Physics 322 Solution to Homework Set #3 Spring 2008

1. Problem 5.19, parts a,b,c in your textbook.

(a)

$$\rho \;=\; \frac{charge}{volume} \;=\; \frac{charge}{atom}\,\frac{atoms}{mole}\,\frac{moles}{gram}\,\frac{grams}{volume}$$

where $charge/atom = 2e = 3.2\times10^{-19} Coul$, $atoms/mole = 6.02\times10^{23}$, $mole/gram = (grams/mole)^{-1} =$ (atomic mass of copper)$^{-1}$ $= (63.54 gram/mole)^{-1}$, and $grams/volume = density = 8.96 g/cm^3$.

$$\rho \;=\; 3.2\times10^{-19} \;\cdot\; 6.02\times10^{23} \;\cdot\; 8.96\;/\;63.54 \;=\; 2.7\times10^{4}\; Coul/cm^3$$

(b)

$$J \;=\; \rho v \;=\; \frac{I}{\pi r^2} \quad => \quad v \;=\; \frac{I}{\rho\pi r^2} \;=\; \frac{1}{2.7\times10^4\;\pi\;(.05cm)^2} \;=\; 4.7\times10^{-3}\; cm/s$$

(c) From Eq. 5.37, the force per unit length, f, is given by:

$$f \;=\; \frac{\mu_0}{2\pi}\Big(\frac{I_1 I_2}{d}\Big) \;=\; \frac{4\pi\times10^{-7}}{2\pi(.01m)} \;=\; 2\times10^{-5}\; N/m$$

2. Problem 5.23 in your textbook.

$$A_\phi \;=\; k \quad => \quad \vec{B} \;=\; \vec{\nabla}\times\vec{A} \;=\; \frac{1}{s}\frac{\partial}{\partial s}(sA_\phi)\;\hat{z} \;=\; \frac{1}{s}\frac{\partial}{\partial s}(sk)\;\hat{z} \;=\; \frac{k\hat{z}}{s}$$

$$\vec{J} \;=\; \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B}) \;=\; \frac{1}{\mu_0}\Big[-\frac{\partial}{\partial s}B_z\Big]\;\hat{\phi} \;=\; \frac{1}{\mu_0}\Big[-\frac{\partial}{\partial s}\Big(\frac{k}{s}\Big)\Big]\;\hat{\phi} \;=\; \frac{k\hat{\phi}}{\mu_0 s^2}$$

3. Problem 5.24 in your textbook.

$$\vec{\nabla}\cdot\vec{A} \;=\; -\frac{1}{2}\vec{\nabla}\cdot(\vec{r}\times\vec{B}) \;=\; -\frac{1}{2}\big[\vec{B}\cdot(\vec{\nabla}\times\vec{r}) - \vec{r}\cdot(\vec{\nabla}\times\vec{B})\big] \;=\; 0$$

because $\vec{B}$ is uniform making $\vec{\nabla}\times\vec{B} = 0$ and $\vec{\nabla}\times\vec{r} = 0$ always.

$$\vec{\nabla}\times\vec{A} \;=\; -\frac{1}{2}\vec{\nabla}\times(\vec{r}\times\vec{B}) \;=\; -\frac{1}{2}\big[(\vec{B}\cdot\vec{\nabla})\vec{r} - (\vec{r}\cdot\vec{\nabla})\vec{B} + \vec{r}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{r})\big]$$

but $\vec{\nabla}\cdot\vec{B} = 0$ always and because $\vec{B}$ is uniform, $(\vec{r}\cdot\vec{\nabla})\vec{B} = 0$.

$$\vec{\nabla}\cdot\vec{r} \;=\; \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} \;=\; 3 \quad \text{and}$$

$$(\vec{B}\cdot\vec{\nabla})\vec{r} \;=\; \left(B_x\frac{\partial}{\partial x} + B_y\frac{\partial}{\partial y} + B_z\frac{\partial}{\partial z}\right)(x\hat{x} + y\hat{y} + z\hat{z}) \;=\; B_x\hat{x} + B_y\hat{y} + B_z\hat{z} \;=\; \vec{B}$$

$$\text{Therefore} \quad \vec{\nabla}\times\vec{A} \;=\; -\frac{1}{2}[\vec{B} - 3\vec{B}] \;=\; \vec{B}$$

The result is not unique because one can add a constant vector to $\vec{A}$ without changing the divergence or curl of $\vec{A}$.

4. Problem 5.35 in your textbook.

For ring of current, $m = I\pi r^2$. For the phonograph surface, a ring of width dr carries a current $I = \sigma v\ dr = \sigma\omega r\ dr$, creating a diople moment $dm = \sigma\omega\pi r^3\ dr$. Therefore,

$$m \;=\; \int_0^R dm \;=\; \sigma\omega\pi \int_0^R r^3\ dr \;=\; \frac{\pi\sigma\omega R^4}{4}$$

5. Problem 5.37 in your textbook.

We can use Eq. 5.35 in the text which gives the field at a point a perpendicular distance s from a straight segment of wire:

$$B \;=\; \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$$

where $\theta_2,\ \theta_1$ are the angles between the perpendicular line, s, and the end points of the current segment.

For a point a distance z above a square loop of side length w, the perpendicular distance from z to any of the sides of the square (the midpoint of a segment) is $s = \sqrt{z^2 + (w/2)^2}$.

The line from z to the corner of the square has a length $h = \sqrt{z^2 + (w/2)^2 + (w/2)^2} = \sqrt{z^2 + w^2/2}$, making $\theta_2 = -\theta_1 = (w/2)/h = (w/2)/\sqrt{z^2 + w^2/2}$.

Therefore, the field from one of the segments is:

$$B \;=\; \frac{\mu_0 I}{4\pi s}\frac{w}{h} \;=\; \frac{\mu_0 I}{4\pi}\frac{w}{\sqrt{z^2 + w^2/4}\ \sqrt{z^2 + w^2/2}}$$

This field has both horizontal and vertical components. The horizontal components of the field cancel when all 4 segments are added (there is no preferred direction in the horizontal plane). The vertical component of the field is $B\sin\phi$ where $\sin\phi = (w/2)/s = (w/2)/\sqrt{z^2 + w^2/4}$. Adding the 4 segments,

$$\vec{B} \;=\; B_{vert}\hat{z} \;=\; \frac{4\mu_0 I}{4\pi}\frac{w(w/2)\ \hat{z}}{(z^2 + w^2/4)\sqrt{z^2 + w^2/2}} \;=\; \frac{\mu_0 I}{2\pi}\frac{w^2\ \hat{z}}{(z^2 + w^2/4)\sqrt{z^2 + w^2/2}}$$

For $z >> w$, we ignore w in the denominator and using $\vec{m} = Iw^2\ \hat{z}$, for the square loop, we have:

$$\text{for} \quad z >> w, \quad \vec{B} \;\rightarrow\; \frac{\mu_0 I}{2\pi}\frac{w^2\ \hat{z}}{z^3} \;=\; \frac{\mu_0 \vec{m}}{2\pi z^3}$$

which is the field from a diople, $\vec{m}$, along the axis of the dipole.

6. Problem 5.55 in your textbook.

Use Eq. 5.86 for the field, $\vec{B}_{dip}$ from a dipole. The total field is then:

$$\vec{B}_{tot} \;=\; B_0\hat{z} + \vec{B}_{dip} \;=\; B_0\hat{z} - \frac{\mu_0 m_0}{4\pi r^3}(2\cos\theta\ \hat{r} + \sin\theta\ \hat{\theta})$$

If no field lines pass through a sphere of radius R, then $\vec{B}_{tot}(R)\cdot\hat{r} = 0$ for all θ:

$$\vec{B}_{tot}(R)\cdot\hat{r} \;=\; B_0(\hat{z}\cdot\hat{r}) - \frac{\mu_0 m_0}{4\pi R^3}(2\cos\theta) \;=\; \left(B_0 - \frac{\mu_0 m_0}{2\pi R^3}\right)\cos\theta$$

$$\vec{B}_{tot}(R)\cdot\hat{r} \;=\; 0 \quad \text{when} \quad B_0 \;=\; \frac{\mu_0 m_0}{2\pi R^3} \quad => \quad R \;=\; \left(\frac{\mu_0 m_0}{2\pi B_0}\right)^{1/3}$$