Instrument characteristics

Dwyer Series 450
Carbon Monoxide Monitor

Range: 0-2000 PPM.
Resolution: 1 PPM.
Accuracy: (using 2000 PPM calibration gas) ±3% of reading, ± the accuracy of the calibration gas.
Response Time: <30 seconds to 90% of reading.
Operating Temperature: 32 to 104°F(0-40)°C).
Humidity Conditions: 0-90% Relative Humidity Non Condensing.
Adjustments: Zero and Span via keypad.

Pressure Range: 5.0 inches w.c.d. (1.24 kPa).
Maximum Working Pressure: 6.89 Bar.
Output Signals: 4-20 mA.
Zero Output: 4 mA.
Span: 16 mA.
Performance @ 70ºF (21.1ºC)
Accuracy: ±0.5% Full Scale (non-linearity, hysterisis, nonrepeatability).
Stability: ±1%/year.
Warm-up Time: 10 minutes.
Operating Temperature: 5 to 50ºC.
Temperature Effects: 0.025%/ºF.

Instrument characteristics

- Range
- Accuracy
- Sensitivity
- Linearity
- Detection limit (threshold)
- Resolution
- Hysteresis and dead band
- Impedance

Input	bounds for measurements
Output	bounds for sensor/transmitter output
Span	difference between upper and lower limits
Zero	lower limit of range

Example: digital thermometer

Input range → -10 to 110ºC

Output range (transmitter)→ 4 - 20 mA

Input span → 120ºC

Output Span → 16 mA

Input Zero →-10ºC

Output Zero →4 mA

Instrument characteristics

- Range
- **Accuracy**
- Sensitivity
- Linearity
- Detection limit (threshold)
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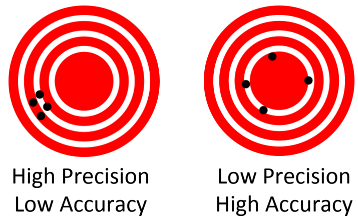
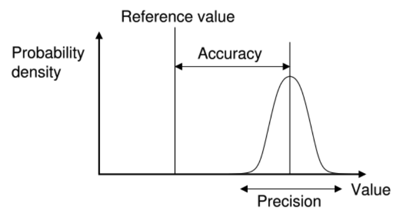
- Impedance

Nearness to the “true” value. Unrelated to precision which is a measure of internal consistency





Instrument characteristics

- Range
- Accuracy
- **Sensitivity**
- Linearity
- Detection limit (threshold)
- Resolution
- Hysteresis and dead band
- Impedance

Change in output over change in input

Example: Hg vs H_2O manometer

Hg manometer provides a 760 mm change over 101 kPa

Sensitivity = 760/101 = 7.52 mm/kPa

H_2O manometer provides a 13.5 mm change over 7.52 kPa

Sensitivity = 13.5/7.52 = 102 mm/kPa

Instrument characteristics

- Range
- Accuracy
- Sensitivity
- **Linearity**
- Detection limit (threshold)
- Resolution
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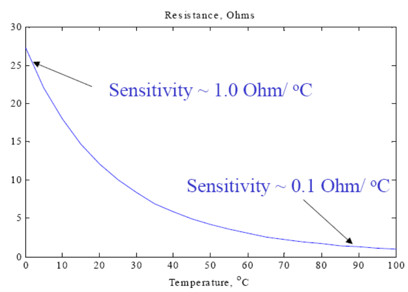
- Impedance

Proportionality of output to input

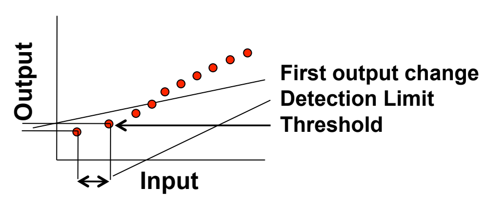
Radio Shack 271-110 thermistor



Instrument characteristics

- Range
- Accuracy
- Sensitivity
- Linearity
- **Detection limit (threshold)**
- Resolution
- Hysteresis and dead band
- Impedance

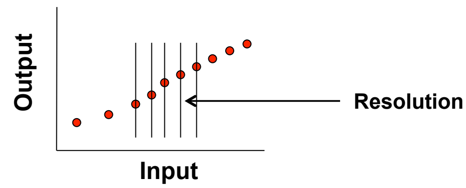
Smallest detectable change in measurement in the low end of range



Instrument characteristics

- Range
- Accuracy
- Sensitivity
- Linearity
- Detection limit (threshold)
- Resolution
- Hysteresis and dead band
- Impedance

Smallest detectable change over the full range of measurements



Instrument characteristics

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- Resolution
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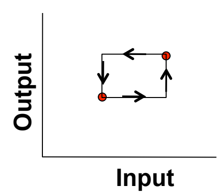
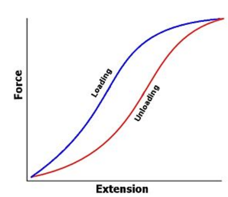
- Impedance

Dead band: region where there is no effective response





Hysteresis: retardation of an effect upon induced change

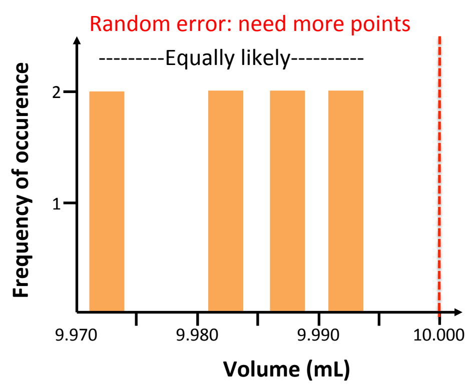
Instrument characteristics

- Range
- Accuracy
- Sensitivity
- Linearity
- Detection limit (threshold)
- Resolution
- Hysteresis and dead band
- Impedance — A measure of the opposition to current in an AC circuit

Assessing Uncertainties

- Goal
  - Determine how much confidence we can place in a number
- Example
  - Pipette calibrated by NIST at 5 decimal places: V = 10.00000 ± 0.000005
- Let's conduct our own experiment

Trial	Volume (ml)
1	9.990
2	9.986
3	9.973
4	9.983
5	9.980
6	9.988
7	9.993
8	9.970



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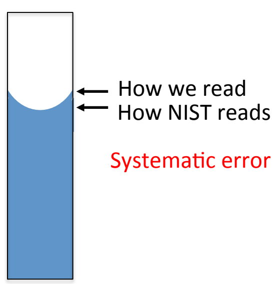

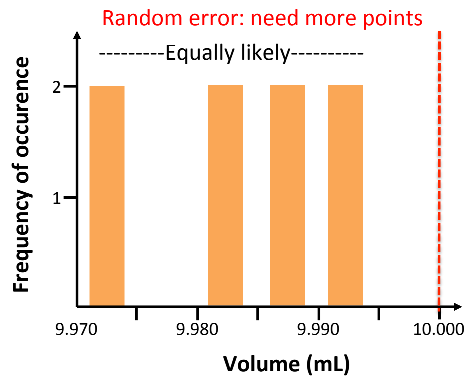

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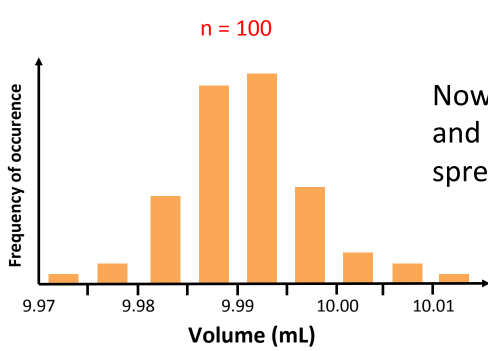
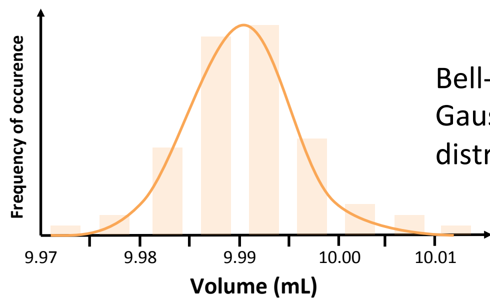


Now can see a mean and characteristic spread in the data

Assessing Uncertainties

- Goal
  - Determine how much confidence we can place in a number
- Example
  - Pipette calibrated by NIST at 5 decimal places: V = 10.00000 ± 0.000005
- Let's conduct our own experiment

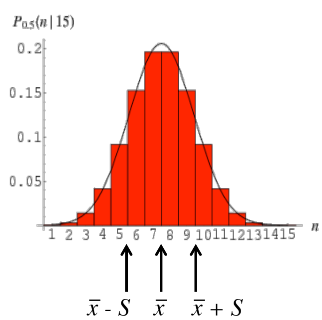
For an infinity of measurements and shrunk intervals



Bell-shaped curve: Gaussian (or normal) distribution

Gaussian distribution

- What good are statistics if I need to make an infinity of measurements?
  - Start by assuming that you have a know distribution (usually Gaussian)
  - Use a small number of measurements to estimate the shape that the distribution would have after an infinity of measurements



Distribution characterized by:

Mean
(average value)
=AVERAGE(numbers) in xls

$$\bar{x} = \frac{\sum_i x_i}{n}$$

Standard deviation
(average deviation from mean)
=STDV(numbers) in xls

$$S = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$$

Variance
(square of standard deviation)

$$S^2 = \frac{\sum_i (x_i - \bar{x})^2}{n-1}$$

Gaussian distribution

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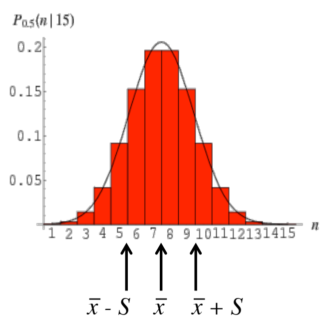


What if I only made 2 measurements?

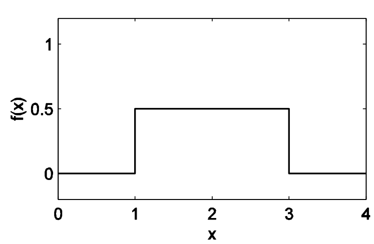
DON'T

As a a last resort, get estimate of S as:

$$S = \sqrt{\frac{\sum (x_1 - x_2)^2}{2}} = \frac{|x_1 - x_2|}{\sqrt{2}}$$

Other distributions exist

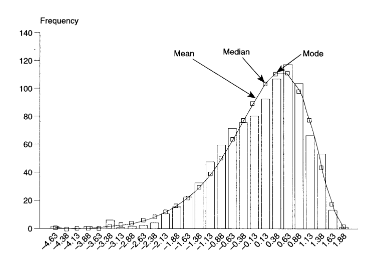
- Can be discrete
  - Measurement falls between 2 graduations of a gage/ruler



$$f(x) = 0 \qquad -\infty < x \le x_1$$
$$f(x) = \frac{1}{x_2 - x_1} \qquad x_1 \le x \le x_2$$
$$f(x) = 0 \qquad x_2 \le x < \infty$$

$$\bar{x} = \frac{x_1 + x_2}{2} \qquad S = \frac{x_2 - x_1}{2\sqrt{3}}$$

- Can be asymmetric
  - Distribution of personal income; house prices, etc



Mode
value occurring the most frequently
Median
Value where the distribution curve is divided into two equal areas

Back to the pipette example

Trial	Volume (ml)
1	9.990
2	9.986
3	9.973
4	9.983
5	9.980
6	9.988
7	9.993
8	9.970

- Mean: $\overline{V}$ = 9.983 mL
- Standard deviation: S = 0.008 mL
- 8 Measurements

- S is small: precision is good
- Good way to report data:

  $\overline{V}$ = 9.983 ± 0.008 mL (n=8)

- How about accuracy?
  - Absolute error in mean:

    ε = 9.983 – 10.000 = - 0.017 mL
  - Relative error in mean:

    ε/V_{true} = - 0.017/10 = -0.17%

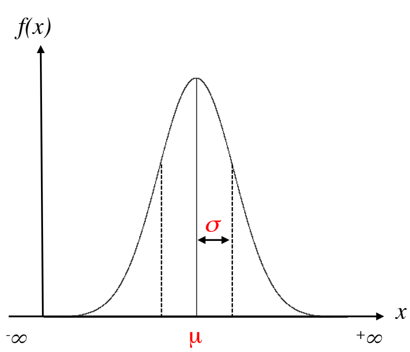
- Suggests systematic error. Better way to assess this later on

Gaussian distribution

- For an infinity of measurements with small, random fluctuations, the probability density function is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ is the *population* mean
σ is the *population* standard deviation



As n -> ∞

Sample mean $\bar{x} = \frac{\sum_i x_i}{n}$ -> μ

Sample SD $S = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$ -> σ

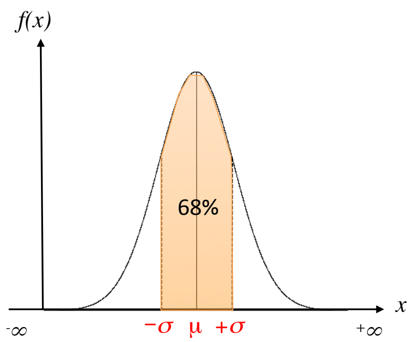
- In practice, we calculate $\bar{x}$ and S and use them as estimates of μ and σ

Gaussian distribution

- The *normal* error probability function allows us to determine how likely an event is to occur

$$P = \int_{-\infty}^{+\infty} f(x) = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 1$$

100% of the variation under the curve



$$\int_{\mu-\sigma}^{\mu+\sigma} f(x) = \int_{\mu-\sigma}^{\mu+\sigma} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 0.68$$

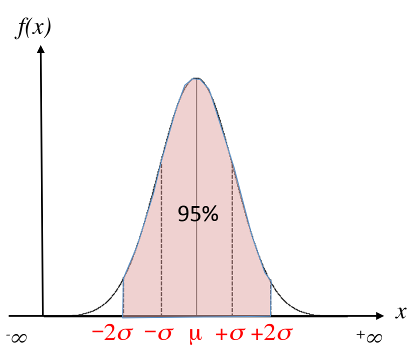
There is a 68% chance that a measurement will fall within one standard deviation of the mean

Gaussian distribution

- The *normal* error probability function allows us to determine how likely an event is to occur

$$P = \int_{-\infty}^{+\infty} f(x) = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 1$$

100% of the variation under the curve



$$\int_{\mu-2\sigma}^{\mu+2\sigma} f(x) = \int_{\mu-2\sigma}^{\mu+2\sigma} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 0.95$$

There is a 95% chance that a measurement will fall within two standard deviations of the mean

- $\mu \pm 3\sigma$ encompasses ≈ 99% of the variation!