ESS / OCEAN / ATMOS 589 -- 2005
Problem Set 2.

1) Using the global 0-D EBM discussed in lecture, determine the sensitivity of global average temperature to

a) an increase in the solar constant of 8 W/m^2 (from an original value of S_0 = 1367).
b) a decrease in effective emissivity of 0.1 (from an original value of 0.76).

The energy balance model is:

$$T_g^{\,4} = \frac{S_0(1-\alpha)}{4\sigma(1-\varepsilon/2)}$$

a) Take the derivative with respect to S_0.

$$T_g = \sqrt[4]{\frac{S_0(1-\alpha)}{4\sigma(1-\varepsilon/2)}}$$

$$T_g = \left(S_0K\right)^{1/4} \text{ where } K = \frac{(1-\alpha)}{4\sigma(1-\varepsilon/2)}$$

$$\frac{\partial T_g}{\partial S_0} = \frac{1}{4}\left(S_0K\right)^{-3/4}K = \frac{1}{4}S_0^{\,-3/4}K^{1/4}$$

$$= \frac{1}{4}\left(S_0\right)^{-3/4}\left(\frac{(1-\alpha)}{4\sigma(1-\varepsilon/2)}\right)^{1/4}$$

For the sensitivity to an 8 W/m^2 increase, take the derivate at S_0+8/2, to get a mid-point value.

$$\frac{\partial T_g}{\partial S_0} = \frac{1}{4}\left(1367+8/2\right)^{-3/4}\left(\frac{(1-0.3)}{4*5.67*10^{-8}(1-0.72/2)}\right)^{1/4}$$

$$= 0.052K/(W/m^2)$$

or 8*0.052 = 0.42°C for an 8 W change.

a) Take the derivative with respect to ε:

$$T_g = \sqrt[4]{\frac{S_0(1-\alpha)}{4\sigma(1-\varepsilon/2)}}$$

$$T_g = \left(\frac{1}{1-x}K\right)^{1/4} \text{ where } K = \frac{S_0(1-\alpha)}{4\sigma},\ x = (\varepsilon/2)$$

$$\frac{\partial T_g}{\partial S_0} = \frac{1}{4}\left(\frac{1}{1-x}K\right)^{-3/4}\left(K\left(\frac{1}{1-x}\right)^2\right)$$

$$= \frac{1}{8}\left(\frac{S_0(1-\alpha)}{4\sigma}\right)^{1/4}\left((1-\varepsilon/2)^{-5/4}\right)$$

$$= \frac{1}{8}\left(\frac{1367(1-0.3)}{4*(5.67*10^{-8}}\right)^{1/4}\left((1-0.755/2)^{-5/4}\right)$$

$$= 57.6K/(\text{unit emissivity})$$

or 5.76 K for a 0.1 change in emissivity.

2) Using the same EBM, find the change in the climate between the last glacial maximum and today, due to the change in the change in the planetary albedo from the presence of the Laurentide ice sheet. Assume that today's global average albedo is 0.3, and that the ice sheets covered 1/30 of the surface area of the globe, and had an albedo of 0.7.

The new albedo is (1/30 * 0.7)+(29/30 * 0.3) = 0.31333…

The temperature change is therefore given by

$$\Delta T_g = \sqrt[4]{\frac{S_0(1-0.3133)}{4\sigma(1-\varepsilon/2)}} - \sqrt[4]{\frac{S_0(1-0.3)}{4\sigma(1-\varepsilon/2)}}$$

= -1.37 °C.

So the presence of the ice sheets can explain roughly 1/5 of the global temperature change of 5°C at LGM.

3) a) Calculate the local temperature in the tropics (20°N to 20°S), and at high latitudes (60°-90°) given that the local insolation at the top of the atmosphere is 1720 W/m^{-2} and 800 W/m^{-2} respectively. Assume the albedo is 0.3 in both cases.

b) Given that the observed mean annual temperature in these latitude bands is +26 C and -10, respectively, calculate the implied meridional heat transport.

c) How does the answer to (b) change if the local albedo is 0.25 and 0.5 in the tropics and high latitudes, respectively?

a,b) The most straightforward way to do this is to recognize that the heat transport arises from an local energy *imbalance*.

The energy balance is given by: $\sigma T_g^{\ 4}(1-\varepsilon/2) = \dfrac{S_0(1-\alpha)}{4}$ which is the same thing as

$$\sigma T_g^{\ 4}(1-\varepsilon/2) - \frac{S_0(1-\alpha)}{4} = 0$$

So the energy imbalance is given by $\sigma T_g^{\ 4}(1-\varepsilon/2) - \dfrac{S_0(1-\alpha)}{4}$

Calculating this with the given values for S_0 and observed T at the high (1720 W/m^2, 26°c) and low (800 W/m^2, -10°C) latitudes, we get 20 W/m^2 for the high latitudes, 28 W/m^2 for the low latitudes. So a rough estimate of the heat transport is 24 W/m^2.

b) Using 0.25 for albedo at low latitudes, 0.5 at high latitudes, these estimates are:

$$\sigma T_g^{\ 4}(1-\varepsilon/2) - \frac{S_0(1-\alpha)}{4}$$

$$5.67 * 10^{-8}(273.15+26)^4(1-0.76/2) - \frac{1760(1-0.25)}{4}$$

=-48 W/m^2 at low latitudes (the negative sign means heat is transported away)

$$\sigma T_g^{\ 4}(1-\varepsilon/2) - \frac{S_0(1-\alpha)}{4}$$

$$5.67 * 10^{-8}(273.15-10)^4(1-0.76/2) - \frac{800(1-0.5)}{4}$$

=68 W/m^2 at high latitudes (heat is transported to the poles)

These values agree well with modern estimates of roughly 50 W/m^2 poleward heat transport.