Numbers

FRACTIONS
i. Add or subtract fractions with the same denominator by adding or subtracting the numerators and keeping the same denominator.
ii. Add or subtract fractions with different denominators by finding the least common denominator, multiplying the numerator and denominator by the necessary factor, and then adding or subtracting.
iii. Subtract one fraction from another by changing the second fraction to an equivalent fraction with the same denominator, and then subtracting the numerators.

MULTIPLICATION
i. Multiply two fractions with the same denominator by multiplying the numerators and denominators.
ii. Multiply two fractions with different denominators by finding the least common denominator, multiplying the numerator and denominator by the necessary factor, and then multiplying the numerators and denominators.

DIVISION
i. Divide two fractions with the same denominator by dividing the numerators and keeping the same denominator.
ii. Divide two fractions with different denominators by finding the least common denominator, multiplying the numerator and denominator by the necessary factor, and then dividing the numerators.

ORDER OF OPERATIONS

VARIABLES, EXPRESSIONS, AND EQUATIONS

1. Real Numbers
   a. Natural Numbers
   i. Whole Numbers
   ii. Integers
   iii. Rational Numbers
   iv. Irrational Numbers
   v. Real Numbers
2. Operations on Real Numbers
   a. Addition
   i. To add two numbers with the same sign, add the numbers and keep the same sign.
   ii. To add two numbers with different signs, subtract their absolute values and keep the sign of the number with the larger absolute value.
   b. Subtraction
   i. To subtract two numbers, add the first number to the additive inverse of the second number.
   ii. To subtract two numbers with different signs, subtract their absolute values and keep the sign of the number with the larger absolute value.
   c. Multiplication
   i. To multiply two numbers, multiply the numbers and keep the sign of the number with the larger absolute value.
   ii. To multiply the product of two numbers, multiply the products and keep the sign of the number with the larger absolute value.
   d. Division
   i. To divide two numbers, divide the numbers and keep the sign of the number with the larger absolute value.
   ii. To divide two numbers with different signs, divide their absolute values and keep the sign of the number with the larger absolute value.

Linear Equations

Derivation of Subtraction
i. Subtracting Real Numbers
   a. Change the subtraction symbol to the addition symbol and the sign of the second number.
   b. Add the two numbers.

Properties of Real Numbers
i. Closure Property
   a. Addition
   i. For any real numbers a and b, a + b is a real number.
   ii. Subtraction
   i. For any real numbers a and b, a - b is a real number.
   iii. Multiplication
   i. For any real numbers a and b, a * b is a real number.
   iv. Division
   i. For any real numbers a and b (b ≠ 0), a / b is a real number.

Exponents (continued)

1. Exponential Rules
   a. Zero Exponent
   i. Any number raised to the power of 0 is 1.
   b. Negative Exponent
   i. A number raised to a negative exponent is the reciprocal of the number raised to the positive exponent.

Polynomials

Factoring
i. Finding the Greatest Common Factor (GCF)
   a. The greatest common factor (GCF) of two or more polynomials is the largest monomial factor that divides each polynomial.
   b. To find the GCF, factor each polynomial completely and then identify the common factors.
ii. Factoring by Grouping
   a. Group the terms.
   b. Factor out the greatest common factor from each group.
   c. Factor the resulting expression.

Simplifying Complex Fractions
i. Simplifying Complex Fractions
   a. Simplify the numerator and denominator separately.
   b. Divide the result of the numerator by the result of the denominator.

Systems of Linear Equations

Two Variables
i. Graphing Method
   a. Graph each equation on the same set of axes.
   b. Find the coordinates of the point of intersection.
ii. Substitution Method
   a. Solve one equation for one variable.
   b. Substitute the expression into the other equation.
   c. Solve the resulting equation for the other variable.
   d. Substitute the solution for one variable into the original equation to find the solution for the other variable.

Graphing Method
i. Elimination Method
   a. Multiply one or both equations so that the coefficients of one variable are opposites.
   b. Add or subtract the equations to eliminate one variable.
   c. Solve the resulting equation for the remaining variable.
   d. Substitute the solution for the remaining variable into one of the original equations to find the solution for the other variable.
**Numbers**

**Fractions**
- **Addition and Subtraction**
  - Add or subtract fractions with the same denominator by adding or subtracting numerators and keeping the denominator.
- **Multiplication**
  - Multiply the numerators and denominators.
- **Division**
  - Divide the fraction by the reciprocal of the divisor.

**Order of Operations**
- **PEMDAS**
  - Parentheses
  - Exponents
  - Multiplication and Division (from left to right)
  - Addition and Subtraction (from left to right)

**Variables, Expressions, and Equations**
- Variables are quantities that can change. An equation is a statement that two expressions are equal.

**Properties of Real Numbers**
- **Commutative Properties**
  - Addition: $a + b = b + a$
  - Multiplication: $ab = ba$
- **Associative Properties**
  - Addition: $(a + b) + c = a + (b + c)$
  - Multiplication: $(ab)c = a(bc)$
- **Distributive Property**
  - $a(b + c) = ab + ac$
- **Identity Properties**
  - Addition: $a + 0 = a$
  - Multiplication: $a \times 1 = a$
- **Inverse Properties**
  - Addition: $a + (-a) = 0$
  - Multiplication: $a \times \frac{1}{a} = 1$

**Rational Expressions**
- **Simplifying Rational Expressions**
  - Factor both the numerator and denominator.
  - Divide out any factors that are common.

**Exponents**
- **Product Rule**
  - $a^m \times a^n = a^{m+n}$
- **Quotient Rule**
  - $\frac{a^m}{a^n} = a^{m-n}$
- **Power Rule**
  - $(a^m)^n = a^{mn}$

**Polynomials**
- **Addition and Subtraction**
  - Combine like terms.
- **Multiplication**
  - Use the distributive property.
  - Use the FOIL method for binomials.

**Graphing Linear Equations**
- **Graphing a Linear Equation**
  - Plot the points.
  - Connect the points with a smooth curve.

**Linear Equations**
- **Solving Linear Equations**
  - Add or subtract to get the variable term alone.
  - Multiply or divide to solve for the variable.

**Rational Equations**
- **Solving Rational Equations**
  - Find a common denominator.
  - Multiply both sides by the common denominator.
  - Solve the resulting equation.

**Systems of Linear Equations**
- **Graphing Method**
  - Graph each equation.
  - Find the point of intersection.
- **Substitution Method**
  - Solve one equation for a variable.
  - Substitute into the other equation.

**Numbers**

**Exponents (continued)**
- **Zero Exponent Rule**
  - $a^0 = 1$
- **Negative Exponent Rule**
  - $a^{-n} = \frac{1}{a^n}$
- **Rational Exponents**
  - $a^{m/n} = (a^{1/n})^m$

**Polynomials (continued)**
- **Factoring**
  - Find the Greatest Common Factor (GCF).
  - Factor out the GCF.
- **Multiplying Polynomials**
  - Use the distributive property.
  - Use the FOIL method.

**Factoring (continued)**
- **Factoring Trinomials**
  - Identify the trial factors.
  - Check by multiplying.
- **Quadratic Equations**
  - Use the quadratic formula.
  - Complete the square.

**Systems of Equations**
- **Graphing Method**
  - Graph each equation.
  - Find the point of intersection.
- **Substitution Method**
  - Solve one equation for a variable.
  - Substitute into the other equation.

**Equations of Lines**
- **Horizontal line**
  - $y = b$
- **Vertical line**
  - $x = a$
- **Slope-intercept form**
  - $y = mx + b$

**Quadratic Equations**
- **Standard Form**
  - $ax^2 + bx + c = 0$
- **Vertex**
  - $x = -\frac{b}{2a}$
- **Axis of Symmetry**
  - $x = -\frac{b}{2a}$
- **Roots**
  - $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Rational Expressions (continued)**
- **Simplifying Complex Fractions**
  - Simplify the numerator and denominator.
  - Divide out the common factors.

**Operations on Rational Expressions**
- **Addition and Subtraction**
  - Add or subtract like terms.
  - Subtract like terms.
- **Multiplication**
  - Multiply the numerators and denominators.

**Applications**
- **Finding the Greatest Common Factor**
  - Find the greatest common factor.
  - Divide each term by the GCF.

**Polynomials**
- **Addition and Subtraction**
  - Combine like terms.
  - Subtract like terms.
- **Multiplication**
  - Use the distributive property.
  - Use the FOIL method.

**Equations of Two Variables**
- **Solving Equations with Two Variables**
  - Substitute one variable into the other equation.
  - Solve for the remaining variable.

**Equations of Lines**
- **Slope-intercept form**
  - $y = mx + b$
- **Point-slope form**
  - $y - y_1 = m(x - x_1)$

**Rational Expressions**
- **Simplifying Expressions**
  - Find the greatest common factor.
  - Divide out the common factors.

**Applications**
- **Finding the Greatest Common Factor**
  - Find the greatest common factor.
  - Divide each term by the GCF.
0 on the number line. Subtract their absolute values. The sum has the same sign as each of the numbers being added. To add two numbers with the same sign, add their absolute values and then apply the sign of the numbers.

THE NUMBER LINE
REAL NUMBERS AND ORDER OF OPERATIONS

In order to simplify expressions, you must know the order of operations. The order of operations is as follows:

i. Simplify any expressions within parentheses.

ii. Evaluate any exponents.

iii. Perform any additions or subtractions from left to right.

iv. Evaluate any multiplications or divisions from left to right.

ORDER OF OPERATIONS

PROPERTIES OF REAL NUMBERS

In this section, we learn about the commutative, associative, identity, and inverse properties of the real numbers.

Commutative Properties

Addition: a + b = b + a

Multiplication: ab = ba

Associative Properties

Addition: (a + b) + c = a + (b + c)

Multiplication: (ab)c = a(bc)

Identity Properties

Additive identity: a + 0 = a

Multiplicative identity: a * 1 = a

Inverse Properties

Additive inverse: a + (-a) = 0

Multiplicative inverse: a * (1/a) = 1

Simplifying Algebraic Expressions

When adding or subtracting algebraic expressions, only like terms can be combined.

Equations

In this section, we learn to solve equations. An equation is a statement that two expressions are equal. To solve an equation, we find the value of the variable that makes the equation true.

Linear Equations

In this section, we learn to solve linear equations. A linear equation is an equation that can be written in the form ax + b = 0, where a and b are constants and a ≠ 0.

1. Simplify the equation by combining like terms.
2. Isolate the variable term on one side.
3. Solve for the variable by using inverse operations.

Rational Equations

In this section, we learn to solve rational equations. A rational equation is an equation that contains a fraction.

1. Find the LCD of all the denominators.
2. Multiply both sides of the equation by the LCD.
3. Solve the resulting equation.

Solving Quadratic Equations

In this section, we learn to solve quadratic equations. A quadratic equation is an equation that can be written in the form ax^2 + bx + c = 0, where a, b, and c are constants and a ≠ 0.

1. Use the quadratic formula: x = [-b ± sqrt(b^2 - 4ac)] / 2a

2. Substitute the known values into the formula.

SIMPLIFYING COMPLEX FRACTIONS

In this section, we learn to simplify complex fractions. A complex fraction is a fraction that contains a fraction in its numerator or denominator.

1. Simplify the numerator and denominator.
2. Divide the numerator and denominator.
3. Evaluate the result.

Equations of Lines

In this section, we learn to write equations of lines. An equation of a line is a statement that two quantities are equal. To find the equation of a line, we need to know the slope and a point on the line.

1. Find the slope.
2. Substitute the known values into the point-slope form.
3. Simplify the equation.

2. Graph each line on the same set of axes.

3. The graphs of the two lines should intersect at the point of solution.
APPLICATIONS

THREE VARIABLES

If the result of step iii is a false statement, then there is no solution.

i. Write the equations in standard form:

Elimination Method

ii. Multiply one or both equations by appropriate numbers so that the coefficients of one variable are additive inverses.

Multiplication by negative numbers:

iii. Any row may be modified by adding to or subtracting from any of the original equations to find the coefficient of any variable from any two of the original equations.

iv. Isolate the variable term on one side.

Multiplying equations:

v. Choose any row to obtain the value of the variable.

Multiplying equations:

vi. Solve each inequality in the compound, inequality symbol when multiplying or dividing by a negative number.

Graphing:

vii. The solution of the compound inequality involves < or >.

To solve:

The imaginary unit is , so .

(continued)

SOLVING NONLINEAR SYSTEMS

FUNCTIONS

Composition of functions:

The logarithmic function is the inverse of the exponential function. The exponential function is the inverse of the logarithmic function. The graph of a function and its inverse are reflections of each other across the line .

The logarithmic function is denoted .

A nonlinear system can be solved by the following methods:

Solving Equations by Completing the Square

Inequality of Ellipse (Standard form):

The equation of an ellipse centered at the origin, whose major axis is the horizontal axis, is of the form:

As before, we consider two separate cases:

To solve:

Inverses, Exponential, and Logarithmic Functions

Conic Sections and Nonlinear Systems

Equation of Hyperbola (Standard Form, Center at the Origin)

The equation of a hyperbola centered at the origin, whose transverse axis is the horizontal axis, is of the form:

Conic Sections

Equation of a Circle: Center-Radius Form

The equation of a circle with center and radius is:

Sequences and Series

A sequence is a list of numbers with an order, such as . A sequence is denoted or simply .

Arithmetic Sequences

An arithmetic sequence is a sequence in which the difference of any two consecutive terms is a constant. The common difference is the difference of any two consecutive terms.

The Binomial Theorem

A binomial coefficient is the coefficient of the term in the expansion of .

The Binomial Theorem
MATRICES

ROW OPERATIONS

If the result of step iii is a false statement, then the solution of the system is empty.

ii. All the elements in any row may be multiplied (or divided) by the same positive real number and the elements of the unknowns.

iii. If the inequalities are joined with the or symbol, then the union of the solution sets is the solution set.

To solve a system of compound inequalities, the solution is the intersection of the solution sets of the two individual compound inequalities.

SOLVING COMPOUND INEQUALITIES

For example, if and are real and positive, then

Functions

Interval notation for a given value of the function follows:

Value: One Variable

ii. Draw the graph of the boundary line, indicate whether the line is solid or dashed, and determine which side of the boundary line should be shaded.

iii. If the inequality involves < or >, then the test point is not in the solution set.

iv. The solution is the set of all values of that make the inequality true.

Quadratic Equations

To solve a quadratic equation, the solutions are the values of that make the equation true.

Inverse Equations, and Functions

i. If and are real and positive, then

where .

ii. If and are real and positive, then

Radical Expressions and Equations

If and are real and positive, then

Logarithmic Functions

For any positive integer and any real number

Operations on Complex Numbers

The operations on complex numbers are similar to those on real numbers.

Roots and Radicals

The solutions of the equation are:

Equations of a Circle: General Form

The equation of a circle in standard form is

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The equation of a circle in standard form is

Equations of a Hyperbola (Standard Form)

The equation of a hyperbola in standard form is

Equations of an Ellipse (Standard Form)

The equation of an ellipse in standard form is

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A sequence is a list of numbers in a certain order, where each number is the next number in the sequence.

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An arithmetic sequence is a sequence in which the difference between any two consecutive terms is constant.

Geometric Sequences

A geometric sequence is a sequence in which the ratio between any two consecutive terms is constant.

The Binomial Theorem

The binomial theorem states that

for any nonnegative integers and , where is the binomial coefficient.

Core ideas of Algebra Review

- Systems of Linear Equations
- Inequalities and Absolute Value: One Variable
- Linear Functions
- Quadratic Equations, Inequalities, and Functions
- Conic Sections and Nonlinear Systems
- Sequences and Series
- Logarithmic Functions
- The Binomial Theorem

Inequalities and Absolute Value: One Variable

i. Add the same quantity to both sides.

ii. Multiply or divide both sides by a positive real number.

iii. Subtract the solution from both sides.

The solution of the equation is the set of all values of that make the equation true.

Quadratic Equations, Inequalities, and Functions

The solutions of the equation are:

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Inequalities and Absolute Value: One Variable

Functions

Operations

Systems of Linear Equations

Operations on Complex Numbers

Applications

Notes:

Functions

Solving Quadratic Equations by Completing the Square

Applications

Applications

Solving Equations involving Radicals

Graphs

Sum of the first $n$ terms of an arithmetic sequence

Conic Sections and Nonlinear Systems

Inverses of Functions

Conic Sections and Nonlinear Systems (continued)

Graphs

Conic Sections and Nonlinear Systems

Graphs

Conic Sections and Nonlinear Systems

Graphs

Conic Sections and Nonlinear Systems

Graphs