1 Collusion in Sealed Bid Auctions

- Sealed bid procurements

- Low Bidder wins

- Firms are heterogeneous

- Differentiated by:
  - location, backlog, political boundaries, other fixed, but persistent factors
• Why is collusion common in sealed bid auctions

- Key problem of cartel- incentives to cheat

- Sealed bid auctions solve problem of monitoring for cartel

- Relatively homogenous product

- Tight profit margins/financial leverage
• Pesendorfer (2000) 55% of the criminal cases filed by Justice Anti-Trust involve bid-rigging.

• Many cities, bid rigging is still a pervasive problem: New York and Chicago.

• Engineering News Record reports recent cases of fraud in industries from paving to interiors to painting.

• Concrete tax in New York, 2% mob tax on concrete laid.

• According to anti-trust officials, economics does not provide useful methodology to detect collusion in bidding.
1.1 Key ideas of reduced form methods

- Auction theory generates some strong restrictions on bids

1. Bids should be conditionally independent

2. Bids should reflect costs

- Use these strong restrictions as a basis for testing

- Empirical studies of known cartels violate 1 and 2.
1.2 Caveats

1. No test for collusion is fool proof: garbage in/garbage out

2. False negatives and false positives
1.3 Benefits

1. The empirical literature suggests clear warning flags

2. Enforcement may prevent future collusion

3. Screening can be the basis for asking “hard questions”

4. Experience is cumulative and ability to detect improves over time

5. Why is no monitoring the optimal regulatory policy?
1.4 Outline

1. Theoretical Model

2. Identification

3. Data

4. Reduced form tests
2 Auctions with Asymmetric Bidders.

- In this model, $N$ firms compete for a contract to build a single and indivisible public works project.

- The cost estimate of each firm is a random variable $c_i$ with cumulative distribution function $F_i(\cdot; \theta_i)$ and probability density function $f_i(\cdot; \theta_i)$.

- $\theta_i$ is firm specific parameter and $\theta \equiv (\theta_1, ..., \theta_N)$.

- The $F_i(\cdot; \theta_i)$ have common support $[c_l, c^u]$.

- Differ by capacity available at time of auction, location and materials prices.
• For example:

\[ c_i = x_i'\theta_i + \varepsilon_i \]

• Where \( x_i \) is vector of cost controls

• \( \theta_i \) parameters

• \( \varepsilon_i \) are iid shocks from a known distribution

• Firm \( i \)'s von Neumann-Morgenstern utility function is:
\[ u_i(b_1, \ldots, b_n, c_i) = \begin{cases} 
 b_i - c_i & \text{if } b_i < b_j \text{ for all } j \neq i \\
 0 & \text{otherwise} 
\end{cases} \]

- Firm \( i \)'s bid function \( b_i = b_i(c_i; \theta) \) is a map from firm \( i \)'s cost draw to set of bids and \( \phi_i(b_i) = b_i^{-1}(b_i; \theta) \).

- If the identity of all other bidders is known in advance then probability of winning is:

\[
Q_i(b; b_{-i}, \theta) \equiv \prod_{j \neq i} 1 - F_j(\phi_j(b; \theta); \theta_j).
\]

and profits are:

\[
\pi_i(b, c_i; b_{-i}, \theta) \equiv (b - c_i)Q_i(b; b_{-i}, \theta) = (b - c_i) \prod_{j \neq i} 1 - F_j(\phi_j(b; \theta); \theta_j)
\]

- Equilibrium exists and is unique.
• Simple example—these models are about spatial competition.

• Inframarginal unit should set the price.

• See example from Antitrust Law Review Paper
3 Identification

- The above model is difficult to study

- Inverse bid functions are a system of $N$ differential equations which are not Lipschitz

- Can only express equilibrium in closed form in special cases

- In general, non-standard numerical methods must be used to compute the model equilibrium
• Key idea- Guerre, Perrigne and Vuong (2000)

• Let $G_i(b|x)$ denote the cdf that $i$ bids $b$ in the auction given cost controls $x = (x_i)$

• Then the probability that $i$ wins the auction with a bid of $b$ is:

$$\prod_{j \neq i} 1 - G_j(b|x)$$

• Bidder $i$ should then maximize:

$$\max_b (b - c_i) \prod_{j \neq i} \left(1 - G_j(b|x)\right)$$

• The first order condition for this model can be
derived as:

\[
\frac{d}{db} \log \left( (b - c_i) \prod_{j \neq i} G_j(b|x) \right) = \\
\frac{1}{(b - c_i)} - \left[ \sum_{j \neq i} \frac{g_j(b|x)}{1 - G_j(b|x)} \right]^{-1} = 0
\]
• Straightforward algebra implies that:

\[ c_i = b - \left[ \sum_{j \neq i} \frac{g_j(b|x)}{1 - G_j(b|x)} \right]^{-1} \]

• At a population level, assume \( G_i(b) \) is known for the purposes of studying identification

• The above equation implies that there is a one to one relationship between bids and private information about costs

• Private information about costs can be reverse engineered by observing the distribution of bids
• Remark: In sealed bid auctions, firms are often capacity constrained

• In this case we can write first order conditions as:

$$c_i + v_{i,t,W}(s) - v_{i,t,L}(s) = b - \left[ \sum_{j \neq i} \frac{g_j(b)}{1 - G_j(b)} \right]^{-1}$$

• $v_{i,t,W}(s)$ option value associated with losing capacity

• $v_{i,t,L}(s)$ option value associated with keeping free capacity.
A1 Let $E_i(b_i|x)$ be firm $i$’s expected bid and let $b_i - E(b_i|x_i)$ be the difference between firm $i$’s actual and expected bid. Then for all $i, j$ with $i \neq j$, $b_i - E(b_i|x)$ and $b_j - E(b_j|x)$ are independently distributed.

A2 The support of each distribution $G_i(b; x)$ is identical for each $i$.

A3. The equilibrium distribution of bids is exchangeable. That is, for all permutations $\pi$ and any index $i$

$$G_i(b; x_1, x_2, x_3, \ldots, x_N) = G_{\pi(i)}(b; x_{\pi(1)}, x_{\pi(2)}, x_{\pi(3)}, \ldots, x_{\pi(N)}).$$

$$\xi_i(b, x) = b_i - \frac{1}{\sum_{j \neq i} \frac{g_i(b; x)}{1 - G_i(b; x)}}. \quad (4.4)$$
A4 For all \( i \) and \( b \) in the support of the \( G_i(b; x) \) the function \( \xi_i(b, x) \) are strictly monotone.

**Theorem 4.** Suppose that the distribution of bids \( G_i(b; x) \) is a Bayes-Nash equilibrium of model M1. Then conditions A1-A4 must hold.

**Theorem 5.** Suppose that the distribution of bids \( G_i(b; x) \) satisfy assumption A1-A4. Then it possible to construct a unique set of \( F_i(c; x) \) such that the \( G_i(b; x) \) are an equilibrium to the game when costs are \( F_i(b; x) \).

- It is worth noting that many of these conditions are likely to hold in more general models or models with common value elements.
• Q: Will cartels generate different bid distributions from competitive bidding?

• A: Sometimes

• Consider a cartel that maximizes joint profits of members

• Also suppose that there is a fringe firm

• Then cartel behavior will differ for two reasons

• First independence A1 will be violated because of need to coordinate actions

• Second, exchangeability A3 will be violated
• An all inclusive cartel could avoid detection by simply shading all bids at 10 percent more than the competitive level

• Empirical studies of cartels involve violations of conditional independence and exchangability

• Porter and Zona: Bids of known cartel members are highly correlated compared to a control group

• Porter and Zona: Distance important in determining low bidders, but not in determining rank of other bids
4 Characterization of competitive bidding models.

**A1** Let $E_i(b_i|z)$ be firm $i$’s expected bid and let $b_i - E(b_i|z_i)$ be the difference between firm $i$’s actual and expected bid. Then for all $i, j$ with $i \neq j$, $b_i - E(b_i|z)$ and $b_j - E(b_j|z)$ are independently distributed.

**A2** The support of each distribution $G_i(b;z)$ is identical for each $i$.

**A3.** The equilibrium distribution of bids is exchangeable. That is, for all permutations $\pi$ and any index $i$

$$G_i(b; z_1, z_2, z_3, \ldots, z_N) = G_{\pi(i)}(b; z_{\pi(1)}, z_{\pi(2)}, z_{\pi(3)}, \ldots, z_{\pi(N)}).$$
\[ \xi_i(b, z) = b_i - \frac{1}{\sum_{j \neq i} \frac{g_i(b; z)}{1 - G_i(b; z)}}. \quad (4.4) \]

**A4** For all \( i \) and \( b \) in the support of the \( G_i(b; z) \) the function \( \xi_i(b, z) \) are strictly monotone.

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**Theorem 5.** Suppose that the distribution of bids \( G_i(b; z) \) satisfy assumption A1-A4. Then it possible to construct a unique set of \( F_i(c; z) \) such that the \( G_i(b; \theta) \) are an equilibrium to the game when costs are \( F_i(b; z) \).

It is worth noting that many of these conditions are likely to hold in more general models or models with common value elements.
**Theorem 6.** If only bid data is observed then it is possible to rationalize the observed bids using an a cartel that is not all inclusive.
5 Sealcoat industry in Upper Midwest.

- All contracts from Minnesota, North Dakota and South Dakota for the years 1994-1998.

- Bought from a firm that sells information about upcoming and completed lettings to construction contractors. Also have internal subjective cost estimates from one firm.

- Seal coating is a maintenance procedure designed to extend the life of a road by adding oil and aggregate to the surface of a road.

- There are $92.8 million dollars of contracts from Jan 1994 through October 1998.
• Firms are heterogenous because of differing capacity utilization, distance, materials prices (particularly oil) and previous experience in local markets.

• Among all jobs, 230 (46.5%) are owned by City, 195 (39.3%) are owned by State, 68 (13.7%) are owned by County, and only 2 owned by the Federal.

• An important control variable for our analysis will be an engineer’s estimate. Now restrict only to projects with such an estimate.

• \( \text{BID}_{i,t} \): The amount bid by firm \( i \) on project \( t \).

• \( \text{EST}_t \): The estimated value of project \( t \).
- $\text{DIST}_{i,t}$: Distance between the location of the firm and the project.

- $\text{LDIST}_{i,t}$: $\log(\text{DIST}_{i,t} + 1.0)$.

- $\text{CAP}_{i,t}$: Used capacity measure of firm $i$ on project $t$.

- $\text{CON}_{i,t}$: Concentration of firm $i$ in market $t$.

- $\text{MAXPER}_{i,t}$: Maximum percentage free capacity of all firms on project $t$, excluding $i$.

- $\text{MINDIST}_{i,t}$: Minimum of distances of all firms on project $t$, excluding $i$.

- $\text{LMINDIST}_{i,t}$: $\log(\text{MINDIST} + 1.0)$.
• NBID\(_t\): Number of bidders.

• MONTH\(_t\): The month project \(t\) is let.

\[
\frac{BID_{i,t}}{EST_t} = \beta_{0,i} + \beta_{1,i} \log(DIST_{i,t} + 1.0) + \beta_{2,i} CAP_{i,t} + \beta_{3,i} MAXPER_{i,t} + \beta_{4,i} LMINDIST_{i,t} + \beta_{5,i} CON_{i,t} \text{ auction dummies}
\]

if \(i\) is one of the largest 8 firms in the market.

\[
\frac{BID_{i,t}}{EST_t} = \alpha_0 + \alpha_1 \log(DIST_{i,t} + 1.0) + \alpha_2 CAP_{i,t} + \alpha_3 MAXPER_{i,t} + \alpha_4 LMINDIST_{i,t} + \alpha_5 CON_{i,t} \text{ auction dummies}
\]

if \(i\) is not one of the largest 8 firms in the market.

• These results are consistent with basic economic intuition and all effects are significant at conventional levels except for \(MAXPER_{i,t}\).
• Bids are an increasing function of distance from the project site and own capacity utilization.

• Firms are clearly heterogenous.
6 Testing conditional independence and exchangeability.

- First test for conditional independence.

- Suppose the coefficient of correlation between firm \(i\) and firm \(j\) \((i \neq j)\) residual is \(\rho_{ij}\)

\[ H_0 : \rho_{ij} = 0 \]

- 4 pairs that fail at 5 percent level are: (Firm 1, Firm 2), (Firm 2, Firm 4), (Firm 5, Firm 14) and (Firm 6, Firm 7).

- The candidate cartel are (Firm 2, Firm 4) since only these firms bid against each other more than a few times per year.
Let $\beta_{i1}, \beta_{i2}, \beta_{i3}, \beta_{i4}, \beta_{i5}$ be the coefficients of CAP1, LDIST1, MAXPER, MINDIST and CON for firm $i$, respectively. Then exchangeability is equivalent to the following hypothesis:

$$H_0: \beta_{ik} = \beta_{jk} \text{ for all } i, j, i \neq j, \text{ and all } k = 1, .., 4.$$

The above tables show that for almost all the tests, we fail to reject the null hypothesis at either 1% or 5% significance level. In particular, we fail to reject the null when we pool all the 11 mail firms.

At 5% significance level, we reject exchangeability for (Firm2, Firm 5).

Another candidate cartel is firm2 and firm5.
• Remark: (Firm 2, Firm 4), (Firm 2, Firm 5) is candidate cartel
7 A Decision Theoretic Approach.

- Decide between three alternative structural models of industry equilibrium.

- We shall refer to the three models as $M1$, $M2$, and $M3$.

- $M1$ is the model of competition.

- $M2$ firms 2 and 4 collude efficiently.

- The cost distribution for the cartel is the minimum of $c_{2,t}$ and $c_{4,t}$.

- This distribution has a probability density function $f_2(c_{c,t}|z, \theta)(1-F_4(c_{c,t}|z, \theta)) + f_4(c_{c,t}|z, \theta)(1-F_2(c_{c,t}|z, \theta))$. 
• Treat the cartel as a single profit maximizing firm.

• $M3$ firm 2 and firm 5 are the cartel.

• In early 1980’s, collusion did occur (and was prosecuted) in the seal coat industry.

• Firms frequently made side payments to each other.

• Let $g_i(b; \theta, z, M)$ be the p.d.f. of firm $i$’s bid and $G_i(b; \theta, z, M)$ the cdf for $M = M1, M2$ or $M3$.

$$c_i^M = b - \frac{1}{\sum_{j \neq i} \frac{g_i(b; \theta, z, M)}{1 - G_i(b; \theta, z, M)}}. \quad (1)$$
8 Testing For Collusion: A Bayesian Framework.

• Given a distribution of latent costs $p(c_{M,EST}^{M})$ from methods of last section.

• Use Bayes Theorem to compute posterior probability for three models.

• The likelihood $p(c_{M,EST}^{M}|z_t, \theta, M)$ is:

$$p(c_{M,EST}^{M}|z_t, \theta, M) = \prod_{t=1}^{T} \prod_{i} p(c_{i,t}|z_t, \theta_i, M) \quad (2)$$

• We shall elicit the prior distribution of the parameters, $p(\theta)$ from industry experts.
• The marginalized likelihood of model $M$ as follows:

$$ML_M = \int \int p(c^{M,EST} | M, \theta) p(\theta) p(c^{M,EST})$$ (3)

• By Bayes Theorem probability of $M = M_i$, $p^*(M_i)$ is:

$$p^*(M_i) = \frac{p(M_i) \cdot ML_i}{p(M_1) \cdot ML_1 + p(M_2) \cdot ML_2 + p(M_3) \cdot ML_3}$$ (4)

### 8.1 Eliciting Prior Beliefs.

• We model costs as:
\[
\frac{c_{i,t}}{E S T_t} = \frac{\tilde{c}_{i,t}(\theta, z_t)}{E S T_t} + \eta_{it}
\]  

(5)

- Model error flexibly as mixture of normals:

\[
\eta_{it} \sim \pi_1 N(\mu_1, \sigma_1) + \pi_2 N(\mu_2, \sigma_2) + \pi_3 N(\mu_3, \sigma_3) + \pi_4 N(\mu_4, \sigma_4)
\]  

(6)

- Must specify prior for \( \theta, \pi = (\pi_1, \pi_2, \pi_3, \pi_4), \mu = (\mu_1, \mu_2, \mu_3, \mu_4) \) and \( \sigma = (\sigma_1, \sigma_2, \sigma_3, \sigma_4) \).

- Let \( m_{i,t} \) be the markup of player \( i \) in auction \( t \) and let \( p(m_{i,t}) \) the prior on markups.

- The views of the experts are averaged:
25^{th} \text{ percentile} = 3\% \quad (7)
50^{th} \text{ percentile} = 5\%
75^{th} \text{ percentile} = 7\%
99^{th} \text{ percentile} = 15\%

\[ p(m_{i,t}) = \begin{cases} 
8.3333 \text{ if } 0 < m_{i,t} < 0.03 \\
12.5 \text{ if } 0.03 < m_{i,t} < 0.05 \\
12.5 \text{ if } 0.05 < m_{i,t} < 0.07 \\
3.125 \text{ if } 0.07 < m_{i,t} < 0.15 
\end{cases} \]

- Markups, bids and costs must satisfy:

\[ c_{i,t} = (1 - m_{i,t}) \ast b_{i,t} \]

- Draw a random vector of markup \( m_T \) for all projects \( t = 1, \ldots, T \) and all bidders \( i \) who bid in the \( t^{th} \) project.
• Then using equation above we can compute a set of latent costs $c_{i,t}$.

• We then estimate the following to get a draw on $\theta$

$$\frac{c_{i,t}}{EST_t} = \beta_{0,i} + \beta_{1,i}LDIST_{i,t} + \beta_{2,i}CAP_{i,t} + \beta_{3,i}LMDIST_{i,t} + \beta_{4,i}CON_{i,t} + \varepsilon_{it}$$

$$\frac{c_{i,t}}{EST_t} = \alpha_0 + \alpha_1LDIST_{i,t} + \alpha_2CAP_{i,t} + \alpha_3LMDIST_{i,t} + \alpha_4CON_{i,t} + \varepsilon_{it}$$

• From equations above we can also find an estimated value $\hat{\sigma}$ for the standard deviation of $\varepsilon_{it}$.

• We must next compute the value of $\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$, $\mu = (\mu_1, \mu_2, \mu_3, \mu_4)$ and $\sigma = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$. 
• We shall assume that the parameters of the mixture of normals are drawn uniformly from all of those distributions satisfying:

\[
\begin{align*}
\pi_1 \mu_1 + \pi_2 \mu_2 + \pi_3 \mu_3 + \pi_4 \mu_4 &= 0 \\
\pi_1 \sigma_1^2 + \pi_2 \sigma_2^2 + \pi_3 \sigma_3^2 + \pi_4 \sigma_4^2 &= \hat{\sigma}^2 \\
\pi_1 + \pi_2 + \pi_3 + \pi_4 &= 1 \\
-0.2 \leq \mu_i \leq 0.2 \\
0.01 \leq \sigma_i \leq 0.2
\end{align*}
\]

• The likelihood for a vector of costs for the competitive model \( M1 \) conditional on \( \theta \) is then:

\[
p\left( \frac{c_i}{E ST_t} | \theta, M1 \right) = \pi_1 N\left( \frac{c_i}{E ST_t} - \tilde{c}_{i,t}, \mu_1, \sigma_1 \right) + \pi_2 N(\cdot, \mu_2, \sigma_2) + \pi_3 N(\cdot, \mu_3, \sigma_3) + \pi_4 N(\cdot, \mu_4, \sigma_4)
\]
Likelihood for costs from the model $M2$ and $M3$ are similar except the cost for the cartel is min of member’s cost.

8.2 Putting It All Together.

Steps needed to compute a simulated value of the likelihood function for model $M$.

1. Draw a random vector $c_{M,EST}^T$ over the distribution of cost estimates from the distribution $p(c_{M,EST}^T)$ that was estimated in Section 6.

2. Draw a random vector $m_T$ from the prior.

3. Given $m_T$, compute fitted values $\tilde{c}_{i,t}$ for all auctions $t$ and all firms $i$. 
4 Draw the parameters of the mixture of normal distribution $\pi, \mu, \sigma$

5 Compute the value of the likelihood function,

$p(c_T|\pi, \mu, \sigma, \alpha, \beta, M1)$

- Using standard methods for computing an integral using simulation, to compute marginalized likelihood.

- Using equation (4), we can compute the posterior beliefs for the 3 competing models.

- Suppose that the anti-trust authority possesses a loss function $u(a, M)$. 
• Here $a$ is the action that the anti-trust authority must take, for example, to prosecute or not and $M$ takes on values $M_1, M_2$ and $M_3$ corresponding to whether or not there is a cartel in the industry.

• The anti-trust authority’s expected loss of an action $a$, conditional on the data is

$$p^*(M_1)u(a, M_1) + p^*(M_2)u(a, M_2) + p^*(M_3)u(a, M_3)$$

(9)

• Given the posterior probabilities over models $p^*(M_1)$, $p^*(M_2)$, $p^*(M_3)$, it is then straightforward for the anti-trust official to choose a loss minimizing action.
9 Results.

- It is clear from Table 9 that markups are consistently higher under the collusive models M2 and M3.

- The difference between these models becomes particularly striking near the 80th and 90th percentiles.

- In model M2, 20 percent of the projects have a markup in excess of 15 percent and 10 percent of the projects have markups in excess of 33 percent.

- In model M3, 20 percent of the projects have markups in excess of 17 percent and 10 percent have markups in excess of 58 percent.

- Models M2 and M3 generate a much higher frequency of markups over 15 percent than the prior beliefs of our industry experts.
Another difference between models M1, M2 and M3 is the percentages of costs that are found to be negative.

### Table 10: Percent of Cost Estimates Found to be Negative.

<table>
<thead>
<tr>
<th>Model</th>
<th>Percentage Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>3%</td>
</tr>
<tr>
<td>M2</td>
<td>5%</td>
</tr>
<tr>
<td>M3</td>
<td>7%</td>
</tr>
</tbody>
</table>

As we can see from Table 11, the competitive model M1 strongly dominates the other two alternatives.

This is because it more closely agrees with the beliefs of the industry insiders.
10 Conclusions.

- Complete characterization of competitive bidding in asymmetric auction models.

- Proposed a two stage approach to identification.

- First stage tests necessary conditions of competitive bidding and helps to identify possible cartels.

- Second stage uses prior information about the cost structure.