Physics 114A
Introduction to Mechanics

(without calculus)

A course about learning basic physics concepts and applying them to solve real-world, quantitative, mechanical problems

Lecture 8

More Kinematics in 2-D

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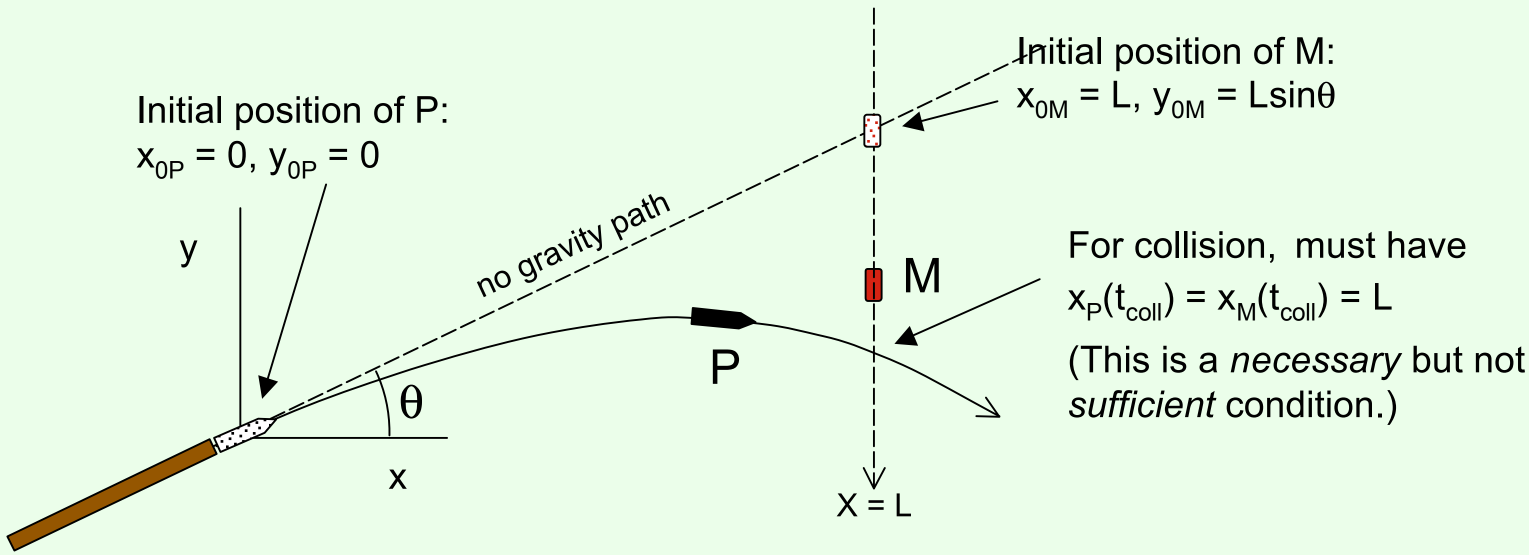
Analysis of Today's Demo:

What conditions must be met to ensure collision?



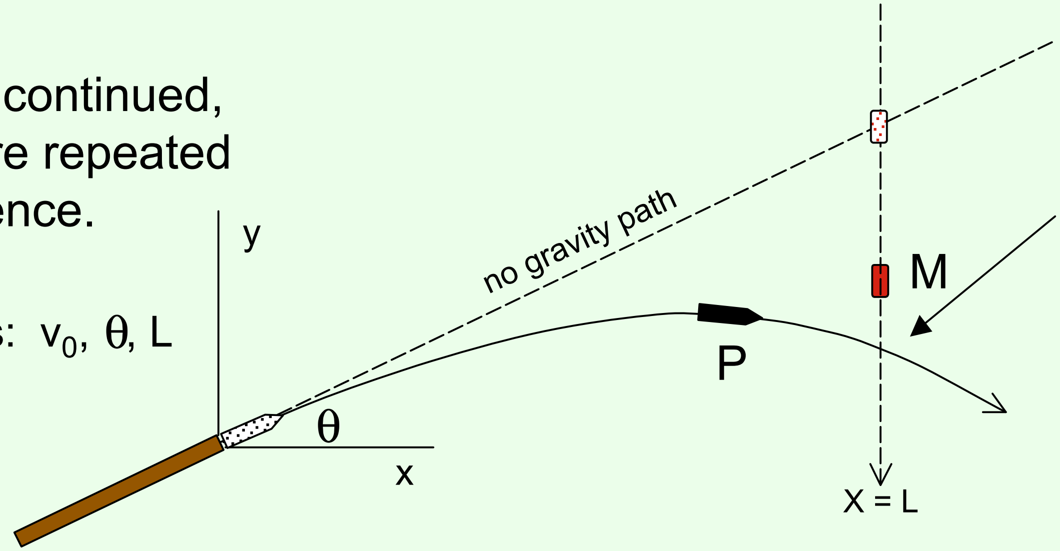
Projectile (P) leaves barrel of gun and Mark (M) is released simultaneously at t = 0. For a collision to occur, it must be true that $y_P(t_{coll}) = y_M(t_{coll})$, as well as $x_P(t_{coll}) = x_M(t_{coll}) = L$.

The question at hand is: How does one meet the second condition* by choosing appropriate values for the only adjustable variables in this arrangement? These variables (aka parameters) are: v_0, θ, L (they determine the kinematics and the geometry).

*The first condition is trivial

Analysis continued, with figure repeated for reference.

Variables: v_0, θ, L



For collision, must have:

1. $x_P(t_{coll}) = x_M(t_{coll}) = L$

2. $y_P(t_{coll}) = y_M(t_{coll})$

How arrange for the second to hold?

Remember the initial position of P: $x_{0P} = 0$, $y_{0P} = 0$

Equations of motion: For P $\quad y_p(t) = v_{0yP}t - (g/2)t^2$, and $x_P(t) = v_{0xP}t$

For M $\quad y_M(t) = y_{0M} - (g/2)t^2$, and $x_M(t) = L$

First, combining the $x_P(t)$ equation above with $x_P(t_{coll}) = L$, we have $t_{coll} = L/v_{0xP}$, so

$v_{0yP}t_{coll} = v_{0yP}(L/v_{0xP})$ in P eq. above. We also can write: $y_{0M} = L\tan\theta = L(v_{0yP}/v_{0xP})$

$$\boxed{\text{Finally, } y_p(t_{coll}) = v_{0yP}(L/v_{0xP}) - (g/2)t_{coll}^2 \text{ , but } y_M(t_{coll}) = L(v_{0yP}/v_{0xP}) - (g/2)t_{coll}^2}$$

Note that the right-hand sides of these two equations are identical. Thus $y_P(t_{coll}) = y_M(t_{coll})$ for almost any values of v_0, θ, L (just need $v_0 > 0$, $\theta < 90°$). In the demo, I had to blow only hard enough to get x_P past L.

Collision is inevitable!

Viewed intuitively, this result makes sense.

If gravity were turned off, the projectile would follow the "no gravity path" and collide with the mark, which would not fall from its initial position.

That is just the point, with no gravity neither projectile nor mark would fall, and the two would clearly collide. When we turn gravity on, both projectile and mark fall exactly the same distance in the same time; so they still collide, just at a lower altitude. With this setup, one can't miss.

Example problem/solution: Chapter 4

36. A passenger on the Ferris wheel described in Problem 18 drops his keys when he is on the way up and at the 10 o'clock position. Where do the keys land relative to the base of the ride?

18. Fairgoers ride a Ferris wheel with a radius of 5.00 m (**Figure 4–19**). The wheel completes one revolution every 32.0 s. **(a)** What is the average speed of a rider on this Ferris wheel? **(b)** If a rider accidentally drops a stuffed animal at the top of the wheel, where does it land relative to the base of the ride? (Note: The bottom of the wheel is 1.75 m above the ground.)

Picture the Problem: The trajectory of the keys is depicted in the first figure. The second figure shows the geometry of the initial position and velocity of the keys.

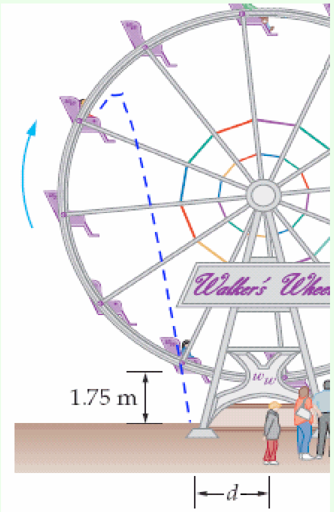


Figure (a): Trajectory of the keys.

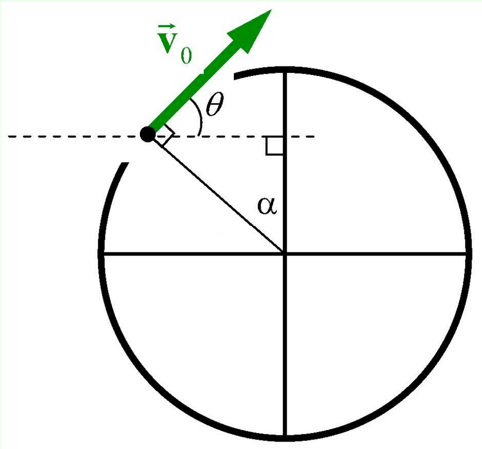


Figure (b): Vector diagram of the release point.

Strategy: Find the initial speed of the keys from the motion of the Ferris wheel. Then use geometry to find the position of the keys when they are released. Find the initial horizontal and vertical components of the key's velocity upon release. Then use equations 4-10 to find the vertical speed of the keys just before they hit the ground. Use the initial and final vertical speeds to find the time of flight, and use the time of flight together with the horizontal velocity of the keys to determine the impact location.

Solution: 1. Find the speed of the keys by finding the speed of the rim of the Ferris wheel.

$$v = \frac{C}{t} = \frac{2\pi r}{t} = \frac{2\pi(5.00\ \text{m})}{32.0\ \text{s}} = \underline{\underline{0.982\ \text{m/s}}}$$

2. Each hour on the clock is 360°/12=30° apart. Therefore the initial position vector of the keys, relative to the center of the wheel, is $\alpha = 60°$ counterclockwise from the vertical position.
The x and y initial positions are:

$$x_0 = -r\sin\alpha = -(5.00\ \text{m})\sin 60° = \underline{\underline{-4.33\ \text{m}}}\ \text{(left of center)}$$

$$y_0 = r\cos\alpha + r + b = (5.00\ \text{m})\cos 60° + 5.00\ \text{m} + 1.75\ \text{m}$$

$$y_0 = \underline{\underline{9.25\ \text{m}}}\ \text{(above the ground)}$$

3. Now find the x and y components of $\vec{\mathbf{v}}_0$ by realizing from the diagram that $\theta = \alpha = 60°$

$$v_{0x} = v_0\cos\theta = (0.982\ \text{m/s})\cos 60° = \underline{\underline{0.491\ \text{m/s}}}$$

$$v_{0y} = v_0\sin\theta = (0.982\ \text{m/s})\sin 60° = \underline{\underline{0.850\ \text{m/s}}}$$

4. Use equations 4-10 to find the final vertical speed:

$$v_y = \pm\sqrt{v_{0y}^2 - 2g\Delta y} = \pm\sqrt{(0.850\ \text{m/s})^2 - 2(9.81\ \text{m/s}^2)(-9.25\ \text{m})}$$

$$= \underline{\underline{-13.5\ \text{m/s}}}\ \ (\text{the keys are traveling downwards})$$

5. Find the time of flight from the initial and final vertical velocities:

$$t = \frac{v_y - v_{0y}}{-g} = \frac{-13.5 - 0.850\ \text{m/s}}{-9.81\ \text{m/s}^2} = \underline{\underline{1.46\ \text{s}}}$$

6. Use the time of flight to find the horizontal position upon landing:

$$x = x_0 + v_{0x}t = -4.33\ \text{m} + (0.491\ \text{m/s})(1.46\ \text{s}) = \boxed{-3.61\ \text{m}}$$, or the keys land 3.61 m left of the base of the Ferris wheel.

Insight: The time of flight can also be found from $y = (v_0\sin\theta)t - \frac{1}{2}gt^2$. Although such an approach would not need step 4 above, it would require the use of the quadratic formula.