Valuation of Timber and Carbon Sequestration: An American Call Option Technique

Stanislav Petrasek John Perez-Garcia Bruce Lippke
B. Bruce Bare

School of Forest Resources
University of Washington, Seattle, WA

Third International Faustmann Symposium, 2009
Darmstadt, Germany

Outline

Problem Statement

- Properties:
  - Classical Faustmann problem: Choose rotation length to maximize bare land value over multiple harvest cycles subject to silvicultural and economic constraints.
  - Modification: Introduce uncertainty via prices of timber and carbon.
- Objectives:
  - Determine the value of bare land value under uncertainty in prices of timber and carbon.
  - Determine optimal harvest strategy in stochastic settings.
  - Determine the impact of carbon sequestration on optimal time of stand harvest.

Stochastic Faustmann Problem as Real Option

IF Land ownership is viewed as the right to exchange timber for harvest cost and sell it in the market at prevailing price.

THEN Valuation of bare land value under price uncertainty parallels the valuation of a multi-period American call option.

American Call	**Bare Land Value**
Underlying Asset	Timber and Carbon
Strike Price	Harvest Cost
Contract Length	Planning Horizon
Exercise Time	Harvest Time

Solution Algorithm I

- Many techniques have been developed for valuation of American options
- Ibáñez and Zapatero algorithm is a good example
  - Allows multiple sources of uncertainty of asset prices
  - Explicitly calculates optimal exercise policy
- Ibáñez and Zapatero algorithm can be applied to the calculation of bare land value
  - Allows timber and CO_2 prices as sources of uncertainty
  - Calculates optimal harvest policy as function of prices and age

Solution Algorithm II

- Unmodified, Ibáñez and Zapatero algorithm would have limited application for bare land valuation:
  - Assumes American option with one exercise opportunity
  - Equivalent to assuming a single harvest cycle
- Extending the algorithm enables broader application:
  - Allows multiple exercise opportunities
  - Equivalent to harvest cycles characteristic of the Faustmann problem
- The extension produces results that are both more realistic and accurate

Basic Assumptions

- `Bare Land Value`: Calculated in USD per acre
- `Price Models`: Timber and carbon prices assumed to follow a logarithmic mean reverting process
- `Harvest Cost`: Fixed and known at all times
- `Discount Rate`: Fixed and known at all times
- `Silviculture`: Douglas fir regime with planting followed by a regeneration clear cut final harvest.
- `Yield Function`: Assumed high yield site in Western Washington State (No risk due to fire, disease or wind.)

Stochastic Price Model

- Many models for price behavior are available

- The model used for this example is a log mean-reverting stochastic process:

$$dS_t = \kappa(\mu - \ln S_t)\, S_t\, dt + \sigma\, S_t\, d\mathbb{W}_t, \qquad (1)$$

Where: S_t is the price at time t, κ is the rate of mean reversion, μ is the log of long term price, σ represents price volatility and $d\mathbb{W}_t$ is an increment of the Wiener process

- The price model in equation 1 was used to model both timber and carbon prices

Carbon Treatment

Three basic carbon pools are considered in this study:

- `Forest Pool:` All carbon contained in a standing forest.
- `Product Pool:` All carbon contained in harvested wood products.
- `Substitution Pool:` All carbon not released into the atmosphere when harvested wood products displace fossil-based alternatives. (Avoided emissions)

Cash Flows

- Profit at time t:

$$\pi_t = \max[CF_T^t + d_t\,\mathcal{F}\mathcal{H};\; CF_C^t + \mathbb{E}[d_t\,\pi_{t+1}]] \tag{2}$$

- Cash flow if harvest does not occur at time t:

$$CF_C^t = \Delta\,Q_t\,P_C \tag{3}$$

- Cash flow if harvest does occur at time t:

$$CF_T^t = Q_t\,(P_T^t - \gamma\,(\alpha_F - \alpha_P - \alpha_S)\,P_C^t - C) \tag{4}$$

- Where: Q_t = timber volume; C = fixed harvest cost; α_F, α_P, α_S = fractions of carbon in forest, product and substitution pools; and γ = scaling parameter that converts carbon in wood to atmospheric CO_2.

Carbon Scenarios

- Scenarios constructed from three sets of values of α_i in equation 4:

$$CF_T^t = Q_t \left(P_T^t - \gamma \left(\alpha_F - \alpha_P - \alpha_S \right) P_C^t - C \right) \tag{4}$$

Scenario	α_F	α_P	α_S
No. 1	0.80	0.2	0.2
No. 2	0.80	0.25	1.0
No. 3	0.80	0.35	2.0

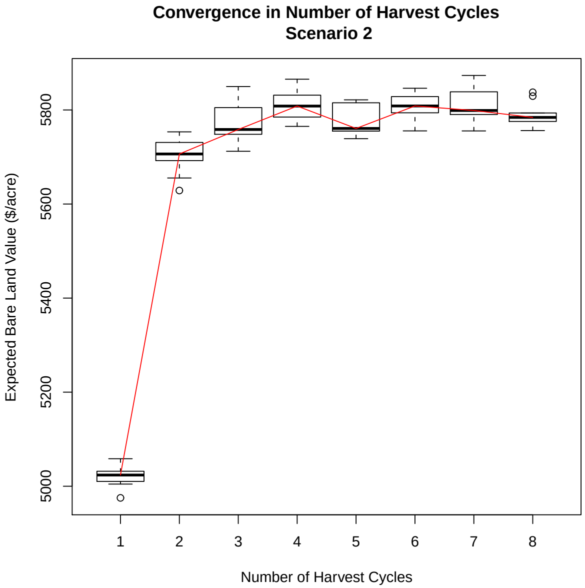
- No. 1: $\alpha_F > \alpha_P + \alpha_S \Rightarrow$ Increased total harvest cost
- No. 2: $\alpha_F < \alpha_P + \alpha_S \Rightarrow$ Increased harvest revenue
- No. 3: $\alpha_F \ll \alpha_P + \alpha_S \Rightarrow$ Increased harvest revenue

Parameter Values

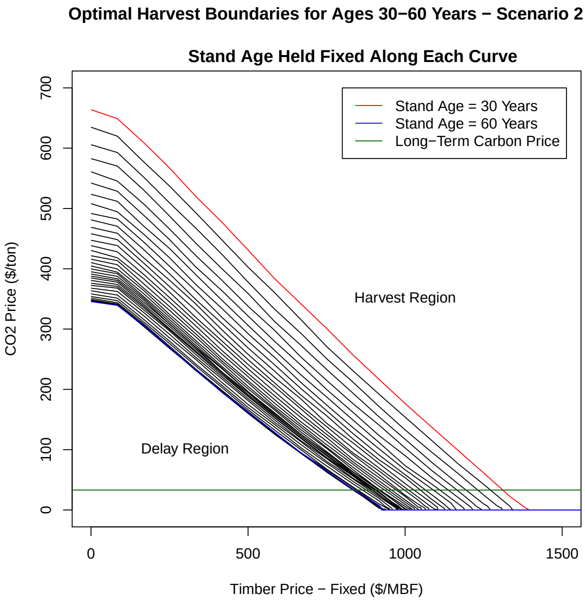
These parameter values were used in all simulations unless stated otherwise.

Parameter	Unit	Timber	Carbon
Initial Price S_0		400 ($/MBF)	25 ($/ton)
Long-term Price	$	665	33
Reversion Rate κ	%/year	0.33	4.0
Volatility σ	%/year	0.25	0.5
Correlation ρ	%	10	
Harvest Cost C	$/MBF	100	
Discount Rate r	%/year	5	
Simulation Horizon T	years	100	
Harvest Time	year	Anytime before T	

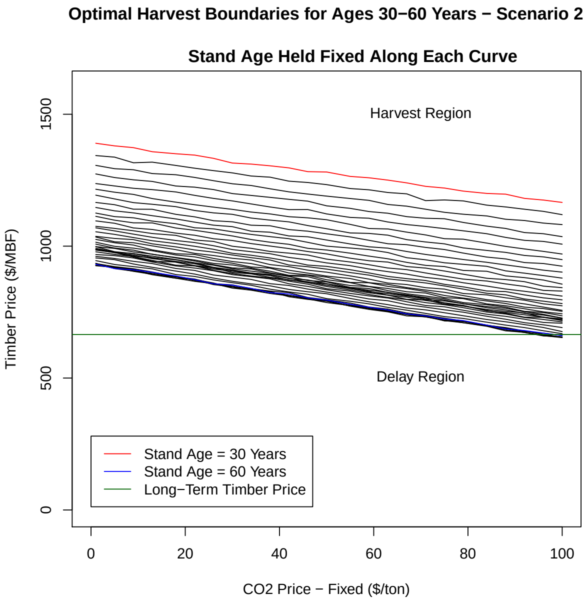
Harvest Cycle Contribution



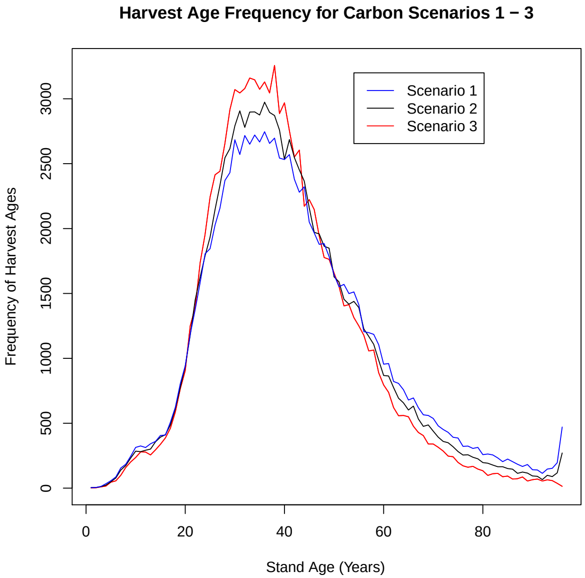
Decision Boundaries - Carbon



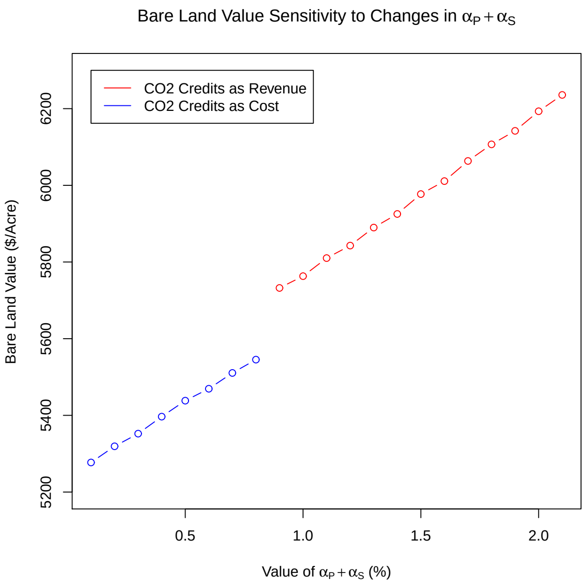
Decision Boundaries - Timber



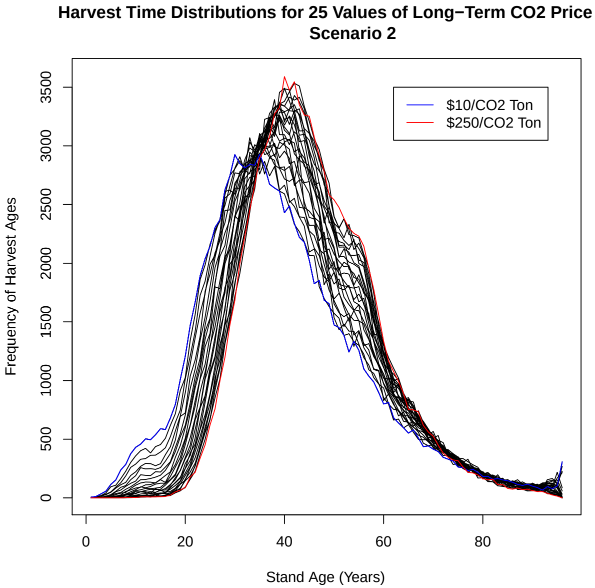
Harvest Age Frequency: Scenarios 1 - 3



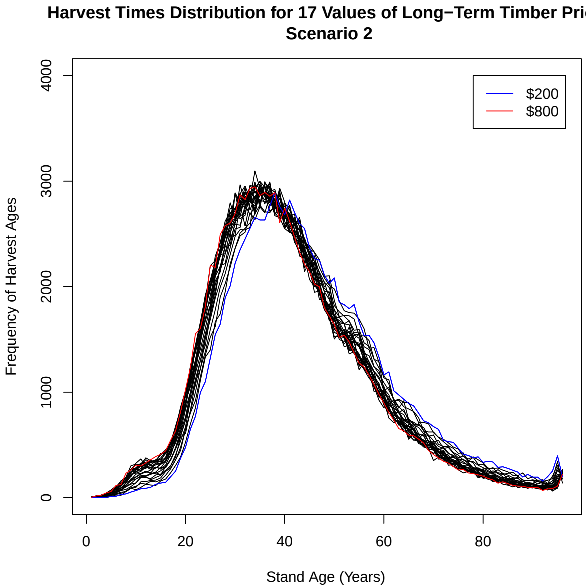
Bare Land Value: Carbon Pool Sensitivity



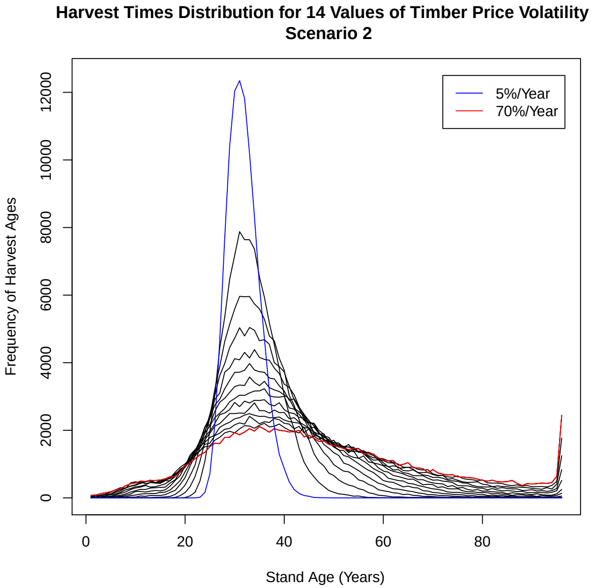
Harvest Time Frequency: CO_2 Price Sensitivity



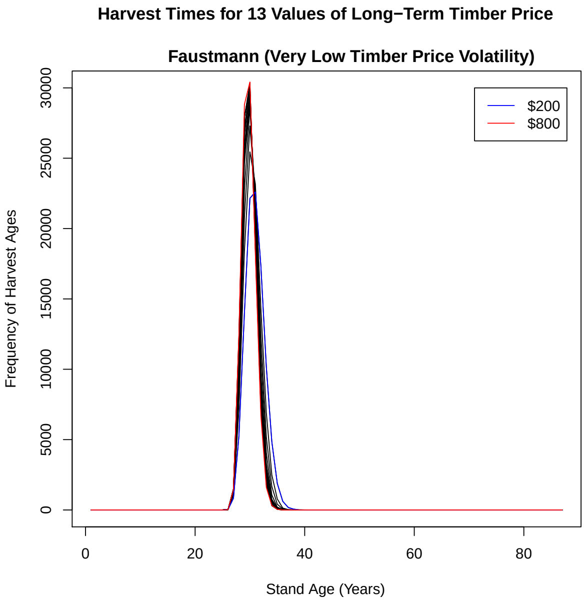
Harvest Time Frequency: Timber Price Sensitivity



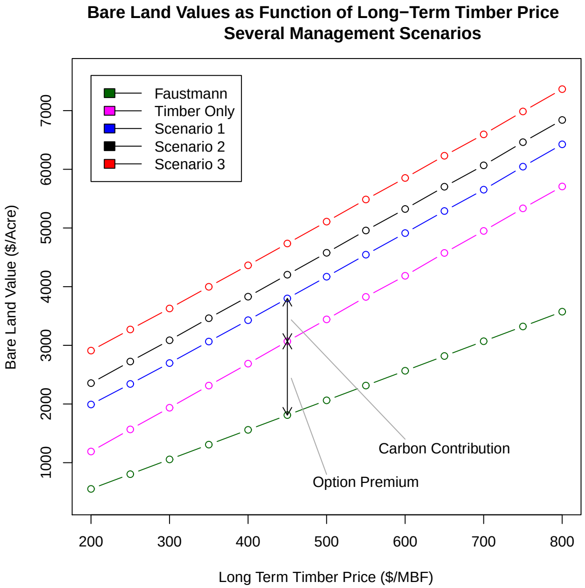
Timber Price Volatility Sensitivity



Harvest Time Frequency: Faustmann



Bare Land Value: Long Term Timber Price



Summary

- Modified Ibáñez and Zapatero algorithm provides a practical methodology for determining expected bare land value under stochastic timber and CO_2 prices.
- Future harvest cycles make a significant contribution to expected bare land value under stochastic timber and CO_2 prices.
- Profitability of carbon sequestration determined jointly by CO_2 price and credit policy.

- Outlook
  - More realistic price models
  - Additional sources of uncertainty
  - Faster, more efficient computation

References I

- Alfredo Ibáñez and Fernando Zapatero.
  Monte Carlo valuation of American options through computation of the optimal exercise frontier.
  Journal of Financial and Quantitative Analysis, 39(2):253-275, 2004.

- John Perez-Garcia, Bruce Lippke, Jeffrey Comnick and Carolina Manriquez.
  An Assessment Of Carbon Pools, Storage, and Wood Products Market Substitution Using Life-Cycle Analysis Results.
  Wood and Fiber Science, 37 CORRIM Special Issue:140-148, 2005.