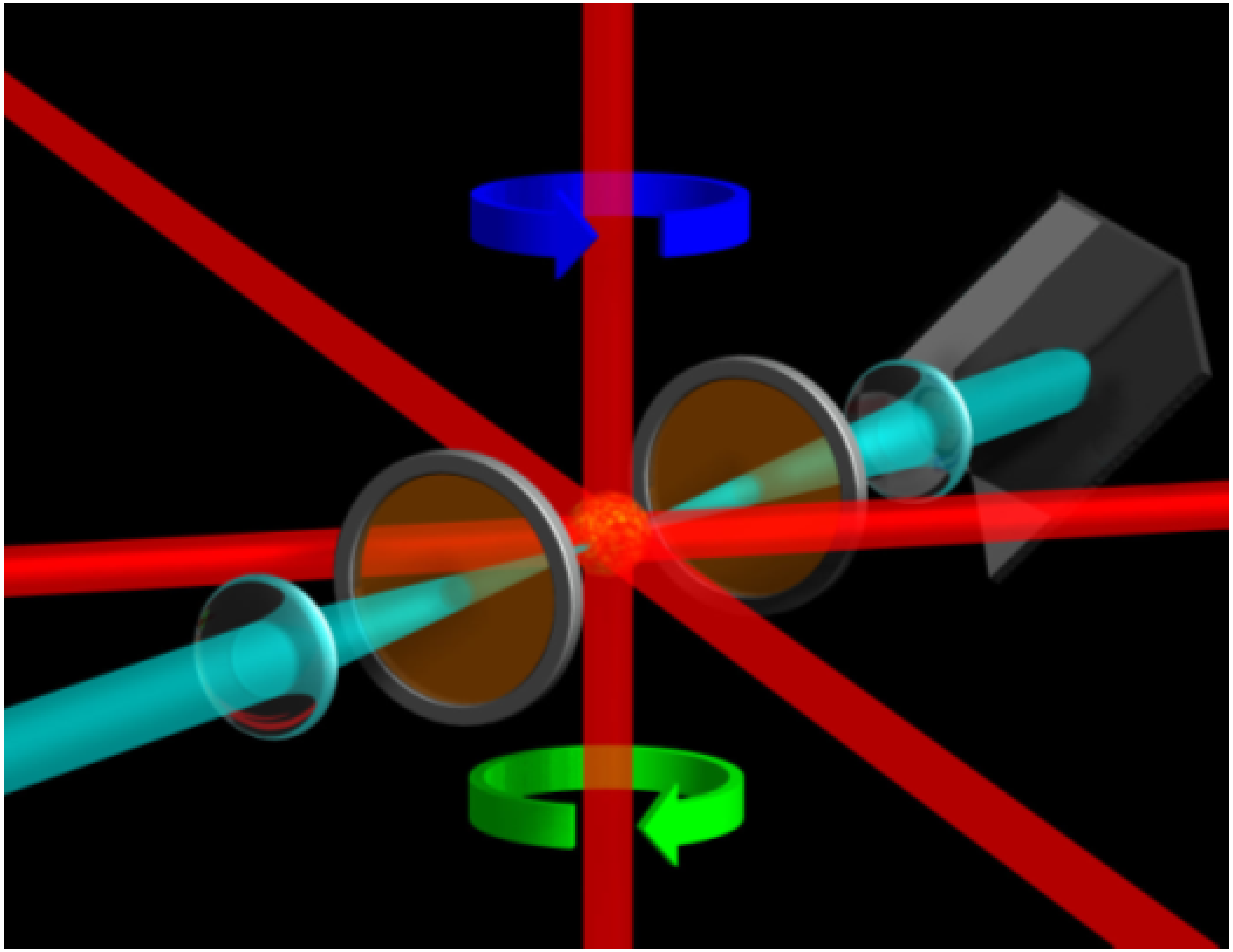
Temperature and Density Independent!

Experiments $\rightarrow$ isentropic behavior

From a talk of J.E. Thomas (Duke)

Optical Trap Loading

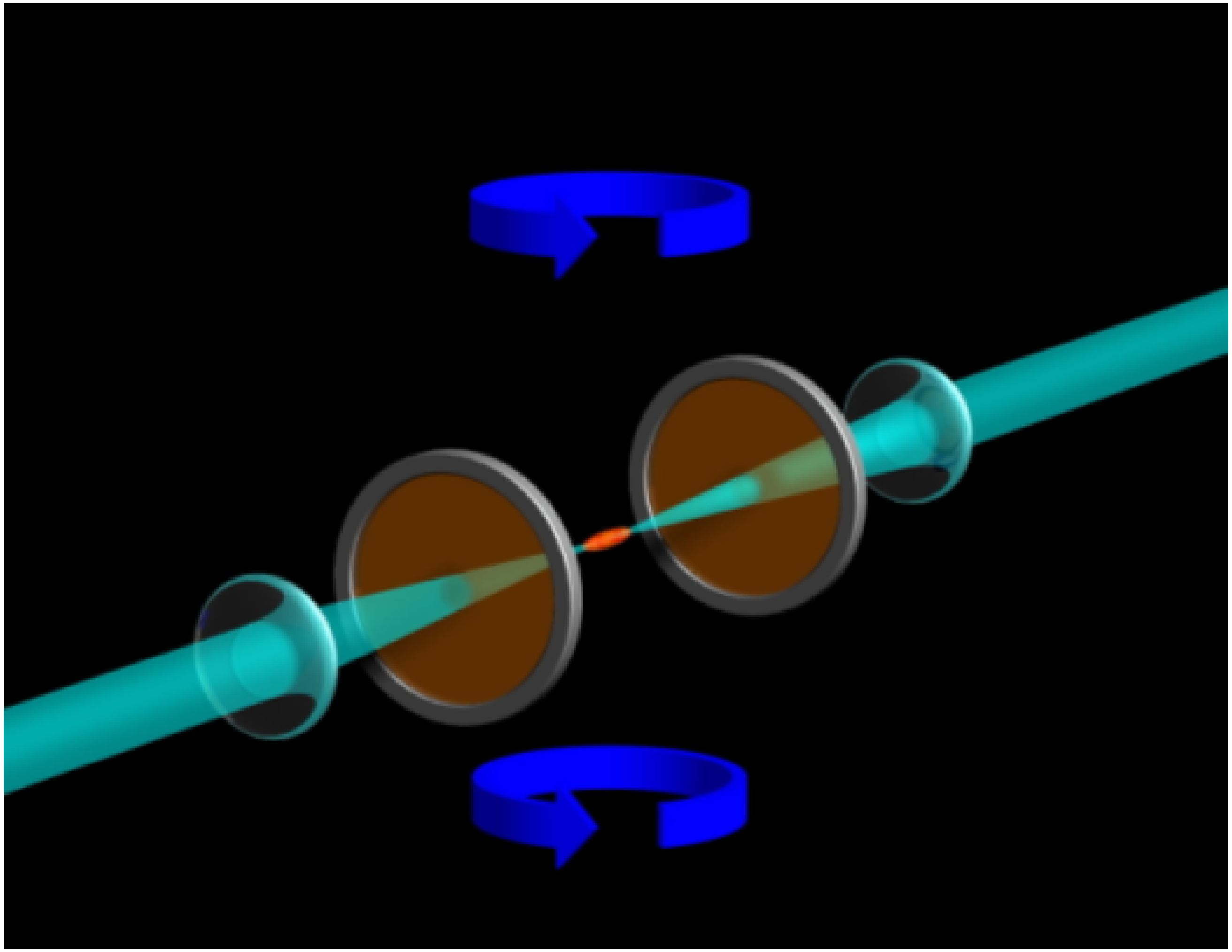
Duke Physics
Atom Cooling and Trapping



From a talk of J.E. Thomas (Duke)

Forced Evaporation

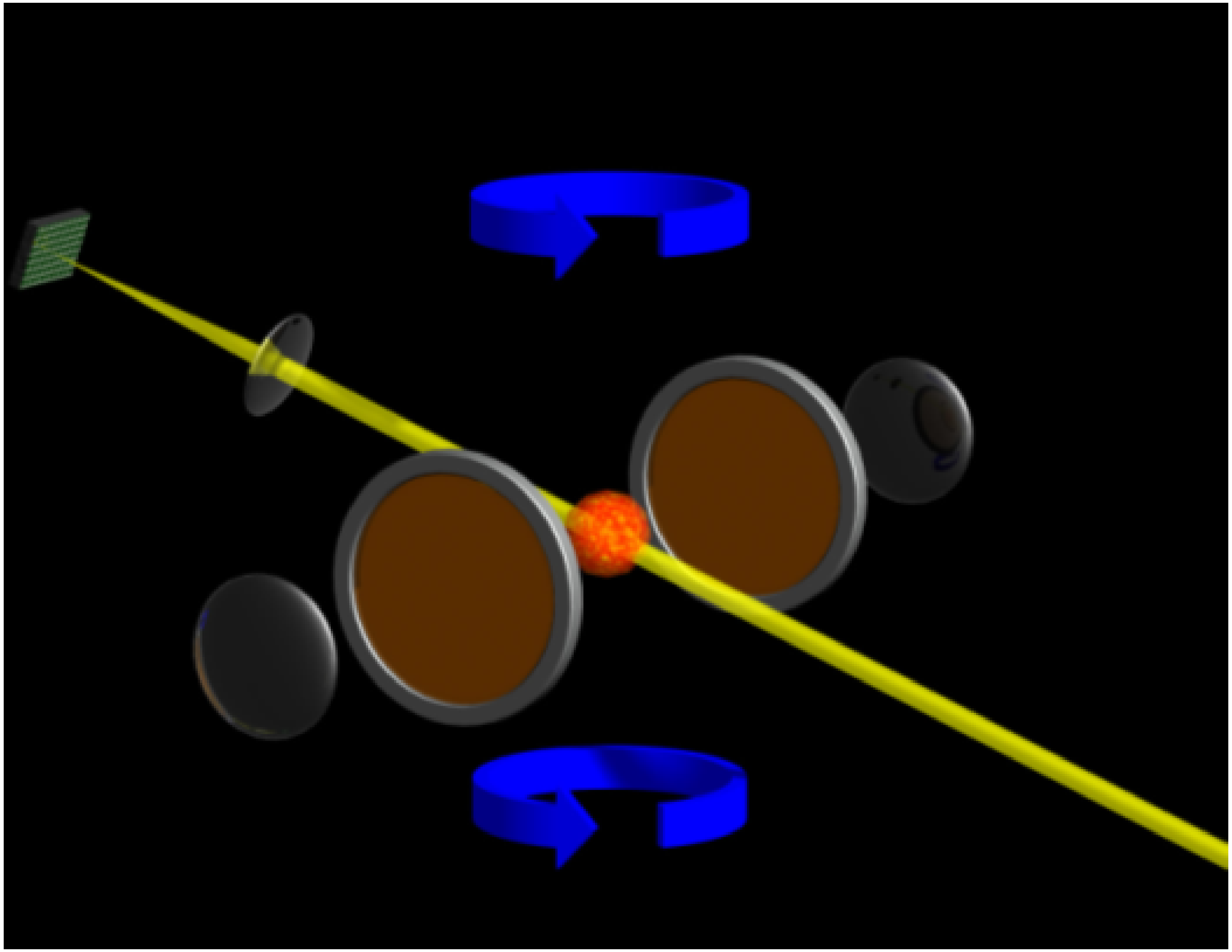
Duke Physics
Atom Cooling and Trapping



From a talk of J.E. Thomas (Duke)

High-Field Imaging

Duke Physics
Atom Cooling and Trapping



From a talk of J.E. Thomas (Duke)

➢ What is the unitary regime?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

The system is very dilute, but strongly interacting!

$$\boxed{n\, r_0^3 \ll 1} \qquad \boxed{n\, |a|^3 \gg 1}$$

n - number density

$$\boxed{r_0 \ll n^{-1/3} \approx \lambda_F/2 \ll |a|}$$

r_0 - range of interaction

a - scattering length

What is the *Holy Grail* of this field?

Fermionic superfluidity!

Superconductivity and superfluidity in Fermi systems

20 orders of magnitude over a century of (low temperature) physics

- Dilute atomic Fermi gases — $T_c \approx$ $10^{-12} - 10^{-9}$ eV
- ✓ Liquid ^{3}He — $T_c \approx$ 10^{-7} eV
- ✓ Metals, composite materials — $T_c \approx$ $10^{-3} - 10^{-2}$ eV
- ✓ Nuclei, neutron stars — $T_c \approx$ $10^{5} - 10^{6}$ eV
- QCD color superconductivity — $T_c \approx$ $10^{7} - 10^{8}$ eV

units (1 eV $\approx$ 10^4 K)

Bertsch Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

Why? Besides pure theoretical curiosity, this problem is relevant to neutron stars!

<u>In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not! A number of people argued that under such conditions fermionic matter is unstable.</u>

- ***systems of bosons are unstable (Efimov effect)***
- ***systems of three or more fermion species are unstable (Efimov effect)***

- **Baker (winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)**

- **Carlson et al (2003) Fixed-Node Green Function Monte Carlo and Astrakharchik et al. (2004) FN-DMC provided the best theoretical estimates for the ground state energy of such systems.**

- **Thomas' Duke group (2002) demonstrated experimentally that such systems are (meta)stable.**

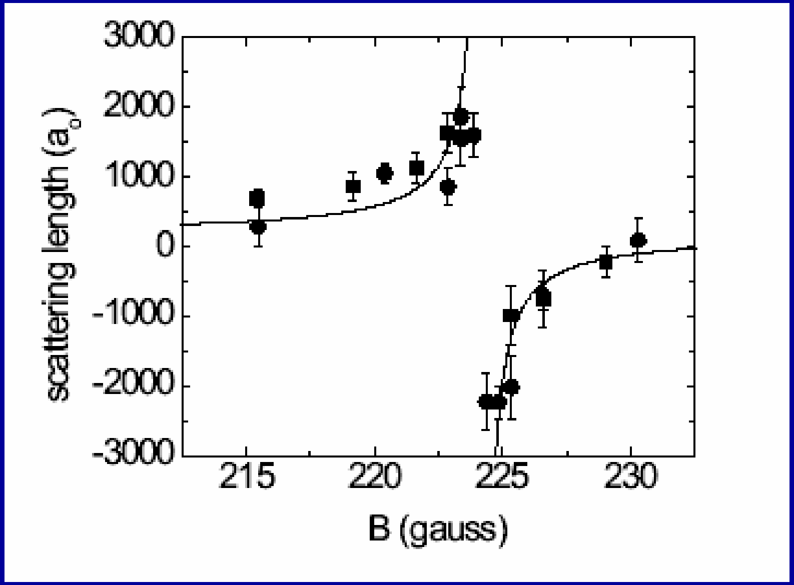
Feshbach resonance

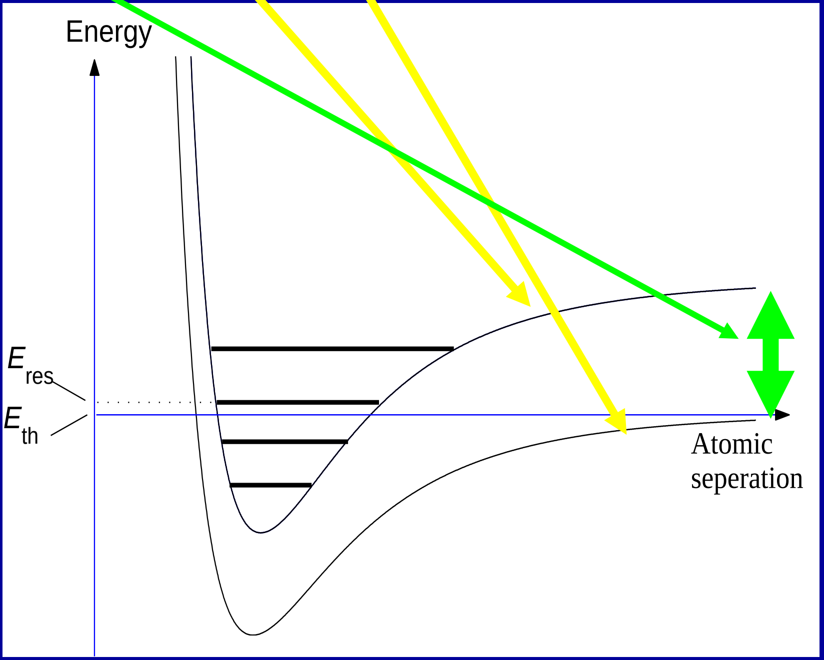
$$H = \frac{\vec{p}^{\,2}}{2\mu_r} + \sum_{i=1}^{2}(V_i^{hf} + V_i^{Z}) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + \cancel{V^d}$$

Channel coupling

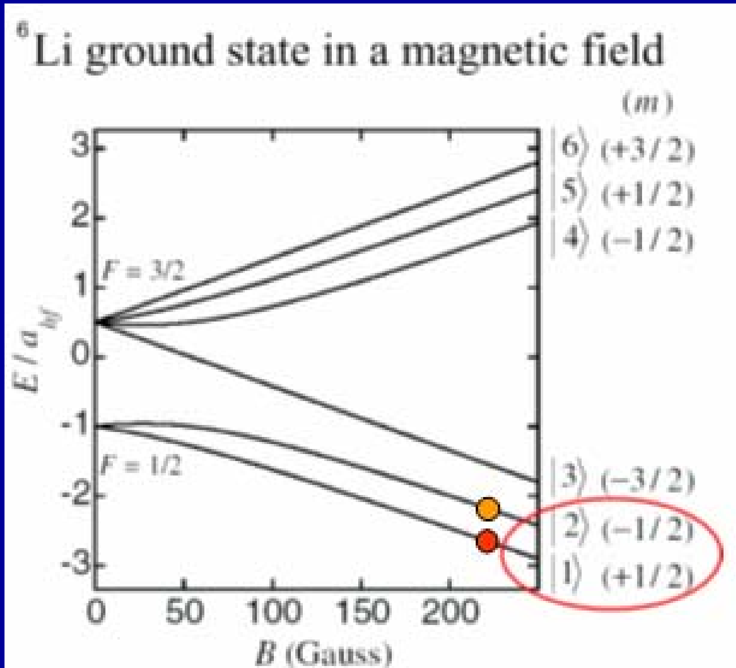
$$V^{hf} = \frac{a_{hf}}{\hbar^2}\vec{S}^e \cdot \vec{S}^n, \; V^Z = (\gamma_e S_z^e - \gamma_n S_z^n)B$$

Tiesinga, Verhaar, Stoof
Phys. Rev. A**47**, 4114 (1993)





Regal and Jin
Phys. Rev. Lett. **90,** 230404 (2003)



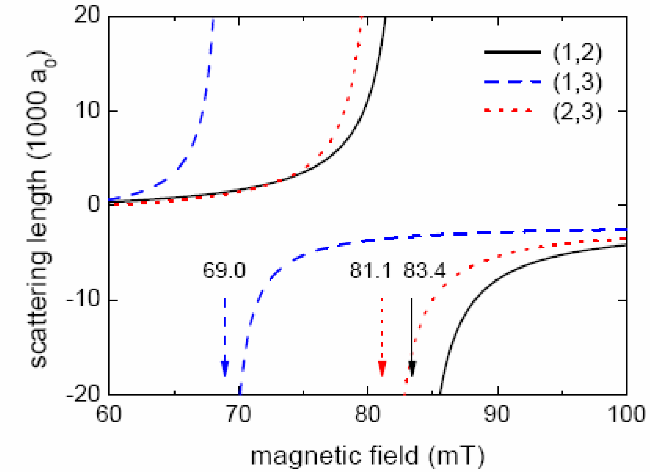
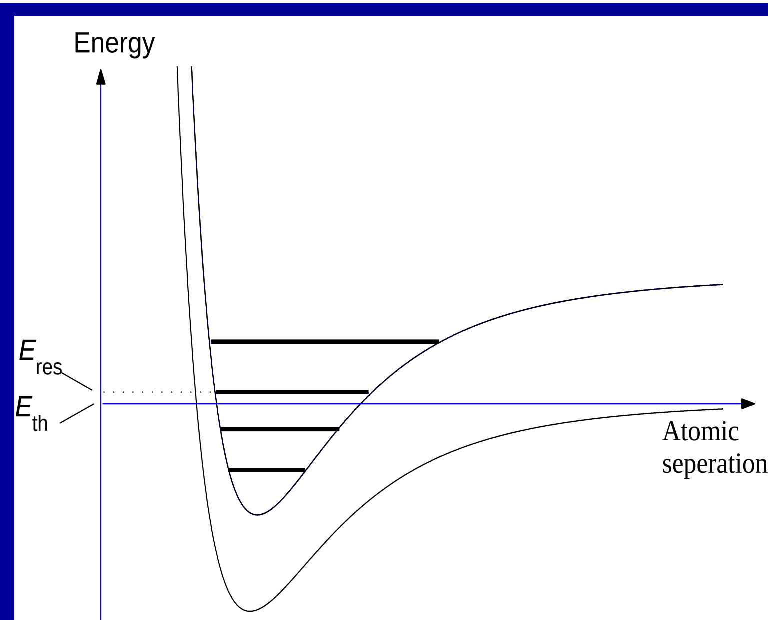
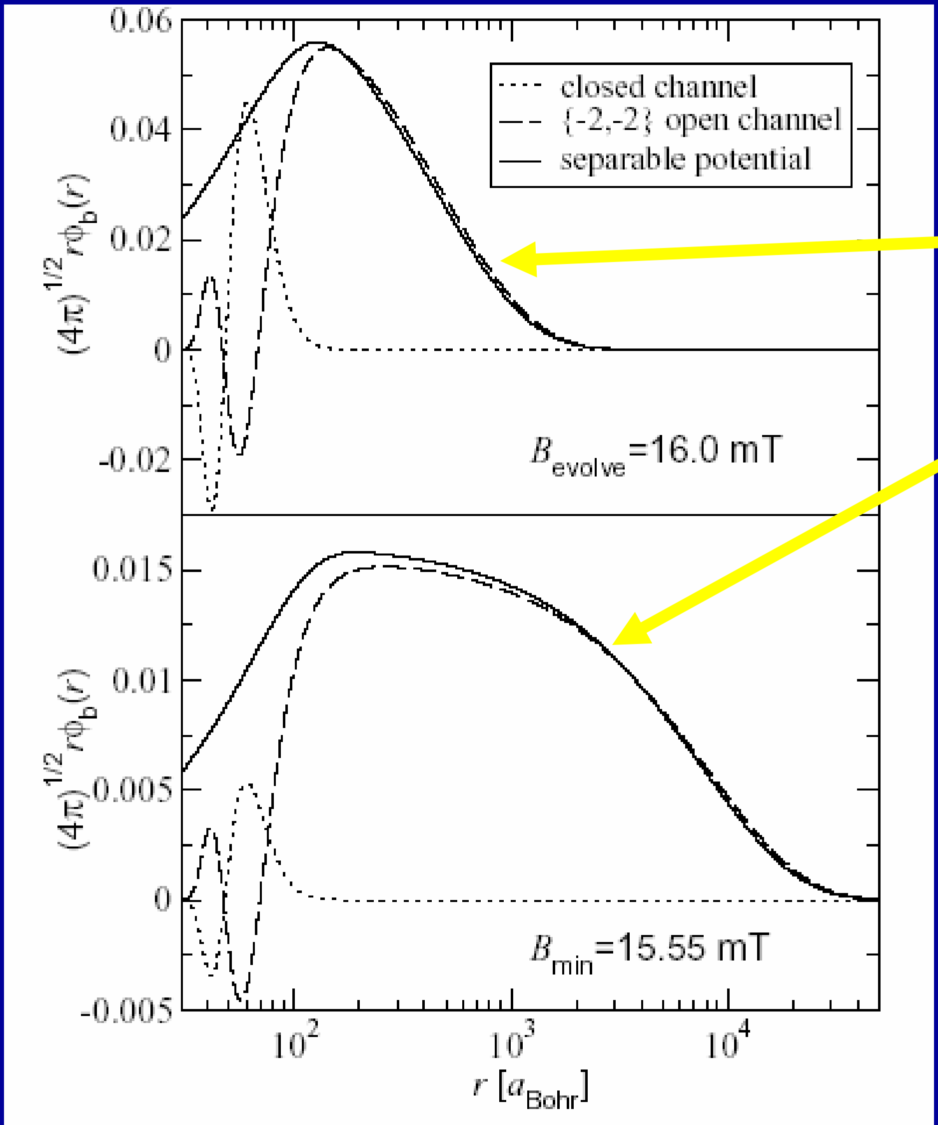


FIG. 4: Scattering lengths versus magnetic field from multi-channel quantum scattering calculations for the (1, 2), (1, 3), and (2, 3) scattering channels. The arrows indicate the resonance positions.



Bartenstein *et al.* Phys. Rev. Lett. **94**, 103201 (2005)

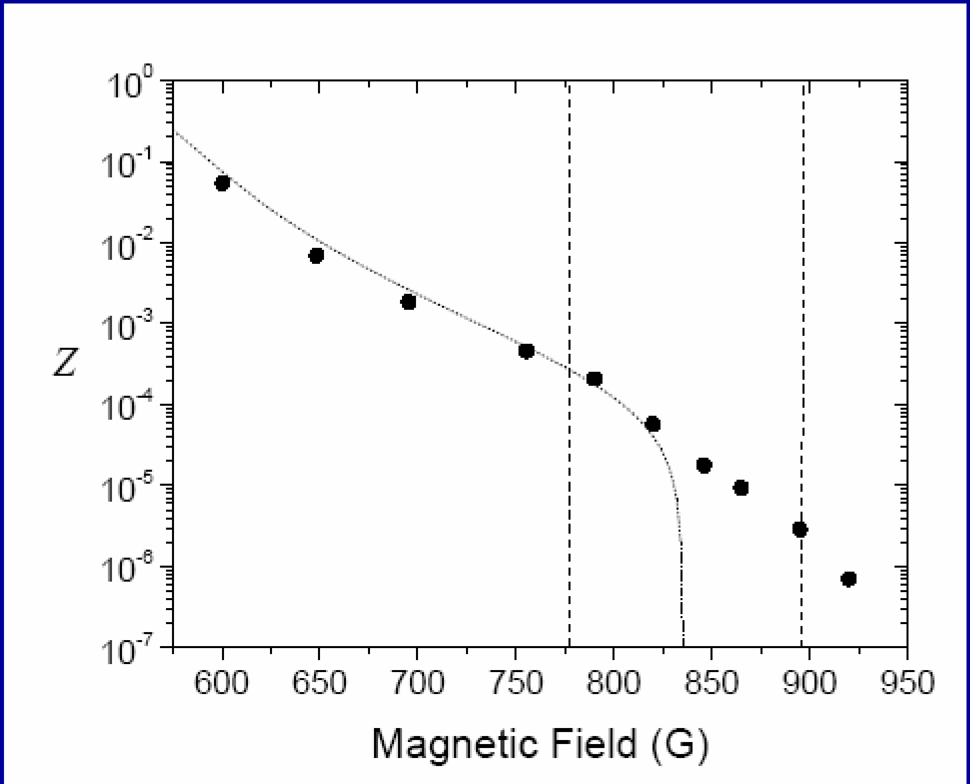


Halo dimer
(open channel)

$$\frac{P(r > r_0)}{P(r < r_0)} \propto \frac{a}{r_0} \quad >> \; 1$$

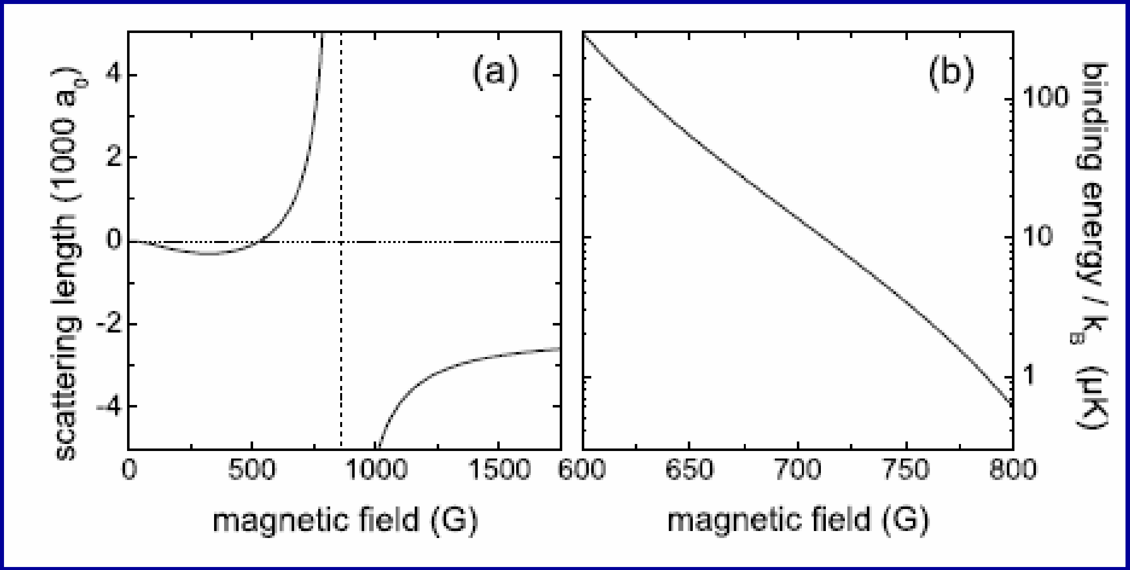
Most of the time two atoms are at distances greatly exceeding the range of the interaction!

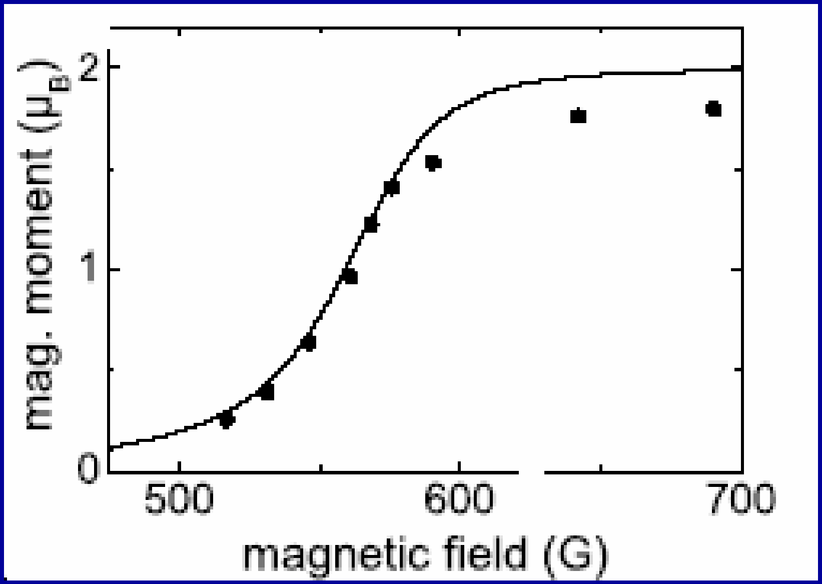
Köhler *et al.* Phys. Rev. Lett. 91, 230401 (2003), inspired by Braaten *et al.* cond-mat/0301489



Z – measured probability to find the two atoms in a singlet state (closed channel)

Dots - experiment of Partridge *et al.* cond-mat/0505353



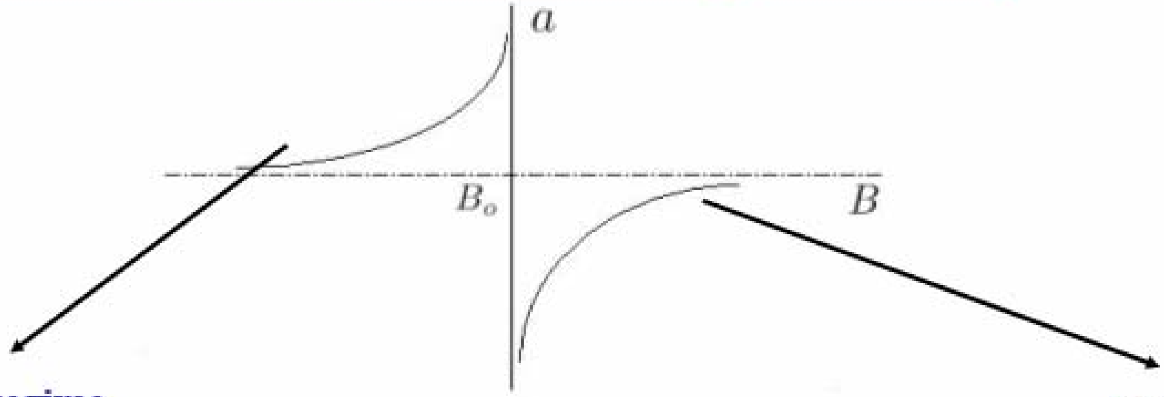


When the system is in the unitary regime the atom pairs are basically pure triplets and thus predominantly in the open channel, where they form spatially large pairs

halo dimers *(if a>0)*

Jochim et al. Phys. Rev. Lett. 91, 240402 (2003)

BCS-BEC crossover in a Fermi gas at T=0



BEC regime

$a > 0 \quad k_F a << 1$

$\mu \approx -|\varepsilon_{bound}|/2$

unitary limit

$k_F|a| >> 1$

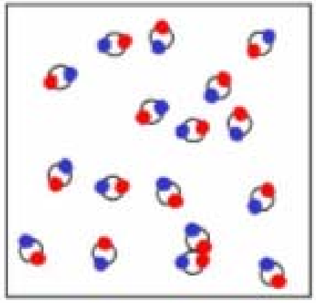
universal regime
(Bertsch-Baker 2001) $\mu = \xi \varepsilon_F$

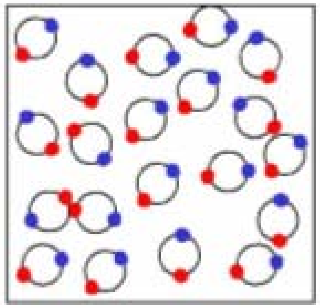
BCS regime

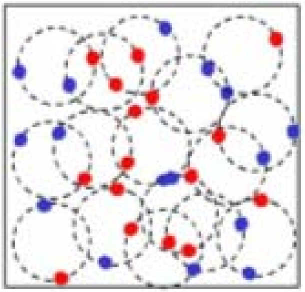
$a < 0 \quad k_F|a| << 1$

$\mu \approx \varepsilon_F$

$\Delta \propto \varepsilon_F \exp(-\pi / 2k_F|a|)$

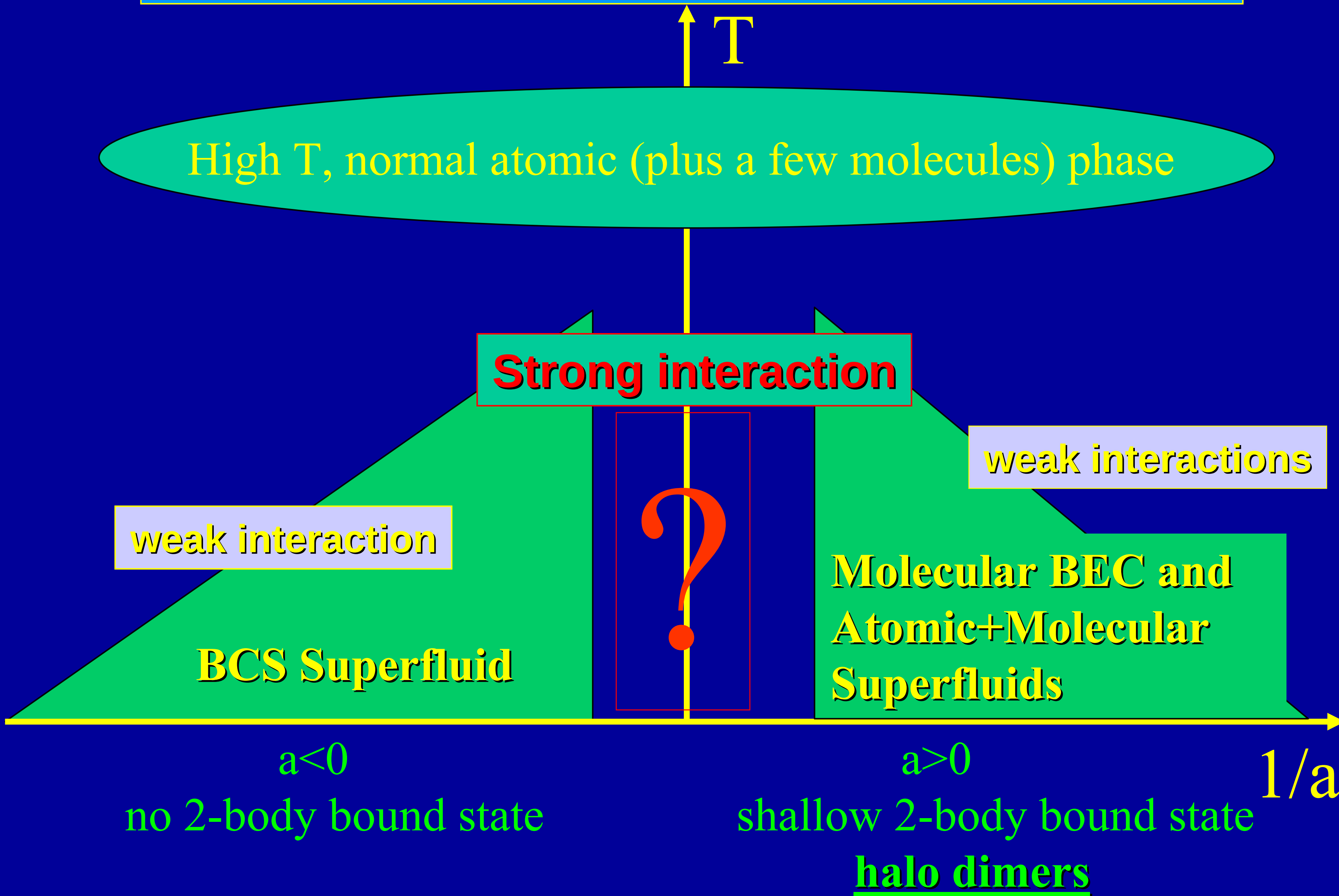






From a talk of Stefano Giorgini (Trento)

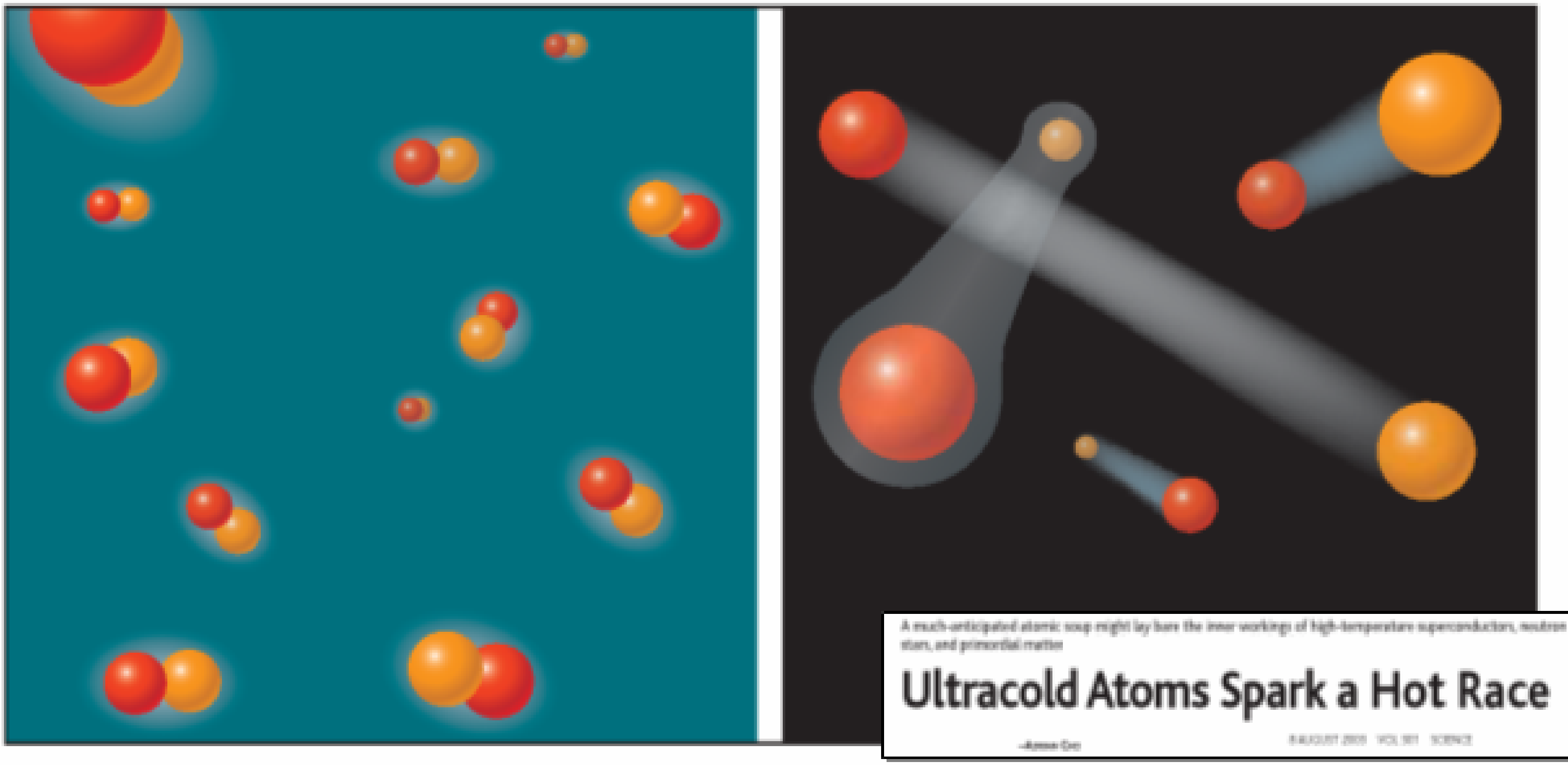
Expected phases of a two species dilute Fermi system
BCS-BEC crossover



molecules
strong coupling

crossover

Cooper pairs
weak coupling



Tango or twist? In a magnetic field, atoms in different spin states can form molecules (*left*). Vary the field, and they might also form loose-knit Cooper pairs.

From a talk of R. Grimm (Innsbruck)
"Original art" from D. Jin (JILA)

Early theoretical approach

Eagles (1969), Leggett (1980) …

$$|gs\rangle = \prod_k \left(\mathrm{u_k} + \mathrm{v_k} a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow}\right)|vacuum\rangle$$ BCS wave function

$$\frac{m}{4\pi\hbar^2 a} = \sum_k \left(\frac{1}{2\varepsilon_k} - \frac{1}{2\mathrm{E}_k}\right)$$ gap equation

$$n = 2\sum_k \left(1 - \frac{\varepsilon_k - \mu}{\mathrm{E}_k}\right)$$ number density equation

$$\Delta \approx \frac{8}{\mathrm{e}^2}\varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$ pairing gap

$$\mathrm{E}_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta^2}$$ quasi-particle energy

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}$$

$$\mathrm{u_k^2} + \mathrm{v_k^2} = 1, \quad \mathrm{v_k^2} = \frac{1}{2}\left(1 - \frac{\varepsilon_k - \mu}{\mathrm{E}_k}\right)$$

Consequences:

- Usual BCS solution for small and negative scattering lengths, with exponentially small pairing gap

- For small and positive scattering lengths this equations describe a gas a weakly repelling (weakly bound/shallow) molecules, essentially all at rest (almost pure BEC state)

$$\Psi\left(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4,\ldots\right)\approx\mathcal{A}\left[\varphi(\vec{r}_{12})\varphi(\vec{r}_{34})\ldots\right]$$

In BCS limit the particle projected many-body wave function has the same structure (BEC of spatially overlapping Cooper pairs)

- For both large positive and negative values of the scattering length these equations predict a smooth crossover from BCS to BEC, from a gas of spatially large Cooper pairs to a gas of small molecules

What is wrong with this approach:

• The BCS gap is overestimated, thus critical temperature and condensation energy are overestimated as well.

• In BEC limit (small positive scattering length) the molecule repulsion is overestimated

• The approach neglects of the role of the "meanfield (HF) interaction," which is the bulk of the interaction energy in both BCS and unitary regime

• All pairs have zero center of mass momentum, which is reasonable in BCS and BEC limits, but incorrect <u>in the unitary regime</u>, where <u>the interaction between pairs is strong !!!</u> (similar to superfluid ^{4}He)

Fraction of non-condensed pairs (perturbative result)!?!

$$\frac{n_{ex}}{n_0} = \frac{8}{3\sqrt{\pi}}\sqrt{n_m a_{mm}^3}, \qquad n_m = \frac{n}{2}, \qquad a_{mm} \approx 0.6a$$

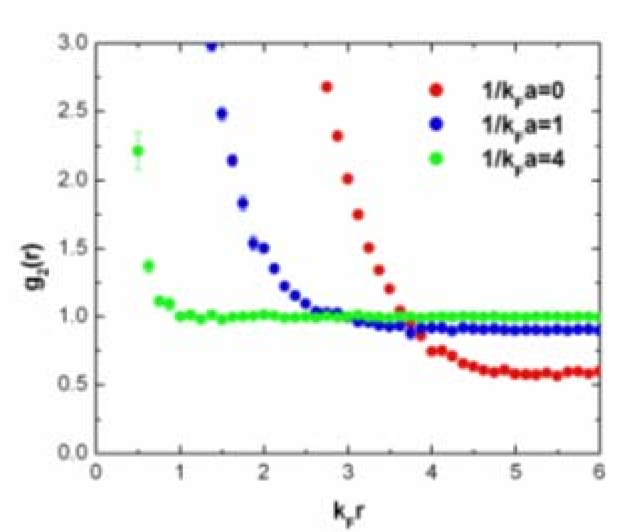
Two-body density matrix and condensate fraction

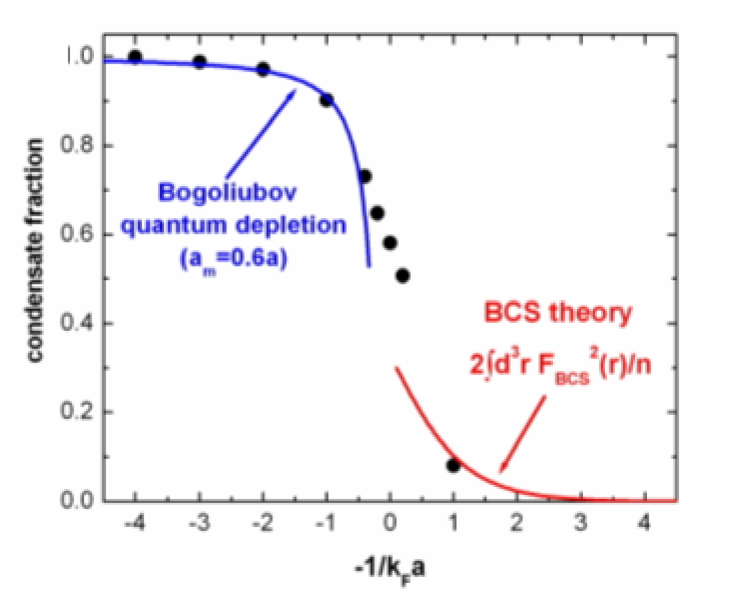
$$\langle \psi_\uparrow^+(\vec{r}_1+\vec{r})\psi_\downarrow^+(\vec{r}_2+\vec{r})\psi_\uparrow(\vec{r}_1)\psi_\downarrow(\vec{r}_2)\rangle \xrightarrow[r\to\infty]{} F^2(|\vec{r}_1-\vec{r}_2|)$$

where

$$F(|\vec{r}_1-\vec{r}_2|)=\langle \psi_\uparrow(\vec{r}_1)\psi_\downarrow(\vec{r}_2)\rangle \quad \textbf{order parameter}$$

$$g_2(r)=\frac{2}{N}\int d^3r_1 d^3r_2 \langle \psi_\uparrow^+(\vec{r}_1+\vec{r})\psi_\downarrow^+(\vec{r}_2+\vec{r})\psi_\uparrow(\vec{r}_1)\psi_\downarrow(\vec{r}_2)\rangle$$





From a talk of Stefano Giorgini (Trento)

What people use a lot ?

(Basically this is Eagles' and Leggett's model, somewhat improved.)

VOLUME 83, NUMBER 14 PHYSICAL REVIEW LETTERS 4 OCTOBER 1999

Rarified Liquid Properties of Hybrid Atomic-Molecular Bose-Einstein Condensates

Eddy Timmermans,[1] Paolo Tommasini,[2] Robin Côté,[2,*] Mahir Hussein,[2,3] and Arthur Kerman[4]

$$\hat{H} = \int d^3r\, \hat{\psi}_a^\dagger \left[-\frac{\hbar^2\nabla^2}{2m} + \frac{\lambda_a}{2} \hat{\psi}_a^\dagger \hat{\psi}_a + \lambda \hat{\psi}_m^\dagger \hat{\psi}_m \right] \hat{\psi}_a + \int d^3r\, \hat{\psi}_m^\dagger \left[-\frac{\hbar^2\nabla^2}{4m} + \frac{\lambda_m}{2} \hat{\psi}_m^\dagger \hat{\psi}_m + \epsilon \right] \hat{\psi}_m + \frac{\alpha}{\sqrt{2}} \int d^3r \{ \hat{\psi}_m^\dagger \hat{\psi}_a \hat{\psi}_a + \hat{\psi}_m \hat{\psi}_a^\dagger \hat{\psi}_a^\dagger \}, \qquad (2)$$

VOLUME 88, NUMBER 9 PHYSICAL REVIEW LETTERS 4 MARCH 2002

Signatures of Resonance Superfluidity in a Quantum Fermi Gas

M. L. Chiofalo,* S. J. J. M. F. Kokkelmans, J. N. Milstein, and M. J. Holland

$$H = \sum_{k\sigma} \epsilon_k a_{k\sigma}^\dagger a_{k\sigma} + \nu \sum_k b_k^\dagger b_k + U \sum_{qkk'} a_{q/2+k\uparrow}^\dagger a_{q/2-k\downarrow}^\dagger a_{q/2-k'\downarrow} a_{q/2+k'\uparrow} + \left(g \sum_{kq} b_q^\dagger a_{q/2-k\downarrow} a_{q/2+k\uparrow} + \text{H.c.} \right),$$

Why?

<u>Everyone likes doing simple meanfield (and sometimes add fluctuations on top) calculations!</u>

Timmermans *et al*. realized that a contact interaction proportional to either a very large or infinite scattering length makes no sense in meanfield approximation.

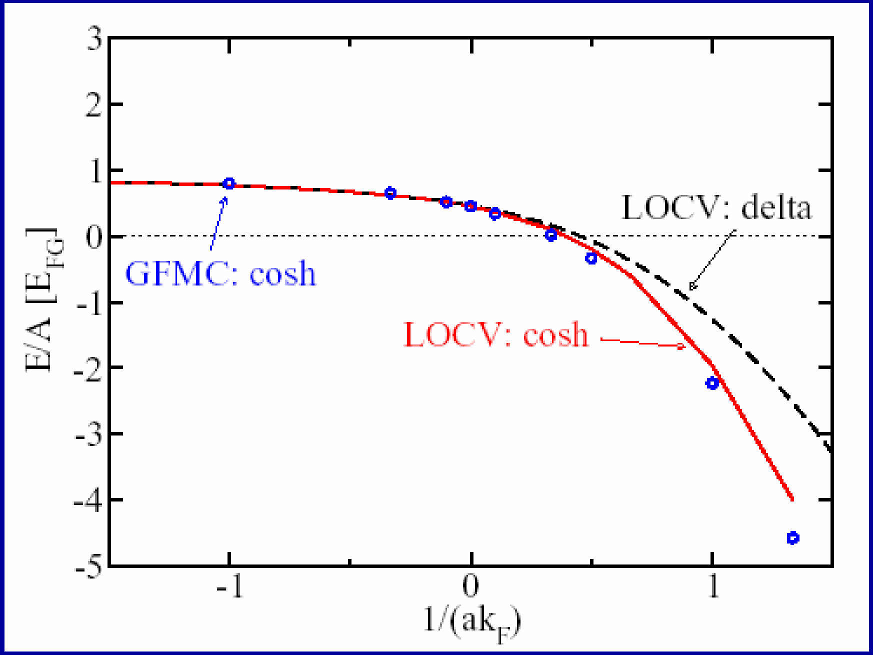
$$U(\vec{r}_1 - \vec{r}_2) = \frac{4\pi\hbar^2 a}{m}\delta(\vec{r}_1 - \vec{r}_2)$$

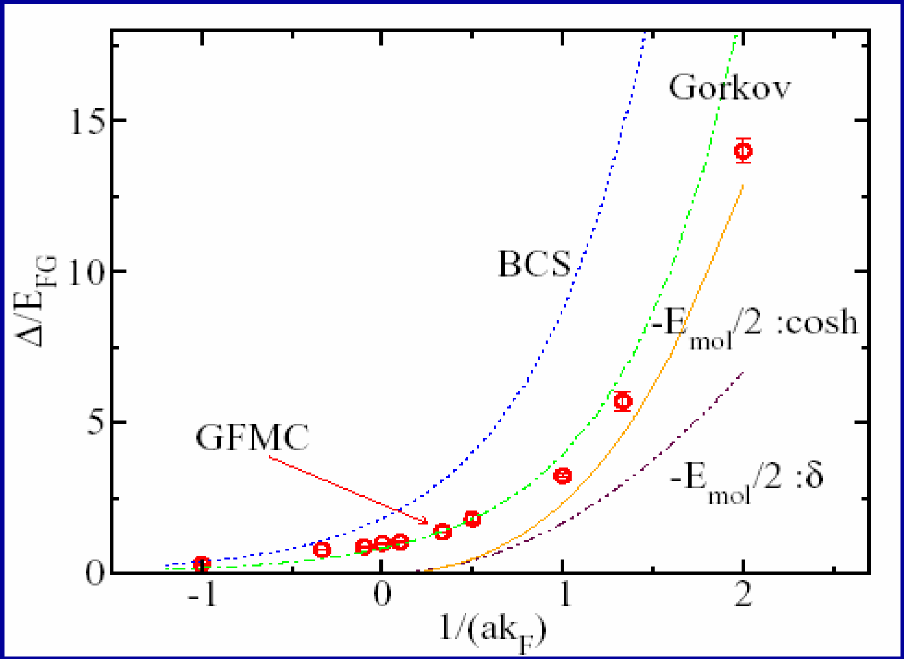
The two-channel approach, which they introduced initially for bosons, does not seem, <u>superficially</u> at least, to share this difficulty. However, one can show that corrections to such a meanfield approach will be governed by the parameter **<u>na^3</u>** anyway, so, the problem has not been really solved.

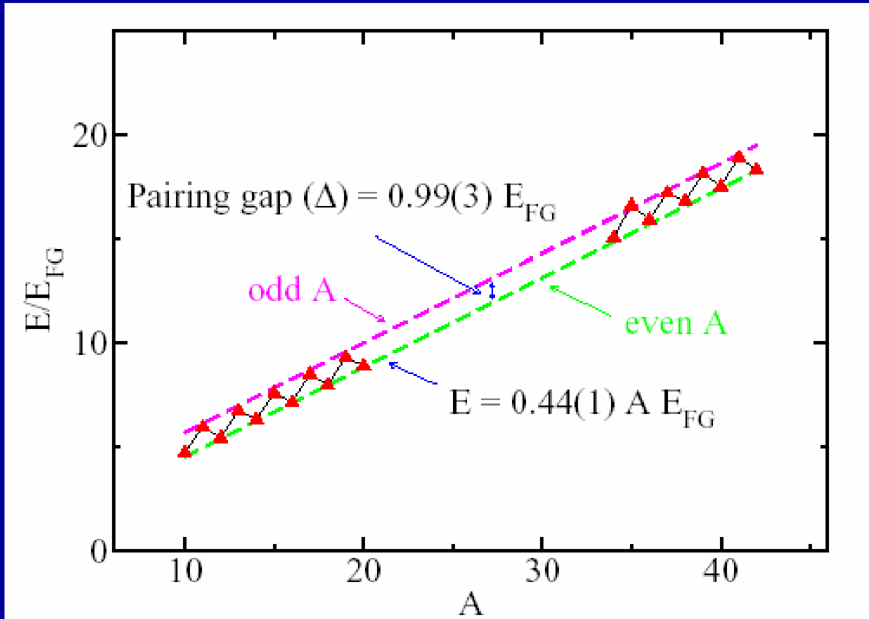
Is there a better approach?

Full blown many body calculations!

Fixed-Node Green Function Monte Carlo approach at T=0





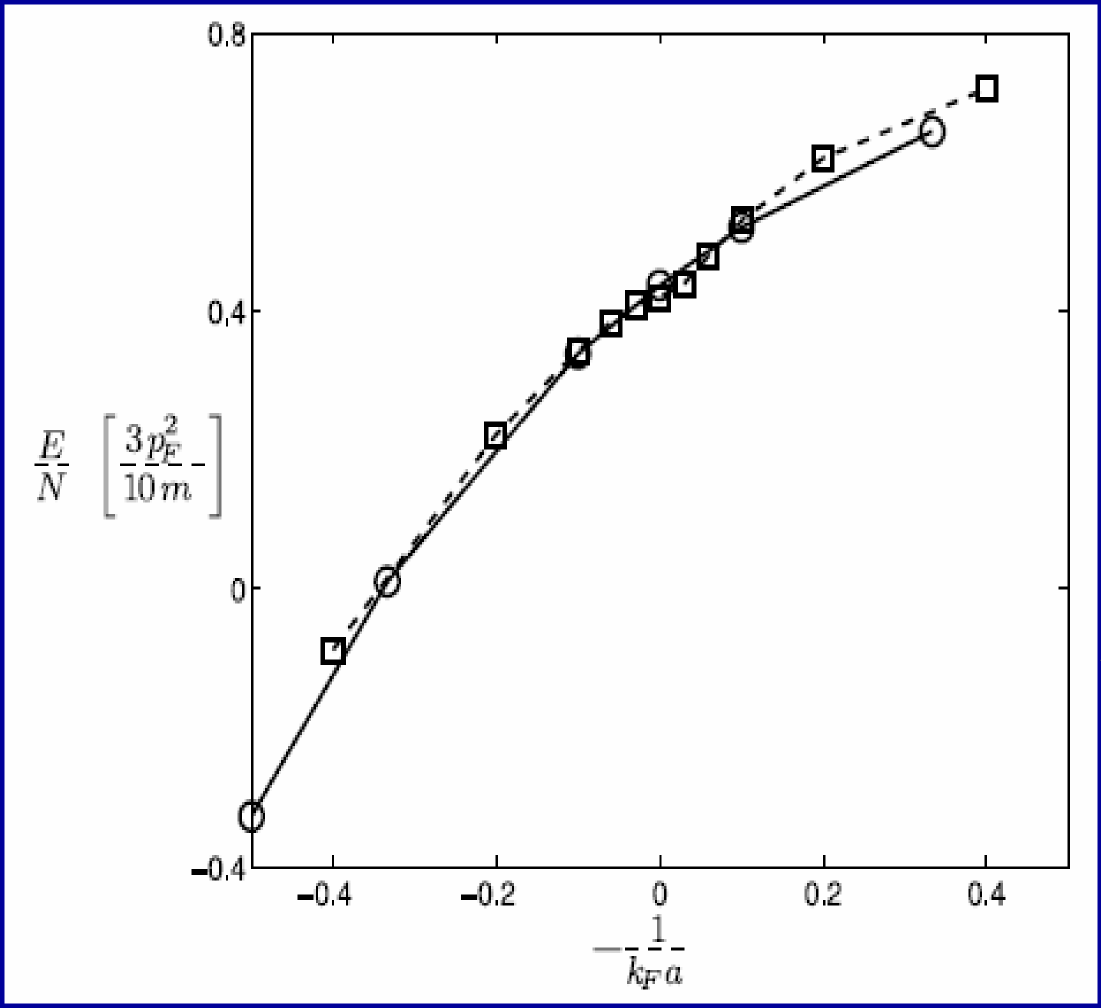


$$\Delta_{\mathrm{BCS}} \approx \frac{8}{\mathrm{e}^2}\varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

$$\Delta_{\mathrm{Gorkov}} \approx \left(\frac{2}{e}\right)^{7/3} \varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

Carlson *et al.* PRL 91, 050401 (2003)
Chang *et al.* PRA 70, 043602 (2004)

Energy per particle near the Feshbach resonance from Fixed Node Green Function/Diffusion Monte Carlo calculations



Solid line with circles
Chang *et al.*
Phys. Rev. A **70**, 043602 (2004)
(both even and odd particle numbers)

Dashed line with squares
Astrakharchik *et al.*
Phys. Rev. Lett. **93**, 200404 (2004)
(only even particle numbers)

$$r_0 \ll \quad \frac{1}{n^{1/3}} \approx \frac{\lambda_F}{2} \quad \ll |a|$$

$$\left.\frac{E}{N}\right|_{GFMC} = \varepsilon[n] \approx \frac{3}{5}\varepsilon_F\left[\xi - \frac{\varsigma}{k_F a} - \frac{5\iota}{3(k_F a)^2}\right], \qquad \xi \approx 0.44, \qquad \varsigma \approx 1, \qquad \iota \approx 1$$

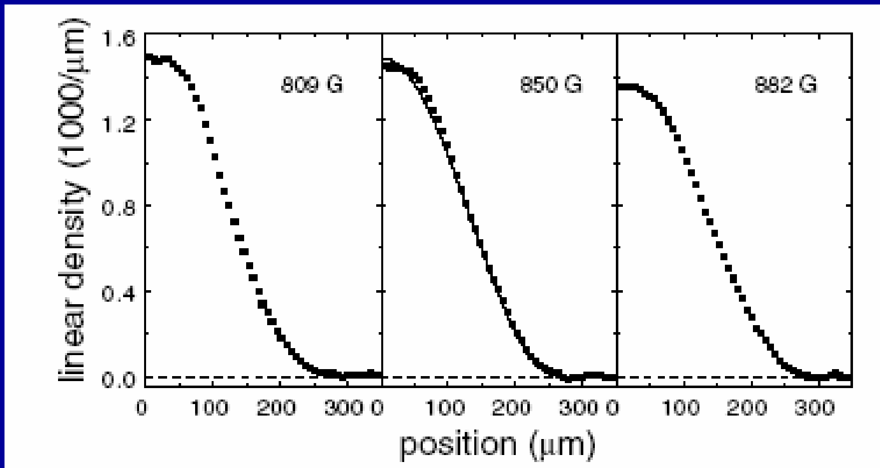
$$\Delta_{GFMC} \approx \varepsilon_F\left(\frac{2}{e}\right)^{7/3}\exp\left(\frac{\pi}{2k_F a}\right), \qquad n = \frac{k_F^3}{3\pi^2}, \quad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}, \quad x = \frac{1}{k_F a}$$

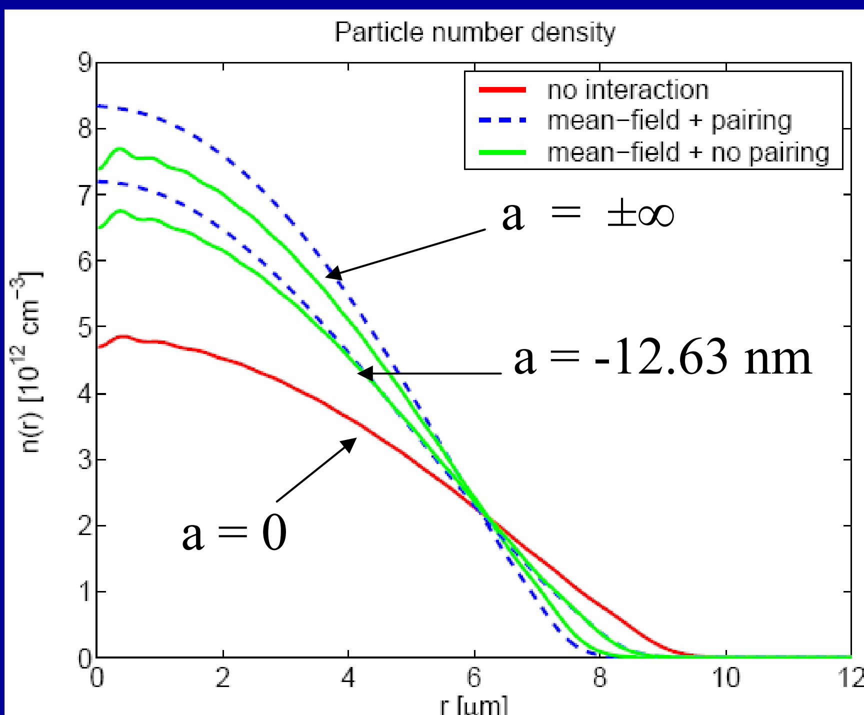
$$\varepsilon_{SLDA}[n]n = \varepsilon_{kin}n + \frac{\hbar^2}{m}\beta[x]\, n^{5/3} + \frac{\hbar^2}{m}\gamma[x]\,\frac{|\nu|^2}{n^{1/3}} \qquad + \quad \text{Renormalization}$$

Dimensionless coupling constants

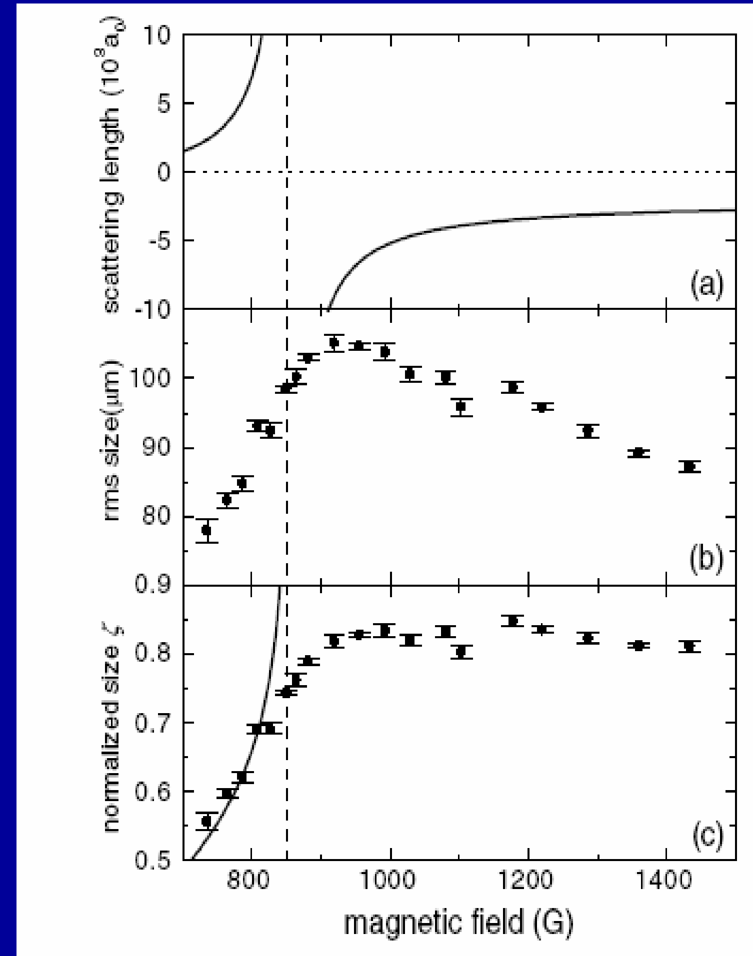
Superfluid LDA (SLDA) is the generalization of Kohn-Sham to superfluid fermionic systems

Jochim et al. Phys.Rev.Lett. 91, 240402 (2003)





Y. Yu, July, 2003, unpublished

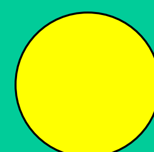
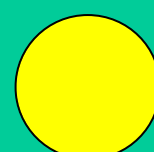
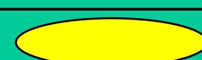


5200 ^{40}K atoms in a spherical trap
$\hbar\omega$=0.568 x 10^{-12} eV

SLDA calculation using
GFMC equation of state of
Carlson *et al.* PRL 91, 050401 (2003)

Sound in infinite fermionic matter

$$\omega = v_s k$$

	Local shape of Fermi surface	Sound velocity	
Collisional Regime - <u>high T!</u> Compressional mode	Spherical 	$v_s \approx \frac{v_F}{\sqrt{3}}$	First sound
Superfluid collisionless- <u>low T!</u> Compressional mode	Spherical 	$v_s \approx \frac{v_F}{\sqrt{3}}$	Bogoliubov-Anderson sound
Normal Fermi fluid collisionless - <u>low T!</u> (In)compressional mode	 Elongated along propagation direction	$v_s = s v_F$ $s > 1$	Landau's zero sound Need repulsion !!!

$$\varepsilon(n) = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m}\left[\xi - \frac{\varsigma}{k_F a} - \frac{5\iota}{(k_F a)^2} + \mathrm{O}\left(\frac{1}{(k_F a)^3}\right)\right]$$

$$\xi \approx 0.44, \qquad \varsigma \approx 1, \qquad \iota \approx 1$$

$$U = \frac{m\omega_0^2\left(x^2+y^2+\lambda^2 z^2\right)}{2}$$

$$\frac{\delta\omega^2}{\omega^2} = \frac{\varsigma}{\xi}\frac{1}{k_F(0)a}K$$

Adiabatic regime
Spherical Fermi surface

Bogoliubov-Anderson modes in a trap

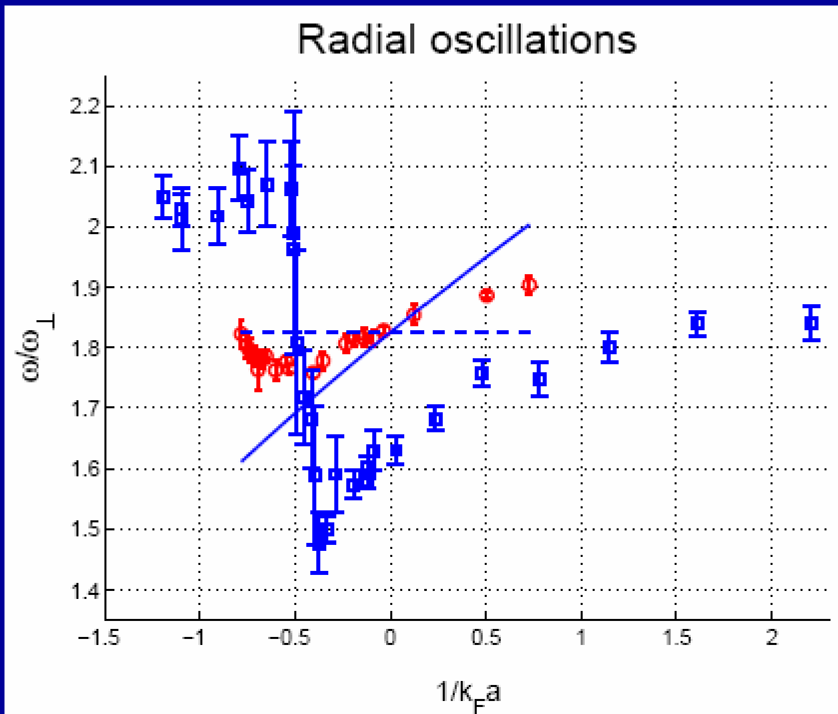
Perturbation theory result using GFMC equation of state in a trap

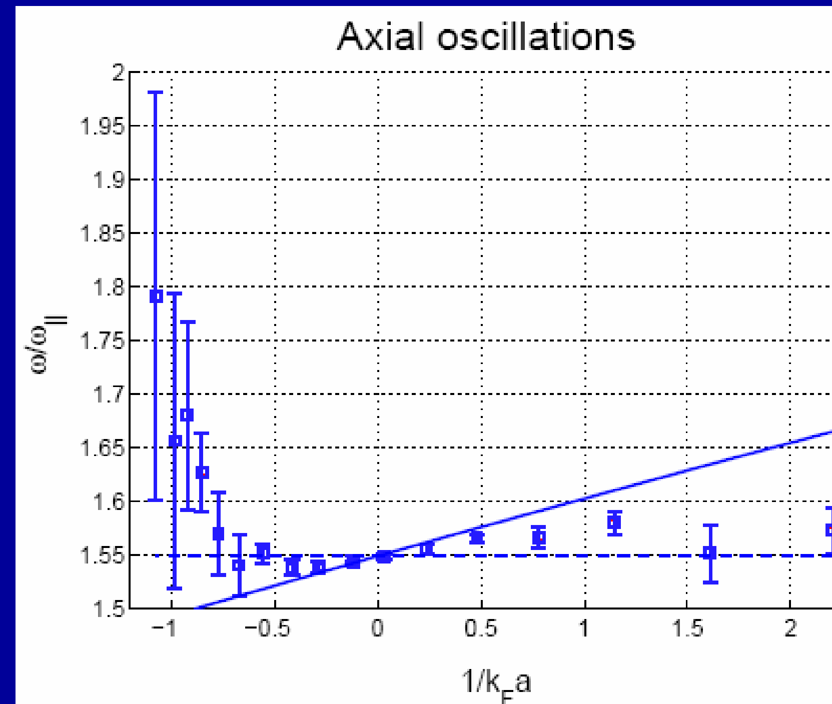
TABLE II: Results for K.

trap type	mode	f_1	ω	K
spherical	dipole	z	ω_0	0
$\lambda = 1$	monopole	$1 - 2r^2$	$2\omega_0$	$\frac{256}{525\pi}$
	quadrupole	xy	$\sqrt{2}\omega_0$	0
axial	$M = \pm 2$	$xy,\ x^2 - y^2$	$\sqrt{2}\omega_0$	0
$\lambda \ll 1$	$M = \pm 1$	$xz,\ yz$	ω_0	0
	radial	$x^2 + y^2 + \frac{2}{5}\lambda^2 z^2 - \frac{2}{5}$	$\sqrt{\frac{10}{3}}\omega_0$	$\frac{1024}{2625\pi}$
	axial	$1 - 6\lambda^2 z^2$	$\sqrt{\frac{12}{5}}\lambda\omega_0$	$\frac{256}{2625\pi}$

Only compressional modes are sensitive to the equation of state and experience a shift!

Innsbruck's results - blue symbols
Duke's results - red symbols





First order perturbation theory prediction (blue solid line)

Unperturbed frequency in unitary limit (blue dashed line)
Identical to the case of non-interacting fermions

If the matter at the Feshbach resonance would have a bosonic character then the collective modes will have significantly higher frequencies!

How should one describe a fermionic system in the unitary regime at finite T?

Grand Canonical Path-Integral Monte Carlo calculations on 4D-lattice

$$H = T + V = \int d^3x \left[\psi_\uparrow^\dagger(\vec{x})\left(-\frac{\hbar^2\Delta}{2m}\right)\psi_\uparrow(\vec{x}) + \psi_\downarrow^\dagger(\vec{x})\left(-\frac{\hbar^2\Delta}{2m}\right)\psi_\downarrow(\vec{x})\right] - g\int d^3x\, n_\uparrow(\vec{x})n_\downarrow(\vec{x})$$

$$N = \int d^3x\, \left[n_\uparrow(\vec{x}) + n_\downarrow(\vec{x})\right]$$

Trotter expansion (*trotterization* of the propagator)

$$Z(\beta) = \mathrm{Tr}\ \exp\left[-\beta\left(H - \mu N\right)\right] = \mathrm{Tr}\ \left\{\exp\left[-\tau\left(H - \mu N\right)\right]\right\}^{N_\tau}, \qquad \beta = \frac{1}{T} = N_\tau \tau$$

Recast the propagator at each time slice and use FFT

$$\exp\left[-\tau\left(H - \mu N\right)\right] \approx \exp\left[-\tau\left(T - \mu N\right)/2\right]\exp(-\tau V)\exp\left[-\tau\left(T - \mu N\right)/2\right] + O(\tau^3)$$

Discrete Hubbard-Stratonovich transformation

$$\exp(-\tau V) = \prod_{\vec{x}} \sum_{\sigma_\pm(\vec{x}) = \pm 1} \left\{1 + \sigma_\pm(\vec{x}) A\left[n_\uparrow(\vec{x}) + n_\downarrow(\vec{x})\right]\right\}, \qquad A = \sqrt{\exp(\tau g) - 1}$$

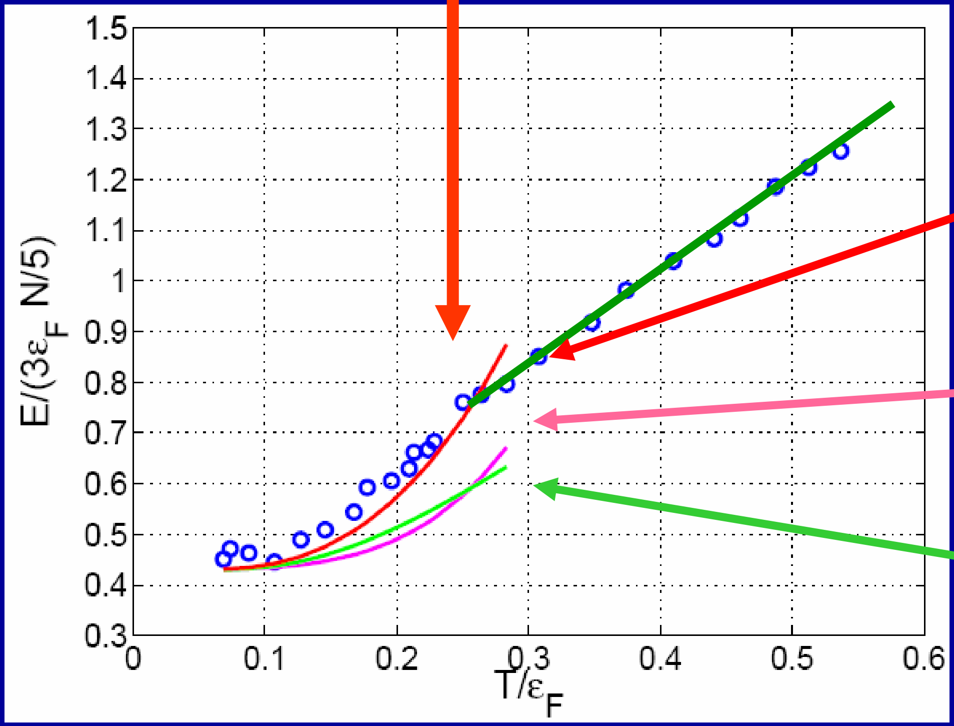
σ-fields fluctuate both in space and imaginary time

$$\frac{m}{4\pi\hbar^2 a} = -\frac{1}{g} + \frac{m k_{cut-off}}{2\pi^2\hbar^2}$$

Running coupling constant g defined by lattice

A. Bulgac, J.E. Drut and P.Magierski

Superfluid to Normal Fermi Liquid Transition



Bogoliubov-Anderson phonons and quasiparticle contribution (red line)

Bogoliubov-Anderson phonons contribution only (magenta line)

People never consider this ???

Quasi-particles contribution only (green line)

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.44$$

$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}}\exp\left(-\frac{\Delta}{T}\right)$$

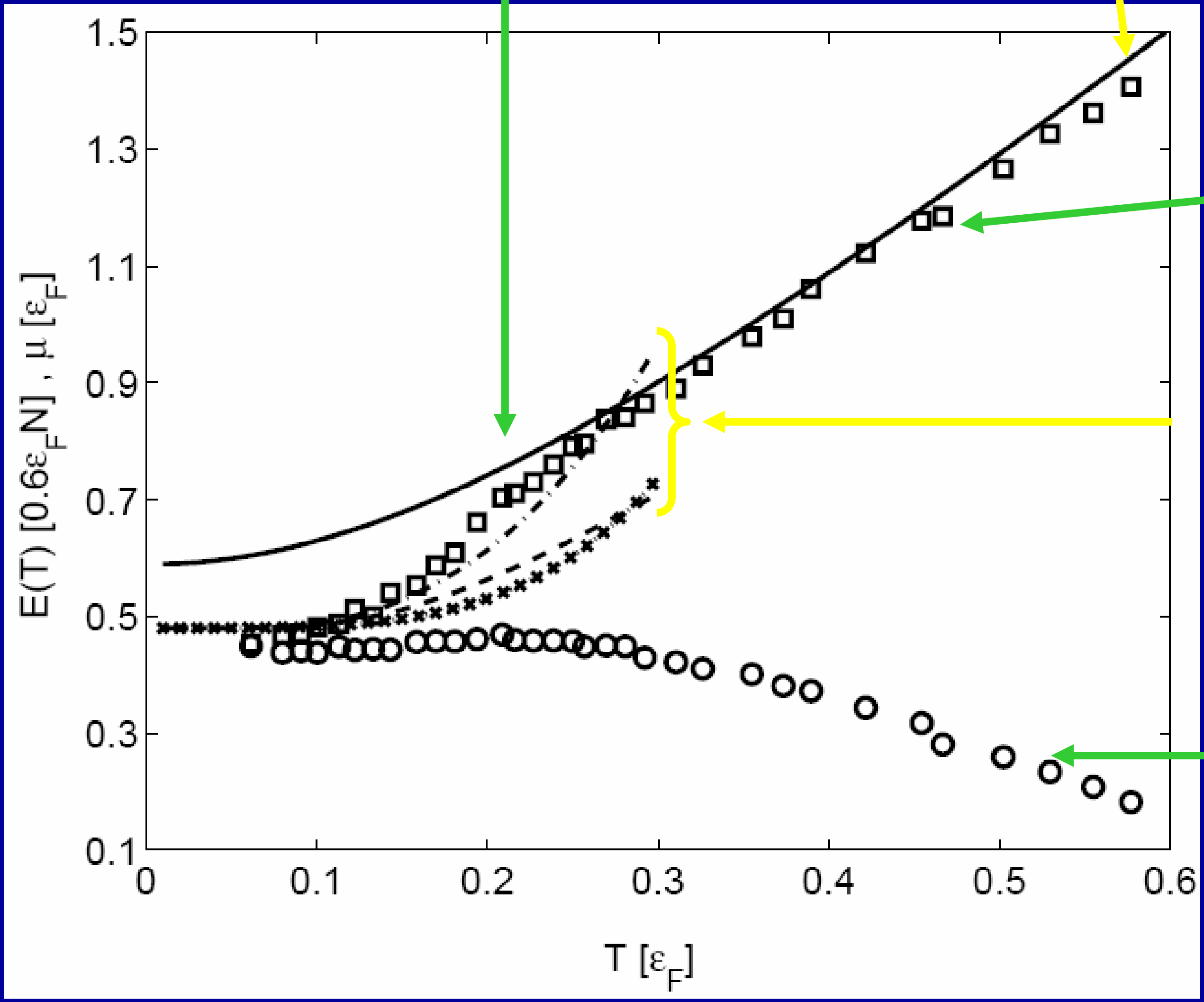
$$\Delta = \left(\frac{2}{e}\right)^{7/3}\varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

- **Lattice size:**
from 6^3 x 112 at low T
to 6^3 x 30 at high T

- **Number of samples:**
Several 10^5's for T
- **Also calculations for 4^3 lattices**
- **Limited results for 8^3 lattices**

T$_c$

non-interacting Fermi gas



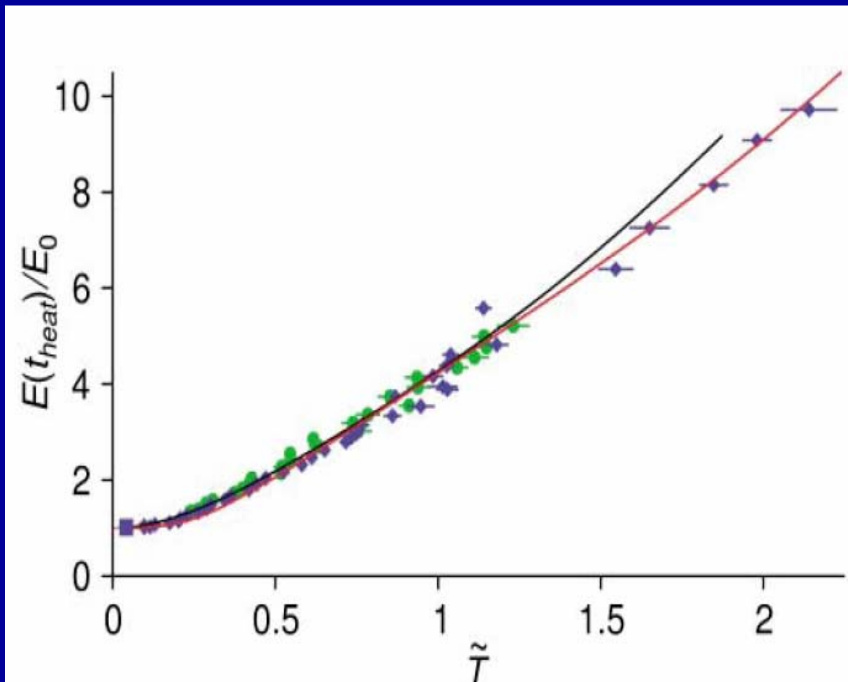
energy per particle

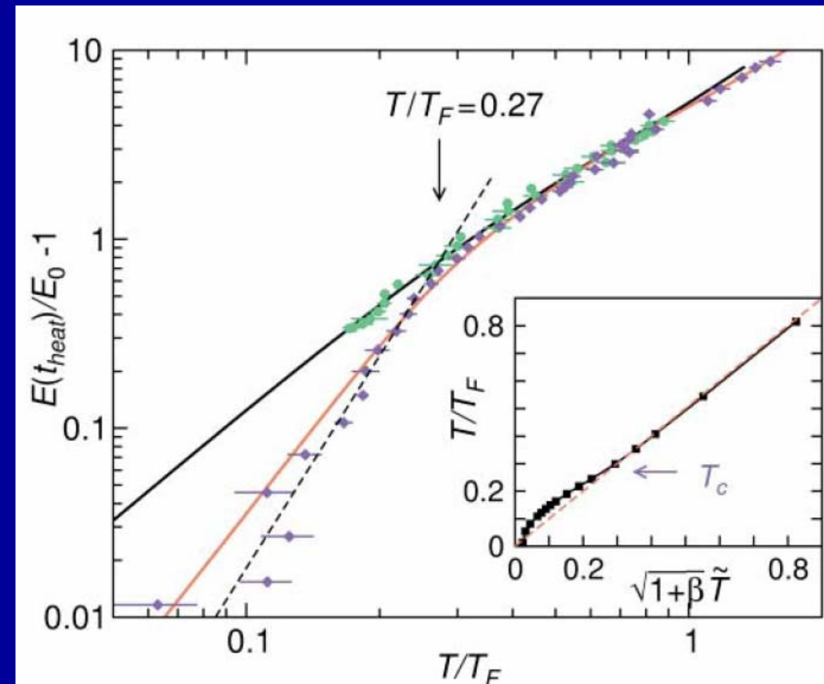
quasi-particle and phonon contributions
in the superfluid phase

chemical potential

Significantly improved statistics and precision

What experiment (with some theoretical input) tells us?





Specific Heat of a Fermi Superfluid in the Unitary Regime

Kinast et al. Science 307, 1296 (2005)
Blue symbols – Fermi Gas in the Unitary Regime
Green symbols – Non-interacting Fermi Gas

Specific heat of a fermionic cloud in a trap

- Typical traps have a cigar/banana shape and one distinguish several regimes because of geometry only!
  - Specific heat exponentially damped if $T \ll \hbar\omega_{\parallel}$
  - If $\hbar\omega_{\parallel} \ll T \ll \hbar\omega_{\perp}$ then $E(T) \approx E_{gs} + \frac{\sqrt{3}\pi^2}{6}\frac{T^2}{\hbar\omega_{\parallel}}$
  - If $T \gg \hbar\omega_{\perp}$ then surface modes dominate

$$E(T) = E_{gs} + \sum_{nl} \frac{(2l+1)\hbar\Omega_{nl}}{\exp(\beta\hbar\Omega_{nl}) - 1} \approx E_{gs} + 142\frac{T^5}{\hbar^4\omega^4}$$

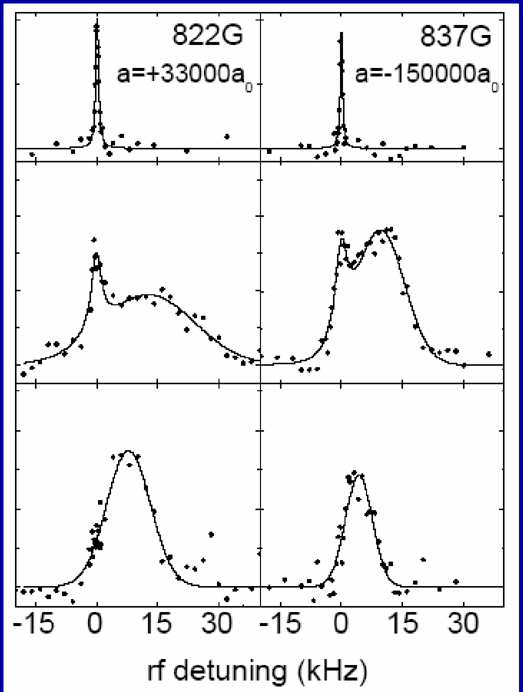
Unexpected!

$$\hbar\Omega_{nl} = \hbar\omega\sqrt{\frac{4}{3}n(n+l+2)+l} \approx \hbar\omega\sqrt{l}$$

$$E(T) = E_{gs} + \frac{3^{1/2}\pi^4}{10}\frac{T^4}{\hbar^3\omega^3}$$

Expected bulk behavior (if no surface modes)

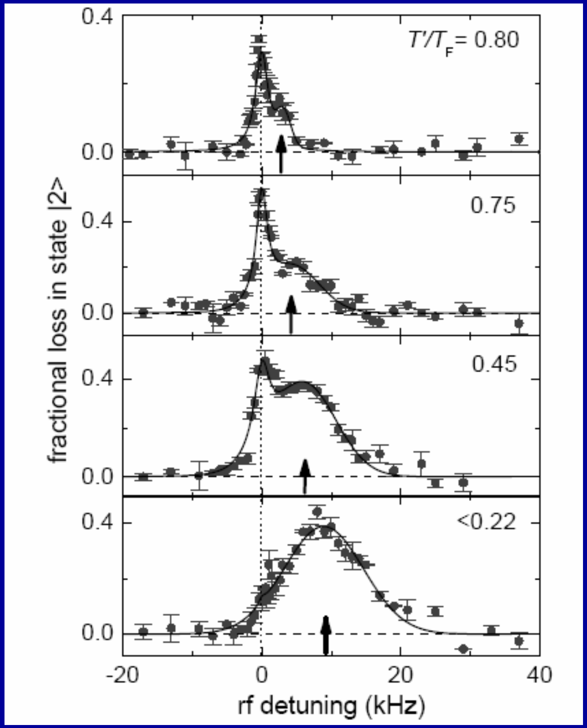
How about the gap?

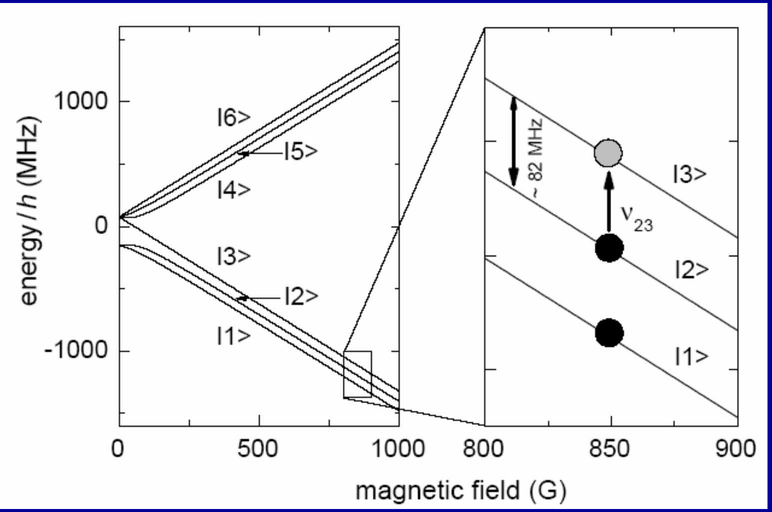


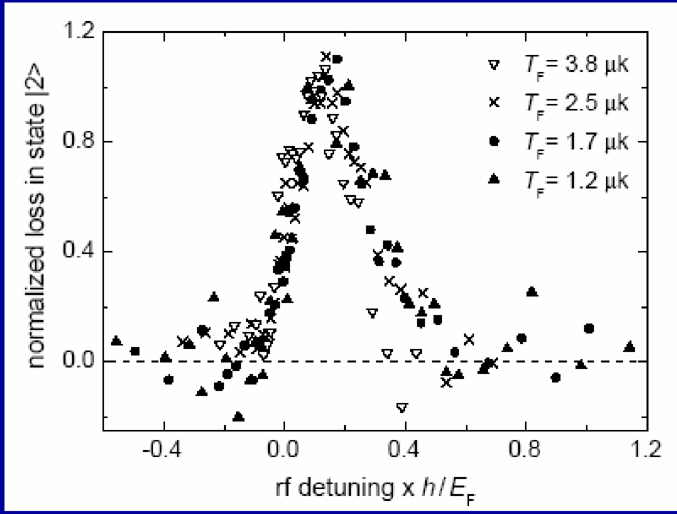
$\Delta_{\text{exp}} \approx 0.2\varepsilon_F$

$\Delta_{\text{theor}} \approx 0.5\varepsilon_F$

$T_c \approx ?$







This shows scaling expected in unitary regime

Chin et al. Science 305, 1128 (2004)

Key experiments seem to confirm to some degree what theorists have expected. <u>However!</u>

✓ **The collective frequencies in the two experiments show significant and unexplained differences.**

✓ **The critical temperature, allegedly determined in the two independent experiments, does not seem to be the same.**

✓ **The value of the pairing gap also does not seem to have been pinpointed down in experiments yet!**

A liberal quote from a talk of Michael Turner of University of Chicago and NSF

No experimental result is definite until confirmed by theory!

Physics aims at understanding and is not merely a collection of facts.

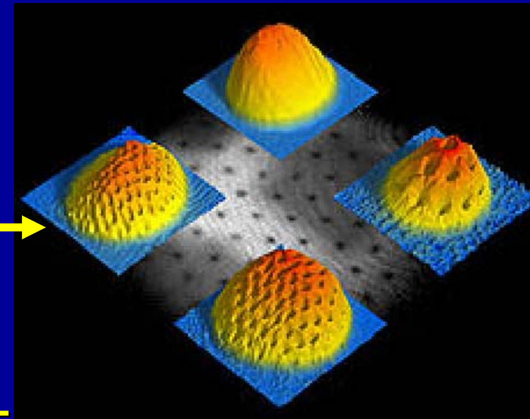
Ernest Rutherford said basically the same thing in a somewhat different form.

If we set our goal to prove that these systems become superfluid, there is no other way but to show it!

Is there a way to put directly in evidence the superflow?

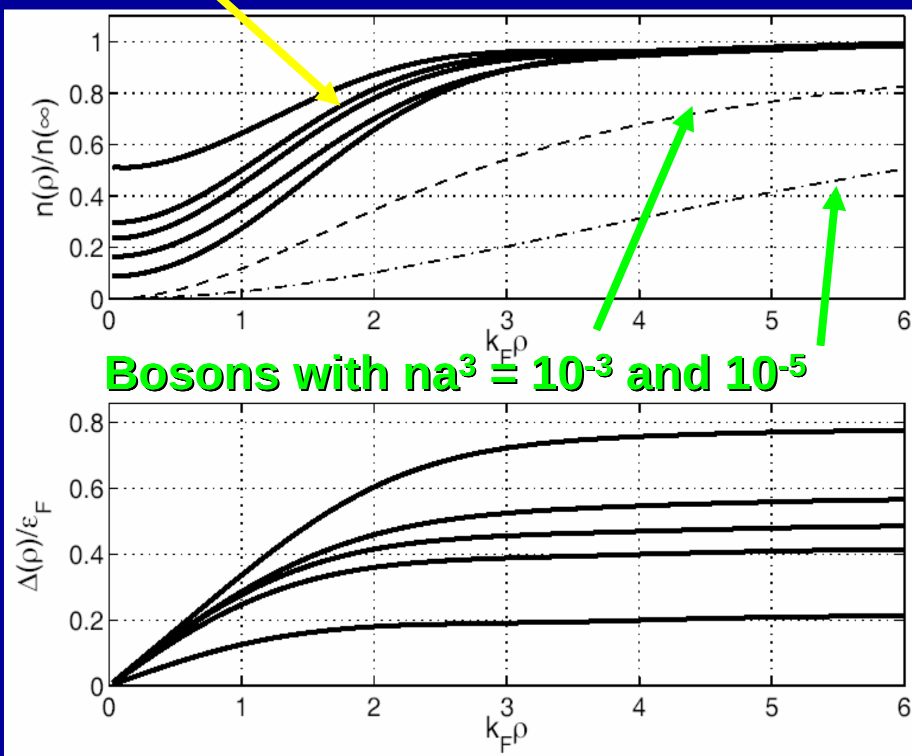
Vortices!

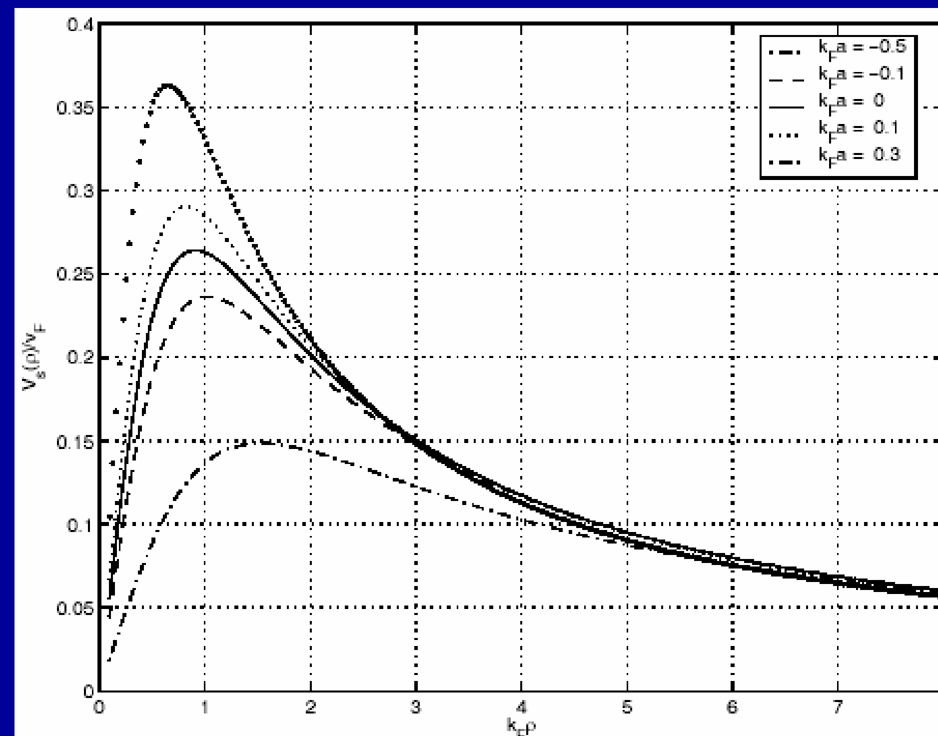
The depletion along the vortex core is reminiscent of the corresponding density depletion in the case of a vortex in a Bose superfluid, when the density vanishes exactly along the axis for 100% BEC.



From Ketterle's group

Fermions with $1/k_F a$ = 0.3, 0.1, 0, -0.1, -0.5





Extremely fast quantum vortical motion!

Number density and pairing field profiles

Local vortical speed as fraction of Fermi speed

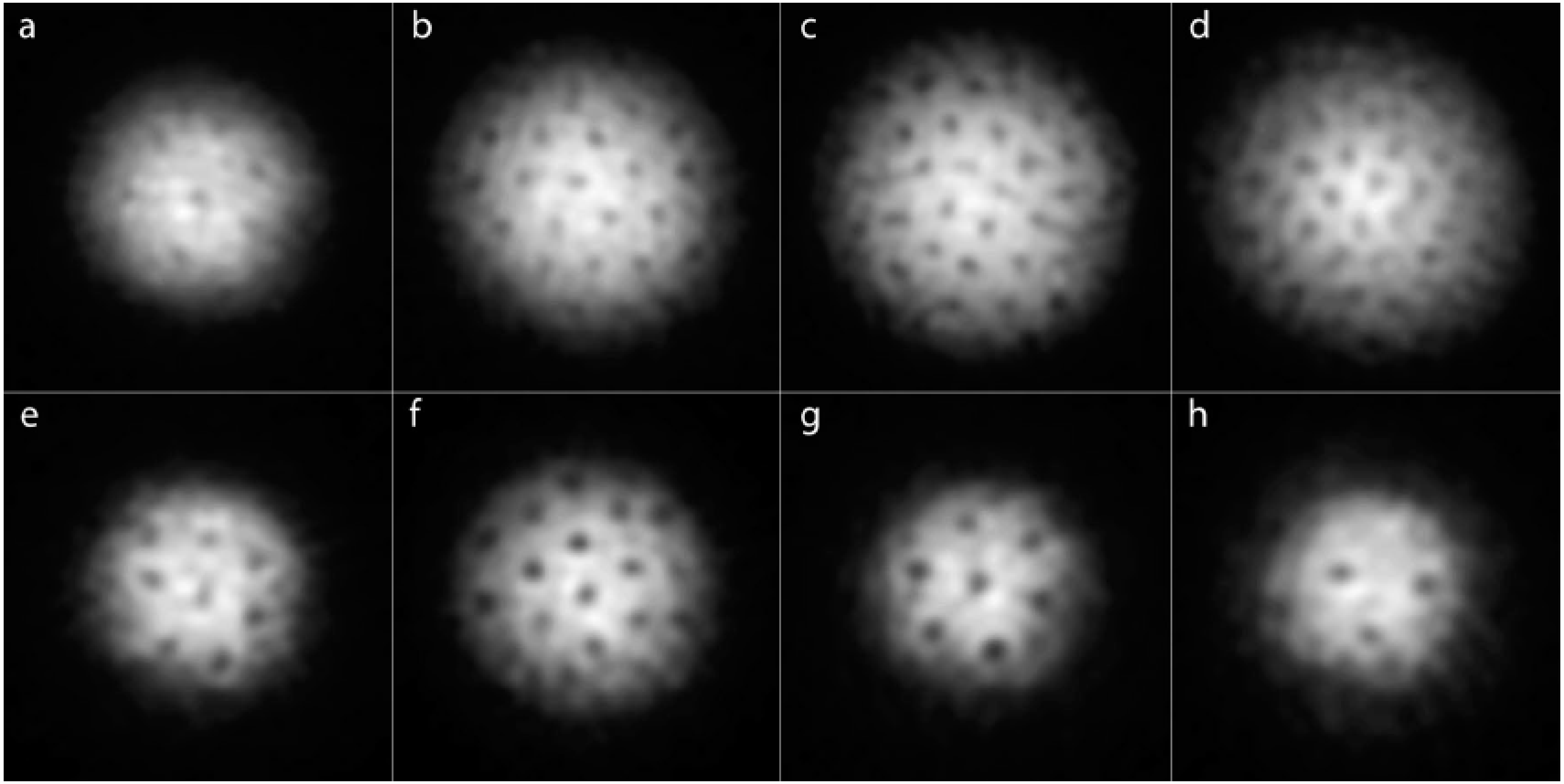


Fig. 2: Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) to 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were (a) 740 G, (b) 766 G, (c) 792 G, (d) 812 G, (e) 833 G, (f) 843 G, (g) 853 G and (h) 863 G. The field of view of each image is $880\ \mu\mathrm{m} \times 880\ \mu\mathrm{m}$.

Zweirlein et al. cond-mat/0505653

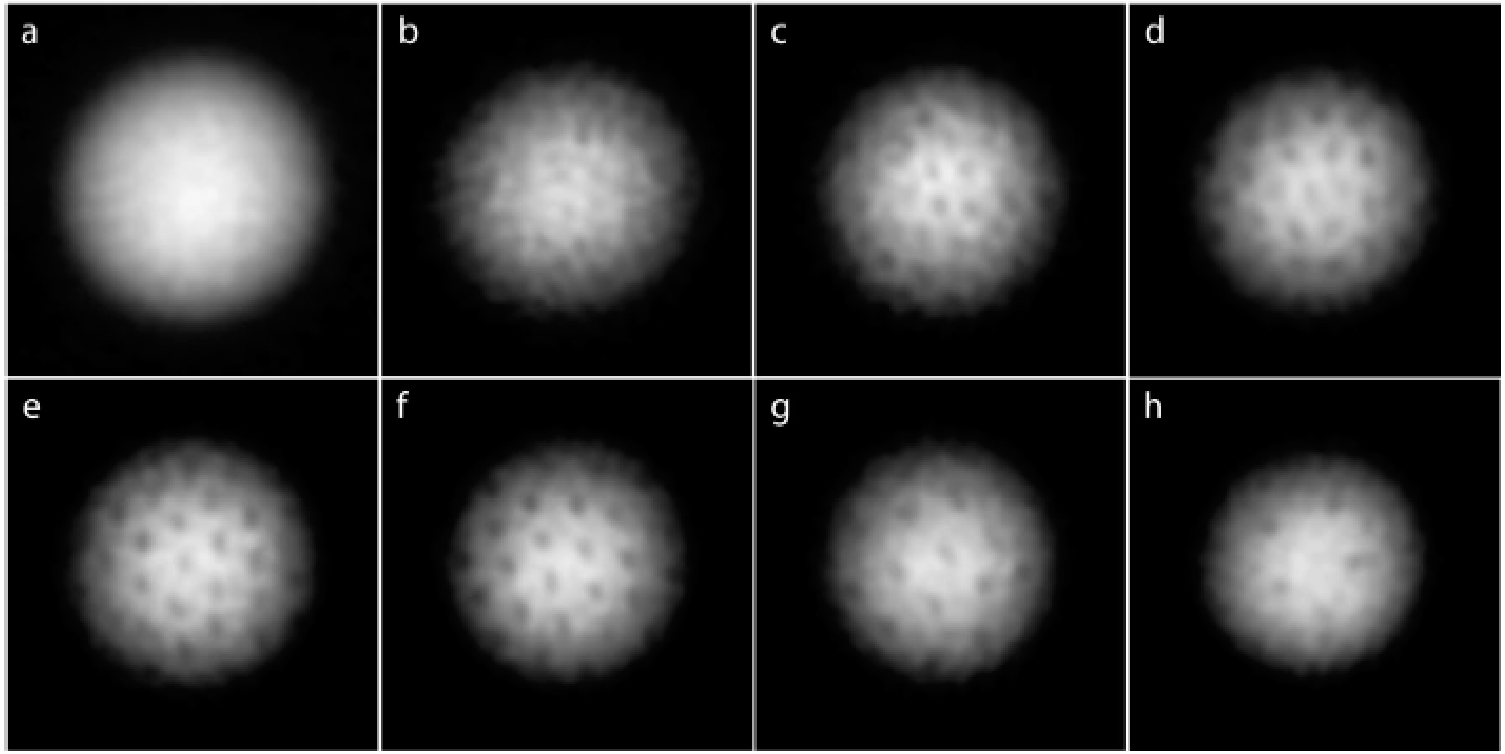


Fig. 6: Formation and decay of a vortex lattice in a fermion pair condensate on the BEC-side close to the Feshbach resonance. A molecular condensate, prepared at 766 G as shown in (a), was stirred for 800 ms. The field was then ramped to 812 G in 20 ms for equilibration. At this field, $1/k_F a = 0.35$, and the condensate was deep in the strongly interacting regime. To observe the vortex lattice, the field was ramped in 25 ms to 735 G ($1/k_F a = 2.3$), where the condensate was released from the trap and imaged after 12 ms time-of-flight. The equilibration times after the end of the stirring were (b) 40 ms, (c) 240 ms, (d) 390 ms, (e) 790 ms, (f) 1140 ms, (g) 1240 ms and (h) 2940 ms. Due to stirring, evaporation and vibrational relaxation, the number of fermion pairs decayed from 3×10^6 (a) to 1×10^6 (b-h). The field of view of each image is $830\ \mu\mathrm{m}\times830\ \mu\mathrm{m}$.

Zweirlein et al. cond-mat/0505653

Superconductivity and superfluidity in Fermi systems

20 orders of magnitude over a century of (low temperature) physics

✓ <u>Dilute atomic Fermi gases</u>	$T_c \approx$ $10^{-12} - 10^{-9}$ eV
✓ Liquid ^3He	$T_c \approx$ 10^{-7} eV
✓ Metals, composite materials	$T_c \approx$ $10^{-3} - 10^{-2}$ eV
✓ Nuclei, neutron stars	$T_c \approx$ $10^{5} - 10^{6}$ eV
• QCD color superconductivity	$T_c \approx$ $10^{7} - 10^{8}$ eV

units (1 eV $\approx$ 10^4 K)

Conclusions

✓ Until recently there was lots of circumstantial evidence and facts in qualitative agreement with theoretical models assuming fermionic superfluidity.

✓ <u>Vortices have been put in evidence. At last!</u>