A Mathematical Model to Determine the Volume of Snail Shells

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Abstract

There are many species of marine snails, each with its own unique exoskeleton shape, or shell shape. What are the tradeoffs that a snail faces as its shell evolves? To answer this question, we need to understand the factors that govern shell shape and the way a snail expends energy during growth. The shell is spiraled; the tighter the spiral, the less material in the shell and the more energy available for other functions such as reproduction. However, if a shell is spiraled too tightly, then the snails' gonads will have to be smaller, so the snail will have fewer offspring. We hypothesize that certain shell shapes have evolved to increase the internal volume (and the space available for gonads) while decreasing the amount of shell material deposited (and the energy expended during growth). To test this, we are developing a model that can form shells of varying shape and thickness from the apertures of real shells. Eventually, we will use this model to compare aperture shapes that are slightly different from biologically real ones to explore how different species balance the tradeoff between growing bigger and minimizing the shell material they need to grow their shells.

Introduction

A number of models have been developed to describe the shape of logarithmically coiled snail shells, some of which have also been used to calculate the internal volume of the shell and the surface area of shell material deposited (e.g., Raup & Graus 1972; Stone 1997). The internal volume of the shell is important because it is correlated to how much space is in a shell for producing eggs and sperm, and, therefore, reproductive fitness (Heath 1985). The surface area of shell material deposited is important because it is correlated to the amount of energy required to grow a shell (Heath 1985). But this traditional approach assumes that shells are infinitely thin. Previous models also assume that the shell's aperture is elliptical in shape. We are developing a model that does not rely on these assumptions. We use the exact shape and thickness of the aperture to accurately predict both the internal volume of the shell and the volume of shell material deposited.

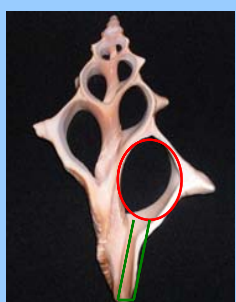


Figure 1 shows the location of the aperture (red) and the siphonal canal (green)

Goal

To develop a model that accurately estimates the internal volume of the shell and the volume of shell material deposited.

Developing the Model

We adapted Stone's (1997) approach for calculating the internal volume of the shell. In his model,

$$V_i = \Sigma\, V_{whorl\_i}$$

where V_i is the internal volume of the whorl and $\Sigma\, V_{whorl\_i}$ is the sum of the volumes for each consecutive whorl.

We also calculate the external volume of the shell, V_e, so that we can estimate the total volume of shell material that a snail deposits, V_{sm}:

$$V_e = \Sigma\, V_{whorl\_e}$$

$$V_{sm} = V_e - V_i$$

Now the trick is to calculate the internal and external volumes for each whorl. To do that, we calculate the area of the aperture in that whorl (A_{whorl_i} and A_{whorl_e} respectively) and multiply it by the distance traversed ($2\pi d_{whorl\_i}$ and $2\pi d_{whorl\_e}$, respectively; notice that these distances are the circumferences for circles that make up a whorl.

$$V_i = A \cdot 2\pi d_i$$

$$V_{sm} = A \cdot 2\pi d_e$$

Our model differs from Stone's, because we directly measure the area of the aperture for each whorl (*A*) instead of assuming the aperture has an elliptical shape. We outline aperture in Adobe Photoshop, and then transfer that outline to a program called TPSDIG (Rohlf 2004). In TPSDIG, we find the coordinates for 100 evenly-spaced points along for the inside of the outline and another 100 evenly-spaced points for the outside of the outline. We input these points into the equations, making our calculations in MATLAB 7.0.

Testing the Model

Without testing our model we would not know if it is an accurate representation of real shells. We have measured the internal volume of the shell and the volume of shell material deposited in 10 shells, each from a different species (Table 1). We measured the volume of shell material deposited (V_{sm}) and the *total* volume (V_{total}) of the shell by displacing the shells in water. Then, we applied the equation

$$V_{total} = V_i + V_{sm}$$

to calculate the internal volume of the shell (V_i).

The volume of the shell material deposited was measured by displacing the shell in water. To ensure that water reached the tip inside the shell, we placed the submerged shell under vacuum. The total volume was measured by displacing a shell that was sealed shut with parafilm. Sealing the shell enables an accurate measurement of total volume because water is not able to enter the shell. Some of the sealed shells floated in water, so we weighed them down with lead fishing weights.

Results

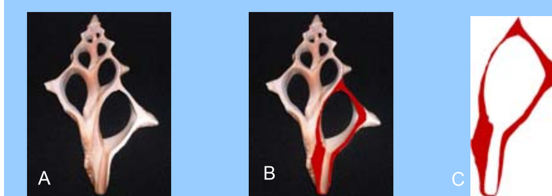
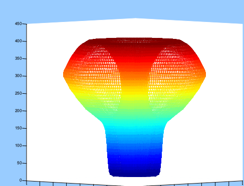
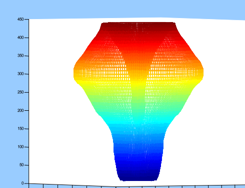


Figure 2 shows the beginning of the modeling process. A, the shell. B, the outline of the aperture drawn on the shell. C, the outline of the shell's aperture





```
xi;
zi;
p=(0:0.01:2)*pi;
c=1;
k=0;
x=(c+xi)*cos(p);
y=(c+xi)*sin(p);
z=zi*ones(size(p))+k;
mesh(x,y,z); axis square; hidden off
```

```
xm;
zm;
p=(0:0.01:2)*pi;
c=1;
k=0;
x=(c+xm)*cos(p);
y=(c+xm)*sin(p);
z=zm*ones(size(p))+k;
mesh(x,y,z); axis square; hidden off
```

Figure 3. The outline of the shell is used to generate the internal structure (left) of the shell and the external structure of the shell (right). To the left of each figure is the MATLAB code used to plot the imported points for xi, zi, xm, and zm.

Table 1. Species used to test the model.

Specimen	Species	Specimen	Species
50	*Fasciolaria hunteria*	55	*Leucozonia nassa*
51	*Strombus alatus*	56	*Littorina irrorata*
52	*Melongena corona*	57	*Pisania tincta*
53	*Terebra dislocata*	58	*Pleuroploca salmo*
54	*Littorina littorea*	59	*Nassarius vibex*

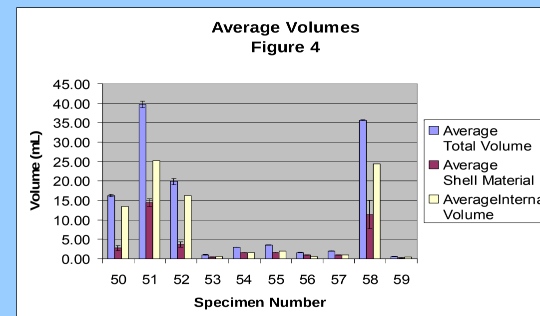


Figure 4. The average total volume and average shell material measured, along with the average internal volume that was calculated. Only specimen 56 had a higher volume of shell material than internal volume.

Discussion and Future Directions

Our model is very close to being finished. We only need to figure out how to position the whorls relative to each other. Once completed, we will be able to compare the ten shells that we measured to the model's output. If the model proves accurate, we will use it to test hypotheses about how energy is allocated during shell growth. Understanding energy allocation will provide insight on the selective pressures important in the evolution of snail shells.

Literature Cited

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Acknowledgements

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