AMATH 301
Homework 2: Spring 2009

DUE: see website for exact time and date. No late assignments accepted.

I Suppose a company has 3 factories $A1, A2, A3$, producing 3 products $B1, B2, B3$. Each day, each of the factories can produce the following number of products:

	A1	A2	A3
B1	11	3	6
B2	3	4	0
B3	2	4	7

Based on the current market situation, the companies plan to produce the following numbers of their products for the following 3 quarters of the year:

	1st quarter	2nd quarter	3rd quarter
B1	391	284	383
B2	131	142	102
B3	282	237	213

For each quarter, let $\vec{x} = (x_1, x_2, x_3)^T$ represent the number of days A1, A2 and A3 need to work. Also let a 3 by 3 matrix M represent the productivity table, where the M_{ij}th entry means that factory Aj can produce M_{ij} number of product Bi in a single day. Finally let $\vec{b} = (b_1, b_2, b_3)^T$ denote the production target in one quarter. For instance, in the 1st quarter $\vec{b} = (391, 131, 282)^T$. In order to know the number of days each factory should work in one quarter, we only need to solve the linear system $M\vec{x} = \vec{b}$. (Note: M is strictly diagonally dominant.)

1. Solve the linear system $M\vec{x} = \vec{b}$ by using the backslash command for the 3 quarters respectively. Export your answer in a 3 by 3 matrix, where the 1st, 2nd and 3rd columns represent the corresponding quarter's production plan, i.e. the number of days each factory should work in this quarter.

ANSWERS: a 3 by 3 matrix in A1.dat

2. Repeat part (1), using LU decomposition, Jacobi iteration and Gauss-Seidel iteration. For the iterative methods, begin with the guess $(x_1, x_2, x_3) = (0, 0, 0)$. This will give you three additional 3 by 3 matrices for the three methods respectively. For the iterative methods, continue iterating until $\|\vec{x}_{k+1} - \vec{x}_k\| < 10^{-6}$.

ANSWERS: The 3 by 3 matrices should be written out as A2.dat - A4.dat

3. For the two iteration methods, calculate the number of iterations it took **on average** for the solution to solve the given equation with accuracy 10^{-6}. The accuracy constraint should be based upon looking at the norm of the difference between successive iterations: $\|\vec{x}_{k+1} - \vec{x}_k\| < 10^{-6}$, as above. Save the two answers (the lowest value of n, **averaged** for the three months for Jacobi first and Gauss-Seidel second) as a row vector with two components.

ANSWERS: Should be written out as A5.dat

II Given the following set of Ordinary Differential Equations (ODE) rewrite the ODE system as a matrix system $\mathbf{Ay} = \frac{\partial \mathbf{y}}{\partial t}$ in order to find the eigenvalues of the system.

$$\begin{aligned}
\frac{\partial y_1}{\partial t} &= 4y_1 + y_2 + 3y_3 \\
\frac{\partial y_2}{\partial t} &= 6y_1 + 7y_2 + 12y_3 \\
\frac{\partial y_3}{\partial t} &= 11y_1 + 2y_2 + 5y_3
\end{aligned}$$

Once the system of ODEs are written as an eigenvalue problem solve for the 3 eigenvalues and output the **absolute values** of the eigenvalues in order from the largest absolute value to the smallest as a column vector. Hint: the MATLAB function 'sort' might be useful.

ANSWERS: Should be written out as A6.dat

III Fit the following data set containing 14 data points with a 13^{th} order polynomial $p_{13}(x) = \sum_{n=0}^{13} a_n x^n$
such that $p_{13}(x_j) = y_j$ for $j = 0, ..., 13$.

x	y
0	−0.0049
0.3077	0.0098
0.6154	0.1274
0.9231	0.0196
1.2308	−0.1667
1.5385	−0.2548
1.8462	−0.4607
2.1538	0.0393
2.4615	0.3821
2.7692	0.3333
3.0769	0.0881
3.3846	−0.5095
3.6923	−0.2060
4.0000	0.9214

1. Without using any of MATLAB's built in fitting functions obtain the fitting polynomial p_{13} coefficients a_n for $n = 0, ..., 13$, and save the coefficients in this order as a column vector.

ANSWERS: Should be written out as A7.dat

2. Once the fitting polynomial has been obtained evaluate the interpolating polynomial at $x = 3$. That is, compute: $p_{13}(3)$.

ANSWERS: Should be written out as A8.dat