THE EFFECT OF COST CHANGES IN AN OLIGOPOLY WITH APPLICATION TO BANK LENDING DISCRIMINATION

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William D. Bradford
School of Business
University of Washington
Seattle, WA 98195

bradford@u.washington.edu
Phone: 206-543-4559
Fax: 206-221-6856
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Abstract

This paper extends previous analyses of the effect of exogenous cost changes on the performance of firms in an oligopoly. The analysis is framed in the context of discrimination in bank lending. Recent U.S. banking studies have assumed that greater lending discrimination (bias) against ethnic minorities results in minorities paying an increasingly higher interest rate than non-minorities as bank concentration rises. Using a framework of asymmetric Cournot-Nash competition, I show how the discrimination itself changes concentration, and that bias and concentration may be positively or negatively associated. Also, the profits and market shares of biased banks may rise or fall as bias increases.

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1. Introduction

The effect of exogenous cost changes on the performance of firms in an oligopoly has

been explored by Kimmel (1992), Lahiri & Ono (1988) (1997), and Zhao (2001). Kimmel

analyzed the outcomes when the costs of all of the N competing firms exogenously change.

Lahiri & Ono and Zhao (the latter for the case of linear demand) analyzed the case when the costs

of one firm (or 1/N firms) change. I extend their analyses in three ways. First, the model

developed herein allows for any subset (*1/N* through *N*) of the firms' costs to change, for both

linear and non-linear demand. Second, I extend the framework to consider the case when

competing firms' products are linearly heterogeneous as well as homogeneous (the papers above

assume homogenous products). Third, I show the implications of the exogenous cost changes for

concentration among the competing firms.[1]

This paper reflects my pursuit of a model that reflects Becker's discrimination coefficient

for banks competing in an oligopoly. In his groundbreaking work on discrimination, Becker

(1971) focused primarily on labor markets, and not the discrimination of sellers against buyers in

product markets.[2] His wage discrimination model concluded that non-economic discrimination

reduces the firm's profits. Thus highly competitive markets would purge discriminatory behavior

from the market place, while non-economic bias could be sustained in

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[1] Other related papers include Levin (1985), Okuguchi (1993) and Salant and Shaffer (1999). These papers
have more restrictive assumptions concerning the distribution of the cost changes than does this paper.
None of these papers relate the nature of market demand to the effect of cost changes, nor do they include
product differentiation in the context of cost changes for oligopolists. Anderson, S., de Palma & Kreider
(2001), Delipalla & Keen (1992) and Stern (1987) examine the effect of costs in the form of taxes on
profits and welfare, but they assume that the competing firms have identical costs; thus they do not
consider volume for individual firms, market shares or concentration.

[2] In Becker's analysis the sellers—laborers—are homogenous and the labor market is competitive. Here the
sellers are banks with asymmetric costs operating in a Cournot-Nash oligopoly.

less competitive markets. Berkovec et al (1998), Cavalluzzo and Cavalluzzo (1998) and Cavalluzzo, Cavalluzzo and Wolken (2001) translated Becker's conclusions into banking markets, concluding that when lending discrimination exists among banks, 1) minority borrowers pay higher interest rates than non-minority borrowers, and 2) the differences in the interest rates paid by minority and non-minority borrowers expand as banking markets become less competitive. These studies test these effects, but do not build a model of banking that provide a theoretical framework for the tests. What does a formal model of banking indicate about these relationships?

In this paper, some banks have a bias against lending in the subject market, where "the market" is defined as loans to a specified group of borrowers (e.g., racial minorities, women). Bank i interprets its bias as a "cost", b_i, in its profit function when it determines how much it will lend in this market. The bias is exogenous to the loan production costs. I examine the impact of the bias on the structure and performance of the competing banks, when b_i is positive for n of the N banks ($0 < n \leq N$) and the banks compete in an asymmetric Cournot-Nash oligopoly.

I find that first, the disfavored group pays a higher rate of interest and receives a lower loan volume when bias increases among the banks. Second, while profits and market share always increase for unbiased banks as bias increases, profits and market share may also increase for some of the biased banks. Third, an increase in bias among the biased banks can increase or reduce the concentration of loans among banks in the market, depending on the size of the biased banks, and the nature of the market demand for loans. Fourth, an increase in the proportion of biased banks can magnify or reduce the changes in profits and market concentration, depending on the distribution of costs of the biased banks among the total group of banks.

Thus in the framework of a Cournot-Nash oligopoly, the disfavored group pays higher interest and receives a lower volume of loans, consistent with the assumptions of the studies on bank lending discrimination. However, in contrast to those studies' assumptions, bank

concentration may increase *or* decline with bias among banks. We can observe markets with higher bias and thus higher interest rates, but lower indices of bank concentration. Finally, an increase in bias *may not* penalize the biased banks in profits and market share. In fact, both may increase.

More generally, these results are inconsistent with both the Structure-Conduct-Performance paradigm that higher concentration is tied to higher interest rates on loans (see Hannan, 1991) and the Relative-Efficiency paradigm that higher concentration would be associated with lower interest rates on loans (Demsetz, 1973). In a Cournot market, when costs (and thus market shares) differ, adding a layer of costs reduces output, increases the price, redistributes market shares and changes the concentration in the market. The redistribution and change in concentration depend on how relative costs change and how demand changes as quantity supplied changes; thus concentration may increase or decrease. The cost changes in this paper relate to bias, but in other cases can reflect other charges, such as taxes and fees.

The remainder of this paper is organized as follows. Part 2 introduces the setting and the basic model. Part 3 examines the impact of bias on the equilibrium interest rate and total loan volume. Parts 4 and 5 discuss the loan volume at individual banks and the profits of banks as bias changes, respectively. Part 6 analyzes the relationship between changes in bias and changes in market shares and concentration among banks. Part 7 examines the impact of changes in bias when bank loans are linearly heterogeneous. Part 8 contains an example using a constant elasticity demand curve, and Part 9 concludes the paper.

2. Basic Approach

Consider a bank that issues deposits, D, and uses deposits and capital funds, K, to make M categories of loans. Variable costs are separable by activity (administering loans or deposits) and by category of loans.   The markets for each type of loan and deposits differ from each other. The profits of the bank can be expressed as

$$(\text{1a}) \ \pi = \sum_{j=1}^{M} (r_j - a_j) L_j - (r_d + a_d) D - F$$

where r and a represent the interest yield and administrative cost rate associated with loans and deposits, and F denotes fixed costs. The borrowers within each loan category have the same credit risk.

At this point I will relate the analyses on endogenous costs to bank lending discrimination. In his analysis of prejudicial discrimination, Becker hypothesized that those who have a taste for discrimination behave as if they were willing to pay something, either directly or in the form of a reduced income, to indulge those tastes (Becker, 1971, p. 14).[3] "The money costs of a transaction do not always completely measure net costs, and a discrimination coefficient acts as a bridge between money and net costs." Assume that market 1 is the disfavored group. Thus define b as the bank's discrimination ("bias") coefficient for lending to customers in market 1, where $b \geq 0$. In this market banks seek to maximize profit, defined as revenues less expenses; and the expenses include a charge for the disutility of lending to the borrowers in this market. Thus the "endogenous cost" analyzed in Lahiri-Ono, Kimmel and Zhou is embodied in the bias coefficient, b. The bank's profits can be restated as[4]

$$(\text{1c}) \ \pi = (r_1 - a_1 - b) L_1 + \sum_{j=2}^{M} (r_j - a_j) L_j - (r_d + a_d) D - F$$

The pecuniary costs of making a loan may differ among banks only due to differences in the administrative costs of lending. When I place b, the bias coefficient, into the profit function in calculating π, the definition of profits here shifts closer to a measure of utility, measured in units of dollars. In other words, b is a monetary equivalent and π is a monetary equivalent to the bank.

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[3] Zizzo and Oswald (2000) provide recent evidence on the willingness of people to pay to reduce others' incomes.
[4] See Freixas and Rochet (1997), chapter 3 for a standard derivation of these relationships. The typical derivation includes securities as an alternate asset. I omit securities since including securities provides no additional insights.

From this point, the analysis focuses only on the market for type 1 loans, and the subscripts refer to the banks that compete in this market. Type 1 loans are assumed to be homogeneous (I drop this assumption later), and the banks in this market compete in a Cournot-Nash oligopoly. There are N banks and no possibility of entry.[5] The total volume of bank loans in this market is L, and the market interest rate on loans is r, where $r = r(L)$ and $r'(L) < 0$. For Nash stability it is assumed that

(1d) $r'(L) + L_i r''(L) < 0$ for all i.

This assumption corresponds to the "normal" case in Seade (1980a) and to strategic substitutes in Bulow et al. (1985). The marginal costs, a_i, are constant for each bank over the relevant range, but vary across banks. Given this framework, bank i's reaction curve is implicitly defined by the first order condition:

(2a) $\dfrac{d\pi_i}{dL_i} = r + r'(L)L_i - a_i - b_i = 0,$

or

(2b) $r + r'(L)L_i = a_i + b_i$

A Cournot-Nash equilibrium in the market for these loans is an output vector such that each bank's loan volume, L_i, is a best response to the vector of all of the other banks' best response. This equilibrium is graphically the intersection point of all reaction curves and algebraically the solution to (2). Thus in equilibrium:

(3) $L_i = \dfrac{-(r - a_i - b_i)}{r'(L)}$

The "costs" of lending for bank i have a technical component and a bias component. Since $r'(L)$

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[5] Entry into banking is regulated and restricted. However, non-bank lenders can compete for loans, depending on the type of loan. Small business lending has historically been very costly, because of the paucity of information about small firms and the high cost of personnel required to obtain and evaluate even that information. Banks dominate that market. The empirical studies on lending discrimination cited above assumed that banks operate in oligopolistic markets.

< 0, the sum $(a_i + b_i)$ and L_i are inversely related: relatively high cost banks have lower loan volumes than low cost banks. But a biased bank with high quality technology, i.e. low a_i, may have a lower loan volume than an unbiased bank with low quality technology. Assume that b_i is positive and equal for n banks: $b_i = b$ for i ∈ n and $b_i = 0$ for i ∉ n. Define $\theta \equiv \dfrac{n}{N}$. The fraction θ of the banks are biased, where $0 \le \theta \le 1$. First, the equilibrium interest rate for loans, r, is derived. Sum L_i over all i to obtain total loan volume, L:

$$(4) \quad L = -\frac{1}{r'(L)}(Nr - \sum_{i=1}^{N} a_i - \sum_{i=1}^{N} b_i)$$

Then r can be expressed as:

$$(5) \quad r = \frac{1}{N}\left\{\sum_{i=1}^{N} a_i + \theta Nb - Lr'(L)\right\}$$

$$= \bar{a} + \theta b - \frac{Lr'(L)}{N}$$

where $\sum_{i=1}^{N} a_i = N\bar{a}$. In this equilibrium the interest rate on loans is a function of both the technology of the N banks, $\bar{a}$, and the bias of the n banks, θb. The market premium for bias is the mean of the bias coefficient over the total number of banks. I now address comparative statics of the model, assuming an equal increase in the bias coefficients of the n (= θN) biased banks.

3. The Interest Rate on Loans and Total Loan Volume

Suppose $db_i = db > 0$ for θN banks and $db_i = 0$ for $(1-\theta)N$ banks. The changes in the interest rate and loan volume resulting from a common change in the bias coefficients of the θN biased banks are as follows:

Proposition 1:

$$(6) \quad \frac{dr}{db} = \frac{\theta Nr'(L)}{(N+1)r'(L) + Lr''(L)} > 0 \text{ and } \frac{dL}{db} = \frac{\theta N}{(N+1)r'(L) + Lr''(L)} < 0.$$

All proofs are in the Appendix. An increase in bias ($db > 0$) increases the interest rate and decreases the total loan volume. As the fraction of biased banks rises, the higher the increase in the interest rate and the greater the decline in loan volume, as measured by expression (6). If only one bank is biased all banks are still affected by the increase in bias, but the impact is smaller.[6] Expression (6) may be compared to expression (A2) in Kimmel (1992), expression (1.2) in Lahiri and Ono (1997), and proposition 1(ii) in Zhao (2001). Kimmel assumes that $\theta = 1$, or that all banks have the same exogenous cost increase. Lahiri and Ono (1997) and Zhao (2001) assume that $\theta = 1/N$ or that only one bank has the cost increase. Their analyses are specific assumptions about θ. One can also express $\dfrac{dr}{db}$ as

$$(7) \quad \frac{dr}{db} = \frac{\theta N}{N + 1 + E(L)}$$

where $E(L)$ is the elasticity of $r'(L)$ with respect to L:

$$(8) \quad E(L) \equiv \frac{dr'(L)}{dL} \frac{L}{r'(L)} \equiv \frac{Lr''(L)}{r'(L)}$$

The change in the interest rate is a function of the fraction of banks that are biased, θ, and $E(L)$. The latter is discussed in Seade (1980b, 1985) and Kimmel (1992). Two standard demand relationships are useful for understanding the relationship between $E(L)$ and $\dfrac{dr}{db}$. First, a constant elasticity inverse demand curve such as $r = AL^{1/\varepsilon}$ results in $E(L) = \dfrac{1}{\varepsilon} - 1$; where ε, the standard measure of demand elasticity, is negative. Exhibit 1 shows the demand curve, M, for the curve defined as $r = 1000L^{1/(-0.35)}$; for L in the 20 – 30 range (this curve will be used later in an

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[6] I assume that θN banks have the same increase in bias. It is well known that in Cournot models, increases in total costs of the same amount will produce the same change in interest rate and output, regardless of the composition of the change (for example, see Bergstrom and Varian (1985)). Thus (6) can reflect the effect of any combination of bias increases among the banks, as long as the changes sum to the same amount as assumed in (6).

example). Line *C* is the slope of *M, r'(L)*, which changes over *L*. Thus *E(L) is the elasticity of C with respect to L*. Since $\varepsilon = -0.35$, *E(L)* is a constant -3.857 over the range of *L*. Expression (7) also translates that $\frac{dr}{db} > 1$ (overshifting occurs) when $E(L) < - N(1 - \theta) - 1$.

If demand is linear, as assumed by Zhao (2001), then *E(L) = 0*, since the slope of a linear demand curve is constant. Here $\frac{dr}{db} = \frac{\theta N}{N+1} < 1$, which approaches 1 as θ and *N* increase.

Finally, *E(L)* is negative (positive) for a demand curve that is convex (concave) at the relevant level of *L*.

4. Loan Volume at Individual Banks.

The impact of bias on the loan volume of individual banks depends on whether or not the bank is a biased bank, as demonstrated in Proposition (Prop.) 2. Define the proportion of total loan volume held by bank i as S_i, where $S_i \equiv \frac{L_i}{L}$;

Proposition 2:

(9a) $\frac{dL_i}{db} = \frac{1}{r'(L)}\left\{1 - \frac{dr}{db}[1 + S_i E(L)]\right\}$ if $b_i = b$

(9b) $\frac{dL_i}{db} = -\frac{1}{r'(L)}\left\{\frac{dr}{db}[1 + S_i E(L)]\right\}$ if $b_i = 0$

and $\frac{dL_i}{db} > 0$ if $b_i = 0$.

Recall that $\sum_{i=1}^{N}\frac{dL_i}{db} = \frac{dL}{db} < 0$. Since $\frac{dL_i}{db} > 0$ for each unbiased bank, the sum of changes in loan volume for biased banks must be negative and larger in absolute size than the sum of changes for unbiased banks. When demand is linear, then *E(L) = 0* and $\frac{dr}{db} < 1$. Here (9a) shows that $\frac{dL_i}{db} < 0$ in this case. All biased banks lose volume when bias increases. But if demand is

not linear, loan volume may increase at some bias banks when bias coefficients increase.[7]

Substitute (7) into (9a) and define $K_0 \equiv \dfrac{1}{\theta N}\left[\dfrac{N(1-\theta)+1+E(L)}{E(L)}\right]$. In the case of biased banks,

(a) $\dfrac{dL_i}{db} > 0$ if $S_i < K_0$ when $E(L) < - N(1- \theta) – 1$, and (b) $\dfrac{dL_i}{db} > 0$ if $S_i > K_0$ when $E(L) >$

$\dfrac{N(1-\theta)+1}{\theta N - 1}$.

Suppose $E(L) < - N(1- \theta) – 1 < 0$.[8] The intuition (see Kimmel, p. 443 for a discussion

when $\theta = 1$) is that from expression (3), $\dfrac{S_i}{S_j} = \dfrac{L_i}{L_j} = \dfrac{r-b-a_i}{r-b-a_j}$. If $\dfrac{dr}{db} > 1$ (which holds when

$E(L) < - N(1- \theta) – 1$), as b increases $r – b$ increases and $\dfrac{S_i}{S_j}$ approaches 1. The market shares of

banks with $S_i < K_0$ increase and offset the decline in the total loan volume, so that the loan

volumes of those small biased banks increase.[9] At the extremes, when $\theta = 1$, $\dfrac{dL_i}{db} > 0$ when $S_i <$

$\dfrac{1+E(L)}{NE(L)}$. However, when $\theta = \dfrac{1}{N}$, the requirement for $\dfrac{dL_i}{db} > 0$ is $S_i < \dfrac{N+E(L)}{E(L)}$. The stability

requirement (1d) translates into $N + E(L) > 0$. Therefore, $\dfrac{N+E(L)}{E(L)}$ is negative, meaning that if

only one bank is biased, it cannot gain volume when its bias coefficient rises; regardless of how

otherwise efficient it is.

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[7] Expressions (9a) and (9b) may be compared to (A2) in Kimmel (1992). Kimmel does not analyze the impact of a change in cost on the volume at individual firms.
[8] Divide (1d) by $r'(L)$ and sum over all i. The stability requirement becomes $N + E(L) > 0$, or that $E(L) > - N$. Thus $E(L) < - N(1- \theta) - 1 < 0$ while $E(L) > -N$ if $-N < E(L) < - N(1- \theta) -1$. This set can be nonempty if $\theta > 1/N$.
[9] In their model of a homogeneous oligopoly, Salant and Shaffer (1999) conclude that volume must decline at firms that have the exogenous cost increase. However, in their model total industry costs are held constant, thus the interest rate is constant; and the impact of the change in the interest rate discussed here does not occur in their model.

Alternatively, suppose $E(L) > \dfrac{N(1-\theta)+1}{\theta N -1} > 0$. From (7) $\dfrac{dr}{db} < 1$ if $E(L) > 0$. As b

increases $r - b$ declines, and the effect of differences in production technology (a_i and a_j) on

market shares increases. The market shares of banks with $S_i > K_0$ increase enough to offset the

decline in total market loan volume. At the extremes, if $\theta = 1$, then $\dfrac{dL_i}{db} > 0$ requires that $S_i >$

$\dfrac{1+E(L)}{NE(L)}$. If $\theta = \dfrac{1}{N}$ then $\dfrac{dL_i}{db} > 0$ requires that $S_i > \dfrac{N+E(L)}{E(L)}$, but $\dfrac{N+E(L)}{E(L)} > 1$, which

exceeds the domain of S_i. As with $E(L) < 0$, when $E(L) > 0$ and only one biased bank exists

($\theta = \dfrac{1}{N}$), that bank will lose volume when its bias coefficient increases, regardless of the bank's

loan production efficiency.

5. Profits

As the lending bias increases, the interest rate on loans increases and total loan volume

declines; but the loan volume at individual banks may increase or decline depending upon the

loan production costs and the bias of the bank. How do these relationships affect the profits of

individual banks? The effect of a common change in the bias of the θN biased banks on bank

profits is as follows:

Proposition 3:

(11a) $\quad \dfrac{d\pi_i}{db} = \dfrac{dr}{db} \dfrac{L_i[E(L)(\theta N S_i - 2) - 2(1 + N(1-\theta))]}{\theta N}$ if $b_i = b$,

(11b) $\quad \dfrac{d\pi_i}{db} = \dfrac{dr}{db} L_i[2 + S_i E(L)] > 0$ if $b_i = 0$.

The profits of unbiased banks increase as the bias coefficient rises. This follows because r

increases for all banks and the loan volume of unbiased banks increases. In the case of biased

banks, $\dfrac{d\pi_i}{db} < 0$ when demand is linear, since $E(L) = 0$ in (11a). When demand is not linear,

$(E(L) \neq 0)$, consider the term inside [ ] in the numerator of (11a). Define $K_1 \equiv$

$$\frac{2}{\theta N}\left[\frac{1+N(1-\theta)+E(L)}{E(L)}\right].$$ In the case of biased banks, (a) $\frac{d\pi_i}{db} > 0$ if $S_i < K_1$ when $E(L) < -$

$N(1-\theta) - 1$, and (b) $\frac{d\pi_i}{db} > 0$ if $S_i > K_1$ when $E(L) > \frac{2[N(1-\theta)+1]}{\theta N - 2}$.

Suppose $E(L) < -N(1-\theta) - 1 < 0$. Here $\frac{d\pi_i}{db} > 0$ if $S_i < K_1$. Notice that $K_1 = 2K_0$. The

range of S_i within which $\frac{d\pi_i}{db} > 0$ is wider than the corresponding range for which $\frac{dL_i}{db} > 0$. This

reflects that profit can increase for some banks with lower loan volume when the increase in the

interest rate, r, is sufficiently large. In the extremes, when $\theta = 1$, $\frac{d\pi_i}{db} > 0$ if $S_i < \frac{2[1+E(L)]}{NE(L)}$.

When $\theta = \frac{1}{N}$, the requirement is $S_i < \frac{2[N+E(L)]}{E(L)}$. But $\frac{2[N+E(L)]}{E(L)}$ is negative, and a profit

increase for one biased bank is not possible.

Suppose $E(L) > \frac{2[N(1-\theta)+1]}{\theta N - 2} > 0$. Here $\frac{d\pi_i}{db} > 0$ when $S_i > K_1$. Here $K_1 > K_0$. The

range of S_i within which $\frac{d\pi_i}{db} > 0$ is smaller than the corresponding range within which $\frac{dL_i}{db} > 0$.

Since r increases at a slower rate than b when $E(L) > 0$, it becomes more difficult to achieve the

volume increases necessary to increase profits. At the extremes, when $\theta = 1$, $\frac{d\pi_i}{db} > 0$ when

$S_i > \frac{2[1+E(L)]}{NE(L)}$. When $\theta = \frac{1}{N}$, $\frac{d\pi_i}{db} > 0$ when $S_i > \frac{2[N+E(L)]}{E(L)}$. Since $\frac{2[N+E(L)]}{E(L)} > 1$ and

$S_i \leq 1$, a profit increase for the one biased bank is not possible.

6. Market Shares and Bank Concentration

The discussion of changes in market shares begins with Prop. 4.

Proposition 4:

(14) $S_i = \dfrac{1}{N} + \dfrac{a_i - \bar{a} + b_i - \theta b}{Lr'(L)}$

where $b_i = b$ for a biased bank and $b_i = 0$ for an unbiased bank. Since $r'(L) < 0$, higher loan

production cost and more bias will reduce market share. But lower production cost can offset

lending bias, and higher production cost may offset a lack of bias, so that high and low market

share banks may include both biased and unbiased banks. Prop. 5 relates to the impact of an

increase in the bias coefficient on the market shares of banks.

Proposition 5:

(15a) $\dfrac{dS_i}{db} = \dfrac{(1-\theta)Lr'(L) + N(\theta - \frac{dr}{db})[\bar{a} - a_i - (1-\theta)b]}{[Lr'(L)]^2}$ if $b_i = b$

(15b) $\dfrac{dS_i}{db} = \dfrac{-\theta Lr'(L) + N(\theta - \frac{dr}{db})[\bar{a} - a_i + \theta b]}{[Lr'(L)]^2}$ if $b_i = 0$

and $\dfrac{dS_i}{db} > 0$ if $b_i = 0$.

Prop. 4 and 5 here are Kimmel's Prop. 3 reformulated to adjust for any fraction of firms

having an exogenous cost increase. Prop. 2 shows that the total loan volume declines with

increases in the bias coefficient, and Prop. 3 shows that the loan volume at each unbiased bank

increases when the bias coefficient increases. Thus one should expect that market shares of

unbiased banks increase as the bias increases (15b).

In the case of biased banks, the first term in the numerator of (15a) is negative, reflecting

the reduction in the demand for loans associated with the higher interest rate. The change in S_i

can be positive only if $N[\theta - \dfrac{dr}{db}][\bar{a} - a_i - b(1-\theta)]$ is positive, or if $[\theta - \dfrac{dr}{db}]$ and

$[\bar{a} - a_i - b(1-\theta)]$ have the same sign. For linear demand curves, $\theta - \dfrac{dr}{db} > 0$ always holds, while

for nonlinear demand, $\theta - \dfrac{dr}{db} \gtreqqless 0$ as $E(L) \gtreqqless -1.$[10] If $\theta - \dfrac{dr}{db} > 0$ then $\dfrac{dS_i}{db} > 0$ only if the bank

is a large "efficient" bank, i.e., $\bar{a} - a_i > b(1-\theta)$. If $\theta - \dfrac{dr}{db} < 0$ then $\dfrac{dS_i}{db} > 0$ only if the bank is

less efficient ($\bar{a} - a_i < (1-\theta)b$).

      A change in bias also affects the concentration ratio, since market shares change when

the bias coefficients change. The measure of concentration used here is the Herfindahl index, H,

where $H \equiv \displaystyle\sum_{i=1}^{N} S_i^2$. H has been used as a measure of concentration by the Federal Reserve in its

decisions on allowing mergers between banks. I will analyze the change in H: $\Delta H = \hat{H} - H$,

resulting from a change in the bias coefficient, where $\hat{H}$ and H are the Herfindahl index after and

before the change, respectively. ΔH can be expressed in component parts as

$$(16)\quad \Delta H = \sum_{i=1}^{N} \hat{S}_i^2 - \sum_{i=1}^{N} S_i^2 ,$$

where $\hat{S}_i \equiv S_i + \Delta S_i$ and ΔS_i is the change in the market share of bank i. Then

$$(17)\quad \Delta H = 2\sum_{i=1}^{N} S_i \Delta S_i + \sum_{i=1}^{N} (\Delta S_i)^2$$

With this relationship in mind, Prop. 6 discusses the effect of a change in bias on the Herfindahl

index of concentration.

***Proposition 6*:**

$$(18a)\quad \frac{dH}{db} = \frac{2N}{Lr'(L)}\left\{ \theta(\bar{S}_b - \frac{1}{N}) - (\theta - \frac{dr}{db})(H - \frac{1}{N}) \right\} \quad \text{if} \quad H > \frac{1}{N},$$

$$(18b)\quad \frac{dH}{db} = \frac{\theta N(1-\theta)}{[Lr'(L)]^2} \quad \text{if} \quad H = \frac{1}{N}$$

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[10] From (7), $\theta - \dfrac{dr}{db} = \dfrac{\theta[(1+E(L)]}{N+1+E(L)}$. For linear demand curves, $E(L) = 0$. The stability conditions require

that $N + 1 + E(L) > 0$; thus for non-linear demand curves, the sign of $\theta - \dfrac{dr}{db}$ comes from $[(1 + E(L)]$.

where $\bar{S}^b$ is the average fraction of the total volume of loans held by the θN biased banks:

$$\bar{S}_b = \frac{\sum S_j}{\theta N} \text{ where } S_j \in n.$$

Expression (17) is the basis for the proof to Prop. 6. When $H > \frac{1}{N}$ the first term in (17) is nonzero and dominates the second term, so that normal calculus techniques (which assume that the second term is zero) can be used, resulting in (18a). When $H = \frac{1}{N}$, the first term in (17) is zero, and (18b) shows the result.[11] Prop. 6 shows that concentration may rise or fall when bias increases, depending on the total volume held by biased banks and the sign and magnitude of

$\theta - \frac{dr}{db}$. Recall that the minimum H is $\frac{1}{N}$ and that $r'(L)$ is negative. Suppose $\bar{S}_b > \frac{1}{N}$, meaning the average biased bank holds more loans than the average unbiased bank in the market.

If $\bar{S}_b > \frac{1}{N}$ and $\theta \leq \frac{dr}{db}$ then $\frac{dH}{db} < 0$. If the biased banks are relatively large and if the interest rate increase is larger than the fraction of biased banks, then concentration falls as the bias increases. Biased banks, which are the larger, more efficient banks, become less relatively

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[11] Since $H = \sum_{i=1}^{N} S_i^2$, then using calculus, $\frac{dH}{db} = 2\sum_{i=1}^{N} S_i \frac{dS_i}{db}$. When $H > \frac{1}{N}$, the second term in (17a) is very small compared to the first term. When $H = \frac{1}{N}$ (which requires that $\bar{S}_b = \frac{1}{N}$), expression (18a), which reflects the first term in (17), is zero. Thus expression (18a) is zero when $H = \frac{1}{N}$. For example, expression (29) in Dixit and Stern (1982) is incorrect when $H = \frac{1}{N}$. The second term in (21) can be used to show the change in H as bias changes. Salant and Shaffer (1999) discuss the impact of cost changes on the Herfindahl Index, but they do not model the specific change; and more important, they assume that the sum of the costs (the sum of production costs and bias in this paper) does not change. If total costs do not change, then the interest rate and total loans do not change. I show that the change in the interest rate produces a change in the distribution of loans amongst the banks. Thus a change in the rate of interest associated with a change in bias causes a change in concentration.

efficient and lose market share. Market shares overall become more evenly distributed, and concentration declines. Here it is unambiguous that concentration declines as bias increases.

Suppose $\bar{S}_b < \dfrac{1}{N}$, or that the average biased bank holds less loans than the average unbiased bank. In this case, when $\theta \geq \dfrac{dr}{db}$, then $\dfrac{dH}{db} > 0$. Since biased banks are smaller and their costs ($a_i + b_i$) are larger, the difference in costs between biased and unbiased banks widens. The increase in bias causes the distribution of loans to become less even, and H increases. In this case concentration increases as the bias of the biased banks increases. These are the only unambiguous cases.

The other possibilities lead to ambiguous outcomes. When $\bar{S}_b < \dfrac{1}{N}$ (biased banks are "small") and $\theta < \dfrac{dr}{db}$ then $\dfrac{dH}{db}$ is positive (negative) if $(\dfrac{dr}{db} - \theta)(H - \dfrac{1}{N})$ is greater (less) than $\theta(\dfrac{1}{N} - \bar{S}_b)$. Or when $\bar{S}^b > \dfrac{1}{N}$ (biased banks are "large") and $\theta > \dfrac{dr}{db}$ then $\dfrac{dH}{db}$ is positive (negative) if $(\theta - \dfrac{dr}{db})(H - \dfrac{1}{N})$ is greater (less) than $\theta(\bar{S}_b - \dfrac{1}{N})$.

The outcome of more bias can be either more concentration or less concentration among banks. I showed earlier that the interest rate on loans increases with more bias: $\dfrac{dr}{db} > 0.$ Thus more bias will always entail higher rates paid by borrowers in the market. On the other hand $\dfrac{dH}{db} > 0$ or $\dfrac{dH}{db} < 0.$ More bias may produce higher concentration or lower concentration, depending upon the cost structure of the biased and unbiased banks, the proportion of banks that are biased, and the amount of loans demanded by borrowers given the rate of interest on loans.

7. Lending Bias in a Model of Differentiated Bank Loans

It may be more realistic to assume that banks have some degree of product differentiation in the market for loans to the disfavored group. This implies a model that allows for heterogeneity among banks. Suppose the loans of the N banks are horizontally differentiated such that $r_i(L)$, the inverse demand function for bank i , is

(19) $r_i = v - \beta L_i - w \sum_{j \neq i} L_j$

where v, β, and w are all positive parameters, with $0 < w < \beta$.[12] I will normalize w and β so that $\beta \equiv 1$, which implies that w $\in (0,1)$. The symmetric degree of loan substitutability between any two banks is measured by *w*. If $w = 0$, the demand for the loans of each bank is independent from that of the other banks, whereas if $w = 1$, the loans are perfect substitutes, and the market for these loans is a homogenous Cournot oligopoly with linear demand. I maintain the assumption that $b_i = b$ for a biased bank and $b_i = 0$ for an unbiased bank. The single period solution for *L*, L_i and π_i under Cournot competition are

(20) $L = \dfrac{N(v - \bar{a} - \theta b)}{2 + w(N-1)}$,

(21) $L_i = \dfrac{1}{2}\left\{ \left(v - w \sum_{j \neq i} L_j - a_i - b_i \right) \right\}$

$= \dfrac{(2-w)v - [2 + w(N-1)](a_i + b_i) + wN(\bar{a} + \theta b)}{(2-w)[2 + w(N-1)]}$, and

(22) $\pi_i = L_i^2$.

Given these relationships, it follows that

(23) $S_i = \dfrac{1}{N} + \dfrac{[(N-1)w + 2][\bar{a} - a_i + \theta b - b_i]}{N(v - \bar{a} - \theta b)(2 - w)}$

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[12] This demand structure for differentiated markets is used by Spence (1976), Majerus (1988), Yuan (1999) and Hackner (2000), among others.

Expressions (20) and (21) show that the effect of b is to reduce to reduce the overall supply of loans and the loans of bank i. Given this framework, Prop. 7 discusses the outcomes when bias changes.

Proposition 7:

When the market for bank loans is differentiated by the function: $r_i = v - \beta L_i - w \sum_{j \neq i} L_j$,

The following relationships hold:

a. Loan Volume

(24a) $\dfrac{dL_i}{db} = \dfrac{-2 - w[N(1-\theta)-1]]}{(2-w)[2+w(N-1)]} < 0$ if $b_i = b$

(24b) $\dfrac{dL_i}{db} = \dfrac{w\theta N}{(2-w)[2+w(N-1)]} > 0$ if $b_i = 0$

b. Profits

(25a) $\dfrac{d\pi_i}{db} = 2L_i \dfrac{w[1-N(1-\theta)]-2}{(2-w)[2+w(N-1)]} < 0$ if $b_i = b$,

(25b) $\dfrac{d\pi_i}{db} = 2L_i \dfrac{w\theta N}{(2-w)[2+w(N-1)]} > 0$ if $b_i = 0$

c. Market Shares

(26) $\dfrac{dS_i}{db} = \dfrac{G[\theta(v-a_i) - p_i(v-\bar{a})]}{(v-\bar{a}-\theta b)^2}$ where $p_i = 1$ if bank i is biased, $p_i = 0$

if bank i is unbiased, $G \equiv \dfrac{2+w(N-1)}{(2-w)N} > 0$, and $\dfrac{dS_i}{db} > 0$ if $b_i = 0$.

d. Concentration

(27a) $\dfrac{dH}{db} = \dfrac{-2\theta}{v-\bar{a}-\theta b} \left\{ NG(\bar{S}^b - \dfrac{1}{N}) - (H - \dfrac{1}{N}) \right\}$ if $H > \dfrac{1}{N}$

(27b) $\dfrac{dH}{db} = \dfrac{\theta N(1-\theta)G^2}{(v-\bar{a}-\theta b)^2}$ if $H = \dfrac{1}{N}$

As mentioned earlier, $E(L) = 0$ when demand is linear; and for linear demand, as bias increases, loan volume and profits always decline for biased firms and increase for unbiased firms. With regard to market shares, the market shares of unbiased firms always increase as bias increases. The market shares of biased firms will increase or decrease depending on sign of the term inside [ ] in (26). When $\theta = 1/N$, this term can be rewritten as follows:

$$(28) \quad \theta(v - a_i) - (v - \bar{a}) = \frac{\sum_{j \neq i} a_j - (N-1)v}{N}$$

From (21), $v > a_i$ for all positive L_i's. There are $(N - 1)$ a_i's and $(N - 1)$ v's. Thus $\theta(v - a_i) - (v - \bar{a})$ is always negative if $\theta = 1/N$: when only one bank is biased, it loses market share if its bias increases. For $\theta > 1/N$, (28) can be rewritten as

$$(29) \quad \theta(v - a_i) - (v - \bar{a}) = (1 - \theta)(\bar{a}_u - v) - \theta(a_i - \bar{a}_b)$$

where $\bar{a}_b$ and $\bar{a}_u$ are the average loan production costs of the biased and unbiased banks, respectively: $\theta\bar{a}_b + (1 - \theta)\bar{a}_u = \bar{a}$. Thus for a biased bank, $\frac{dS_i}{db} > 0$, if

$$(30) \quad a_i - \bar{a}_b < \frac{(1 - \theta)}{\theta}(\bar{a}_u - v)$$

Since $v > \bar{a}_u$, the RHS of 30 is negative. Thus a biased bank's production cost must be lower than the average production cost of biased banks in order for the bank's market share to increase as bias increases. If $\theta = 1$, the requirement for $\frac{dS_i}{db} > 0$ is $a_i < \bar{a}$. Efficient (and large) banks gain market share with more bias.

In terms of concentration, similar to the case with homogeneous banks, H may increase or decline with more bias. The key term is $\bar{S}_b - \frac{1}{N}$. If this term is sufficiently positive, then concentration falls as bias increases, and if it is negative, concentration rises as bias increases. If biased banks hold a sufficiently high market share (reflecting lower costs), then the distribution of loans becomes more evenly distributed when bias increases. But if biased banks are small

(reflecting higher costs), their loss of loan volume as bias increases can lead to more concentration of loans held by unbiased banks.

8. Example

Exhibit 2 shows the equilibrium outcomes in interest rate, loan volume, market shares, concentration, and profits under the constant elasticity demand structure, $r = GL^{-1/u}$; where G = 1,000, u = 0.35 and N = 5 banks. Here $E(L) = \frac{1}{-.35} - 1 = -3.857$. The exhibit shows the equilibrium outcomes of six different cost structures under this demand function. Case 0 is the base case. Here $b_i = 0$ for all banks. Bank 1 has the highest loan production cost (a_i), banks 2, 3 and 4 have equal production costs and bank 5 has the lowest production cost. In cases 1 through 5, an increasing number of the banks discriminate, and the bias coefficient is the same for all biased banks. In case 1 only bank 1 has a positive bias coefficient. Banks 1 and 2 are biased in case 2. Banks 1,2 and 3 are biased in case 3. In case 4, banks 1, 2, 3 and 5 are biased. All of the banks have a positive bias coefficient in case 5. Comparison of the outcomes in r, L, L_i, S_i, H and π_i across the demand structures demonstrate several of the propositions above. They show that *as the degree of bias in the market increases*:

1. The interest rate increases and total loan volume declines. The equilibrium in case 0, the no bias case, has the lowest interest rate and highest total loan volume. Borrowers are worse off in all cases of lending bias.

2. For individual banks, market share may increase (compare market shares in case 0 with those in case 5 for banks 1 through 4) or decline (compare case 0 with case 4 for the banks 1, 2, 3 and 5); market share always increases for unbiased banks (compare the market share of bank 4 in case 0 to its market share in cases 1 through 4).

3. Concentration (Herfindahl index) may increase (compare case 0 with cases1 through 3) or decrease (compare case 0 with cases 4 and 5).

4. At individual banks, profits may increase or decrease. Consider bank 1, which is biased in cases 1 through 5. Profits are higher in cases 3, 4 and 5 than in case 0, and lower in cases 1 and 2 than in case 0.

I should add that the empirical studies have differed in their measurement of concentration. Berkovic et al use the Herfindahl index of bank loans, while Cavalluzzo et al use the Herfindahl index of bank deposits. The issue behind the use of concentration measures is the extent to which a few banks exhibit control over interest rates on loans.[13] Thus the use of deposit concentration is a proxy for the concentration of loans in the market. Deposit concentration might not correlate with control of prices and quantities in specific loan markets, and thus may be only a noisy measure of control in loan markets. But bank regulators use deposit concentration in their analysis of control in banking markets.

9. Conclusion

A Cournot model has been extended to consider the impact of lending bias on the performance of commercial banks. The loan volume, profits and market shares of biased banks may increase as they increase their degree of bias, even when there are competing unbiased banks. In addition, higher bias leads to higher interest rates on loans but loan concentration may increase or decline. This latter finding conflicts with the assumption used by several empirical studies that if bias exists, interest rates should increase as concentration increases.

The result that higher bias can lead to profit increases for some banks begs the question of how prejudicial discrimination is defined. Becker (1971, p. 14) states

"If an individual has a 'taste for discrimination' he must act *as if* he were willing to pay something, either directly or in the form of a reduced income, to be associated with some persons instead of others. When actual discrimination occurs, he must, in fact <u>either pay or forfeit income</u> for this privilege."

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[13] See Geroski (1983).

I underlined the last sentence to emphasize that in Becker's framework, discrimination occurs only when the discriminator suffers a loss.[14] I argue that discrimination should include cases in which the discriminator gains from bias, if the discriminator was acting "as if he were willing to pay something" along the lines considered here.   When banks add a non-economic charge to their profit functions for lending to disfavored customers, some of those banks may increase profits while others suffer profit reductions. But the cost of borrowing always increases and the amount of loans declines—the borrower always loses. Because of the negative impact of the bias, all who add the charge should be considered as practicing prejudicial discrimination.

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[14] Becker (1993, p. 18) writes that "discrimination in the marketplace consists of voluntarily relinquishing profits, wages, or income in order to cater to prejudice".

APPENDIX

Proof of Proposition 1

Proof: Totally differentiate (2b) to obtain:

(A1) $r'(L)dL + L_i r''(L)dL + r'(L)dL_i = da_i + db_i$

In the case of an increase in the bias coefficients of the n biased banks, $da_i = 0$, $db_i = db \; \forall \; i \in n$

and $db_i = 0$ for $i \notin n$. Sum (A1) over all i and make use of the relationship that $\sum_{i=1}^{N} L_i = L$. The

result is

(A2) $\dfrac{dL}{db} = \dfrac{\theta N}{(N+1)r'(L) + Lr''(L)}$

By the stability assumption, the denominator of (A2) is negative so $\dfrac{dL}{db} < 0$. With regard to the

change in the interest rate on loans as bias changes,

(A3) $\dfrac{dr}{db} = r'(L)\dfrac{dL}{db}$

$\quad = r'(L)\dfrac{\theta N}{(N+1)r'(L) + Lr''(L)}$.

Since $r'(L) < 0$ and $\dfrac{dL}{db} < 0$ then $\dfrac{dr}{db} > 0$. Q.E.D.

Proof of Proposition 2

Proof: Restate Expression (A1) with $da_i = 0$ as

(A4) $r'(L)\dfrac{dL}{db} + L_i r''(L)\dfrac{dL}{db} + r'(L)\dfrac{dL_i}{db} = \dfrac{db_i}{db}$

For biased banks $\dfrac{db_i}{db} = 1$. Use (A2) to substitute for $\dfrac{dL}{db}$ and (8) to derive (9a). For unbiased

banks $\dfrac{db_i}{db} = 0$, and use (A2) to substitute for $\dfrac{dL}{db}$ and (8) to derive (9b). In (9b), $-\dfrac{1}{r'(L)}\dfrac{dr}{db}$ is

positive and from the stability requirement (1d), $1 + S_i E(L)$ is positive; therefore (9b) is positive.

Q.E.D.

Proof of Proposition 3:

For biased banks, from (1c)

(A5) $\dfrac{d\pi_i}{db} = \dfrac{dL_i}{db}(r - a_i - b) + L_i(\dfrac{dr}{db} - 1)$

As b changes, L_i responds so that $\dfrac{dL_i}{db}$ remains 0. Thus use the first order condition (2b) to

replace $(r - a_i - b)$ and (9a) to replace $\dfrac{dL_i}{db}$. The result is

(A6) $\dfrac{d\pi_i}{db} = L_i \dfrac{\dfrac{dr}{db}[N(2\theta - 1) - 1 - E(L) + \theta N S_i E(L)] - \theta N}{\theta N}$

From (7) the following holds:

(A7) $\theta N = \dfrac{dr}{db}[N + 1 + E(L)]$

Use (A7) to substitute for the last θN in the numerator of (A6) and (11a) follows.

For unbiased banks, from (1c)

(A2) $\dfrac{d\pi_i}{db} = \dfrac{dL_i}{db}(r - a_i) + L_i \dfrac{dr}{db}$

Use (2b) to replace $r - a_i$ (set $b_i = 0$) and (9b) to replace $\dfrac{dL_i}{db}$, and (11b) follows. From the

stability assumption (1c) it follows that $1 + S_i E(L) > 0$; thus $\dfrac{d\pi_i}{db} > 0$ for unbiased banks. Q.E.D.

Proof of Proposition 4

Proof: Divide L_i in (3) by L to obtain:

(A8) $S_i \equiv \dfrac{L_i}{L} = \dfrac{-(r - a_i - b_i)}{Lr'(L)}$

Use (5) to substitute for r, which results in (14).[15] Q.E.D.

Proof of Proposition 5.

For biased banks, from (14) the derivative of S_i with respect to b is

$$(A9) \quad \frac{dS_i}{db} = \frac{(1-\theta)Lr'(L) + \frac{d[Lr'(L)]}{db}[\bar{a} - a_i - b(1-\theta)]}{[Lr'(L)]^2}$$

The proof is completed by showing that $\frac{d[Lr'(L)]}{db} = N(\theta - \frac{dr}{db})$:

$$\frac{d[Lr'(L)]}{db} = \frac{dL}{db}r'(L)[1 + \frac{Lr''(L)}{r'(L)}] = \frac{dr}{db}[1 + E(L)]. \quad \text{From (A7)},$$

$$\frac{dr}{db}[1 + E(L)] = N(\theta - \frac{dr}{db}). \quad \text{This completes the proof for biased banks.}$$

For unbiased banks,

$$(A10) \quad \frac{dS_i}{db} = \frac{-\theta Lr'(L) + \frac{d[Lr'(L)]}{db}[\bar{a} - a_i + \theta b]}{[Lr'(L)]^2}.$$

Use the relationship that $\frac{d[Lr'(L)]}{db} = N(\theta - \frac{dr}{db})$, and (15b) follows. To show that $\frac{dS_i}{db} > 0$

always holds for unbiased banks, first note from (A7) that

$$N(\theta - \frac{dr}{db}) = \frac{\theta N[1 + E(L)]}{N + 1 + E(L)}, \text{ and } [Lr'(L)]^2 > 0. \quad \text{Thus given (A3), } \frac{dS_i}{db} > 0 \text{ if}$$

$$(A11) \quad \theta N[1 + E(L)][\bar{a} + \theta b - a_i] - \theta Lr'(L)[N + 1 + E(L)] > 0.$$

From (5), $a + \theta b = r + \frac{Lr'(L)}{N}$, and from (2b), $r - a_i = -r'(L)L_i$, since $b_i = 0$ for an unbiased

bank. Substitute these into (A11) and the requirement becomes

$$(A12) \quad \theta N[1 + E(L)][\frac{Lr'(L)}{N} - Lr'(L)S_i] - \theta Lr'(L)[N + 1 + E(L)] > 0.$$

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[15] See Kimmel (1992) for an alternate proof of Prop. 4 for the case in which $\theta = 1$.

Divide by $-\theta Lr'(L)$, which is positive, collect terms and cancel out the N. The requirement for a

positive $\dfrac{dS_i}{db}$ is

(A13) $1 + S_i[1 + E(L)] > 0$.

The stability assumption (1c) translates into $1 + S_i E(L) > 0$. Thus $\dfrac{dS_i}{db} > 0$ always holds when b_i

= 0, or when the bank is unbiased. Q.E.D.

Proof of Proposition 6.

When $H > \dfrac{1}{N}$, from (17),

(A14) $\dfrac{dH}{db} = 2\sum_{i=1}^{N} S_i \dfrac{dS_i}{db}$

Note that consistent with calculus methodology, the second term in (21) is assumed to be zero.

By using (15a) and (15b), (A7) can be restated as

(A15) $\dfrac{dH}{db} = 2\sum_{i=1}^{N} S_i \dfrac{N(\theta - \dfrac{dr}{db})[\bar{a} - a_i - b_i + \theta b] - \theta Lr'(L)}{[Lr'(L)]^2} + \dfrac{2\theta N \bar{S}_b}{Lr'(L)}$

where $b_i = b$ for a biased bank and $b_i = 0$ for an unbiased bank; and $\bar{S}_b$ is the mean market share

of the θN biased banks ($\sum_{i \in n} S_i / \theta N$). The second term in (A15) reflects that the θN biased banks

have a $(1 - \theta)Lr'(L)$ term in (15a) while the unbiased banks have a $-\theta Lr'(L)$ term in (15b).

Simplify (A15) by separating out the $\theta Lr'(L)$ term from the first part of (A15):

(A16) $\dfrac{dH}{db} = 2\sum_{i=1}^{N} S_i \dfrac{N(\theta - \dfrac{dr}{db})[\bar{a} - a_i - b_i + \theta b]}{[Lr'(L)]^2} - \dfrac{2\theta \sum_{i=1}^{N} S_i}{Lr'(L)} + \dfrac{2\theta N \bar{S}_b}{Lr'(L)}$

Using $\sum_{i=1}^{N} S_i = 1$, and further refinement brings

$$\text{(A17)} \quad \frac{dH}{db} = \frac{2N(\theta - \frac{dr}{db})}{Lr'(L)} \sum_{i=1}^{N} S_i \left\{ \frac{[\bar{a} - a_i - b_i + \theta b]}{Lr'(L)} \right\} - \frac{2\theta(1 - N\bar{S}_b)}{Lr'(L)}$$

The term inside the { } brackets is equal to $\frac{1}{N} - S_i$. This and the facts that $\sum_{i=1}^{N} S_i = 1$ and

$H \equiv \sum_{i=1}^{N} S_i^2$ produce

$$\text{(A18)} \quad \frac{dH}{db} = \frac{2N(\theta - \frac{dr}{db})}{Lr'(L)} \frac{1}{N} - \frac{2N(\theta - \frac{dr}{db})}{Lr'(L)} H - \frac{2\theta(1 - N\bar{S}_b)}{Lr'(L)}$$

$$= \frac{2N[\theta(\bar{S}^b - \frac{1}{N}) - (\theta - \frac{dr}{db})(H - \frac{1}{N})]}{Lr'(L)}$$

In order to obtain (18b), when $S_i = \frac{1}{N}$ for all i, then $H = \frac{1}{N}$ and (18a) is 0; thus the change in H is calculated as

$$\text{(A19)} \quad \frac{dH}{db} = \sum_{i=1}^{N} \left(\frac{dS_i}{db} \right)^2 = \sum_{i=1}^{N} \left\{ \frac{b_i - \theta}{Lr'(L)} + \frac{N(\theta - \frac{dr}{db})}{Lr'(L)} \frac{-(a_i - \bar{a} + b_i - \theta b)}{Lr'(L)} \right\}^2$$

where $b_i = b$ for a biased bank , and $b_i = 0$ for an unbiased bank. But in (A30)

$$\frac{-(a_i - \bar{a} + b_i - \theta b)}{Lr'(L)} = \frac{1}{N} - S_i. \quad \text{When } S_i = \frac{1}{N}, \text{ (A30) becomes } \frac{dH}{db} = \frac{\theta N(1-\theta)}{[Lr'(L)]^2}.$$

Q.E.D.

Proof of Proposition 7

a. Loan Volume

$$\text{(A20)} \quad \frac{dL_i}{db} = \frac{-p_i[2 + w(N-1)] + \theta Nw}{(2 - w)[2 + w(N-1)]} \quad \text{where } p_i = 1 \text{ for a biased bank, and } p_i = 0 \text{ for an unbiased}$$

bank. This expression results in (24a) and (24b) for biased and unbiased banks, respectively.

a. Profits

Since $\pi_i = L_i^2$, then

(A21) $\dfrac{d\pi_i}{db} = 2L_i \dfrac{dL_i}{db}$.

Plug (A20) into (A21), which produces:

(A22) $\dfrac{d\pi_i}{db} = 2L_i \dfrac{w\theta N - p_i[2 + w(N-1)]}{(2-w)[2+w(N-1)]}$

where $p_i = 1$ for a biased bank and $p_i = 0$ for an unbiased bank.

b. Market Shares

Differentiation of (23) produces

(A23) $\dfrac{dS_i}{db} = G\left\{ \dfrac{(\theta - p_i)[v - \bar{a} - \theta b] + \theta[\bar{a} - a_i + \theta b - b_i]}{(v - \bar{a} - \theta b)^2} \right\}$

where $p_i = 1$ and $b_i = b$ for a biased bank; and $p_i = 0$ and $b_i = 0$ for an unbiased bank.

Collect terms and (26) is obtained separately for biased and unbiased banks.

c. Concentration

(A24) $\dfrac{dH}{db} = 2\sum_{i=1}^{N} S_i \dfrac{dS_i}{db}$

From substitute for $\dfrac{dS_i}{db}$ from (A23), which results in

(A25) $\dfrac{dH}{db} = 2\sum_{i=1}^{N} S_i \dfrac{G(\theta - p_i)}{v - \bar{a} - \theta b} + 2\sum_{i=1}^{N} S_i \dfrac{\theta G(\bar{a} - a_i + \theta b - b_i)}{(v - \bar{a} - \theta b)^2}$

Use the relationship that

$\dfrac{\theta G(\bar{a} - a_i + \theta b - b_i)}{(v - \bar{a} - \theta b)^2} = \dfrac{\theta}{v - \bar{a} - \theta b}\left[S_i - \dfrac{1}{N} \right]$ to obtain expression (27a). In order to obtain

(27b), recall that when $S_i = \dfrac{1}{N}$ for all i, then $H = \dfrac{1}{N}$ and (27a) is 0; thus when $S_i = \dfrac{1}{N}$ the change in H is calculated as

(A26) $\dfrac{dH}{db} = \sum_{i=1}^{N}\left(\dfrac{dS_i}{db} \right)^2 = \sum_{i=1}^{N}\left\{ \dfrac{G(\theta - p_i)}{v - \bar{a} - \theta b} \right\}^2 = \dfrac{\theta NG^2(\theta - 1)^2}{(v - \bar{a} - \theta b)^2} + \dfrac{(1 - \theta)NG^2\theta^2}{(v - \bar{a} - \theta b)^2}$

The first term reflects the θN biased banks and the second term reflects the

$(1-\theta)N$ unbiased banks. Canceling out the redundant terms results in (27b). Q.E.D.

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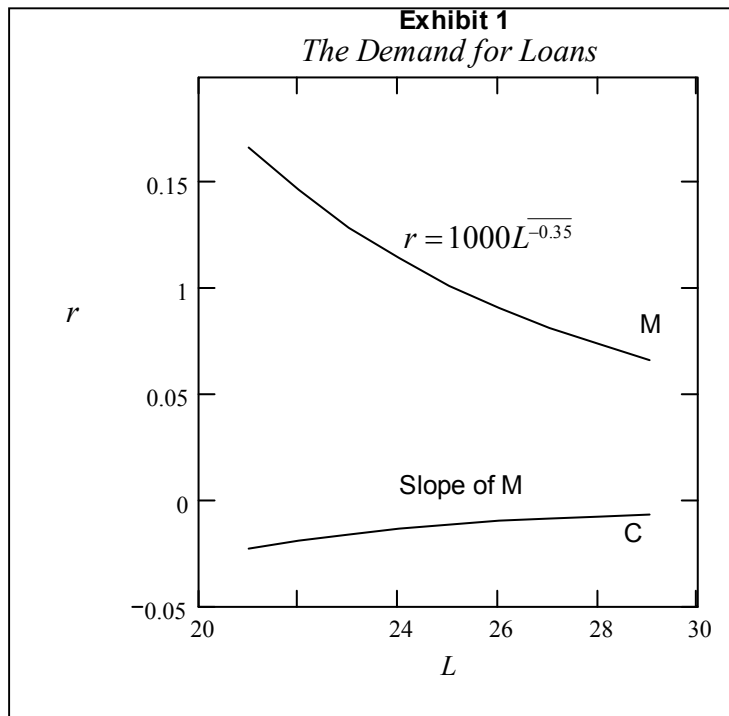
Exhibit 1
The Demand for Loans

EXHIBIT 2: EXAMPLE USING CONSTANT ELASTICITY DEMAND

Iseolastic Demand: $r = GL^{-1/u}$ Example: G = 1,000 u = 0.35   N = 5 banks

Costs ($a_i + b_i$):	Base: a_i Structr 0	With bias coefficient: $a_i + b_i$ Structr 1	Structr 2	Structr 3	Structr 4	Structr 5
Bank 1	0.0700	0.0725	0.0725	0.0725	0.0725	0.0725
Bank 2	0.0550	0.0550	0.0575	0.0575	0.0575	0.0575
Bank 3	0.0550	0.0550	0.0550	0.0575	0.0575	0.0575
Bank 4	0.0550	0.0550	0.0550	0.0550	0.0550	0.0575
Bank 5	0.0350	0.0350	0.0350	0.0350	0.0375	0.0375
Total Costs	0.2700	0.2725	0.2750	0.2775	0.2800	0.2825
Interest Rate (r)	0.1260	0.1272	0.1283	0.1295	0.1307	0.1318
Tot Loan Vol (L)	23.167	23.092	23.019	22.946	22.874	22.803
Loan Volume (L_i)						
Bank 1	3.6038	3.4745	3.5051	3.5349	3.5639	3.5920
Bank 2	4.5691	4.5867	4.4468	4.4652	4.4829	4.5001
Bank 3	4.5691	4.5867	4.6037	4.4652	4.4829	4.5001
Bank 4	4.5691	4.5867	4.6037	4.6202	4.6361	4.5001
Bank 5	5.8561	5.8578	5.8593	5.8605	5.7083	5.7108
Total	23.1670	23.0924	23.0187	22.9459	22.8740	22.8030
Market Shares (S_i)						
Bank 1	0.15556	0.15046	0.15227	0.15405	0.15580	0.15752
Bank 2	0.19722	0.19862	0.19318	0.19459	0.19598	0.19735
Bank 3	0.19722	0.19862	0.20000	0.19459	0.19598	0.19735
Bank 4	0.19722	0.19862	0.20000	0.20135	0.20268	0.19735
Bank 5	0.25278	0.25367	0.25455	0.25541	0.24955	0.25044
Total	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
Herfindahl (H)	0.20478	0.20534	0.20530	0.20524	0.20445	0.20437
Profits$_i$						
Bank 1	0.202	0.190	0.196	0.201	0.207	0.213
Bank 2	0.324	0.331	0.315	0.321	0.328	0.335
Bank 3	0.324	0.331	0.338	0.321	0.328	0.335
Bank 4	0.324	0.331	0.338	0.344	0.351	0.335
Bank 5	0.533	0.540	0.547	0.554	0.532	0.539
Total	1.708	1.723	1.733	1.742	1.746	1.755

Calculations: r = Total Costs/(N - 1/u)   Bank Loan Volume = $S_i L$

Total Loan Volume = $(G/r)^u$ Market Share (S_i) = $\{[r - (a_i + b_i)]u\}/r$

$H = S_1^2 + S_2^2 + \ldots + S_5^2$ Bank profit$_i$ = $[r - (a_i + b_i)]S_i L$