Physics 114A
Introduction to Mechanics
(without calculus)

A course about learning basic physics concepts and applying them to solve real-world, quantitative, mechanical problems

Lecture 34

Einstein's Gravity, Black Holes, and Neutron Stars

March 13, 2008

Einstein's Principle of Equivalence—

A 20th c. rethinking of how to view gravity

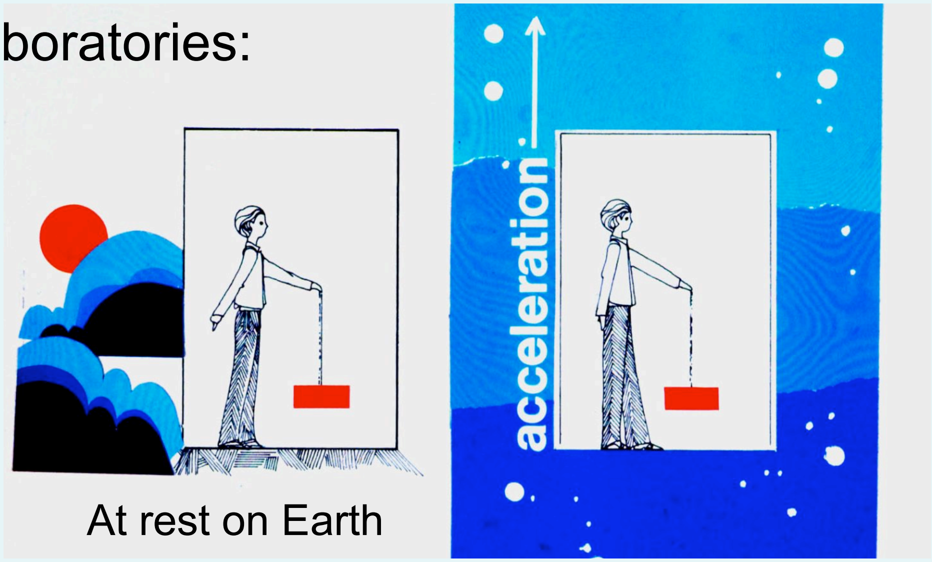
"One cannot, through local measurements, distinguish between effects arising from the presence of mass (gravitational force) and the effects of being in an accelerated frame of reference (inertial force)."

Consider two small, "enclosed" laboratories:

Let brick fall in each, motion is precisely the same according to the *Principle of Equivalence of gravity and inertia*.

Experimenter cannot discern in which lab it is working.

Cannot discern by any possible measurement carried out within the lab—this is a "principle."

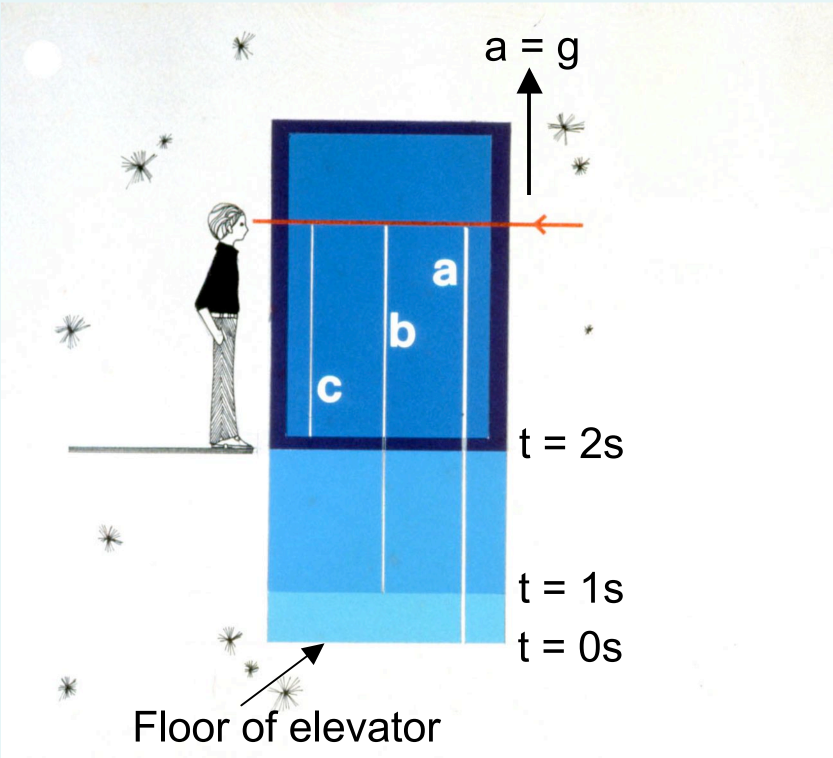


At rest on Earth

Accelerated in outer space a = g

What are the implications of this principle?

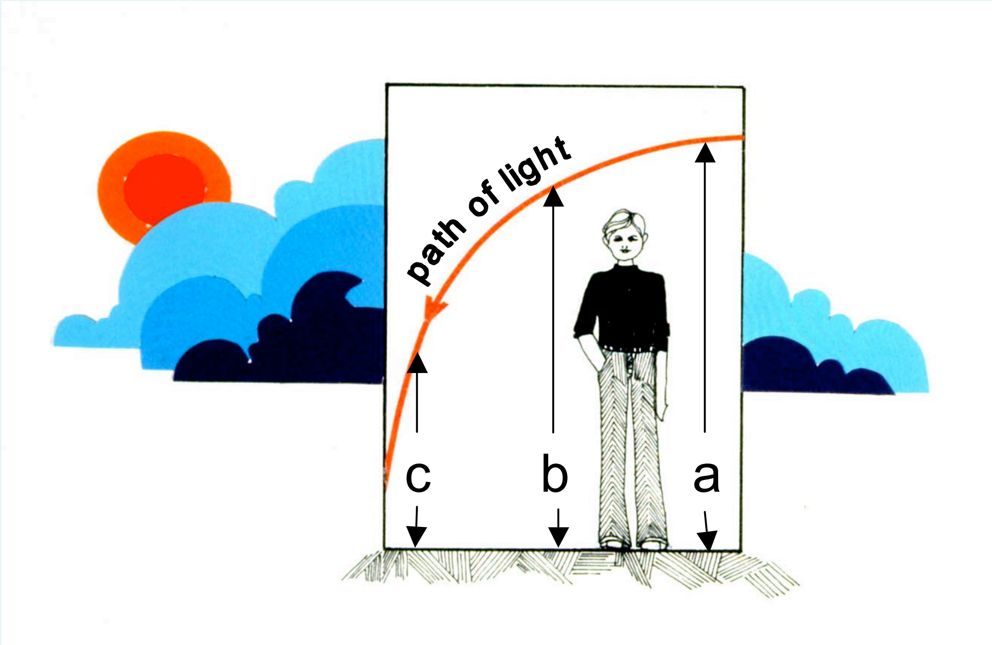
- Can produce the effect of gravity or transform it away with an accelerated frame of reference
- Light travels in straight lines—except when path lies near mass



First, consider a specially constructed, accelerated lab in outer space (a = g). The non-accelerated external observer sees light beam from distant source traveling in straight line.

Occupant of lab sees beam of light enter high above floor and exit nearer floor. Path of light is manifestly not a straight line for the accelerated observer, who sees the light beam to be "bent."

By the <u>Principle if Equivalence</u>, the **same** phenomenon must be observed in *earth-bound* lab!



Gravity places the observer in an accelerated frame of reference

Is path of light bent by gravity?

Einstein says “Yes!”

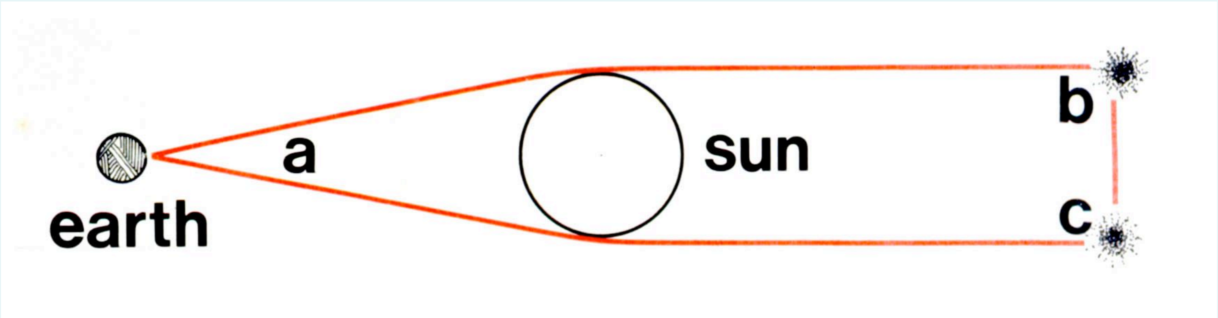
Logical steps

1. Light travels along a *geodesic* path—the “straightest”* line between points
2. Path of light is observed to be curved in an accelerated frame of reference
3. Accelerated frame is equivalent to gravity frame—the Principle of Equivalence
4. Therefore, path of light is curved by gravity
5. Finally, the <u>space</u> through which light travels must be <u>curved by the presence of mass</u>

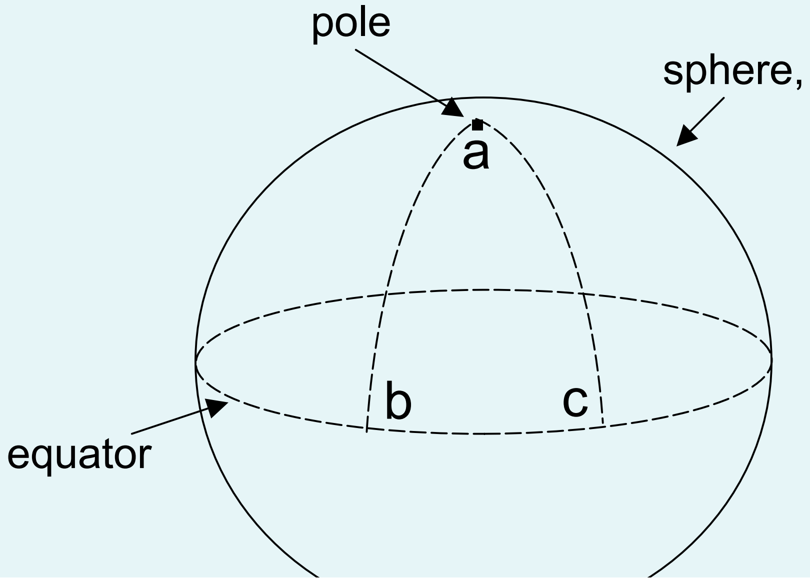
*shortest or longest

Why does it follow that the space through which light travels must be curved by the presence of mass?

Consider path of light near sun



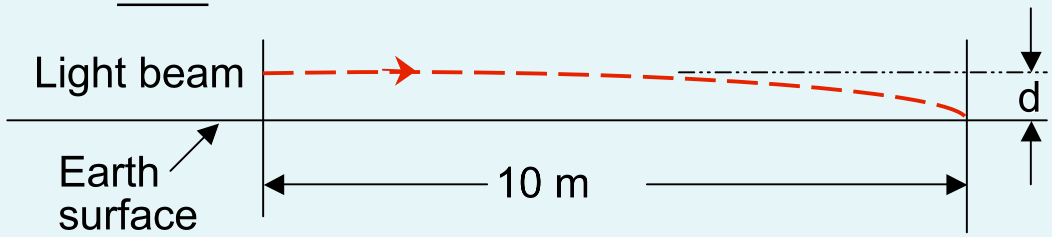
Reminds us of triangle drawn on a curved surface



Sum of interior angles is greater than 180º, surface of positive curvature

Viewed Einstein's way—light falls at surface of Earth just like a brick does (the Earth rises up to meet it!)

This means curvature of light path must be very small because light travels so fast. What does this mean?



Consider how far light falls over a path 10m long. First: How long a time does it have to fall? Let Newton/Galileo guide the way:

Distance = rate x time ($d = vt$), or $t = d/v$ But $v = c = 3 \times 10^8$ m/s

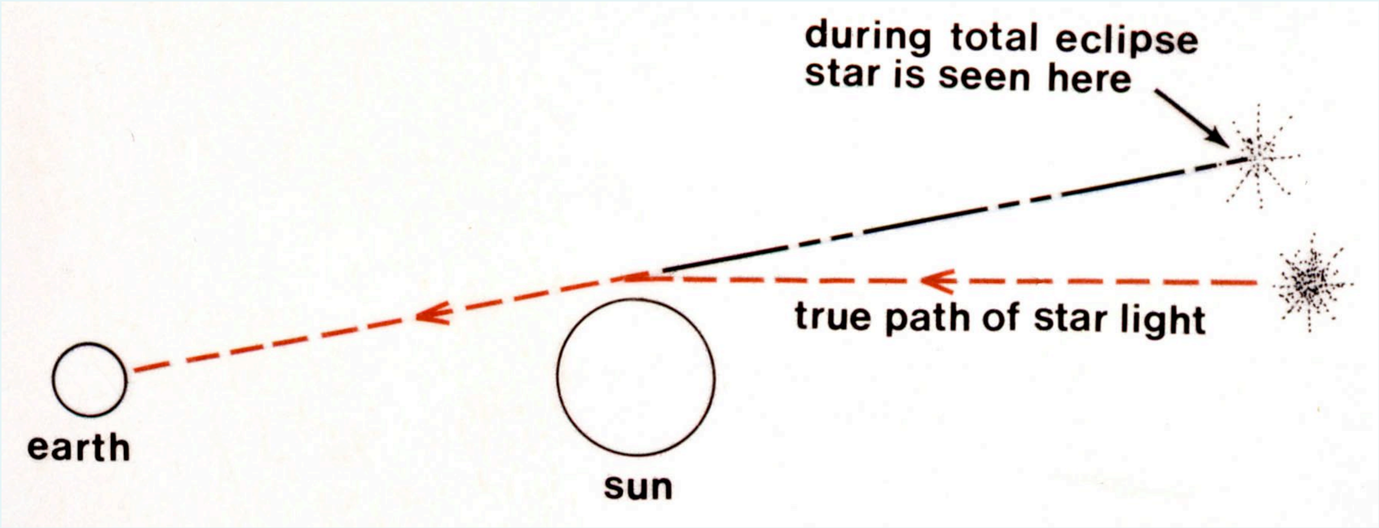
So, $t = 10\text{m}/3\text{x}10^8\text{m/s} = 3.3\text{x}10^{-8}\text{s}$ (33 nanoseconds)

How far fall in 33ns?

$$d = 1/2\, gt^2 = 1/2\,(10\ \text{m/s}^2)\,(3.3\text{x}10^{-8})^2 = 1.7 \times 10^{-15}\text{m}$$

This is roughly the diameter of a proton (100,000 times smaller than an atom— incredibly small!)

Based on his 1915 General Theory of Relativity (GTR) Einstein predicted that this effect would deflect light by 1.75 arc seconds upon passing close to surface of sun



Claimed to be confirmed (was big deal!) in 1919
(established to 10% In 1970s)

So we argue that mass effectively curves space—
why is that interesting or useful?

For Newton, there were two kinds of motion under influence of gravity force:

$F_g = 0$ straight-line motion, constant speed
$F_g \neq 0$ may be curved path, varying speed

For Einstein—only ***one*** kind of motion (for $F = 0$):

Motion along a straightest line (geodesic) in a curved space (when no "*non-gravity* forces" are acting). *There is no gravity force!*

This is Einstein's description of motion (under influence of "gravity")—*freely falling* bodies or light beams move along geodesic paths in a space curved by the presence of spectator masses

No action-at-a-distance (no "occult" force)

This point of view leads to a description of motion under influence of gravity alone:

- Mass curves space (that curvature manifests what Newton called “gravity”)
- In this curved space, in the absence of forces (gravity is not a force now), *bodies and light* follow geodesic paths (straightest lines between two points)
- For technical reasons, all this works only if Einstein’s formulation of gravity is viewed in terms of motion taking place in a 4-D *space-time* formed from 3 spatial dimensions combined with one time dimension.

Einstein's 4-D representation provides consistent picture

"Gravity" is a manifestation of space-time curved by the presence of mass, and motion under gravity is "freely falling" motion along geodesics in that curved space-time

familiar, tiny light deflection → **Weak gravity — strong gravity** ← unfamiliar, large light deflection

For large deflection of light need frame with large acceleration

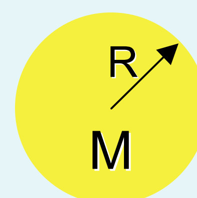
Consider Newtonian "approximation" to calculate acceleration

$a = F_g/m = \cancel{m}MG/\cancel{m}R^2 = MG/R^2$

For Sun: $M = 2 \times 10^{30}$ kg,

$R = 7 \times 10^5$ km

a (at surface) = 27 g $\cong$ 270 m/s^2



Sun

This gives only a tiny effect—need big M and small R

➔ *compact object* ⇐

If could squeeze Sun down to fit into 7 km radius, would increase acceleration by a factor of 10^{10} at surface of this compact object.

[Because $a \propto R^{-2}$, $\dfrac{a_{co}}{a_{Sun}} = \dfrac{R^2_{sun}}{R^2_{co}} = \dfrac{(7 \times 10^5)^2}{(7)^2} = (10^5)^2 = 10^{10}$]

So, $a_{co} \cong 3 \times 10^{11} g$ at its surface!

Now get *substantial* deflection of light—explore “strong” gravity

Revisit mountaintop-cannon experiment:

For cannonball—we let radius be fixed, <u>determined v</u> of circular orbit

For light beam—now let v = c (fixed the speed), <u>find radius</u> of orbit

(Take previous Newtonian approach just substitute c for v —
will not give accurate result, but “relativistic correction” is not huge)

Newton's mountaintop-*lantern* experiment*

What is radius r for circular orbit if we let $v = c$?

Recall that Newton's cannon ball moves in circle because gravity provides constant force perpendicular to direction of motion, thereby constantly accelerating it toward center of Earth.

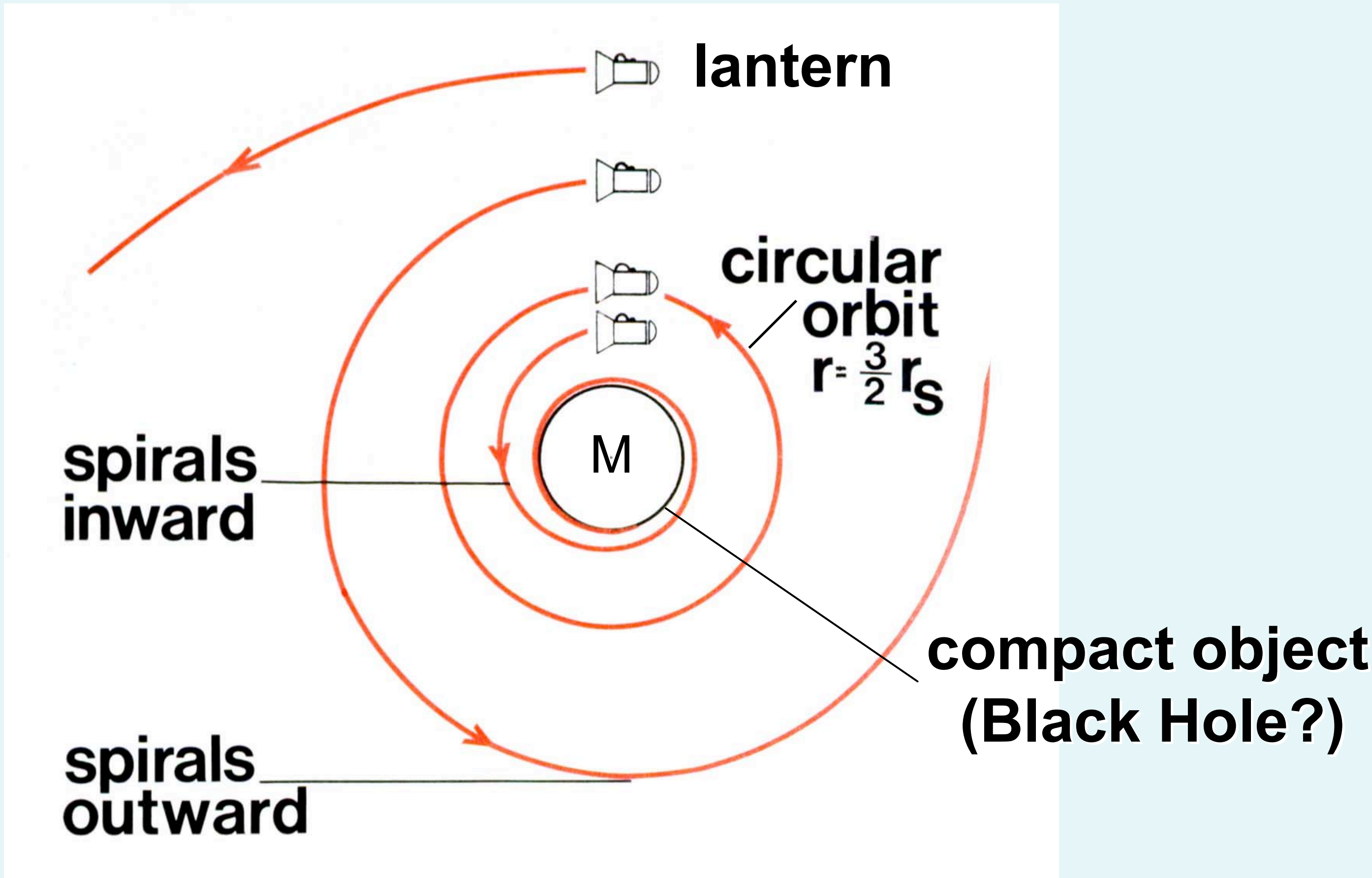
For any constant-speed circular motion, acceleration toward the center of the circle is $a_{c(entripetal)} = v^2/r$. For an orbit, this acceleration must arise from gravity.

We also know that $a_c = F_g/m = MG/r^2$, so finally we have $v^2/r = MG/r^2$, or $r = MG/v^2$. Now, for $v = c$, $r = MG/c^2$ —voilà!

Actually, Einstein finds $r_{orb} = 3MG/c^2$ for light (the correct relativistic result for light moving initially in the direction tangent to circle)

*Still puzzled why light falls even though it has no mass?
Remember, falling is independent from m—that's the way gravity works, especially for Einstein—key is to recognize accelerated frame of reference.

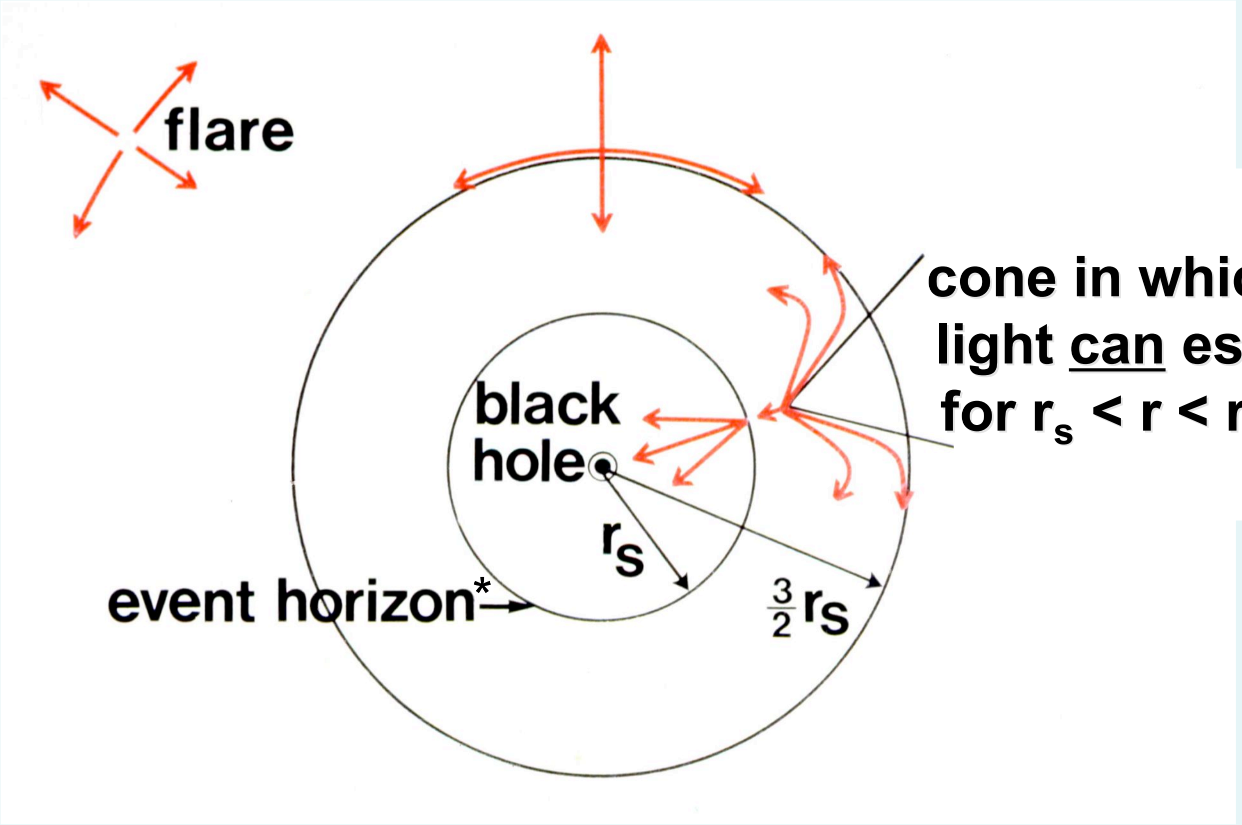
What If move lantern closer to M, or farther away?



Even if point lantern radially outward (use flare),

light <u>cannot</u> escape for $r \leq 2MG/C^2 \equiv r_{g(ravitation)}$

Where also, $r_{s(chwarzschild)} \equiv r_g$



*event horizon defined by $r = r_s$

Local strong gravity—Neutron stars and Black Holes

So Einstein says:

Light orbits mass M with $r_{orbit} = \frac{3MG}{c^2}$

Light can't escape for $r < r_g = \frac{2MG}{c^2}$

For $M = 1M_{\odot} = 2 \times 10^{30}$ kg, $r_{orbit} = 4.5$ km, $r_g = 3$ km

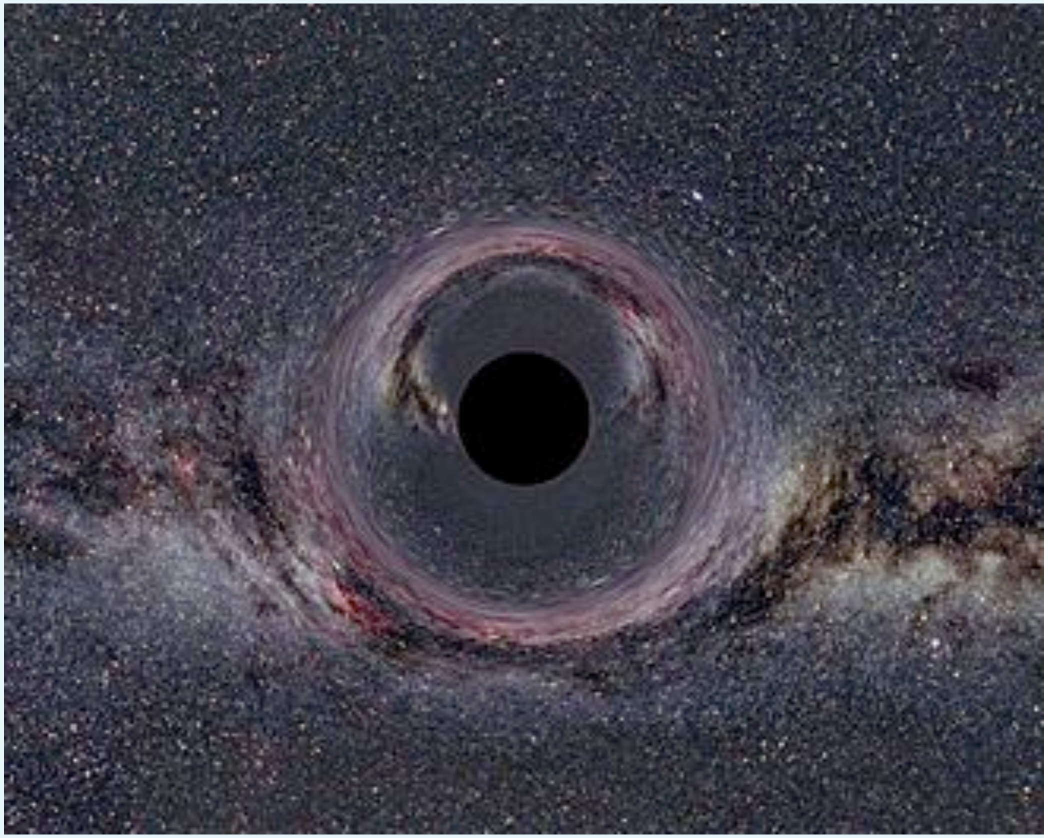
If can stuff M within r_g, Black Hole forms.

How accomplish that? Gravity knows. . .

Stellar collapse: fuel-depleted star cools, internal pressure drops, star shrinks catastrophically and gravity definitely wins if $M > 3M_{\odot}$

Non-rotating Black Hole phenomenology

- Event Horizon, $r = r_g$ — no info escapes from within (one-way membrane)
- Gravitational redshift — extreme at $r = r_g$, time dilation, “frozen star” (замороженные звезды, черная дыра)
- Tidal stress — making spaghetti
- Mass accretion —fierce gravitational engine



Nearby, large, non-accreting Black Hole in our line of sight to the Milky Way (simulation!)