Applied Calculus Review

PEARSON

Linear Functions

- **Formula for slope:** If the two points (x_1, y_1) and (x_2, y_2) are on a line, the slope of the line is
$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$
- **Slope of a horizontal line:** Slope is zero.
- **Slope of a vertical line:** Slope is undefined.

EQUATIONS OF LINES

1. Slope–Intercept Form: $y = mx + b$
(The line has slope m and y-intercept b.)
2. Point–Slope Form: $y - y_1 = m(x - x_1)$
(The point (x_1, y_1) is on the line, and m is the slope.)
3. $x = k$ (vertical line)
4. $y = k$ (horizontal line)

$\left.\right\}$, k is a real number.

PARALLEL AND PERPENDICULAR LINES

- **Parallel lines:** Two nonvertical lines are parallel if and only if they have the same slope, $m_1 = m_2$.
- **Perpendicular lines:** Two lines, neither of which is vertical, are perpendicular if and only if their slopes have a product of -1. (In other words, the slopes are negative reciprocals of one another, $m_1 = -\frac{1}{m_2}$.)

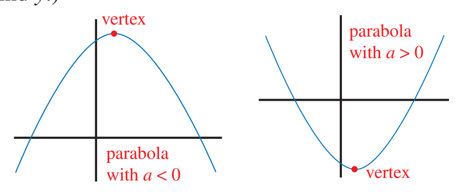
FUNCTIONS

- **Definition of a function:** A function is a rule that assigns to each element in one set exactly one element from another set. In other words, for each input (independent variable) there is exactly one output (dependent variable).
- **Definition of domain:** The domain of a function is the set of all possible real values for the independent variable (or in other words, the set of all allowable values, or meaningful replacements for x).
- **Definition of range:** The resulting set of all possible output values for the dependent variable (or y) is called the range.

Quadratic Functions

- **Quadratic functions:** $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.
- **Properties of quadratic functions:**

1. If $a > 0$, the parabola opens up. If $a < 0$, the parabola opens down.
2. Parabolas have either a highest or lowest point. This point is called the **vertex.**
3. The x-coordinate of the vertex is $x = \frac{-b}{2a}$, and the corresponding y-coordinate is $f\left(\frac{-b}{2a}\right)$. (Find the value for x first, and then substitute that number into the equation to find y.)



Translations and Reflections of Functions

Let $f(x)$ be any function, and let h and k be positive constants. Then,

1. $y = f(x) + k$ shifts the graph of $f(x)$ up by k.
2. $y = f(x) - k$ shifts the graph of $f(x)$ down by k.
3. $y = f(x + h)$ shifts the graph of $f(x)$ to the left by h.
4. $y = f(x - h)$ shifts the graph of $f(x)$ to the right by h.
5. $y = -f(x)$ reflects the graph of $f(x)$ vertically (across the x-axis).
6. $y = f(-x)$ reflects the graph of $f(x)$ horizontally (across the y-axis).

Polynomial and Rational Functions

- **Polynomials:** A polynomial of degree n, where n is a nonnegative integer, is defined by
$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$.
$a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers called **coefficients**.
a_n is called the **leading coefficient**.

The domain of any polynomial function is all real numbers.

Rational Functions

Rational functions are so named because they are a **ratio** of polynomial functions.

Rational functions are functions of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.

- **Asymptotes:** If a function grows without bound as x approaches some number k, then $x = k$ is a **vertical asymptote**.

To find vertical asymptotes for rational functions:
Set the denominator equal to 0 and solve for x.

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Exponential Functions

Exponential Functions are functions of the form $f(x) = a^x$, where $a > 0$, and $a \neq 1$.

PROPERTIES OF EXPONENTIAL FUNCTIONS

1. Domain: $(-\infty, \infty)$
2. Range: $(0, \infty)$ (But be careful—translations and reflections will alter the range.)

EXPONENTIAL GROWTH AND DECAY

Let y_0 be the amount or number of some quantity initially present ($t = 0$). Then the amount present at any time t is $y = y_0 e^{kt}$.

If $k > 0$, this is exponential growth and k is called the **growth constant**.

If $k < 0$, this is exponential decay and k is called the **decay constant**.

Logarithmic Functions

For $a > 0$, $a \neq 1$, and $x > 0$, $y = \log_a x$ means $a^y = x$. The logarithm, y, is the exponent to which you must raise a to produce x.

PROPERTIES OF LOGARITHMIC FUNCTIONS

1. Domain: $(0, \infty)$
2. Range: $(-\infty, \infty)$

PROPERTIES OF LOGARITHMS

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a x^r = r\log_a x$
4. $\log_a a = 1$
5. $\log_a 1 = 0$
6. $\log_a a^r = r$
7. $a^{\log_a x} = x$

SPECIAL NOTATIONS

- **The common log:** $\log_{10} x \Rightarrow \log x$
- **The natural log:** $\log_e x \Rightarrow \ln x$

CHANGE OF BASE FOR LOGS

If x, a, and b are positive, $a \neq 1$, $b \neq 1$, then $\log_a x = \frac{\log_b x}{\log_b a}$.

In particular, $\log_a x = \frac{\ln x}{\ln a}$.

LOGARITHMIC EQUATIONS

For $x > 0$, $y > 0$, $b > 0$, $b \neq 1$, if $x = y$, then $\log_b x = \log_b y$.
And if $\log_b x = \log_b y$, then $x = y$.

Limits

$\lim_{x\to a} f(x) = L$ is read as "the limit of $f(x)$ as x approaches a is L."

This means:
1. As x takes on values closer and closer (but not equal) to a on both sides of a, the corresponding values of $f(x)$ get closer and closer to L.
2. The value of $f(x)$ can be made as close to L as desired by taking values of x close enough to a.

ONE SIDED LIMITS

$\lim_{x\to a^-} f(x)$ is the limit as x approaches a from the left ($x < a$).

$\lim_{x\to a^+} f(x)$ is the limit as x approaches a from the right ($x > a$).

$\lim_{x\to a} f(x)$ only exists if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$.

RULES FOR LIMITS

Let a, A, and B be real numbers and let f and g be continuous functions such that $\lim_{x\to a} f(x) = A$ and $\lim_{x\to a} g(x) = B$.

1. If k is a constant, then $\lim_{x\to a} k = k$.
2. $\lim_{x\to a} k \cdot f(x) = k \lim_{x\to a} f(x) = kA$
3. $\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = A \pm B$
4. $\lim_{x\to a} [f(x) \cdot g(x)] = \left[\lim_{x\to a} f(x)\right] \cdot \left[\lim_{x\to a} g(x)\right] = A \cdot B$
5. $\lim_{x\to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{A}{B}$, $B \neq 0$.
6. If $p(x)$ is a polynomial, then $\lim_{x\to a} p(x) = p(a)$.
7. For any real number k, $\lim_{x\to a} \left[f(x)\right]^k = \left[\lim_{x\to a} f(x)\right]^k = A^k$, provided this limit exists.
8. $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$ if $f(x) = g(x)$, for all $x \neq a$.
9. For any real number b, $b > 0$, $\lim_{x\to a} b^{f(x)} = b^{\left[\lim_{x\to a} f(x)\right]} = b^A$.
10. For any real number b, $0 < b < 1$, or $b > 1$,
$\lim_{x\to a} \log_b(f(x)) = \log_b\left(\lim_{x\to a} f(x)\right) = \log_b A$; $A > 0$.

LIMITS AT INFINITY

For any positive real number n, $\lim_{x\to\infty} \frac{1}{x^n} = 0$ and $\lim_{x\to-\infty} \frac{1}{x^n} = 0$.

(If x is negative, x^n does not exist for certain values of n, so the second limit is undefined.)

Let $p(x)$ and $q(x)$ be polynomials, $q(x) \neq 0$. To find
$$\lim_{x\to\infty} \frac{p(x)}{q(x)} \text{ or } \lim_{x\to-\infty} \frac{p(x)}{q(x)}:$$

1. Divide the numerator and denominator by x raised to the highest power of x appearing in either polynomial.
2. Then find the limit of the result from Step 1 by using the rules for limits, including the rules
$\lim_{x\to\infty} \frac{1}{x^n} = 0$ and $\lim_{x\to-\infty} \frac{1}{x^n} = 0$.

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CONTINUITY AT $x = c$

$f(x)$ is continuous at $x = c$ if
1. $f(c)$ is defined. 2. $\lim_{x\to c} f(x)$ exists. 3. $\lim_{x\to c} f(x) = f(c)$.

Rate of Change and the Derivative

- **Average rate of change:** $\frac{f(b) - f(a)}{b - a}$ is the average rate of change of the function f between $x = a$ and $x = b$.
- **Instantaneous rate of change:** $\lim_{h\to 0} \frac{f(a + h) - f(a)}{h}$ is the instantaneous rate of change of $f(x)$ at $x = a$.

A function which describes the instantaneous rate of change (or slope of tangent line) of $f(x)$ at any point x is called the derivative.

- **Derivative:** $f'(x) = \lim_{h\to 0} \frac{f(x + h) - f(x)}{h}$

TECHNIQUES FOR FINDING DERIVATIVES

- **Notation:** $f'(x)$, $\frac{dy}{dx}$, $\frac{d}{dx}[f(x)]$, $D_x[f(x)]$

1. Constant Rule: If $f(x) = k$, then $f'(x) = 0$.
2. Power Rule: If $f(x) = x^n$, then $f'(x) = nx^{n-1}$.
3. Constant Times a Function: $\frac{d}{dx}[kf(x)] = k \cdot f'(x)$
4. Sum or Difference:
If $f(x) = u(x) \pm v(x)$, then $f'(x) = u'(x) \pm v'(x)$.
5. Product Rule:
If $f(x) = u(x) \cdot v(x)$, then $f'(x) = u(x)v'(x) + v(x)u'(x)$.
6. Quotient Rule: If $f(x) = \frac{u(x)}{v(x)}$ and $v(x) \neq 0$, then
$f'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{[v(x)]^2}$.
7. Chain Rule: If $y = f(u)$ and $u = g(x)$ so that $y = f(g(x))$,
then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ or $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$.
8. $\frac{d}{dx}[e^x] = e^x$
9. $\frac{d}{dx}[a^x] = a^x \cdot \ln a$
10. $\frac{d}{dx}[a^{g(x)}] = (\ln a)a^{g(x)} \cdot g'(x)$
11. $\frac{d}{dx}\left[\ln |g(x)|\right] = \frac{g'(x)}{g(x)}$

USES OF THE FIRST DERIVATIVE

- Instantaneous rate of change
- Slope of the tangent line
- Critical numbers, intervals of increase and decrease of a function, and relative extrema
- Marginal revenue, marginal profit, and marginal cost
- Velocity, $v(t)$

Relative (Local) Extrema

TEST TO FIND INTERVALS WHERE $f(x)$ IS INCREASING/DECREASING

Suppose a function has a derivative at each point in the open interval.

1. If $f'(x) > 0$ for each x in the interval, then $f(x)$ is increasing on the interval.
2. If $f'(x) < 0$ for each x in the interval, then $f(x)$ is decreasing on the interval.
3. If $f'(x) = 0$ for each x in the interval, then $f(x)$ is constant on the interval.

CRITICAL NUMBERS

A critical number is any number c for which $f'(c) = 0$ or $f'(c)$ is undefined.

RELATIVE EXTREMA

1. $f(c)$ is a relative maximum on (a, b) if $f(x) \leq f(c)$ for all x in (a, b).
2. $f(c)$ is a relative minimum on (a, b) if $f(x) \geq f(c)$ for all x in (a, b).

The term relative extremum refers to either a maximum or a minimum.

If f has a relative extremum at $x = c$, then c is a critical number of $f(x)$.

FIRST DERIVATIVE TEST

Let c be a critical number for a function. Suppose that $f(x)$ is continuous on (a, b) and differentiable on (a, b) except possibly at c and that c is the only critical number on (a, b).

1. $f(c)$ is a relative maximum if f' changes from positive to negative at $x = c$.
2. $f(c)$ is a relative minimum if f' changes from negative to positive at $x = c$.

Absolute Extrema of a Function f on an Interval $[a, b]$

Absolute extrema only occur at critical values of f or at the endpoints of the interval, a or b.

FINDING ABSOLUTE EXTREMA FOR f ON $[a, b]$

1. Find all critical numbers for f in (a, b).
2. Evaluate $f(x)$ for all critical numbers in (a, b) (ignore any critical numbers not within the given interval).
3. Evaluate $f(x)$ at the given endpoints of the interval, namely a and b.
4. The largest value for $f(x)$ is the absolute maximum and the smallest value for $f(x)$ is the absolute minimum.

Applied Calculus Review

Higher Derivatives

Notation: 2^{nd} Derivative: $f''(x)$, $\dfrac{d^2y}{dx^2}$, $D_x^2(f(x))$, y''

3^{rd} Derivative: $f'''(x)$, $\dfrac{d^3y}{dx^3}$, $D_x^3(f(x))$, y'''

For derivatives $\geq 4^{\text{th}}$, the nth derivative is $f^{(n)}(x)$.

The Second Derivative and Its Meaning

The second derivative determines **concavity**.

CONCAVITY

A function is concave up on an interval if f' is increasing on the interval.

A function is concave down on an interval if f' is decreasing on the interval.

A function is concave up on an interval (a, b) if the graph of $f(x)$ lies above the tangent line for each point in (a, b).

A function is concave down on an interval (a, b) if the graph of $f(x)$ lies below the tangent line for each point in (a, b).

TEST FOR CONCAVITY

Let $f(x)$ be a function with derivatives $f'(x)$ and $f''(x)$ existing at all points in an interval (a, b).

Then $f(x)$ is concave up on (a, b) if $f''(x) > 0$ and $f(x)$ is concave down on (a, b) if $f''(x) < 0$.

Point of inflection: A point of inflection is a point at which concavity changes. At any point of inflection, $f''(x) = 0$ or $f''(x)$ does not exist.

SECOND DERIVATIVE TEST TO DETERMINE RELATIVE (LOCAL) EXTREMA

Let $f(x)$ exist on some open interval containing c and let $f'(c) = 0$ (c is a critical value), then

1. If $f''(c) > 0$, then a relative minimum occurs at c.
2. If $f''(c) < 0$, then a relative maximum occurs at c.
3. If $f''(c) = 0$, then the test gives no information.

USES OF THE SECOND DERIVATIVE

- Concavity
- Point(s) of inflection
- Acceleration (second derivative of position)
- Point of diminishing returns

Implicit Differentiation

When $y = f(x)$, y is said to be "explicitly defined in terms of x." Functions can also be defined implicitly.

The process of implicit differentiation:

- Treat y like it is some unknown function of x. To find the derivative of the unknown function, you must use the chain rule. The derivative of the "unknown" function is $\frac{dy}{dx}$.
- Simplify to solve for $\frac{dy}{dx}$.

Related Rates

1. Identify all given quantities; draw a sketch.
2. Write an equation involving all variables in the problem.
3. Use implicit differentiation to find derivatives on both sides of the equation.
4. Solve for the derivative giving the unknown rate of change and substitute the given quantities. (Only substitute in the numbers AFTER taking the derivatives.)

Antiderivatives

THE INDEFINITE INTEGRAL: RULES FOR INTEGRATION (INDEFINITE INTEGRALS)

1. Power Rule: $\int u^n du = \dfrac{u^{n+1}}{n+1} + C \qquad (n \neq -1)$
2. Constant Multiple Rule: $\int k \cdot f(x)\, dx = k \int f(x)\, dx$
3. Sum/Difference Rule: $\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$
4. $\int e^u\, du = e^u + C$
5. $\int e^{ku}\, du = \dfrac{e^{ku}}{k} + C$
6. $\int u^{-1}\, du = \ln|u| + C$

THE DEFINITE INTEGRAL

The **Fundamental Theorem of Calculus:** Let f be continuous on $[a, b]$ and let F be any antiderivative of f, then $\int_a^b f(x)\, dx = F(x)\big|_a^b = F(b) - F(a)$

Properties of the definite integral:

1. $\int_a^a f(x)\, dx = 0$
2. $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$
3. $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

FINDING AREA UNDER $f(x)$ FROM $x = a$ TO $x = b$

To determine the area bounded by f, $x = a$, $x = b$, and the x-axis

1. Sketch the graph.
2. Find any x-intercepts of f in $[a, b]$. Consider each subregion defined by these x-intercepts individually.
3. The definite integral will be positive for subregions above the x-axis, and will be negative for subregions below the x-axis. Use separate integrals to compute these areas. Area can only be positive, so take the absolute value of the integral for regions lying below the x-axis.
4. The total area from $x = a$ to $x = b$ will be the sum of the areas of all the subregions.

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AREA BETWEEN TWO CURVES

If $f(x)$ and $g(x)$ are continuous, and $f(x) \geq g(x)$ on $[a, b]$, then the area between $f(x)$ and $g(x)$ from $x = a$ to $x = b$ is $\int_a^b [f(x) - g(x)]\, dx$ or $\int_a^b$[top − bottom] dx.

INTEGRATION BY PARTS

$\int u dv = uv - \int v du$

VOLUME

If f is a non-negative function and R is a region between f and the x-axis from $x = a$ to $x = b$, then the volume formed by rotating R about the x-axis is $V = \pi \int_a^b [f(x)]^2\, dx$.

AVERAGE VALUE OF A FUNCTION

The average value of $f(x)$ on $[a, b]$ is $\dfrac{1}{b-a}\int_a^b f(x)\, dx$.

IMPROPER INTEGRALS

$$\int_a^\infty f(x)\, dx = \lim_{b\to\infty}\int_a^b f(x)\, dx$$

$$\int_{-\infty}^b f(x)\, dx = \lim_{a\to -\infty}\int_a^b f(x)\, dx$$

$$\int_{-\infty}^\infty f(x)\, dx = \lim_{a\to -\infty}\int_a^c f(x)\, dx + \lim_{b\to\infty}\int_c^b f(x)\, dx$$

If the limits exist, then the integrals are called "convergent." If not, they are "divergent."

Multivariable Calculus

FUNCTIONS IN THREE DIMENSIONAL SPACE

Equation of a plane: $ax + by + cz = d$ is a plane if a, b, and c are not all zero.

The graph of $z = f(x)$ is a surface in three dimensional space.

QUADRIC SURFACES

1. Paraboloid: $z = x^2 + y^2$

   Ellipsoid: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$
2. Hyperbolic Paraboloid: $x^2 - y^2 = z$
3. Hyperboloid of Two Sheets: $-x^2 - y^2 + z^2 = 1$

PARTIAL DERIVATIVES

The partial derivative of $f(x, y)$ with respect to x is obtained by treating y as a constant and x as a variable. The partial derivative of $f(x, y)$ with respect to y is obtained by treating x as a constant and y as a variable.

NOTATION

The partial derivative of $f(x, y)$ with respect to x is written $f_x(x, y)$, $\dfrac{\partial f}{\partial x}$ or $\dfrac{\partial}{\partial x}f(x, y)$.

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The partial derivative of $f(x, y)$ with respect to y is written $f_y(x, y)$, $\dfrac{\partial f}{\partial x}$ or $\dfrac{\partial}{\partial y}f(x, y)$.

SECOND ORDER PARTIAL DERIVATIVES

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = f_{xx} = \frac{\partial^2 z}{\partial x^2} = z_{xx} \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = f_{yy} = \frac{\partial^2 z}{\partial y^2} = z_{yy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = f_{yx} = \frac{\partial^2 z}{\partial x \partial y} = z_{yx} \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = f_{xy} = \frac{\partial^2 z}{\partial y \partial x} = z_{xy}$$

RELATIVE MAXIMA AND MINIMA

If $z = f(x, y)$ has a relative maximum or minimum at (a, b) and $f_x(a, b)$ and $f_y(a, b)$ exist, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

TEST FOR RELATIVE EXTREMA

For $z = f(x, y)$, let f_{xx}, f_{yy}, and f_{xy} all exist in a circular region in the xy-plane with center (a, b). Let $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Define D by $D = f_{xx}(a, b) \cdot f_{yy}(a, b) - [f_{xy}(a, b)]^2$.

1. $f(a, b)$ is a relative maximum if $D > 0$ and $f_{xx}(a, b) < 0$.
2. $f(a, b)$ is a relative minimum if $D > 0$ and $f_{xx}(a, b) > 0$.
3. $f(a, b)$ is a saddle point if $D < 0$.
4. If $D = 0$, the test is inclusive.

DOUBLE INTEGRALS

Properties:

Let $z = f(x, y)$ be a nonnegative function on the region R defined by $c \leq x \leq d$ and $a \leq y \leq b$. Then the volume over the rectangular region is given by $\int_a^b \int_c^d f(x, y)\, dx\, dy = \int_c^d \int_a^b f(x, y)\, dy\, dx$.

Double integrals over variable regions:

Let $z = f(x, y)$ be a nonnegative function of two variables. If R is defined by $c \leq x \leq d$ and $g(x) \leq y \leq h(x)$, then the volume over R is obtained by $\int_c^d \int_{g(x)}^{h(x)} f(x, y)\, dy\, dx$.

Let $z = f(x, y)$ be a nonnegative function of two variables. If R is defined by $g(y) \leq x \leq h(y)$ and $a \leq y \leq b$, then the volume over R is obtained by $\int_a^b \int_{g(y)}^{h(y)} f(x, y)\, dx\, dy$.

Differential Equations

A differential equation is an equation that contains an unknown function $y = f(x)$ and a finite number of derivatives. The solution to a differential equation is a function.

SOLVING DIFFERENTIAL EQUATIONS—GENERAL SOLUTIONS VERSUS PARTICULAR SOLUTIONS

To solve differential equations, you must integrate. Whenever a problem involves integration, a "C" must be added. When no additional information is given to allow you to find the numerical value for C, a "general solution" is obtained.

If you are given additional information about the problem, such as the value of $f(x)$ when $x = 0$ (called the **initial value**), then you will be able to find the numerical value for C. This will yield a "particular solution."

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SEPARATION OF VARIABLES

Separation of variables is used to solve a differential equation problem of this type: $\dfrac{dy}{dx} = \dfrac{f(x)}{g(y)}$, if $g(y) \neq 0$.

Cross multiply and integrate both sides: $\int g(y)\, dy = \int f(x)\, dx$.

Sequences and Series

Geometric sequence: A sequence in which each term after the first term is found by multiplying the preceding term by some number, r, (called the common ratio) is a geometric sequence.

THE GENERAL TERM OF A GEOMETRIC SEQUENCE

$a_n = ar^{n-1}$

SUM OF THE FIRST n TERMS OF A GEOMETRIC SEQUENCE

If a geometric sequence has a first term a and a common ratio r, then the sum of the first n terms is $s_n = \sum_{i=1}^{n} ar^{i-1} = \dfrac{a(r^n - 1)}{r - 1}$, $r \neq 1$.

SUM OF AN INFINITE GEOMETRIC SEQUENCE

A geometric series converges to $\dfrac{a}{1-r}$ if r is in $(-1, 1)$ and diverges if r is not in $(-1, 1)$.

INFINITE SERIES

An infinite series is the sum of an infinite sequence of numbers, called "terms:" $a_1 + a_2 + a_3 + \cdots = \sum_{i=1}^{\infty} a_i$.

The nth partial sum of an infinite series is the sum of the first n terms of the series $S_n = a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^{n} a_i$.

Suppose $\lim_{n\to\infty} S_n = L$ for some real number L. Then L is the sum of the infinite series $a_1 + a_2 + a_3 + \cdots$, and the series converges. If no such limit exists, the series diverges and has no sum.

Taylor Polynomials and Taylor Series

Taylor polynomials are polynomial approximations to non-polynomial functions close to particular values for x.

TAYLOR SERIES

If all derivatives of $f(x)$ exist at $x = 0$, then the Taylor series for $f(x)$ centered at $x = 0$ is

$P(x) = f(0) + \dfrac{f'(0)}{1!}x + \dfrac{f^{(2)}(0)}{2!}x^2 + \cdots$.

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COMMON TAYLOR SERIES

Function	Taylor Series	Interval of Convergence
e^x	$1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n + \cdots$	$(-\infty, \infty)$
$\ln(1 + x)$	$-x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots - \frac{x^n}{n} - \cdots$	$[-1, 1)$
$\frac{1}{1-x}$	$1 + x + x^2 + x^3 + \cdots + x^n + \cdots$	$(-1, 1)$

L'Hospital's Rule

Used for indeterminate forms (limits which produce $\frac{0}{0}$).

To use L'Hospital's rule:

1. Be sure $\lim_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{0}{0}$.
2. Take the derivative of the numerator and the derivative of the denominator independently.
3. Find $\lim_{x\to a} f'(x)$ and $\lim_{x\to a} g'(x)$ and look at the ratio $\dfrac{\lim_{x\to a} f'(x)}{\lim_{x\to a} g'(x)}$. If this limit exists, then it is $\lim_{x\to a}\dfrac{f(x)}{g(x)}$.
4. If $\dfrac{\lim_{x\to a} f'(x)}{\lim_{x\to a} g'(x)} = \dfrac{0}{0}$, then use L'Hospital's rule again.

Derivatives and Antiderivatives of Trigonometric Functions

Derivatives	Antiderivatives
$\frac{d}{dx}[\sin x] = \cos x$	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}[\csc x] = -(\csc x \cot x)$	$\int \csc^2 x\, dx = -\cot x + C$
	$\int \sec x \tan x\, dx = \sec x + C$
	$\int \csc x \cot x\, dx = -\csc x + C$