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Computing Maximum Willingness to Pay with Faustmann's Formula: Some Special Situations from New Zealand[1]

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Abstract

Faustmann's theory of discounted cash flow analysis, developed using perpetually recurring before-tax cash flows, facilitates the determination of the maximum willingness to pay (WTP) (i.e., economic) value of bare forestland and immature timber. He describes three valuation approaches: (1) compounding costs, (2) compounding/discounting annuities, and (3) discounted future cash flows. If non-recurring costs (or benefits) are incurred in any rotation or if the current stand is non-optimal with respect to future rotations, the discounting of future cash flows is the preferred approach. When after-tax cash flows are considered, four categories of tax treatment must be addressed: (1) immediately deductible expenditures, (2) capitalized expenditures which are deducted against future harvest revenue, (3) capitalized expenditures which are depreciated or amortized, and (4) non-deductible expenditures. Illustrations of the consequences of the four categories of tax treatment are shown with respect to New Zealand pine plantations. Extension to the tax policies of other countries is straightforward.

Keywords: Forest valuation, forest economics, income tax, bare land value, stand value, discount rate.

Introductory Background

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Faustmann (1849) published one of the earliest known applications of the principle of discounted cash flow analysis. His analysis considered: 1) the calculation of the value of bare forestland when in forestry use; 2) intermittent and sustained yield management scenarios; 3) procedures for valuing bare land as well as the immature timber growing thereon using compound interest formulae; and 4) normal and non-normal existing immature timber stands. The analysis assumed: 1) before-tax cash flows arising from the growing and harvesting of even-aged timber crops in perpetuity; 2) constant timber growing costs and prices known with certainty; 3) a price taking timber producer; 4) that only timber values are relevant; and 5) no economies of scale.

Assuming intermittent management, Faustmann (1849) illustrated how to calculate the value of bare forestland using two different approaches that led to the same result. His first approach converts all incomes and expenditures to annuities, with the difference between the two defined as the annual land rental. Capitalization of this quantity yields the bare forestland value. The second approach uses a discounted cash flow analysis wherein all incomes and expenditures occurring until infinity are reduced to their present value equivalents. The value of bare forestland is calculated as the difference between discounted incomes and discounted expenditures.

Faustmann (1849) also considers the situation where land currently carries a timber stand. Assuming normal yields, he calculates the maximum willingness to pay (WTP) (i.e., economic) value of a stand using three different methods: (a) compounding costs, (b) compounding/discounting annuities, and (c) discounting future cash flows.

Special Conditions

Faustmann (1849) also considers the case where the current stand is non-normal. He calculates the value of this abnormal (i.e., understocked) stand by discounting future cash flows. In doing so, he takes account of the depressed yields during the first rotation and assumes that the normal (i.e., optimal) stand is created for all subsequent rotations. He demonstrates that the land value under the abnormal stand is the same as calculated when imagining normal yields, because “one need only fell and regenerate the present stand in order to create fully stocked conditions on the land immediately” (Faustmann 1849). From this he concludes that “the land value remains the same, whether the area carries a stand or not, whatever the age of the stand, and no matter whether it is fully stocked or abnormal; the difference [in the value of the forest] is attributable solely to differences in the stand value” (Faustmann 1849).

In addition to the case where the current crop is sub-optimal (i.e., abnormal), another special case (not explicitly discussed by Faustmann) involves the occurrence of non-recurring costs. Here, as with the abnormal stand case, the discounting of future cash flows is the preferred valuation strategy, as the other two valuation approaches do not reliably yield the correct answer. This conclusion is illustrated via a simple illustrative example later in this paper.

Although Faustmann (1849) does not consider the effects of income taxes, in many situations it is desirable to base the calculations on after-tax cash flows. One such case is the recently adopted New Zealand Institute of Forestry Forest Valuation Standards (1999), which adopts the convention that when valuation is based on the expectation approach, after-tax cash flows should be used. In particular, taxable income is determined in accordance with the provisions of the Income Tax Act of 1994 as they apply to forestry and the tax rate is at the corporate rate current at the date of valuation. In this case, the expectation approach refers to the discounted value of the cash flows that the crop and/or land generate in the future. Three specific terms are defined:

- Forest Expectation Value (FEV)
  The present value of cash flows arising from both the land and the crop in perpetuity.
- Land Expectation Value (LEV)
  The present value of a perpetual series of crop rotations on the land, the land being bare of the crop at the commencement of the series.
- Crop Expectation Value (CEV)

The present value of cash flows arising from the crop where the cost of land is included as a notional rent calculated as the discount rate applied to the LEV. The CEV may be alternatively calculated as CEV = FEV - LEV. It is assumed that the owner of the crop also owns the land. Therefore the land rental is a notional rather than actual payment – it represents the opportunity cost of the land by using it for the current crop.

In contrast to expectation values, a comparable set of market values realized by the sale of the crop and/or the land between a willing buyer and willing seller in an arm's length transaction may be used if suitable transaction evidence is available for the purpose in terms of reliability, comparability, and volume of transactions. However, throughout this paper, we rely on the expectation approach to valuation under intermittent management as defined above.

When income taxes are considered, four categories of tax treatment must be addressed: (1) immediately deductible expenditures, (2) capitalized expenditures which are deducted against future harvest revenue, (3) capitalized expenditures which are depreciated or amortized, and (4) non-deductible expenditures.

Objectives

This paper reports the results of a study to test, for a number of different cases, the hypothesis that the economic value for an immature timber stand determined by discounting future cash flows is identical to that determined by the compounding of costs method. The hypothesis is tested for the following cases:

- before-tax cash flows (classical Faustmann)
- before-tax cash flows with non-recurring costs
- before-tax cash flows with a non-optimal current stand
- after-tax cash flows considering, in turn, each of the four categories of tax treatment
- after-tax cash flows with non-recurring costs

Appendix 1 gives a proof of the equivalence between the compounding costs method and the discounting of future cash flows method under the classical Faustmann assumptions.[4] This paper illustrates, by reference to the proof in Appendix 1 and by the use of an illustrative example, the conditions under which the hypothesis can and cannot be rejected.

In the New Zealand context, calculation of the economic stand value is generally for the purpose of estimating the market value of a tree crop. However, the results discussed here also hold when the purpose is to estimate an investor's WTP for a stand or the "value-in-use" of a crop to a current owner is desired.

Illustrative Example

Before-tax Calculations

Consider the simple classical Faustmann forest investment example shown in Table 1. The details of the regime and the absolute costs and revenues assumed are not important for the paper. It is assumed that a normal (i.e., optimal) stand is developed by planting bare land at time 0 (i.e., the end of year 0) at a cost of $1000/ha. Subsequent silvicultural and overhead costs and clear fell revenues all occur at the end of the specified year. For example, clear felling occurs at the end of year 28 (stand age 28). Replanting is assumed to occur immediately after harvesting (regeneration lag of 0 years), with the timing and cost of operations for the second and subsequent rotations identical to that of the first rotation. All revenues and costs are expressed in real terms and remain fixed in subsequent rotations.

The LEV shown in Table 1 is calculated from the net present value (NPV) of the first rotation owing to the above assumption of consistency in the timing and cost of operations for all rotations.

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[4] Except for one instance (i.e., Table 3) we do not utilize the compounding/discounting of annuities approach in our paper. Thus, this method is not included in Appendix 1.

This standard calculation uses before-tax cash flows and a real annual discount rate of 9%. As defined, the NPV covers only one rotation and does not include the notational land rent. However, the LEV represents the maximum WTP for bare land over an infinite series of rotations. Thus, it accounts for the cost of land. The two quantities (i.e., NPV and LEV) are not equivalent in concept or numeric value.

The CEV may be determined using any of the three valuation approaches developed by Faustmann. As an example, we calculate the CEV for a stand at age 5 (immediately after costs at time 5 years have been incurred). Table 2 shows the detailed calculation of CEV for the 5 year-old stand by compounding costs (incurred from time 0 to time 5) forward at the 9% before-tax discount rate. Note that the calculation also includes the notional land rent as a cost. Table 3 details the compounding/discounting annuities approach and in Table 1, the CEV is calculated using the discounted cash flow approach as the difference between FEV - LEV. The FEV is calculated following the procedure described by Davis and Johnson (1987) as:

$$FEV_n = (FV_k \text{ (remainder of current rotation)} + LEV) / 1.0p^{k-n}$$

where FV_k = Future value for remainder of current rotation if harvested at age k
n = Current stand age; k = Stand age at harvest; p = Percent annual interest rate

For our example, FEV at age n=5 is computed as:

$$FEV_5 = (70000 - 450*1.09^{22} - 450*1.09^{20} - 400*1.09^{18} - 100*((1.09^{23} -1)/.09) + 3585.57) / 1.09^{23}$$

thus	FEV = 8160.40 ($/ha)
less	LEV = 3585.57 ($/ha)
equals	CEV = 4574.83 ($/ha)

All three approaches yield identical numerical results when applied to the before-tax classical Faustmann situation. Thus, for this case we are unable to reject the hypothesis that the CEV is the same no matter which of three valuation approaches is selected.

Two situations occur wherein precautions must be taken if the CEV calculated by Faustmann's three approaches are to produce equivalent results: (a) non-recurring costs and (b) abnormal stand conditions. These two cases are examined below.

Before-tax cash flows with non-recurring costs

Suppose that owing to the condition of the land, the non-recurring establishment cost for the first rotation is $2000/ha but that the establishment cost for all subsequent rotations remains at $1000/ha. Immediately prior to stand establishment, the NPV of future cash flows in perpetuity is $2585.57/ha. Not surprisingly this value is $1000/ha less than the LEV calculated in Table 1. This NPV should not be interpreted as an LEV (which assumes the same cash flows in perpetuity) but is an estimate of the current forest value. In this case, the current condition of the land is such that a one-time expenditure of $1000/ha is required before the first crop can be initiated. Once the non-recurring cost is incurred, the LEV is easily shown to be $3585.57/ha.

As a consequence of the non-recurring establishment cost, the economic value for the 5 year-old stand should be based on the LEV of $3585.57/ha. The CEV for the 5 year-old stand is calculated in Table 4 using the discounted cash flow approach. Note that this calculation produces the same numeric value for the CEV as previously shown in Tables 1 –3. However, as shown in Table 5, if one uses the compounded costs method to calculate the CEV in the presence of the non-recurring establishment cost, an incorrect CEV of $6113.46/ha is obtained. The difference of $1538.63/ha between the two CEV estimates represents the value of the non-recurring portion of the establishment costs (i.e., $1000/ha) compounded forward for 5 years at 9%. In this case, the CEV is correctly determined by compounding forward only recurring costs. Non-recurring costs associated with crop establishment should not be included in the LEV as, by definition, it is based on future recurring costs. However, such costs are included in the calculation of the forest value prior to stand initiation. Once they are incurred they are sunk and ignored in the calculation of LEV, FEV and CEV. Thus, the CEV for the 5 year-old stand remains at $4574.83/ha.

With reference to the proof in Appendix 1, we can consider that there is an additional non-recurring establishment cost C_{nr}. The first term in the expression for the sum of compounded costs (V_c) then becomes $(C+ C_{nr})(1.0p)^n$ and so V_c will increase by $C_{nr}(1.0p)^n$. As the non-recurring cost is not included in the calculation of the sum of discounted future cash flows, V_R will remain unchanged. Consequently V_C will exceed V_R by $C_{nr}(1.0p)^n$. The hypothesis is rejected.

Before-tax cash flows with a non-optimal current stand

Now consider the case where the current 5 year-old crop is abnormal (i.e., sub-optimal) for the site. Since the LEV is based on optimal stand management for the site, the current stand is sub-optimal relative to all future stands to be grown in perpetuity. Assume that all costs are unchanged but because of inferior volume and quality (compared to what could be achieved on the site in subsequent rotations) the current crop is only expected to produce $50,000/ha at harvest (compared to $70,000/ha for subsequent rotations). The LEV remains at $3585.57/ha because it is based on optimal forestry for the site. However, the CEV, calculated by discounting future cash flows to the present, now equals $1819.20/ha. The CEV is calculated by subtracting the LEV from the reduced FEV of $5404.77/ha. Use of the compounding costs method will not yield the correct CEV because this method compounds all sunk or historical costs incurred. As such, it estimates the CEV as $4574.83/ha because these costs and the LEV are unchanged from the initial classical Faustmann case– only the expected harvest yield of the abnormal crop is reduced. Since expected returns from the sub-optimal stand are expected to produce a revenue of only $50,000/ha when harvested at age 28, the 5 year-old stand has to be worth less than this amount. Thus, the discounting approach must be used to obtain the correct stand value in this situation.

With reference to the proof in Appendix 1, V_C (sum of compounded costs) and LEV will remain unchanged. The first term in the expression for V_R (sum of discounted future cash flows) becomes $R_S/1.0p^{u-n}$, where R_S is the net clear fell revenue for the current sub-optimal crop. V_R will therefore be less than V_C by $(R-R_S)/1.0p^{u-n}$. The hypothesis is rejected.

The two cases just discussed (i.e., before-tax cash flows with non-recurring costs or a non-optimal current stand) lead us to reject the hypothesis that the CEV produced by the compounding costs and the discounted cash flow methods are the same. Our example shows that in both cases the discounted cash flow approach should be used as only it produces the correct economic value.

This example also illustrates the fundamental principle enunciated by Faustmann – that LEV is always to be used when computing land rents (assuming continued forest production is the highest and best use for the site). Thus, in the above example, we use the optimal LEV of $3585.57/ha and not a land value based on sub-optimal management. Although stand age 28 is used as the rotation age for the sub-optimal stand, it is likely that this is too long. Thus, one should determine the optimal holding time for the sub-optimal stand. The higher the LEV relative to reduced revenues (or higher costs) of the sub-optimal stand, the shorter the holding period. This is a direct consequence of the land rent that accrues as a result of deferring initiation of the optimal crop.

After-tax Calculations

In this section of the paper, all cash flows are adjusted to reflect the tax treatment of expenditures following one of the four categories previously listed. All after-tax cash flows are discounted using an appropriate after-tax discount rate. Cash flow and discount rate adjustments from the before-tax case may alter the attractiveness of a given forest investment opportunity. Campbell and Colletti (1990) investigate the accuracy of using the following rule-of-thumb relationship between before-tax and after-tax discount rates:

$$r_a = r_b (1-t)$$

where r_a = real after-tax discount rate

r_b = real before-tax discount rate

t = marginal income tax rate

They found that the relationship between the before-tax and after-tax discount rates, for investments with depreciable, depletable, or tax-deferred assets, varies with the rate of return of the investment, the marginal tax rate, the level of inflation and the investment period. Klemperer (1996), in discussing use of the above formula, notes that it only applies to annual yield properties. For a series of cash flows with different investment lengths no simple rule-of-thumb exists. And, as correctly noted by Klemperer, the longer the time between investment and return, "the less an income tax will reduce the before-tax rate of return."

Under current New Zealand tax legislation, the "equivalent" discount rate to be applied to before-tax cash flows (i.e., the discount rate applied to before-tax cash flows that yields the same value as that calculated by discounting after-tax cash flows by the required after-tax rate of return) varies with stand age as well as the other factors identified by Campbell and Colletti (1990). If all forestry revenues are immediately taxable and all forestry costs (including the purchase price) are immediately deductible then the "equivalent" discount rate to be applied to before-tax cash flows equals that applied to after-tax cash flows. However, in New Zealand and other countries, the purchase price for timber investments is not immediately deductible and must be carried forward and deducted against future harvest revenues. Consequently, the "equivalent" discount rate to apply to before-tax cash flows is different (i.e., larger) than the discount rate to apply to after-tax cash flows to give the same present value. To simplify the presentation of our example, we use a real 9% after-tax discount rate regardless of the tax treatment of forestry expenditures.

As previously listed, forestry expenditures can be divided into four categories on the basis of tax treatment (McSoriley and Herrington 1994):

- Immediately deductible expenditures. This includes planting and tending expenses, annual operating expenses, harvest expenses, and post-harvesting expenses.
- Expenditures that have to be capitalized and deducted against future revenue. This includes the cost of purchasing a crop of trees that goes into a "cost of bush" or "cost of timber" account and is deducted against harvest revenue.
- Expenditures that are capitalized and depreciated. This includes land development expenditures such as the construction of roads.
- Non-deductible expenditures. This includes the cost of land contouring or other permanent improvements to land.

1. Immediately deductible expenditures:

Initially we assume that all forestry costs are immediately deductible. Since the purchase price of a crop is not deductible it is temporarily ignored. All depreciable costs are also assumed immediately deductible.

After-tax cash flows associated with our prior example are shown in Table 6 assuming the current (1998) New Zealand corporate tax rate of 33%. The current NZIF Forest Valuation Standards (1999) taxation convention is that, "tax should be assumed payable/claimable in the period in which the liability/credit arises" because it is assumed that the owner has, from other sources, "sufficient assessable income to fully utilize any tax losses in the year they occur." For example, the after-tax cash flow associated with stand establishment at time 0 is -$670/ha. This is the net of the establishment cost of -$1000/ha and the tax credit of $330 that arises. It is assumed that this tax credit is immediately deductible. For immediately deductible expenses, the after-tax cash flow is computed as:

After-tax cash flow = Before-tax cash flow *(1 - tax rate)

For the example, the after-tax LEV is calculated in Table 6 as $2402.33/ha. As all costs and revenues are reduced by 0.67 it is not surprising that the LEV calculated from after-tax cash flows is exactly 0.67 times the LEV calculated from before-tax cash flows.

The after-tax CEV for any stand age, calculated using any of Faustmann's three methods, will also equal 0.67 of the CEV calculated using before-tax cash flows. For example, as shown in Table 7 using the compounding costs method, after-tax CEV for the 5 year-old stand is $3065.14/ha. Note that land rent of $216.21/ha (0.09 * $2402.33) is included as an opportunity cost. This notional land rent is not tax deductible - however it is based on an LEV that is calculated from after-tax cash flows. The key point is that, if all costs and revenues are immediately deductible, the before-tax observations of Faustmann (1849) continue to hold.. Thus, with reference to the proof in Appendix 1, on an after-tax basis each of the costs and revenues in the expressions for V_C and V_R will be reduced to (1-tax rate) of the before-tax level. Therefore V_C will still equal V_R. The hypothesis cannot be rejected.

2. Capitalized, depreciated and non-deductible expenditures:

Next we turn to situations where forestry costs fall into the remaining three categories of tax treatment: (i) non-deductible, (ii) capitalized and recovered through depreciation, or (iii) capitalized and recovered at the time of harvest. Each situation is discussed independently for ease of exposition.

(i) Non-deductible expenditures

Suppose that $400/ha of the establishment costs incurred at time 0 are not deductible for tax purposes. Further assume that this situation prevails for reestablishment costs for all subsequent rotations. Non-deductible forestry costs are not common but when they occur it is more normal for them to be associated only with the first rotation. However this case is trivial and is dealt with below in the section on non-recurring costs.

As shown in Table 8, the after-tax cash flow at time 0 becomes -$802/ha (i.e., $600* (1 - .33) + $400) instead of -$670/ha as when the full cost was immediately deductible. After-tax LEV is consequently reduced from $2402.33/ha to $2257.35/ha. Table 8 also shows the calculation of CEV using the compounded costs method. The after-tax CEV at age 5 increases from $3065.14/ha (Table 7) to $3190.15/ha (Table 8). This is because: (i) in the case of discounting future cash flows, the reduction in the notional land rent (based on LEV) has a greater effect than the increased cost incurred upon reestablishment some 23 years into the future and (ii) in the case of compounding costs, the increase in establishment costs incurred 5 years ago has a greater effect than the reduction in land rental. Note that the after-tax FEV of the 5 year-old stand has decreased from $5467.47/ha (Table 7) to $5447.50/ha (Table 8).

We can break the establishment cost (C) referred to in Appendix 1 into two components: a deductible component (C_D) and a non-deductible component (C_{ND}). The after-tax establishment cost will increase by (tax rate)* C_{ND} (relative to the immediately deductible after-tax case). The expression for LEV will be reduced (relative to the immediately deductible after-tax case) by (tax rate)* C_{ND}*$(1.0p)^u/(1.0p^u - 1)$. Both V_C and V_R will be increased by (tax rate)*C_{ND}*$[(1.0p)^u - (1.0p)^n]/(1.0p^u - 1)$. The hypothesis cannot be rejected.

(ii) Capitalized expenditures recovered through depreciation

Suppose that $400/ha of the establishment costs incurred at time 0, previously considered non-deductible, is depreciable (at the rate of 5% of the diminishing depreciable value (DV)). Again assume that this situation prevails for reestablishment costs for all subsequent rotations. In this situation, the after-tax cash flow at time 0 is still -$802 but now there is a series of positive cash flows (which extend past the rotation age) relating to the tax deductibility of the depreciation expense. For example, at time 1 the nominal depreciation expense is $20 (5% of $400) that has a tax shield value of $6.60 (33% of $20). At time 2, the nominal depreciation expense is $19 (5% of the DV of $380) that has a tax shield value of $6.27.

As there is no "inflation-proofing" of the future tax deductions associated with depreciation, they must be converted from nominal to real dollars by dividing by $(1.0i)^n$ where i is the annual inflation rate and n is the stand age. For example, assuming an annual inflation rate of i = 3%, the real value of the tax shield at time 1 is $6.41 and at time 2 it is $5.91. Based upon these values, the LEV for the example forest is $2299.32/ha. This is $41.97/ha greater than the LEV of $2257.35/ha (Table 8) calculated for the situation where $400/ha of establishment costs was neither immediately deductible nor depreciable. This difference is equal to the present value of the depreciation tax deductions associated with establishment of all future rotations. It can be calculated by scaling up the present value of the depreciation tax deductions for the first rotation (i.e., $38.22/ha) using the periodic payments formula.[5] The present value of depreciation tax deductions may be calculated directly as:

$$PV(\text{depreciation tax deductions}) = [t*d*(C/(1.0i)^n)]/(1.0p*1.0i - (1-d))$$

where t = marginal income tax rate

d = depreciation rate (diminishing value basis)

C = current value of depreciable cost [in real terms (i.e.,current dollars)]

i = annual inflation rate

n = years since cost was incurred

p = real annual discount rate

For example, consider the present value at time 0 of the future tax deductions arising from the depreciable first rotation establishment costs of $400/ha. By setting t=0.33, d=0.05, C=400, i=3, n=0, p=9, the PV is calculated as $38.22/ha.

Table 9 shows the calculation of CEV for the example 5 year-old stand using the compounding costs method. The depreciation tax shield is calculated by deflating the depreciation expense associated with $400/ha even though the $400/ha is a real cost in today's dollars. Although not shown, the CEV of $3179.46/ha for the 5 year-old stand shown in Table 9 can also be calculated by discounting future cash flows.

We can break the establishment cost (C) referred to in Appendix 1 into two components: a deductible component (C_D) and a non-deductible but depreciable component (C_{DP}). The after-tax

[5] PV = $38.22 * [1.09^{28}/(1.09^{28} - 1)]$ = $41.97/ha

establishment cost will increase by the following amount: (tax rate)* C_{PR} - PV_D (the present value of the depreciation tax deductions) relative to the immediately deductible after-tax case. And, the expression for LEV will be reduced (relative to the immediately deductible after-tax case) by the amount: $[(\text{tax rate})^* C_{DP} - PV_D](1.0p)^u/(1.0p^u - 1)$. Both V_C and V_R will be increased by the amount: $[(\text{tax rate})^* C_{DP} - PV_D]^*[(1.0p)^u - (1.0p)^n]/(1.0p^u - 1)$ + PV of future tax deductions. The hypothesis cannot be rejected.

(iii) Capitalized expenditures recovered at time of harvest

Next we consider the current New Zealand tax situation with respect to the deductibility of the purchase price of the crop. In this case, the cost is not deductible until the crop is harvested. We adopt the convention of the NZIF Forest Valuation Standards (1999) and calculate CEV from the perspective of a purchaser. (Note that, because of the asymmetry of the tax position, CEV will be different from the perspective of a seller (i.e., tax is payable at the time of sale)). It is assumed throughout that the purchase price of the crop is equivalent to the CEV

When calculating CEV by discounting future cash flows, additional revenue at the time of harvest must be added to equal the value of the tax deduction associated with the crop purchase price. This is computed as (CEV * tax rate). As there is no "inflation-proofing" of this future tax deduction, it must be converted from nominal to real dollars by dividing by $(1.0i)^{u-n}$ where i is the annual inflation rate, u is the rotation age and n is the stand age. Then it is discounted using the real after-tax discount rate. The present value of the tax deduction associated with the purchase price is calculated as:

$$PV(\text{purchase price tax deduction}) = 0.33^*CEV/(1.0p * 1.0i)^{u-n}$$

where all terms are as previously defined.

A practical difficulty arises in the calculation of CEV because it includes the present value of the tax deduction that is itself calculated from CEV. This circularity can be overcome using an iterative approach. As shown in Table 10, for the example 5 year-old stand, the CEV can be estimated as $3137.42/ha. The present value of the tax deduction adds $72.28/ha to the CEV of $3065.14/ha as shown in Table 7 where no tax deduction is permitted for the crop purchase price. It is also important to recognize that since the cost of purchase is a non-recurring item, its presence does not affect the calculation of the LEV. Thus, for the case under consideration, the LEV remains at $2402.33/ha as shown in Tables 6 and 7.

When CEV is calculated by discounting future cash flows it includes the value of this tax deduction. However the compounding costs method will give only the crop value exclusive of the tax deduction. In order for the compounding costs method to provide the correct CEV, the PV (purchase price tax deduction) must be added to the crop value exclusive of the tax deduction. The cost of purchase deduction is not anticipated by the compounding cost method because the future is captured in the LEV and the LEV does not have any cost of purchase deduction because: (a) it assumes bare land and, hence, a crop value of zero and/or (b) it takes a transaction to trigger the cost of purchase and this transaction has not been anticipated.

With reference to the proof in Appendix 1, V_C (sum of compounded costs) and LEV will remain unchanged. However to V_R must be added the expression $0.33 * V_R / (1.0p * 1.0i)^{u-n}$. The hypothesis can be rejected.

3. Non-recurring costs:

Earlier we discussed the proper before-tax treatment for non-recurring costs associated with bare land and stand valuation. Similarly, on an after-tax basis, non-recurring costs (either because of the activity being non-recurring or the first rotation tax treatment being different from that of subsequent rotations) do not affect the calculation of the LEV. This is true because the LEV calculation is always based on future recurring cash flows. Such cash flows should, however, be included in the calculation of the current forest value of a project if they are anticipated but not yet incurred. Once incurred, they are sunk and ignored in the calculation of LEV, FEV and CEV.

(i) Non-recurring forestry costs that are non-deductible

The example considered earlier when non-deductible costs were discussed assumes that the non-deductible costs incurred at time 0 are also incurred in all subsequent rotations. This is contrary to the current New Zealand tax situation where it is more normal for non-deductible costs to be associated only with the first rotation (e.g., land contouring costs). Since these costs do not recur, calculations of LEV, FEV and CEV, for any stand after establishment, should exclude these costs. It follows that the calculation of the CEV using either compounded costs or discounted future cash flows will yield consistent results.

(ii) Non-recurring depreciable forestry costs

There are examples where costs are non-recurring because the first rotation tax treatment is different from that of subsequent rotations. For example, expenditure on "the destruction, to enable the planting of trees on the land, of weeds or plants detrimental to the land" (CITATION??) is depreciable at 5% DV whereas expenditures incurred for the purpose of replanting are immediately deductible. Once the first rotation cost has been incurred, LEV, FEV and CEV should be calculated 'looking forward' assuming the costs and tax treatment applicable to the second rotation. In this case, the accumulation of compounded costs will still be equal to CEV calculated by discounting future cash flows.

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Appendix 1
Proof of Equivalence between Compounding Costs Method and Discounting Future Cash Flows Method for Estimating CEV

The purpose of this appendix is to show that the compounding costs and discounting of future cash flow approaches produce equivalent numeric results under the classical Faustmann (1849) assumptions identified in his paper.

Let,

n = current stand age (yrs)
u = rotation age (yrs)

C = establishment cost ($/ha)
A = annual overhead cost ($/ha)
S = silvicultural costs at time s (s < n) ($/ha)
T = silvicultural costs at time t (t > n) ($/ha)
R = net clear fell revenue ($/ha)

L = LEV ($/ha)
p = real annual discount rate

Value of stand at age n:

(a) Compounding costs method:

(i) Establishment cost
$C\,(1.0p)^{n}$

(ii) Past Silvicultural cost
$S\,(1.0p)^{n-s}$

(iii) Annual overhead cost
$A * (1.0p^{n} - 1)/0.0p$

(iv) Annual Land rental
$(L * 0.0p) * (1.0p^{n} - 1)/\ 0.0p$

$L * (1.0p^{n} - 1)$

Sum of compound costs at age n:

$V_C \quad = \quad C\,(1.0p)^n + S\,(1.0p)^{n-s} + A * (1.0p^n - 1)/0.0p + L\,(1.0p^n - 1)$

$\quad\quad\; = \quad C\,(1.0p)^n + S\,(1.0p)^{n-s} + A * (1.0p^n - 1)/0.0p + L\,(1.0p^n) - L$

(b) Discounting future cash flows method:

(i) Net clear fell revenue
$R/(1.0p)^{u-n}$

(ii) Future silvicultural cost
$T/(1.0p)^{t-n}$

(iii) Annual overhead cost
$A * (1.0p^{u-n} - 1)/(0.0p * 1.0p^{u-n})$

(iv) Annual Land rental
$L * 0.0p * (1.0p^{u-n} - 1)/(0.0p * 1.0p^{u-n})$

$L * (1.0p^{u-n} - 1)/(1.0p)^{u-n}$

$L - L/(1.0p)^{u-n}$

Sum of discounted future cash flows at age n:

$$V_R = R/(1.0p)^{u-n} - T/(1.0p)^{t-n} - A * (1.0p^{u-n} - 1)/(0.0p * 1.0p^{u-n}) - L + L/(1.0p)^{u-n}$$

But we know from Faustmann (1849) that LEV can be calculated as:

$$L = [R - C\,(1.0p)^u - S\,(1.0p)^{u-s} - T\,(1.0p)^{u-t} - A * (1.0p^u - 1)/0.0p]\,/(1.0p^u - 1)$$

Thus,

$$R = L * (1.0p^u - 1) + C\,(1.0p)^u + S\,(1.0p)^{u-s} + T\,(1.0p)^{u-t} + A * (1.0p^u - 1)/0.0p$$

Substituting this value of R back into the discounted future cash flow equation:

$$V_R = [L * (1.0p)^u - 1)]\,/(1.0p)^{u-n} + C\,(1.0p)^n + S\,(1.0p)^{n-s} + T\,(1.0p)^{n-t}$$
$$+ A * (1.0p^u - 1)/(0.0p * (1.0p)^{u-n}) - T/(1.0p)^{t-n} - A * (1.0p^{u-n} - 1)/(0.0p * 1.0p^{u-n}) - L$$
$$+ L/(1.0p)^{u-n}$$

$$= L * (1.0p)^u/(1.0p)^{u-n} - L/(1.0p)^{u-n} + C\,(1.0p)^n + S\,(1.0p)^{n-s} + T\,(1.0p)^{n-t}$$

$+ A * (1.0p^{u} - 1)/(0.0p * 1.0p^{u-n}) - T/(1.0p)^{t-n} - A * (1.0p^{u-n} - 1)/(0.0p * 1.0p^{u-n}) - L$
$+ L/(1.0p)^{u-n}$

$= C (1.0p)^{n} + S (1.0p)^{n-s} + A * (1.0p^{n} - 1)/0.0p - L + L * (1.0p)^{n} = V_{C}$

This shows the equivalence of the two approaches for calculating CEV.