Rotation Dynamics

Last time:

- Defined torque, $\tau = r \times F = r \times \overrightarrow{F} = rF \sin \theta$

- $r = rs \sin \theta$

- If $F$ acts directly through axis/pivot, $\tau = 0$

From this definition, formulated Newton's 2nd law for rotation: $\Sigma \tau_i = \tau_{\text{net}} = I \alpha$

where the sign convention for $\tau$ must be same as for $\alpha$.

First and third laws follow directly:

Law 1: $\tau_{\text{net}} = 0 \Rightarrow \alpha = 0, \omega = \text{const.}$

Law 3: Action-reaction torque results between members of system, $\Sigma \tau_i (\text{internal}) = 0$

So $\tau_{\text{net}}$ is $\tau_{\text{net}} (\text{external})$. 
Application of $\tau_{\text{net}} = I \alpha$ to systems in static equilibrium, $\tau_{\text{net}} = 0$

→ Return to problem 30 (ch 11) — like Tycho's "hanging sign" problem

On to Dynamical application of $\tau_{\text{net}} = I \alpha$

→ Solve problem 61 (ch 10) dynamically — compare to solution using E conservation.

→ Try prob 43 (ch 11) — good exercise

→ Also in prob 44 (study question) there is an important lesson:

\[ T_2 \neq T_1 \]

not ideal pulley ($M \neq 0$)

If $\alpha \neq 0$, $T_2 \neq T_1$ even if string is ideal!
30. A rigid, vertical rod of negligible mass is connected to the floor by a bolt through its lower end, as shown. The rod also has a wire connected between its top end and the floor. If a horizontal force \( F \) is applied at the midpoint of the rod, find (a) the tension in the wire, and (b) the horizontal and (c) the vertical components of force exerted by the bolt on the rod.

First, set \( \sum \tau = 0 \) to find \( T \)

\[
\sum \tau = T\cos(45^\circ) - \frac{1}{2}k(F) = 0
\]

\[
T = \frac{F}{2\cos 45^\circ} = \frac{F}{\sqrt{2}}
\]

Second, set \( \sum F_x = 0 \)

\[
\sum F_x = F + F_{bx} - T\cos 45^\circ = 0
\]

\[
F_{bx} = T\cos 45^\circ - F = \left(\frac{F}{2\cos 45^\circ}\right)\cos 45^\circ - F = -\frac{F}{2}
\]

Third, set \( \sum F_y = 0 \)

\[
\sum F_y = F_{by} - T\sin 45^\circ = 0
\]

\[
F_{by} = \left(\frac{F}{2\cos 45^\circ}\right)\sin 45^\circ = \frac{1}{2}F \left(\tan 45^\circ\right) = \frac{F}{2}
\]

\( F_{by} \) has magnitude \( |F_{by}| = \frac{F}{\sqrt{2}} \), and points in direction shown.
A 1.3-kg block is tied to a string that is wrapped around the rim of a pulley of radius 7.2 cm. The block is released from rest. (a) Assuming the pulley is a uniform disk with a mass of 0.31 kg, find the speed of the block after it has fallen through a height of 0.50 m. (b) If a small lead weight is attached near the rim of the pulley and this experiment is repeated, will the speed of the block increase, decrease, or stay the same? Explain.

\[ E_i = E_f \]
\[ U_i + K_i = U_f + K_f \]
\[ \frac{1}{2} I \omega^2 \]
\[ mgh + 0 = 0 + \frac{1}{2} mu^2 + \frac{1}{2} \left( \frac{MR^2}{2} \right) \left( \frac{u}{R} \right)^2 \]
\[ mgh = \frac{1}{2} mu^2 + \frac{1}{4} M u^2 \]
\[ mgh = u^2 \left( \frac{1}{2} m + \frac{1}{4} M \right) \]
\[ u = \left( \frac{4 mgh}{2m + M} \right)^{1/2} = 3.0 \text{ m/s} \]

(b) small mass on disk increase \( I \) (equiv increase in \( M \)) — \( u \times \frac{1}{(2m+M)^{1/2}} \)
so \( u \) be decreased at every point.
\[ \text{If } M = 0, \quad u = \sqrt{2gh} = 3.1 \text{ m/s} \quad h = 0.5 \text{ m} \]
A 1.3-kg block is tied to a string that is wrapped around the rim of a pulley of radius 7.2 cm. The block is released from rest. Assuming the pulley is a uniform disk with a mass of 0.31 kg, find the speed of the block after it has fallen through a height of 0.50 m.

**Elements of Problem**

- Disk, $M = 0.31 \text{ kg}$
- Weight, $m = 1.3 \text{ kg}$

**Free-body Diagram**

- $I = \frac{1}{2} MR^2$
- $T = -RT = I\alpha$
- $a < 0$
- $T = \frac{mg}{2}
- $T - mg = ma$

**Solution:**

- Disk and weight connected by ideal string under tension $T$.
- For the Disk, have $T = -RT = I\alpha$, where $\alpha = \frac{a}{R}$
- So $-RT = I\frac{a}{R} \Rightarrow T = -I\frac{a}{R^2}$. Substitute $T = m(a + g)$
- Find $ma = -I\frac{a}{R^2} - mg \Rightarrow a = -\frac{g}{1 + \frac{M\omega^2}{m}}$
- So $a = \frac{-g}{1 + \frac{1}{2} \frac{M}{m}} = \frac{-10}{1 + \frac{1}{2} \frac{0.31}{1.3}} = 8.93 \text{ m/s}^2$
- $\omega = (2a \Delta y)^{\frac{1}{2}} = (8.93 \text{ m/s}^2)^{\frac{1}{2}} = 3.0 \text{ m/s}$
Angular Momentum

Define \( L = I \omega = rP \cdot i = rP \sin \theta \)

Same geometrical argument as for defining \( \tau \).

Just as with torque, angular momentum is computed about some axis.

Why is \( L \) important?

\[
L = I \omega \quad \text{(suggested by \( L = rP = rmw^2 \))}
\]

but \( \omega = wr \), so \( L = \frac{r^2 mw^2}{I_{x + m}} \)

\[
L = I \omega \rightarrow \frac{\Delta L}{\Delta t} = I \frac{\Delta \omega}{\Delta t} \rightarrow \frac{dL}{dt} = I \alpha = \tau
\]

(analog of \( \frac{d\vec{p}}{dt} = m \vec{a} = \vec{F} \))

Torque changes angular momentum (just as force changes translational momentum)

\( \tau_{\text{net}} \rightarrow \Delta \tau \) and \( \tau_{\text{net}} \rightarrow \Delta L \)

Thus, if \( \tau_{\text{net}} = 0 \), \( L = L_i \) and angular momentum is conserved.