AMATH 301
Homework 3: Spring 2009

DUE: see website for exact time and date. No late assignments accepted.

I Consider the following data set

x	0	0.5	1	1.5	2	2.5	3	3.5
y	1	2.4	3.1	5.0	7	11	17	24

1. Fit the data with the parabolic fit

$$f(x) = Ax^2 + Bx + C$$

and calculate the E_2 measure of error over the given $n = 8$ data points:

$$E_2 = (\frac{1}{n}\sum_{k=1}^{n}(f(x_k) - y_k)^2)^{1/2}$$

Use POLYFIT and POLYVAL to get your results. Evaluate the curve $f(x)$ for $x = (0 : 0.01 : 3.5)$ and save this in a column vector. Plot the data and the interpolated values on the same figure.

ANSWERS: The error and curve should be written out as A1.dat and A2.dat respectively.

2. Use the INTERP1 (use both default *linear* option and *nearest* option) and SPLINE command to generate an interpolated approximation to the data for $x = 0 : 0.01 : 3.5$. Save these three results as column vectors in the order listed (*linear*, *nearest* and *spline*).

Plot the three sets of data on the same graph.

ANSWERS: Three column vectors should be written out as A3.dat - A5.dat,

3. Calculate the E_2 measure of error over the given $n = 8$ data points for each option in part 2 (*linear*, *nearest* and *spline*). Save the three errors into one **row** vector with each component corresponding to the *linear*, *nearest* and *spline* cases in this order.

ANSWERS: The row vector should be written out as A6.dat

II Consider the Van der Pol differential equation

$$y'' + \epsilon(y^2 - 1)y' + y = 0$$

which has the nonlinear damping term $\epsilon(y^2 - 1)y'$.

(1) With $\epsilon = 0.1$, solve the equation for $t \in [0 : 0.5 : 30]$ for initial conditions $y(0) = 0.1$ and $y'(0) = -1$. Repeat with $\epsilon = 1$ and $\epsilon = 20$. (Use the command ode45 with default settings, but `TSPAN` set to $[0 : 0.5 : 30]$, in usage `ODE45(ODEFUN,TSPAN ...)` – use `help ode45` for more on this.)

ANSWERS: $y(t)$, for $t \in [0 : 0.5 : 30]$, should be written out as a $61X1$ column vector for $\epsilon = 0.1, 1$ and 20 respectively. Save these as A7.dat - A9.dat respectively.

(2) With $\epsilon = 1$, $t \in [0, 30]$ (let MATLAB pick step-size – do NOT use `TSPAN` set to $[0 : 0.5 : 30]$ as above) and initial conditions $y(0) = 5$ and $y'(0) = 0$, to solve the equation

with two different integration methods: ode45 and ode23. For each method, use the *diff* and *mean* command to calculate the average step-size Δt taken to solve the problem over $t \in [0, 30]$ for a given tolerance. Control the error tolerance, TOL, in the ode solvers with commands of the general form

```
TOL=1e-4;
OPTIONS = odeset('RelTol',TOL,'AbsTol',TOL);
[T,Y] = ODE45('F',TSPAN,Y0,OPTIONS);
```

Use the following tolerance values: $10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}, 10^{-9}, 10^{-10}$. Plot on a log-log scale the average step-size (x-axis) and the tolerance (y-axis) for the given tolerance values. Calculate the slopes of these lines (i.e., the slope of the log of tolerance vs log of stepsize) with the *polyfit* command and a first-order polynomial. Note that the local error should be $O(\Delta t^5)$ and $O(\Delta t^3)$ for ode45 and ode23 respectively – is this at least very roughly consistent with your findings?

ANSWERS: Slope for ode45 should be written out as a scalar number A10.dat; slope for ode23 should be written out as a scalar number A11.dat.