AMATH 301
Homework 5: Spring 2009

DUE: see website for exact time and date. No late assignments accepted.

I Solve the nonlinear Schrödinger equation

$$i\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + |u|^2 u$$

on a domain $x \in [-10, 10]$ for time $t \in [0, 10]$, with periodic boundary conditions and initial condition $u(x, 0) = sech(x)$ {Note: you will need a second initial condition after one time step for leap-frog (2,2), use: $sech(x+dt)$}. Solve the equation numerically using the leap-frog (2,2) method with the CFL condition $\lambda = 0.05$, using n = 50 nodes in the spatial domain (including the left boundary point). Graph the absolute value of u using the waterfall command.

ANSWERS: Save the length of your vector for time t in the file A1.dat. Save the length of your spatial domain vector x in the file A2.dat. Save the absolute value of your solution $|u|$ as a matrix with 'length(x)' number of rows and 'length(t)' number of columns in the file A3.dat.

II Consider the partial differential equation (PDE)

$$u_{tt} = c^2 u_{xx} \tag{1}$$

where $c^2 = 1$, $x \in [-10, 10]$. Assume periodic boundary conditions and use $n = 128$ for the number of discrete points. For initial conditions start with the $x = 1$-centered Gaussian , $u(x, 0) = e^{-(x-1)^2}$ and $u'(x, 0) = 0$. Fourier transform the PDE and use **ode45** to advance the solution in time for *tspan=0:.1:6*, then make use of ifft in order to get the solution of the PDE in the original spatial domain.

ANSWER: Save the frequency components 'k' of the FFT as a row vector of length 128 in A4.dat. The inverse transform for each time slice of the ode45 solution outputs should be saved as a matrix with 128 rows and 61 columns saved in A5.dat.