Physics 322 Second Midterm Exam Spring 2008

NAME SOLUTION

Part I. (17 pts) A thin disk of radius a and thickness h has uniform magnetization, $\vec{M} = M_0 \hat{z}$.

1. (3 pts) What is the bound volume current density, $\vec{J}_b$, within the disk?

$$\vec{J}_b \;=\; \vec{\nabla} \times \vec{M} \;=\; 0 \quad \text{because} \quad \vec{M} \quad \text{is uniform.}$$

2. (3 pts) What is the bound surface current density, $\vec{K}_b$, on the disk?

$$\vec{K}_b \;=\; \vec{M} \times \hat{n} \;=\; M\hat{\phi}$$

around the outer rim of the disk. $\vec{K}_b = 0$ on the top and bottom of the disk where $\vec{M}$ is parallel to $\hat{n}$.

3. (3 pts) What is the magnetic dipole moment of the disk (in terms of a, h and M)?

$$\vec{m} \;=\; I\vec{A} \;=\; I\pi a^2 \hat{z} \quad \text{where} \quad I \;=\; Mh \quad => \quad \vec{m} \;=\; Mh\pi a^2 \hat{z}$$

4. (4 pts) For $h << a$, what is the magnetic field $\vec{B}$ at the center of the disk (in terms of a, h, M and μ_0)?

For $h << a$, the surface current is a circular ring of current for which we know the field at the center is given by: $B = \mu_0 I/(2a)$. Here, $I = K_b h = Mh =>$

$$\vec{B} \;=\; \frac{\mu_0 Mh}{2a}\hat{z}$$

5. (4 pts) What is the H field, $\vec{H}$, at the center of the disk?

$$\vec{H} \;=\; \frac{\vec{B}}{\mu_0} - \vec{M} \;=\; \frac{Mh}{2a}\hat{z} - M\hat{z} \;=\; -M\left(1 - \frac{h}{2a}\right)\hat{z}$$

Part II. (19 pts) Linear Media: A small sphere of radius a is placed at the origin, in the presence of an external magnetic field, $\vec{B}_{ext}(\vec{r})$. The sphere is a linear magnetic media with a magnetic susceptibility, $\chi_m << 1$. $\vec{B}_{ext}$ induces a magnetization, $\vec{M}(\vec{r})$ within the sphere.

1. (4 pts) For a **paramagnetic sphere**, is $|\vec{B}| > \mu_0|\vec{H}|$, $|\vec{B}| < \mu_0|\vec{H}|$, or $|\vec{B}| = \mu_0|\vec{H}|$ inside the sphere ? Explain your answer.

$$\vec{B} \;=\; \mu_0(\vec{H} + \vec{M}) \;=\; \mu_0(\vec{H} + \chi_m \vec{H}) \;=\; \mu_0 \vec{H}(1 + \chi_m)$$

For a paramagnetic substance, $\chi_m > 0 \;=>\; |\vec{B}| > \mu_0|\vec{H}|$.

2. (3 pts) For a uniform external field, $\vec{B}_{ext}(\vec{r}) = B_0\hat{z}$, what is the magnetic dipole moment, $\vec{m}$, of the sphere in terms of its magnetization, $\vec{M}$?

For a uniform external field, $\vec{M}$ is uniform in the sphere and $\vec{m} = \vec{M} \times Vol$. Therefore,

$$\vec{m} \;=\; \frac{4\pi a^3}{3}\vec{M}$$

3. (4 pts) For a **diamagnetic sphere**, does the tip of a small bar magnet placed near the sphere attract, repel, or exert no force on the sphere ? Explain your answer.

The diamagnetic sphere is repelled from the magnet pole. The field from the magnet induces a magnetization in the sphere that is opposite the magnetic field (because $\chi_m < 0$ for a diamagnetic substance). This is like placing two like magnet poles near to one another, leading to a repulsive force.

4. (5 pts) Now consider a non-uniform external field, $\vec{B}_{ext}(\vec{r}) = (bs)\hat{s} + (B_0 - 2bz)\hat{z}$ (in cylindrical coordinates). For small a and the sphere centered at $\vec{r}$, we can assume that the magnetic dipole moment of the sphere, $\vec{m}(\vec{r}) = \gamma\vec{B}_{ext}(\vec{r})$. Derive an expression for F_z, the $\hat{z}$ component of the magnetic force acting on the sphere, in terms of γ, b and B_0.

$$\vec{F} \;=\; \vec{\nabla}(\vec{m}\cdot\vec{B}_{ext}) \quad => \quad F_z \;=\; \frac{\partial}{\partial z}(\gamma B_{ext}^2) \;=\; \frac{\partial}{\partial z}\Big[\gamma(b^2s^2 + B_0^2 - 4bzB_0 + 4b^2z^2)\Big] \;=\; -4\gamma b B_0$$

when the sphere is at the origin ($z = 0$).

5. (4 pts) A "perfect diamagnet" is a material in which $\vec{B}$ vanishes, similar to how $\vec{E}$ vanishes in a perfect conductor. Is a perfect diamagnet a linear magnetic medium? If so, what value must χ_m have ?

$$\vec{B} \;=\; \mu_0(\vec{H} + \vec{M}) \;=\; \mu_0\vec{H}(1 + \chi_m)$$

For $\chi_m = -1$, $\vec{B} = 0$, and the material is still linear because $\vec{M} = \chi_m\vec{H}$.

Part III. (14 pts) Consider a uniformly magnetized cylinder (permanent magnet, $\vec{M} = M_0\hat{z}$), and the path, C, indicated to the right.

1. (4 pts) What is $\int_C \vec{H} \cdot d\vec{l}$ around path C?

$$\int_C \vec{H} \cdot d\vec{l} \;=\; I_{free,enc} \;=\; 0$$

because there are no free currents on the disk.

2. (4 pts) What is $\int_C \vec{B} \cdot d\vec{l}$ around path C?

$$\int_C \vec{B} \cdot d\vec{l} \;=\; \mu_0 I_{enc} \;=\; \mu_0 M h$$

because the bound surface current, $\vec{K}_b = M_0\vec{\phi}$ contributes to $I_{enc} = M_0 h$.

3. (3 pts) Where is $\vec{\nabla} \cdot \vec{H} \neq 0$? Explain your answer.

$\vec{\nabla} \cdot \vec{H} \;=\; -\vec{\nabla} \cdot \vec{M} \neq 0$ on the top and bottom surfaces of the disk where $\vec{M}$ goes from non-zero to zero.

4. (3 pts) If the cylinder was placed in a uniform magnetic field $\vec{B} = B_0(\hat{x} + \hat{z})$, in what direction is the magnetic torque that acts on the cylinder?

$$\vec{N} \;=\; \vec{m} \times \vec{B} \;\propto\; \hat{z} \times (\hat{x} + \hat{z}) \;=\; \hat{y}$$

The torque is along the $+\hat{y}$ axis.