

## **Study Questions/Problems Week 8**

Chapters 11 Formulates and apply Newton's laws to rotating systems, defines angular momentum, and illustrates how conservation of angular momentum is a powerful problem-solving tool. Again, this chapter covers many aspects of rotational statics and dynamics; hence, another long list of problems.

Chapter 11:

Conceptual Questions 1, 4, 5, 6, 7, 11, 14

Conceptual Exercises 1, 2, 5, 6, 10, 12, 13

Problems 1, 6, 7, 11, 14, 16, 20, 38, 41, 44, 51, 54, 59, 60, 61, 65, 66

Answers/solutions for all even numbered CQs and CEs, and for each of the problems listed above, are on the following pages—don't peek until you have done your best to solve a problem.

## Chapter 11: Rotational Dynamics and Static Equilibrium

### Answers to Even-Numbered Conceptual Questions

2. As a car brakes, the forces responsible for braking are applied at ground level. The center of mass of the car is well above the ground, however. Therefore, the braking forces exert a torque about the center of mass that tends to rotate the front of the car downward. This, in turn, causes an increased upward force to be exerted by the front springs, until the net torque acting on the car returns to zero.
4. The force that accelerates a motorcycle is a forward force applied at ground level. The center of mass of the motorcycle, however, is above the ground. Therefore, the accelerating force exerts a torque on the cycle that tends to rotate the front wheel upward.
6. Consider an airplane propeller or a ceiling fan that is just starting to rotate. In these cases, the net force is zero because the center of mass is not accelerating. However, the net torque is nonzero and the angular acceleration is nonzero.
8. A car accelerating from rest is not in static equilibrium because its center of mass is accelerating. Similarly, an airplane propeller that is just starting up is not in static equilibrium because it has an angular acceleration.
10. Yes. When an airplane's engine starts up from rest the propeller has a nonzero rotational acceleration, though its translational acceleration is zero.
12. The tail rotor on a helicopter has a horizontal axis of rotation, as opposed to the vertical axis of the main rotor. Therefore, the tail rotor produces a horizontal thrust that tends to rotate the helicopter about a vertical axis. As a result, if the angular speed of the main rotor is increased or decreased, the tail rotor can exert an opposing torque that prevents the entire helicopter from rotating in the opposite direction.
14. No. If the diver's initial angular momentum is zero, it must stay zero unless an external torque acts on her. A diver needs to start off with at least a small angular speed, which can then be increased by folding into a tucked position.

### Answers to Even-Numbered Conceptual Exercises

2. The moment of inertia is greatest when more mass is at a greater distance from the axis of rotation, and the greater the moment of inertia the smaller the angular acceleration produced by a given torque. With these observations in mind, we arrive at the following ranking: case C < case B < case A.
4. As the person climbs higher on the ladder, the torque exerted about the base of the ladder increases. To counter this torque, the wall must exert a greater horizontal force, and the floor must exert the same increased horizontal force in the opposite direction. Therefore, the ladder is more likely to slip as the person climbs higher.
6. The answer is position (1), not yet vertical. It might seem that moving the mass to the end of the rod will cause a more rapid rotation, because of the increased torque. On the other hand, moving the mass to the end of the rod increases the moment of inertia, which tends to slow the rotation. The greater effect is the moment of inertia, since it depends on the square of the distance from the axis of rotation, whereas the torque depends only on the first power of the distance. As a result, the rod to the right rotates more slowly, and is not yet vertical.
8. (a) The force exerted by the ground on the rear tire is in the forward direction. It is this force that causes the motorcycle to accelerate. (b) The force exerted by the ground on the front tire is in the backward direction. This force causes the front tire to rotate in such a way that the bottom part of the tire moves in the backward direction and the upper part moves in the forward direction. (c) Increasing the moment of inertia of the front tire makes it harder to rotate—thus, the ground must exert a larger force in the backward direction to cause the rotation. This results in a decrease of the motorcycle's acceleration.

10. As the string is pulled downward it exerts a force on the puck that is directly through the axis of rotation. Therefore, the string exerts zero torque on the puck, which means that the puck's angular momentum will remain constant. **(a)** From  $L = r p = r m v$ , and the fact that  $L$  is a constant, we see that the puck's linear speed must increase as  $(\text{constant})/r$ . **(b)** Similarly, from the relation  $L = I\omega = (mr^2)\omega$ , we see that the puck's angular speed must increase as  $(\text{constant})/r^2$ . **(c)** As mentioned at the outset, the angular momentum will remain the same, since the net torque exerted on the puck is zero.
12. As the beetle begins to walk, it exerts a force and a torque on the turntable. The turntable exerts an equal but opposite force and torque on the beetle. Therefore, the system consisting of the beetle and turntable experiences no net change in its linear or angular momentum. **(a)** If the turntable is much more massive than the beetle, it will barely rotate backward as the beetle moves forward. The beetle, then, will begin to circle around the perimeter of the turntable almost the same as if it were on solid ground. **(b)** If the turntable is virtually massless, on the other hand, it will rotate backward with a linear speed at the rim that is almost equal to the forward linear speed of the beetle. The beetle will progress very slowly relative to the ground in this case—though as far as it is concerned, it is running with its usual speed. In the limit of a massless turntable, the beetle remains in the same spot relative to the ground.
14. If the Earth were to expand, keeping the same mass and the same mass distribution, its moment of inertia would increase. Since the angular momentum of the Earth would stay the same (no external torques), its angular speed would decrease, like a skater extending her arms. As a result, the length of the day would increase.

**See following pages for Problem Solutions**

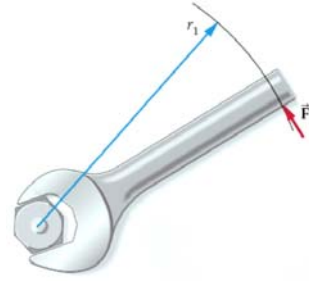
1. **Picture the Problem:** The force is applied in a direction perpendicular to the handle of the wrench and at the end of the handle.

**Strategy:** Use equation 11-1 to find the force from a knowledge of the torque and the length of the wrench.

**Solution:** Solve equation 11-1 for  $F$ :

$$\tau = r(F \sin \theta)$$

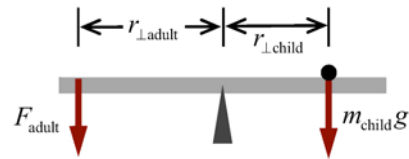
$$F = \frac{\tau}{r \sin \theta} = \frac{15 \text{ N} \cdot \text{m}}{(0.25 \text{ m}) \sin 90^\circ} = \boxed{60 \text{ N}}$$



**Insight:** A longer wrench can exert a larger torque for the same amount of force.

6. **Picture the Problem:** The adult pushes downward on the left side of the teeter-totter and the child sits on the right side as depicted in the figure:

**Strategy:** Calculate the torques exerted by the weight of the child and the force of the parent's hands and sum them. The sign of the net torque indicates the direction in which the teeter-totter will rotate.



**Solution: 1. (a)** Find the torque the child exerts on the teeter-totter.

$$\tau_{\text{child}} = r_{\perp \text{child}} m_{\text{child}} g$$

$$= -(1.5 \text{ m})(16 \text{ kg})(9.81 \text{ m/s}^2)$$

$$\tau_{\text{child}} = \underline{-235 \text{ N} \cdot \text{m}}$$

2. Find the torque exerted by the parent and sum the torques to find the direction of travel:

$$\tau_{\text{adult}} = r_{\perp \text{adult}} F_{\text{adult}} = (3.0 \text{ m})(95 \text{ N}) = \underline{285 \text{ N} \cdot \text{m}}$$

Here  $\tau_{\text{adult}} + \tau_{\text{child}} > 0$  so the teeter-totter will rotate counterclockwise and the child will move up.

3. (b) Repeat step 2 with the new  $r_{\perp}$  for the adult:

$$\tau_{\text{adult}} = r_{\perp \text{adult}} F_{\text{adult}} = (2.5 \text{ m})(95 \text{ N}) = \underline{238 \text{ N} \cdot \text{m}}$$

Here  $\tau_{\text{adult}} + \tau_{\text{child}} > 0$  so the teeter-totter will rotate counterclockwise and the child will move up.

4. (c) Repeat step 2 with the new  $r_{\perp}$  for the adult:

$$\tau_{\text{adult}} = r_{\perp \text{adult}} F_{\text{adult}} = (2.0 \text{ m})(95 \text{ N}) = \underline{190 \text{ N} \cdot \text{m}}$$

Here  $\tau_{\text{adult}} + \tau_{\text{child}} < 0$  so the teeter-totter will rotate clockwise and the child will move down.

**Insight:** The parent would have to exert the 95-N force exactly 2.48 m from the pivot point in order to balance the teeter-totter. We bent the rules for significant figures slightly to more easily compare the magnitudes of the torques.

7. **Picture the Problem:** The torque applied to the bicycle wheel causes it to rotate with constant angular acceleration.

**Strategy:** Calculate the moment of inertia of the wheel using  $I = mr^2$  (table 10-1) and then use Newton's Second Law for rotation (equation 11-4) to determine the angular acceleration.

**Solution:** Solve equation 11-4 for  $\alpha$ :

$$\alpha = \frac{\tau}{I} = \frac{\tau}{mr^2} = \frac{0.97 \text{ N} \cdot \text{m}}{(0.75 \text{ kg})(0.35 \text{ m})^2} = \boxed{11 \text{ rad/s}^2}$$

**Insight:** To exert a 0.97 N·m torque on a 0.35-m wheel you need only apply a tangential force of 2.8 N or 10 ounces.

11. **Picture the Problem:** The wheel rotates about its axis, decreasing its angular speed at a constant rate, and comes to rest.

**Strategy:** Use table 10-1 to find the moment of inertia of a uniform disk and calculate  $I$ . Then use equation 10-11 to find the angular acceleration from the initial angular speed and the angle through which the wheel rotated. Use  $I$  and  $\alpha$  together in equation 11-4 to find the torque exerted on the wheel.

**Solution: 1. (a)** Use table 10-1 to find  $I = \frac{1}{2} MR^2$ : 
$$I = \frac{1}{2} MR^2 = \frac{1}{2}(6.4 \text{ kg})(0.71 \text{ m})^2 = \underline{\underline{1.6 \text{ kg} \cdot \text{m}^2}}$$

2. Solve equation 10-11 for  $\alpha$ : 
$$\alpha = \frac{\omega^2 - \omega_0^2}{2\Delta\theta} = \frac{0^2 - (1.22 \text{ rad/s})^2}{2(0.75 \text{ rev} \times 2\pi \text{ rad/rev})} = \underline{\underline{-0.158 \text{ rad/s}^2}}$$

3. Apply equation 11-4 directly: 
$$\tau = I\alpha = (1.6 \text{ kg} \cdot \text{m}^2)(-0.158 \text{ rad/s}^2) = \underline{\underline{-0.25 \text{ N} \cdot \text{m}}}$$

4. (b) If the mass of the wheel is doubled and its radius is halved, the moment of inertia will be cut in half (doubled because of the mass, cut to a fourth because of the radius). Therefore the magnitude of the angular acceleration will increase if the frictional torque remains the same, and the angle through which the wheel rotates before coming to rest will decrease.

**Insight:** If the moment of inertia is cut in half, the angular acceleration will double to  $-0.32 \text{ rad/s}^2$  and the angle through which the wheel rotates will be cut in half to 0.38 rev. This is because the wheel has less rotational inertia but the frictional torque remains the same. We bent the rules for significant figures in step 2 to avoid rounding error in step 3.

14. **Picture the Problem:** The fish exerts a torque on the fishing reel and it rotates with constant angular acceleration.

**Strategy:** Use table 10-1 to determine the moment of inertia of the fishing reel assuming it is a uniform cylinder ( $\frac{1}{2} MR^2$ ). Find the torque the fish exerts on the reel by using equation 11-1. Then apply Newton's Second Law for rotation (equation 11-4) to find the angular acceleration and equations 10-2 and 10-10 to find the amount of line pulled from the reel.

**Solution: 1. (a)** Use table 10-1 to find  $I$ : 
$$I = \frac{1}{2} MR^2 = \frac{1}{2}(0.84 \text{ kg})(0.055 \text{ m})^2 = \underline{\underline{0.00127 \text{ kg} \cdot \text{m}^2}}$$

2. Apply equation 11-1 directly to find  $\tau$ : 
$$\tau = rF = (0.055 \text{ m})(2.1 \text{ N}) = \underline{\underline{0.12 \text{ N} \cdot \text{m}}}$$

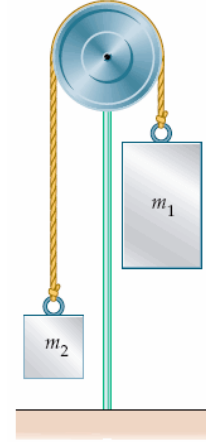
3. Solve equation 11-4 for  $\alpha$ : 
$$\alpha = \frac{\tau}{I} = \frac{0.12 \text{ N} \cdot \text{m}}{0.0013 \text{ kg} \cdot \text{m}^2} = \underline{\underline{92 \text{ rad/s}^2}}$$

4. (b) Apply equations 10-2 and 10-10: 
$$s = r\theta = r\left(\frac{1}{2}\alpha t^2\right) = (0.055 \text{ m})\frac{1}{2}(92 \text{ rad/s}^2)(0.25 \text{ s})^2 = \underline{\underline{0.16 \text{ m}}}$$

**Insight:** This must be a small fish because it is not pulling very hard; 2.1 N is about 0.47 lb or 7.6 ounces of force. Or maybe the fish is tired?

16. **Picture the Problem:** The two masses hang on either side of a pulley.

**Strategy:** Use Newton's Second Law for rotation (equation 11-4) to find the frictional torque  $\tau_{\text{fr}}$  that would make the angular acceleration of the system equal to zero. In each case the torque exerted on the pulley by the hanging masses is the weight of the mass times the radius of the pulley. Let  $m_1 = 0.635 \text{ kg}$  and  $m_2 = 0.301 \text{ kg}$ . The torque due to  $m_1$  is clockwise and therefore taken to be in the negative direction.



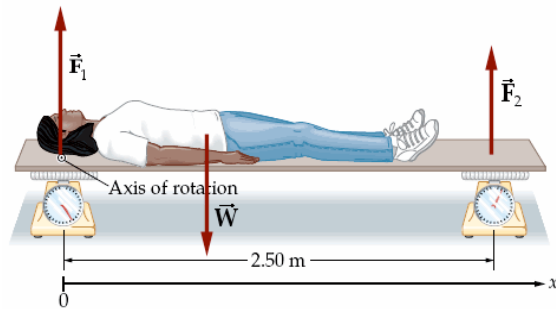
**Solution:** Write Newton's Second Law for rotation and solve for  $\tau_{\text{fr}}$

$$\begin{aligned}\sum \vec{\tau} &= -r(m_1g) + r(m_2g) + \tau_{\text{fr}} = 0 \\ \tau_{\text{fr}} &= rg(m_1 - m_2) \\ &= (0.0950 \text{ m})(9.81 \text{ m/s}^2)(0.635 - 0.301 \text{ kg}) \\ \tau_{\text{fr}} &= \boxed{0.311 \text{ N}\cdot\text{m}}\end{aligned}$$

**Insight:** This frictional torque represents a static friction force. If a little bit of mass were added to  $m_1$ , the system would begin accelerating clockwise and the frictional torque would be reduced to its kinetic value.

20. **Picture the Problem:** The person lies on a lightweight plank that rests on two scales as shown in the diagram at right.

**Strategy:** Write Newton's Second Law in the vertical direction and Newton's Second Law for rotation to obtain two equations with two unknowns,  $m$  and  $x_{\text{cm}}$ . Solve each to find  $m$  and  $x_{\text{cm}}$ . Using the left side of the plank as the origin, there are two torques to consider: the positive torque due to the right hand scale and the negative torque due to the person's mass.



**Solution: 1. (a)** Write Newton's Second Law in the vertical direction to find  $m$ :

$$\begin{aligned}\sum F_y &= F_1 + F_2 - mg = 0 \\ m &= \frac{F_1 + F_2}{g} = \frac{290 + 122 \text{ N}}{9.81 \text{ m/s}^2} = \boxed{42 \text{ kg}}\end{aligned}$$

**2. (b)** Write Newton's Second Law for rotation and solve for  $x_{\text{cm}}$ :

$$\begin{aligned}\sum \tau &= r_2 F_2 - x_{\text{cm}} mg = 0 \\ x_{\text{cm}} &= \frac{x_2 F_2}{mg} = \frac{(2.50 \text{ m})(122 \text{ N})}{(42 \text{ kg})(9.81 \text{ m/s}^2)} = \boxed{0.74 \text{ m}}\end{aligned}$$

**Insight:** The equation in step 1 does not depend on the axis of rotation that we choose, but the equation in step 2 does. Nevertheless, we find exactly the same  $x_{\text{cm}}$  if we choose the other scale, near her feet, to be the axis of rotation.

38. **Picture the Problem:** The necklace hangs on one end of the meter stick and the balance point is found to be 9.5 cm from the 50.0 cm mark.

**Strategy:** Write Newton's Second Law for torque with the necklace at 100 cm and the pivot point at the 59.5-cm mark. Solve the resulting expression for the mass  $m$  of the necklace. Let  $M$  be the mass of the meter stick.

**Solution: 1. (a)** The mass of the necklace is less than the meter stick's, because the moment arm for the necklace

(50.0 – 9.5 cm = 40.5 cm) is greater than the moment arm for the mass of the meter stick (9.5 cm).

2. (b) Set  $\sum \tau = 0$  and solve for  $m$ :  $\sum \tau = d(Mg) - r(mg) = 0$

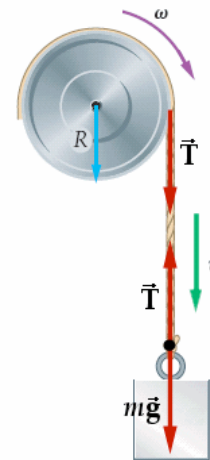
$$m = \frac{dM}{r} = \frac{(9.5 \text{ cm})(0.34 \text{ kg})}{(50.0 - 9.5 \text{ cm})} = \boxed{0.080 \text{ kg}}$$

**Insight:** If the necklace were heavier than the meter stick, the balance point would move more than 25.0 cm from the center.

41. **Picture the Problem:** The mass falls straight down, its speed reduced due to the rotation of the disk. The physical situation is depicted at right.

**Strategy:** Write Newton's Second Law for torque about the axis of the pulley, and Newton's Second Law in the vertical direction for the bucket. Combine those two equations together with the relation,  $a = r\alpha$ , which comes from the fact that the rope does not slip along the rim of the pulley, in order to find the linear acceleration, angular acceleration, and distance traveled in 1.50 s. Let  $m$  = bucket mass,  $M$  = pulley mass,  $R$  = pulley radius,  $T$  = rope tension, and note that for the pulley,

$I = \frac{1}{2}MR^2$ . Let downward be the positive direction for the bucket.



**Solution: 1. (a)** Set  $\sum \tau = I\alpha$   $\sum \tau = rT = I\alpha$

for the pulley and solve for  $T$ :

$$T = \frac{I\alpha}{r} = \frac{\left(\frac{1}{2}MR^2\right)(a/R)}{R} = \underline{\underline{\frac{1}{2}Ma}}$$

2. Set  $\sum F_y = ma_y$  for the bucket and

$$-T + mg = ma$$

substitute the expression

for  $T$  from step 1:

3. Now solve for  $a$ :

$$mg = \left(m + \frac{1}{2}M\right)a$$

$$a = \left(\frac{m}{m + \frac{1}{2}M}\right)g = \left[\frac{2.85 \text{ kg}}{2.85 + \frac{1}{2}(0.742) \text{ kg}}\right](9.81 \text{ m/s}^2) = \boxed{8.68 \text{ m/s}^2}$$

4. (b) Now set  $a = R\alpha$  and solve for  $\alpha$ :

$$\alpha = \frac{a}{R} = \frac{8.68 \text{ m/s}^2}{0.121 \text{ m}} = \boxed{71.7 \text{ rad/s}^2}$$

5. (c) Use equation 4-3(b) with

$$v_{0y} = 0$$

to find  $\Delta y$ .

$$\Delta y = 0 + \frac{1}{2}a_y t^2 = \frac{1}{2}(8.68 \text{ m/s}^2)(1.50 \text{ s})^2 = \boxed{9.77 \text{ m}}$$

**Insight:** The relatively small mass of the pulley (it weighs 1.6 lb) doesn't slow down the heavy (6.3 lb) bucket very much. The bucket accelerates at 0.885g and travels 9.77 m (32.1 ft) in just 1.50 s.

44. **Picture the Problem:** You pull straight downward on a rope that passes over a disk-shaped pulley and then supports a weight on the other side. The force of your pull rotates the pulley and accelerates the mass upward.

**Strategy:** Write Newton's Second Law for the hanging mass and Newton's Second Law for torque about the axis of the pulley, and solve the two expressions for the tension  $T_2$  at the other end of the rope. We are given in the problem that  $T_1 = 25 \text{ N}$ . Let  $m$  be the mass of the pulley,  $r$  be the radius of the pulley, and  $M$  be the hanging mass. For the disk-shaped pulley the moment of inertia is  $I = \frac{1}{2}mr^2$ .

**Solution: 1. (a)** The tension in the rope on the other end of the rope accelerates the hanging mass, but the tension on your side both imparts angular acceleration to the pulley and accelerates the hanging mass. Therefore, your end of the rope has the greater tension.

**2. (b)** As stated in the problem,  $T_1 = \boxed{25 \text{ N}}$ .

**3.** Set  $\sum \vec{F} = m\vec{a}$  for the hanging  $\sum F_y = T_2 - Mg = Ma$   
mass:

**4.** Set  $\sum \tau = I\alpha$  for the pulley:  $\sum \tau = rT_1 - rT_2 = I\alpha = \left(\frac{1}{2}mr^2\right)(a/r) \Rightarrow a = \underline{\underline{2(T_1 - T_2)/m}}$

**5.** Substitute the expression for  $a$   $T_2 - Mg = M\left[2(T_1 - T_2)/m\right]$   
from step 4 into the one from  
step 3, and solve for  $T_2$ :

$$mT_2 - mMg = 2MT_1 - 2MT_2$$

$$T_2 = \frac{M(2T_1 + mg)}{2M + m}$$

$$= \frac{(0.67 \text{ kg})\left[2(25 \text{ N}) + (1.3 \text{ kg})(9.81 \text{ m/s}^2)\right]}{2(0.67 \text{ kg}) + 1.3 \text{ kg}} = \boxed{16 \text{ N}}$$

**Insight:** The net force on the hanging mass is thus  $T_2 - Mg = 16 - 6.6 \text{ N} = 9.4 \text{ N}$ , enough to accelerate it upward at

$14 \text{ m/s}^2$ . The angular acceleration of the pulley is thus  $a/r = (14 \text{ m/s}^2)/(0.075 \text{ m}) = 187 \text{ rad/s}^2$ .

51. **Picture the Problem:** Jogger 1 runs in a straight line at constant speed in the manner indicated by the figure at right.

**Strategy:** Use  $p = mv$  (equation 9-1) and  $L = rmv$  (equation 11-12) to find the linear and angular momenta, respectively.

**Solution: 1. (a)** Apply equation 9-1 directly:

$$p = mv = (65.3 \text{ kg})(3.35 \text{ m/s})$$

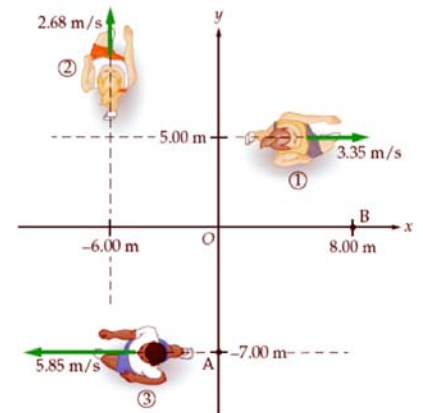
$$= \boxed{219 \text{ kg} \cdot \text{m/s}}$$

**2. (b)** Apply equation 11-12 directly:

$$L = r_{\perp}mv$$

$$= (5.00 \text{ m})(63.5 \text{ kg})(3.35 \text{ m/s})$$

$$L = \boxed{1.09 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{s}}$$



**Insight:** Note that the angular momentum of the jogger depends upon the location of the origin of the coordinate system.

54. **Picture the Problem:** The egg beater rotates about its axis with constant angular acceleration due to the applied torque.



**Strategy:** Use equation 11-14 to find the change in angular momentum due to the applied torque. Then use equation 11-11 to find the angular speed of the egg beater.

**Solution: 1. (a)** Solve equation 11-14 for  $\Delta L$ :  $\Delta L = \tau \Delta t = (0.12 \text{ N} \cdot \text{m})(0.50 \text{ s}) = \boxed{0.060 \text{ kg} \cdot \text{m}^2/\text{s}}$

**2. (b)** Solve equation 11-11 for  $\omega$ :  $\omega = \frac{L}{I} = \frac{0.060 \text{ kg} \cdot \text{m}^2/\text{s}}{2.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2} = \boxed{24 \text{ rad/s}}$

**Insight:** As long as the torque is applied to the egg beater, its angular speed and angular momentum will increase linearly with time.

59. **Picture the Problem:** In midair the diver pulls her arms and legs in, decreasing her moment of inertia and increasing her angular speed.

**Strategy:** The angular momentum of the diver remains the same throughout the dive because there is assumed to be no torque of any kind acting on her body. Use the conservation of angular momentum (equation 11-15) together with equation 11-11, to find the ratio  $\omega_f/\omega_i$ .

**Solution:** Set  $L_i = L_f$  and solve for  $\omega_f/\omega_i$ :  $I_i \omega_i = I_f \omega_f \Rightarrow \frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{I_i}{\frac{1}{2} I_i} = \boxed{2}$  Her angular speed doubles.

**Insight:** Her angular momentum actually decreases ever so slightly due to the effect of air friction.

60. **Picture the Problem:** In midair the diver pulls her arms and legs in, decreasing her moment of inertia and increasing her angular speed.

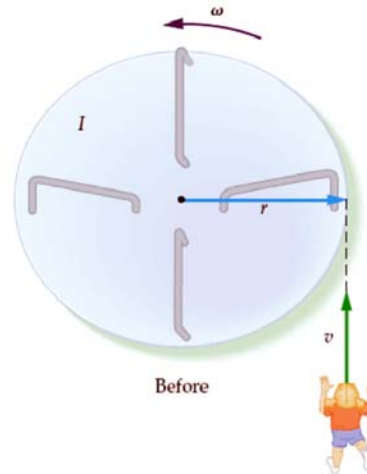
**Strategy:** Find the ratio of  $K_f/K_i$  by using equation 10-17, the given ratio  $I_f = \frac{1}{2} I_i$ , and the result of the previous problem,  $\omega_f = 2\omega_i$ .

**Solution: 1. (a)** The diver's kinetic energy will increase, because the diver does work in going into her tuck.

**2. (b)** Use  $K = \frac{1}{2} I \omega^2$  to find  $K_f/K_i$ :  $\frac{K_f}{K_i} = \frac{\frac{1}{2} I_f \omega_f^2}{\frac{1}{2} I_i \omega_i^2} = \frac{\left(\frac{1}{2} I_i\right) (2\omega_i)^2}{I_i \omega_i^2} = \boxed{2}$  Her kinetic energy doubles.

**Insight:** One way to picture the work the diver must do is to realize that in her frame of reference, centrifugal force tries to push her arms and legs outwards, away from the axis of rotation. She must do work against that force to enter the tuck position, and the work her muscles do increases her kinetic energy.

61. **Picture the Problem:** The person runs tangentially to the rotating merry-go-round and hops on.



**Strategy:** Use conservation of angular momentum because there is no net torque on the system as long as the system includes both the person and the merry-go-round. Find the moments of inertia of the disk-shaped merry-go-round,

$$I_{\text{mgr}} = \frac{1}{2} M r^2, \text{ and the system after the person hops on}$$

$I_f = \frac{1}{2} M r^2 + m r^2$ , where  $M$  is the mass of the merry-go-round,  $m$  is the mass of the person, and  $r$  is the radius of the merry-go-round. Set  $L_i = L_f$  and solve for the final angular speed  $\omega_f$ , where the initial angular speed

$$\text{is: } \omega_i = (0.641 \text{ rev/s})(2\pi \text{ rad/rev}) = 4.03 \text{ rad/s.}$$

**Solution:** Set  $L_i = L_f$  and  $L_{\text{disk}} + L_{\text{person}} = L_{\text{final}}$

solve for  $\omega_f$ :

$$\begin{aligned} \left(\frac{1}{2} M r^2\right)\omega_i + m v r &= \left(\frac{1}{2} M r^2 + m r^2\right)\omega_f \\ \omega_f &= \frac{\frac{1}{2} M r^2 \omega_i + m v r}{\frac{1}{2} M r^2 + m r^2} = \frac{M r \omega_i + 2 m v}{M r + 2 m r} \\ &= \frac{(155 \text{ kg})(2.63 \text{ m})(4.03 \text{ rad/s}) + 2(59.4 \text{ kg})(3.41 \text{ m/s})}{(155 \text{ kg})(2.63 \text{ m}) + 2(59.4 \text{ kg})(2.63 \text{ m})} \\ \omega_f &= \boxed{2.84 \text{ rad/s}} \end{aligned}$$

**Insight:** The merry-go-round has slowed down because the initial linear speed of the person (3.41 m/s) is less than the initial linear speed of the rim of the merry-go-round (10.6 m/s).

65. **Picture the Problem:** The mouse on the freely rotating turntable walks to the rotation axis.

**Strategy:** The moment of inertia of the turntable-mouse system will decrease as the mouse walks toward the axis, but the angular momentum of the system will remain the same because there is no external torque. Use conservation of angular momentum together with equation 11-11 to find the final angular speed of the system. The initial angular speed is

$$\omega_i = \left(33\frac{1}{3} \text{ rev/min}\right)(2\pi \text{ rad/rev})\left(1 \text{ min}/60 \text{ s}\right) = 3.49 \text{ rad/s.}$$

**Solution: 1. (a)** Angular momentum is conserved as the moment of inertia decreases, so the turntable rotates **faster** according to the equation,  $I_i \omega_i = I_f \omega_f$ .

**2. (b)** Set  $L_i = L_f$  and  $I_i \omega_i = I_f \omega_f$

solve for  $\omega_f$ :

$$\begin{aligned} \omega_f &= \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{I + m r^2}{I + 0}\right)\omega_i \\ &= \frac{5.4 \times 10^{-3} \text{ kg} \cdot \text{m}^2 + (0.036 \text{ kg})(0.15 \text{ m})^2}{5.4 \times 10^{-3} \text{ kg} \cdot \text{m}^2} (3.49 \text{ rad/s}) = \boxed{4.0 \text{ rad/s}} \end{aligned}$$

**Insight:** A heavier mouse would have an even larger effect upon the final angular speed because it would create a larger change in the moment of inertia.

66. **Picture the Problem:** The student rotates freely on the piano stool with outstretched arms holding the masses away from the axis of rotation, and then pulls the masses inward toward his body.

**Strategy:** The moment of inertia of the student-masses system will decrease as he brings the masses inward toward the rotation axis, but the angular momentum of the system will remain the same because there is no external torque. Use conservation of angular momentum together with equations 10-18 and 11-11 to find the final distances of the masses from the rotation axis. Then use equation 10-17 to find the initial and final kinetic energies of the system.

**Solution: 1. (a)** Find  $I_i$  for the system using equation 10-18:

$$I_i = I_{s-s} + 2mr_i^2$$

$$= 5.43 \text{ kg} \cdot \text{m}^2 + 2(1.25 \text{ kg})(0.759 \text{ m})^2 = \underline{\underline{6.87 \text{ kg} \cdot \text{m}^2}}$$

2. Set  $L_i = L_f$  and solve for  $I_f$ :

$$I_i \omega_i = I_f \omega_f$$

$$I_f = I_i \left( \frac{\omega_i}{\omega_f} \right) = (6.87 \text{ kg} \cdot \text{m}^2) \left( \frac{2.95 \text{ rev/s}}{3.54 \text{ rev/s}} \right) = \underline{\underline{5.73 \text{ kg} \cdot \text{m}^2}}$$

3. Use equation 10-18 to find  $r_f$ :

$$I_f = I_{s-s} + 2mr_f^2$$

$$r_f = \sqrt{\frac{I_f - I_{s-s}}{2m}} = \sqrt{\frac{5.73 - 5.43 \text{ kg} \cdot \text{m}^2}{2(1.25 \text{ kg})}} = \underline{\underline{0.35 \text{ m}}}$$

4. (b) Use equation 10-17 to find  $K_i$ :

$$\frac{1}{2} I_i \omega_i^2 = \frac{1}{2} (6.87 \text{ kg} \cdot \text{m}^2) (2.95 \text{ rev/s} \times 2\pi \text{ rad/rev})^2 = \underline{\underline{1.18 \text{ kJ}}}$$

5. Use equation 10-17 to find  $K_f$ :

$$\frac{1}{2} I_f \omega_f^2 = \frac{1}{2} (5.73 \text{ kg} \cdot \text{m}^2) (3.54 \text{ rev/s} \times 2\pi \text{ rad/rev})^2 = \underline{\underline{1.42 \text{ kJ}}}$$

**Insight:** The kinetic energy increases because the student must do work to pull in the masses against the centrifugal force. This student is probably pretty dizzy after rotating 3 times per second! The required centripetal force on the masses at the original rotation rate and distance is 26.6 times the weight of the masses. That means the 2.75 lb exercise weights now feel like 73 lbs each!