Physics 114A
Introduction to Mechanics

(without calculus)

A course about learning basic physics concepts and applying them to solve real-world, quantitative, mechanical problems

Lecture 9

Projectile and Relative Motion Review

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Projectile Equations of Motion:

(scalar equations for vector components)

$$x(t) = x_0 + v_{0x}t \qquad v_x = \text{const.}$$

$$y(t) = y_0 + v_{0y}t + \tfrac{1}{2} a_y t^2$$

$$v_y(t) = v_{0y} + a_y t$$

$$v_y^2(\Delta y) = v_{0y}^2 + 2 a_y \Delta y$$

Where: $(a_y = -g = -9.8 \text{ m/s}^2)$

$v_{0x} = v_0 \cos\theta$, $v_{0y} = v_0 \sin\theta$

and $\boldsymbol{\theta} \equiv$ Launch Angle w/ Horizontal

(Note: Textbook substitutes -g for a_y)

Horizontal Range, $R = (v_0^2/a_y) \sin 2\theta$ (only if $y_f = y_i$)

Maximum Height, $y_{max} = v_0^2 \sin^2\theta/(2a_y)$

Q 1: What launch angle gives the Maximum Range for a projectile of initial speed 3 m/s ?

A. 19° B. 27° C. 33° D. 38° E. 42° F. 45° G. 49°

From $R = (v_0^2/a_y) \sin 2\theta$ (and $y_f = y_i$), we see that the dependence of range R on launch angle is through $\sin 2\theta$. Since $\sin\theta$ has a maximum at $\theta = 90°$, it follows that $\sin 2\theta$ has a maximum at $\theta = 45°$. The information regarding initial speed is superfluous.

Q 2: A player throws a 50 m pass that travels for 2.8 s. It is launched and caught at the same height above ground. At what angle must the ball be thrown?

A. 19° B. 27° C. 33° D. 38° E. 42° F. 45° G. 49°

The launch angle is determined by the direction of the initial velocity, $\mathbf{v}_0$, and can be expressed as $\theta = \tan^{-1}(v_{0y}/v_{0x})$. So the task is then to solve for v_{0y} and v_{0x} from the information provided.

We can immediately write v_{0x} = range / time = 50m / 2.8s = 17.9s. Regarding the vertical component of the ball's motion, we know that $v_y(t)= v_{0y} - gt$, and by the symmetry of the trajectory, that the ball's maximum altitude, $v_{ymax}(t')$, occurs at the flight mid-point in both x(t) and t, so t' = 2.8s/2 = 1.4s. That is, the ball rises for 1.4s and falls for 1.4s. A projectile rising to rest ($v_{ymax}(t') = 0$) in 1.4s must have v_{0y} given by:

$$v_{0y} - gt' = v_y(t')= 0 \text{ or, } v_{0y} = gt' = (9.8\text{m/s2})(1.4\text{s}) = 13.7\text{s}.$$

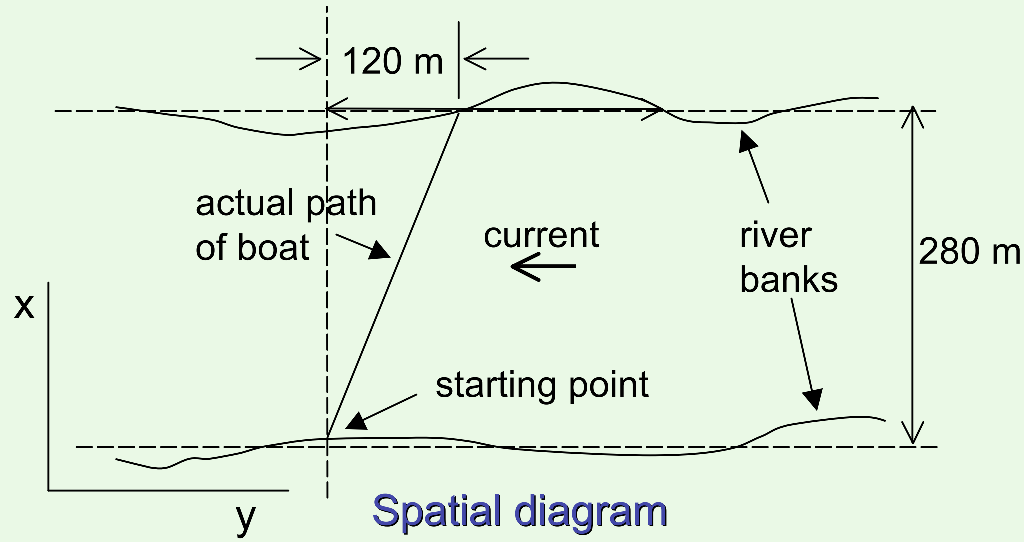
Finally, $\theta = \tan^{-1}(v_{0y}/v_{0x}) = \tan^{-1}(13.7\text{s}/17.9\text{s}) = 38°$ (approximately)

Q 3. A boat, whose speed in still water is 2.40 m/s, must cross a 280 m -wide river and arrive at a point 120 m upstream from where it starts. To do this, the boat must be headed at a 45.0° upstream angle. What is the speed of the current?

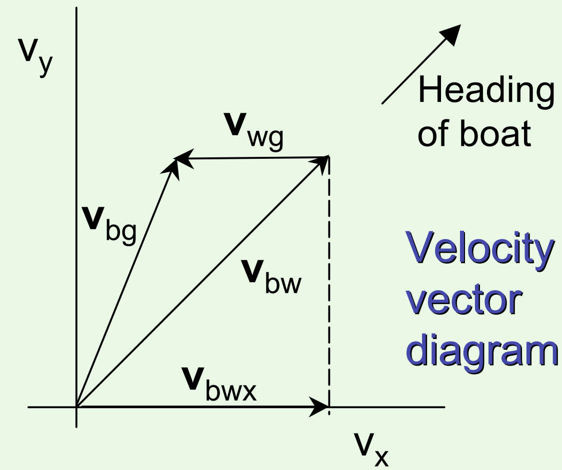
A. 1.83 m/s   B. 1.46 m/s   C. 1.19 m/s   D. 0.970 m/s   E. 0.742 m/s

For relative-motion problems, we begin as usual with a vector diagram of the addition of the velocity of an object moving in reference frame #1, to the velocity of that frame as seen from reference frame #2, thereby arriving at the velocity of the object as observed in frame #2.

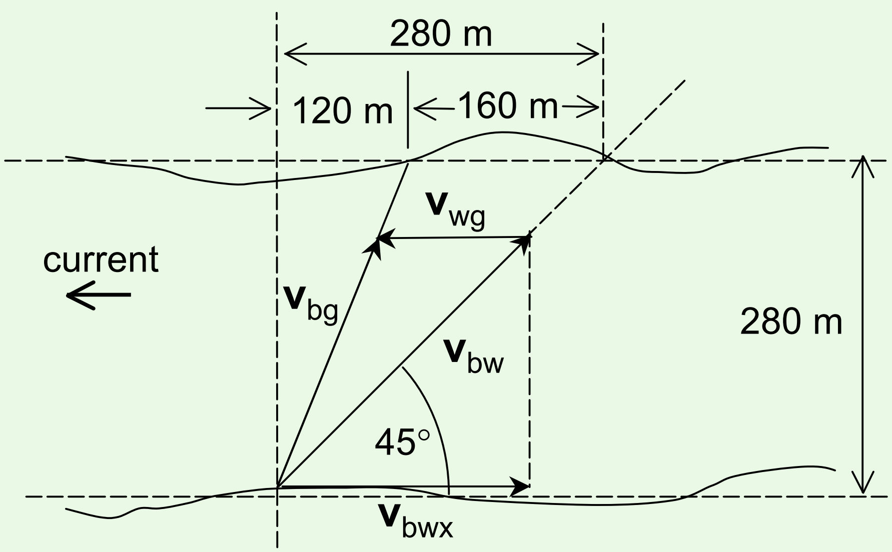
As shown in the next slide, the velocity of boat in still water, $\mathbf{v}_{bw}$, added to the velocity of the water relative to the ground, $\mathbf{v}_{wg}$, yields the velocity of the boat relative to the ground, $\mathbf{v}_{bg}$. As we have noted before, the diagram of this vector sum in *velocity space* is geometrically similar to the *spatial* diagram showing the river, the actual trajectory of the boat, and a line indicating the boat's heading. We exploit this similarity to arrive at a value for the missing magnitude of the river current (also in next slide).



Spatial diagram



Velocity vector diagram



Superposition of Velocity Vector Diagram and Spatial Diagram

We are given the directions of $\mathbf{v}_{wg}$ and $\mathbf{v}_{bg}$, plus the direction and magnitude of $\mathbf{v}_{bw}$, ($|\mathbf{v}_{bw}| = 2.4$ m/s). This is sufficient information to determine the magnitudes of both $\mathbf{v}_{wg}$ and $\mathbf{v}_{bg}$. By superposing the two diagrams (at left), $|\mathbf{v}_{wg}|$ may be determined without resorting to trigonometry. Because of the addition information about the spatial configuration of the river and course of the boat, we can simply write:

$|\mathbf{v}_{wg}| / |\mathbf{v}_{bwx}| = 160 \text{ m} / 280 \text{ m}$

or $|\mathbf{v}_{wg}| = [|\mathbf{v}_{bw}|\cos(45°)][160/280]$

which gives the speed of the current

$|\mathbf{v}_{wg}| = 0.97$ m/s