Study Questions/Problems Week 9

Chapter 12. The concept of Universal gravitation was formulated by Newton in the latter half of the 17th century. It is still THE theory of gravity for describing the motion of objects subject to the gravitational force as long as it is not too strong and objects are not moving too fast. It fails near neutron stars and black holes and for objects traveling near the speed of light—circumstances that require the use of Einstein’s 20th century theory of gravity. Newton’s application of universal gravitation to the motion of celestial bodies, made sense of the celestial mechanism, and culminated a two-millennia quest to rationalize the cosmos, and made him a legend in his own time (to borrow a trite phrase).

Chapter 11:

  Conceptual Questions  1, 2, 4, 6, 8, 11
  Conceptual Exercises  2, 3, 6, 7, 8, 9, 10, 12, 14
  Problems  2, 4, 6, 14, 18, 23, 28, 37, 38, 39, 41, 49

Answers/solutions for all even numbered CQs and CEs, and for each of the problems listed above, are on the following pages—don’t peek until you have done your best to solve a problem.
Chapter 12: Gravity

Answers to Even-Numbered Conceptual Questions

2. A person passing you on the street exerts a gravitational force on you, but it is so weak (about $10^{-7}$ N or less) that it is imperceptible.

4. As the tips of the fingers approach one another, we can think of them as two small spheres (or we can replace the finger tips with two small marbles if we like). As we know, the net gravitational attraction outside a sphere of mass is the same as that of an equivalent point mass at its center. Therefore, the two fingers simply experience the finite force of two point masses separated by a finite distance.

6. No. A satellite must be moving relative to the center of the Earth to maintain its orbit, but the North Pole is at rest relative to the center of the Earth. Therefore, a satellite cannot remain fixed above the North Pole.

8. Yes. The rotational motion of the Earth is to the east, and therefore if you launch in that direction you are adding the speed of the Earth’s rotation to the speed of your rocket.

10. As the astronauts approach a mascon, its increased gravitational attraction would increase the speed of the spacecraft. Similarly, as they pass the mascon, its gravitational attraction would now be in the backward direction, which would decrease their speed.

12. It makes more sense to think of the Moon as orbiting the Sun, with the Earth providing a smaller force that makes the Moon “wobble” back and forth in its solar orbit.

Answers to Even-Numbered Conceptual Exercises

2. Yes, there is a slight difference. If you fly to the east, which is the direction of the Earth’s rotation, you have a greater speed relative to the center of the Earth than if you fly to the west. As a result, the centripetal force required to maintain your circular motion is greater, and your apparent weight is less.

4. First finding the net force, then dividing by the mass, yields the following ranking: $B < C < A$.

6. The Earth moves faster the closer it comes to the Sun. Thus the Earth is closest to the Sun around January 4 and farthest from the Sun around July 4.

8. The satellite drops into an elliptical orbit that brings it closer to the Earth. The situation is similar to that illustrated in Figure 12-13(a). (a) The apogee distance remains the same. (b) The perigee distance is reduced.

10. More energy is required to go from the Earth to the Moon. To see this, note that you must essentially “escape” from the Earth to get to the Moon, and this takes much more energy than is required to “escape” from the Moon, with its much weaker gravity. This is why an enormous Saturn V rocket was required to get to the Moon, but only a small rocket on the lunar lander was required to lift off the lunar surface.

12. Skylab’s speed increased as its radius decreased. This can be seen by recalling that $T = (constant) r^{3/2}$ (Kepler’s third law) and that $v = 2\pi r / T$ (circular motion). It follows that $v = (constant) r^{-1/2}$, and therefore the speed increases with decreasing radius. You might think that friction would slow Skylab—just like other objects are slowed by friction—but by dropping Skylab to a lower orbit, and freeing up gravitational potential energy, friction is ultimately responsible for an increase in speed.

14. More energy is required to put the satellite in orbit because, not only must you supply enough energy to get to the altitude $h$, you must also supply the kinetic energy the satellite will have in orbit.
See following pages for Problem Solutions
2. **Picture the Problem**: The two bowling balls attract each other gravitationally.

**Strategy**: Use the Universal Law of Gravity (equation 12-1) to find the force between the bowling balls, then solve the same equation for distance to answer part (b).

**Solution**: 1. (a) Apply equation 12-1:

\[ F = G \frac{m_1 m_2}{r^2} = \left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right) \frac{(6.1 \text{ kg})(7.2 \text{ kg})}{(0.75 \text{ m})^2} = 5.2 \times 10^{-9} \text{ N} \]

2. (b) Solve equation 12-1 for \( r \):

\[ r = \sqrt{\frac{G m_1 m_2}{F}} = \sqrt{\frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \cdot (6.1 \text{ kg})(7.2 \text{ kg})}{2.0 \times 10^{-9} \text{ N}}} = 1.2 \text{ m} \]

**Insight**: Increasing the distance between the balls from 0.75 m to 1.2 m decreased the force from 5.2 nN to 2.0 nN.

4. **Picture the Problem**: You and the asteroid attract each other gravitationally.

**Strategy**: Estimate that your mass = 70 kg. Apply equation 12-1 to find the gravitational force between you and Ceres.

**Solution**: Apply equation 12-1:

\[ F = G \frac{m_1 m_2}{r^2} = \left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right) \frac{\left(8.7 \times 10^{20} \text{ kg}\right)(70 \text{ kg})}{\left(12 \times 10^6 \text{ m}\right)^2} = 0.028 \text{ N} \]

**Insight**: If you stood on the surface of Ceres (radius 500 km) the force would be 16 N (3.6 lb).

6. **Picture the Problem**: The spaceship is attracted gravitationally to both the Earth and the Moon.

**Strategy**: Use the Universal Law of Gravity (equation 12-1) to relate the attractive forces from the Earth and the Moon. Set the force due to the Earth equal to twice the force due to the Moon when the spaceship is at a distance \( r \) from the center of the Earth. Let \( R = 3.84 \times 10^8 \text{ m} \), the distance between the centers of the Earth and Moon. Then solve the expression for the distance \( r \).

**Solution**: 1. (a) Set \( F_E = 2F_M \) using equation 12-1 and solve for \( r \):

\[ G \frac{m_E m_1}{r^2} = 2G \frac{m_M m_1}{(R - r)^2} \]

\[ m_E (R - r)^2 = 2m_M r^2 \]

\[ R - r = \sqrt{\frac{2m_M}{m_E}}r \]

\[ r = \frac{R}{1 + \sqrt{\frac{2m_M}{m_E}}} = \frac{3.84 \times 10^8 \text{ m}}{1 + \sqrt{\frac{2 \times 7.35 \times 10^{22} \text{ kg}}{5.97 \times 10^{24} \text{ kg}}}} = 3.32 \times 10^8 \text{ m} \]

2. (b) The answer to part (a) is independent of the mass of the spaceship because the spaceship’s mass is included in the force between it and both the Moon and the Earth, and so its value cancels out of the expression.

**Insight**: The distance in part (a) is the same for any mass, and corresponds to about 52 Earth radii or about 86% of the distance \( R \) between the Earth and the Moon. The two forces are equal at \( 3.46 \times 10^8 \text{ m} \) or about 90% of \( R \).

14. **Picture the Problem**: The acceleration of gravity at an altitude \( h \) above the Earth’s surface is reduced due to the increased distance from the center of the Earth.
Strategy: Set the acceleration of gravity at an altitude $h$ equal to one-half the acceleration at $h = 0$ using equation 12-4, and solve for $h$.

Solution: Set $g_h = g_0$ and solve $\frac{GM_E}{(R_E + h)^2} = \frac{1}{2} \left( \frac{GM_E}{R_E^2} \right)$ for $h$:

$$(R_E + h)^2 = 2R_E^2$$

$$h = (\sqrt{2} - 1)R_E = (\sqrt{2} - 1)(6.37 \times 10^6 \text{ m}) = 2.64 \times 10^6 \text{ m}$$

Insight: This altitude is over ten times higher than the 200-km altitude of the International Space Station.

18. **Picture the Problem:** The mass experiences a gravitational attraction to the Earth that depends upon its distance from the center of the Earth.

**Strategy:** Use Newton’s Universal Law of Gravitation (equation 12-1) to find the distance from Earth’s center that would produce the given force (weight) for a 4.0-kg mass. Then use Newton’s Second Law to find the acceleration.

**Solution:**

1. (a) Solve equation 12-1 for $r$:

$$r = \sqrt{\frac{GM_Em}{F}}$$

$$= \sqrt{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right) \left(5.97 \times 10^{24} \text{ kg}\right) \left(4.0 \text{ kg}\right) \over 2.0 \text{ N}} = 2.8 \times 10^7 \text{ m}$$

2. (b) Solve Newton’s Second Law for $a$:

$$a = \frac{F}{m} = \frac{2.0 \text{ N}}{4.0 \text{ kg}} = 0.50 \text{ m/s}^2$$

3. (c) Since the gravitational force is inversely proportional to $r^2$, doubling $r$ reduces $F$ by a factor of 4.

4. (d) Doubling $r$ also reduces the acceleration by a factor of 4 since the force has decreased by that factor and the mass has not changed.

**Insight:** The distance $2.8 \times 10^7 \text{ m}$ is about $4.4R_E$. If the mass were on the Earth’s surface, it would weigh 39 N (8.8 lb).

23. **Picture the Problem:** The Apollo capsule orbited the Moon at an altitude of 110 km above the Moon’s surface.

**Strategy:** Use equation 12-7 to determine the period of orbit using the mass and radius of the Moon from the inside back cover of the text.

**Solution:** Apply equation 12-7:

$$T = \left(\frac{2\pi}{\sqrt{GM_M}}\right)r^{3/2}$$

$$= \left[\frac{2\pi}{\sqrt{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right) \left(7.35 \times 10^{22} \text{ kg}\right)}}\right] \left(1.74 \times 10^9 + 110 \times 10^3 \text{ m}\right)^{3/2}$$

$$T = 7140 \text{ s} = 1.98 \text{ h}$$
Insight: This period turns out to be a bit larger than the 1.44-h orbit period of a satellite that is 110 km above the Earth’s surface.

28. Picture the Problem: The tiny moon Dactyl travels around 243 Ida in an approximately circular orbit.

Strategy: Solve Kepler’s Third Law (equation 12-7) for the mass of 243 Ida, using the orbit distance and period given in the problem.

Solution: 1. (a) Solve Kepler’s Third Law (equation 12-7) for the mass of 243 Ida, using the orbit distance and period given in the problem.

2. (b) Solve equation 12-7 for $M_{243 \text{ Ida}}$:

$$M_{243 \text{ Ida}} = \left(\frac{2\pi}{T}\right)^2 \frac{r^3}{G} = \left(\frac{2\pi}{19 \text{ h} \times 3600 \text{ s/h}}\right)^2 \frac{\left(89 \times 10^3 \text{ m}\right)^3}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = 8.9 \times 10^{16} \text{ kg}$$

Insight: This asteroid is a little bit bigger than Manhattan. The moon Dactyl is only about 1.4 km in diameter.

37. Picture the Problem: The object is located at the surface of the Earth and later at an altitude of 350 km.

Strategy: Use equation 12-8 to find the gravitational potential energy of the object as a function of its distance $r = R_E + h$ from the center of the Earth. Then take the difference between the values at $h = 0$ and $h = 350$ km and compare it with the approximate change in potential, $\Delta U = mgh$.

Solution: 1. (a) Calculate $U = -G \frac{M_E m}{R_E + h}$ at $h = 0$:

$$U_0 = -\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right) \frac{\left(5.97 \times 10^{24} \text{ kg}\right) \left(7.7 \text{ kg}\right)}{6.37 \times 10^6 \text{ m}} = -4.81 \times 10^8 \text{ J}$$

2. (b) Calculate $U$ at $h = 350$ km:

$$U_h = -\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right) \frac{\left(5.97 \times 10^{24} \text{ kg}\right) \left(7.7 \text{ kg}\right)}{6.37 \times 10^6 + 350 \times 10^3 \text{ m}} = -4.56 \times 10^8 \text{ J}$$

3. (c) Take the difference $\Delta U = U_h - U_0 = \left(-4.56 \times 10^8 \text{ J}\right) - \left(-4.81 \times 10^8 \text{ J}\right) = 2.5 \times 10^7 \text{ J}$

4. Compare with $mgh$: $\Delta U = mgh = \left(7.7 \text{ kg}\right) \left(9.81 \text{ m/s}^2\right) \left(350 \times 10^3 \text{ m}\right) = 2.6 \times 10^7 \text{ J}$

Insight: The two calculations of $\Delta U$ differ by about 4%. The $mgh$ calculation is an approximation because it assumes the value of $g$ is constant over the 350 km, when in fact it gets smaller as the distance from the Earth’s center increases.

38. Picture the Problem: The two basketballs are initially touching and are then separated from each other.

Strategy: When the basketballs are touching, their centers are one diameter or 0.24 m apart. Find the difference in gravitational potential energy between when they are touching and when they are separated by using equation 12-9.

Solution: 1. (a) Determine an expression for $\Delta U$:

$$\Delta U = U_2 - U_1 = -G \frac{m^2}{r_2} - \left(-G \frac{m^2}{r_1}\right) = Gm^2 \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
2. Calculate $\Delta U$ for $r_2 = 1.0$ m:

$$\Delta U = \left( \frac{1}{0.24 \text{ m}} - \frac{1}{1.0 \text{ m}} \right) \left( 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \right) (0.59 \text{ kg})^2 = 7.4 \times 10^{-11} \text{ J}$$

3. (b) Calculate $\Delta U$ for $r_2 = 10.0$ m:

$$\Delta U = \left( \frac{1}{0.24 \text{ m}} - \frac{1}{10.0 \text{ m}} \right) \left( 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \right) (0.59 \text{ kg})^2 = 9.4 \times 10^{-11} \text{ J}$$

**Insight:** The work required to separate these basketballs is tiny, only 94 pJ to separate them by 10.0 m. This is equivalent to raising the center of mass of one of the basketballs by 16 pm, less than the 100-pm diameter of a typical atom!

39. **Picture the Problem:** The rocket is given enough kinetic energy to completely escape the Earth or the Moon.

**Strategy:** The rocket completely escapes the Earth when it is infinitely far away, which is when its gravitational potential energy is zero. Set the kinetic energy equal to the magnitude of the initial (negative) gravitational potential energy in order to find the energy needed to escape. Use the radius and mass data for the Earth and Moon given in the inside back cover of the book.

**Solution:**

1. (a) Calculate $K = -U_i = G \frac{M_M m}{R_M}$ for the Moon:

$$K = \left( 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \right) \left( 7.35 \times 10^{22} \text{ kg} \right) \left( 39,000 \text{ kg} \right) \left( \frac{1.74 \times 10^6 \text{ m}}{6.37 \times 10^6 \text{ m}} \right) = 1.1 \times 10^{11} \text{ J}$$

2. (b) Calculate $K = -U_i = G \frac{M_E m}{R_E}$ for the Earth:

$$K = \left( 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \right) \left( 5.97 \times 10^{24} \text{ kg} \right) \left( 39,000 \text{ kg} \right) \left( \frac{6.37 \times 10^6 \text{ m}}{6.37 \times 10^6 \text{ m}} \right) = 2.4 \times 10^{12} \text{ J}$$

**Insight:** It takes 22 times more energy to escape the Earth because of its larger mass. The larger radius of the Earth actually makes it a bit easier to escape, but in the end the fact that the Earth has 81 times more mass makes it harder to escape the Earth than the Moon.

41. **Picture the Problem:** The impact on Mars gives the meteorite enough kinetic energy to escape the planet.

**Strategy:** Use an analog to equation 12-13 to find the escape speed from the surface of Mars. Use the mass and radius information given in Appendix C.

**Solution:**

Apply equation 12-13 for Mars:

$$v_e = \sqrt{ \frac{2GM_M}{R_M} } = \sqrt{ \frac{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(0.108 \times 5.97 \times 10^{24} \text{ kg})}{3.394 \times 10^6 \text{ m}}} = 5030 \text{ m/s} = 5.03 \text{ km/s}$$

**Insight:** The smaller radius of Mars makes it a bit more difficult to escape that planet when compared with the Earth, but its smaller mass more than compensates and its 5.03 km/s escape speed is much smaller than the 11.2 km/s for the Earth.

49. **Picture the Problem:** The planet is ten times more massive and has one-tenth the radius of Earth. A projectile at its surface is given sufficient kinetic energy to escape the planet.

**Strategy:** Use a ratio to compare the escape speed on the new planet with the escape speed on Earth. Use equation
Solution: Write a ratio of the escape speeds:

\[
\frac{v_{e,\text{new}}}{v_{e,\text{Earth}}} = \sqrt{\frac{2GM_{\text{new}}}{R_{\text{new}}}} = \sqrt{\frac{M_{\text{new}} R_{\text{E}}}{M_{\text{E}} R_{\text{new}}}} = \sqrt{\frac{(10M_{\text{E}})R_{\text{E}}}{M_{\text{E}} \left( \frac{1}{10} R_{\text{E}} \right)}} = \sqrt{100} = 10
\]

**Insight:** Although the speed only needs to be increased by a factor of 10, the initial kinetic energy of a projectile needs to be 100 times larger on this new planet in order for the projectile to escape that planet’s gravity.