1. [6 points] Two small spheres, each with mass \( m = 5.0 \, \text{g} \) and charge \( q \), are suspended from a point by threads of length \( L = 0.30 \, \text{m} \). What is the charge on each sphere if the threads make an angle \( \theta = 20^\circ \) with the vertical?

\[
\begin{align*}
\text{A) } & 7.9 \times 10^{-7} \, \text{C} \\
\text{B) } & 2.9 \times 10^{-7} \, \text{C} \\
\text{C) } & 7.5 \times 10^{-2} \, \text{C} \\
\text{D) } & 6.3 \times 10^{-13} \, \text{C} \\
\text{E) } & 1.8 \times 10^{-7} \, \text{C}
\end{align*}
\]

2. [5 points] Two positive charges \((+8.0 \, \text{mC} \text{ and } +2.0 \, \text{mC})\) are separated by 300 m. A third charge is placed at distance \( r \) from the \(+8.0 \, \text{mC}\) charge in such a way that the resultant electric force on the third charge due to the other two charges is zero. The distance \( r \) is

\[
\begin{align*}
\text{A) } & 0.25 \, \text{km} \\
\text{B) } & 0.20 \, \text{km} \\
\text{C) } & 0.15 \, \text{km} \\
\text{D) } & 0.13 \, \text{km} \\
\text{E) } & 0.10 \, \text{km}
\end{align*}
\]
3. [6 points] Two charges $Q_1$ and $Q_2$ are 8.0 cm apart. Charge $Q_1 = 5.0$ nC and $Q_2 = -5.0$ nC. The magnitude of the electric field at point P, 3.0 cm from the midpoint of the line joining $Q_1$ and $Q_2$, is

A) $2.9 \times 10^5$ N/C  
B) $2.9 \times 10^4$ N/C  
C) $3.6 \times 10^5$ N/C  
D) 0.29 kN/C  
E) $3.6 \times 10^6$ N/C

4. [5 points] Charges $q_1$ and $q_2$ exert repulsive forces of 10 N on each other. What is the repulsive force when their separation is decreased so that their final separation is 80% of their initial separation?

A) 16 N  
B) 12 N  
C) 10 N  
D) 8.0 N  
E) 6.4 N
5. [6 points] A negatively charged particle moving with speed \( v \) enters a region of uniform electric field \( E \). If the charge \( q = 1 \text{ nC} \), mass \( m = 1 \times 10^{-14} \text{ kg} \), speed \( v = 10^5 \text{ m/s} \), the electric field strength \( E = 2 \times 10^5 \text{ V/m} \), and width, \( w \), of the electric field is 0.2 m, what is the speed of the particle when it emerges from the other side?

A) \( 1.0 \times 10^5 \text{ m/s} \)  
B) \( 4.0 \times 10^4 \text{ m/s} \)  
C) \( 1.08 \times 10^5 \text{ m/s} \)  
D) \( 1.4 \times 10^5 \text{ m/s} \)  
E) \( 1.8 \times 10^5 \text{ m/s} \)

6. [6 points] An infinite line charge of linear density \( \lambda = 0.30 \mu\text{C/m} \) lies along the \( z \) axis and a point charge \( q = 6.0 \mu\text{C} \) lies on the \( y \) axis at \( y = 2.0 \text{ m} \). The \( x \) component of the electric field at the point \( P \) on the \( x \) axis at \( x = 3.0 \text{ m} \) is approximately

A) \( 1.8 \text{ kN/C} \)  
B) \( 4.2 \text{ kN/C} \)  
C) \( 0.96 \text{ kN/C} \)  
D) \( 5.2 \text{ kN/C} \)  
E) \( 0.64 \text{ mN/C} \)
7. [5 points] A solid sphere of radius $a$ is concentric with a hollow sphere of radius $b$, where $b > a$. If the solid sphere has a uniform charge distribution totaling $+Q$ and the hollow sphere a charge of $-Q$, the electric field at radius $r$, where $r < a$, is which of the following, in terms of $k = (4\pi\varepsilon_0)^{-1}$?

A) $kQ/r^2$  B) $kQr/a^3$  C) $kQ/a^2$  D) $kQ/b^2$  E) zero

8. [6 points] An infinitely long cylinder of radius 4.0 cm carries a uniform volume charge density $\rho = 200 \text{ nC/m}^3$. What is the electric field at $r = 2.0$ cm?

A) zero  B) 0.11 kN/C  C) 57 N/C  D) 0.44 kN/C  E) 0.23 kN/C
9. [5 points] An infinitely long cylindrical shell of radius 6.0 cm carries a uniform surface charge density $\sigma = 12 \text{ nC}/\text{m}^2$. The electric field at $r = 10 \text{ cm}$ is approximately

A) $12 \text{ kN/C}$  B) $0.56 \text{ kN/C}$  C) $1.3 \text{ kN/C}$  D) **0.81 \text{ kN/C}**  E) zero

10. [5 points] An infinite plane of surface charge density $\sigma = +8.00 \text{ nC}/\text{m}^2$ lies in the $yz$ plane at the origin, and a second infinite plane of surface charge density $\sigma = -8.00 \text{ nC}/\text{m}^2$ lies in a plane parallel to the $yz$ plane at $x = 4.00 \text{ m}$. The electric field at $x = 3.50 \text{ m}$ is approximately

A) $226 \text{ N/C}$  B) $339 \text{ N/C}$  C) **904 \text{ N/C}**  D) $452 \text{ N/C}$  E) zero
IV. [25 points total] An aluminum rod is on an insulating stand. Then: (1) A negatively charged Teflon rod is scraped along the aluminum rod and then removed. (2) An uncharged metal ball hanging from an insulating string is brought near so the ball touches the aluminum rod. (3) The stand is moved so the ball is near but not touching the aluminum rod.

A. [7 pts] Would the ball hang straight down, to the right of vertical, or to the left of vertical? Draw the ball and string on the diagram to illustrate your answer. Sketch the final charge distributions on the aluminum rod and the ball. Explain.

When the conducting ball touches the aluminum rod, some of the negative charge is transferred from the rod to the ball. The rod and the ball now both have the same type of net charge, so they will repel.

B. [9 pts] A positively charged acrylic rod is now held near the aluminum rod as shown. The magnitude of the charge on the acrylic rod is much greater than that on the aluminum rod. Would the angle at which the ball hangs: increase, decrease, or stay the same? Draw the ball and string on the diagram and sketch the charge distributions. Explain.

The positively charged acrylic rod will attract the negative charge inside the aluminum rod, drawing it away from the conducting ball. Additionally, the positive charge inside the aluminum rod will be repelled by the acrylic rod, and thus will be closer to the ball. This results in a decreased repulsion as well as an increased attraction between the rod and the ball. Additionally, the acrylic rod attracts the ball. Thus the angle at which the ball hangs decreases.

C. [9 pts] While the positively charged rod is held in place, someone touches the aluminum rod. Would the angle at which the pith ball hangs: increase, decrease, or stay the same? Draw the ball and string on the diagram and sketch the charge distributions.

When the person touches the rod, charges are free to move between the person and the rod. The positive acrylic rod will attract negative charges and repel positive charges while the conducting ball will attract positive charges and repel negative charges. Since the acrylic rod has much more charge than the conducting ball, more negative charge will remain on the aluminum rod than positive charges. If the net negative charge on the rod increases as a result of touching it, the angle at which the ball hangs increases. If the net negative charge on the rod decreases as a result of touching it, the angle at which the ball hangs decreases.
V. [20 pts.]

A. A point charge $+Q$ sits at the exact center of a very thin charged, insulating ring. The ring has radius $R$, and total charge $+Q$ is distributed uniformly around the ring. The ring is composed of two arcs labeled A and B.

i. [6 pts.] What are the magnitude and direction of the net force exerted on the point charge by the ring? Explain your reasoning.

Consider the forces on the point charge due to one point on the ring and the point diametrically opposite to that point. These two points on the ring have the same charge, since the charge is distributed uniformly. They are also the same distance from the center of the circle, so they will exert forces with equal magnitude on the point charge. These forces will point in opposite directions, so the net force on the point charge due to two diametrically opposed points on the ring is zero. Since there is a point diametrically opposite to every point on the ring, the net force on the point charge is zero.

ii. [4 pts.] How does the magnitude of the force exerted on the point charge by arc A compare to the magnitude of the force exerted on the point charge by arc B? Explain.

Since the net force on the point charge is zero, the forces on the point charge due to arc A and arc B must cancel exactly. This can only occur if the magnitudes are equal.

B. [5 pts.] Suppose now that each arc has a charge $+Q$ distributed uniformly on it.

How does the magnitude of the force exerted on the point charge by arc A compare to the magnitude of the force exerted on the point charge by arc B? Explain.

When arcs A and B have the same charge density, we saw above that they exert forces with equal magnitude on the point charge. Since arc A is smaller, giving each arc a charge $+Q$ will give arc A a larger charge density. Therefore, the magnitude of the force exerted by arc A is greater than the magnitude of the force exerted by arc B.

C. [5 pts.] Arc A is moved so that its center is a distance $2R$ away from the point charge.

Is the magnitude of the force exerted on the point charge by arc A after it is moved less than $1/4$, exactly $1/4$, or greater than $1/4$ the magnitude just before it was moved? Explain your reasoning.

If every point on arc A were moved to be twice as far from the center as it was originally without changing the direction from the point to the center, the force would be exactly $1/4$ what it was just before being moved. However, every point except for the center of arc A has been moved slightly less than twice as far away, and their orientations have changed so that the forces point slightly more horizontal. Both of these effects increase the force making it greater than $1/4$ the magnitude just before it was moved.