1. Sets and Functions

You have probably seen a lot of these things in intro to proofs, but it is worth reviewing.

Definition 1. Two sets A and B are said to be equal (notated as $A = B$) if they contain the same elements.

We showed examples of finite sets in class.

To prove $A = B$, for simple sets we can simply match the elements, however for more complex sets we need to show $A \subseteq B$ (proper subset) and $B \subseteq A$. To prove $A \subseteq B$, we suppose $x \in A$ and show $x \in B$.

1.1. Some important sets.

$$\mathbb{N} := \{1, 2, 3, \ldots, n, \ldots\}$$
$$\mathbb{Z} := \{0, \pm 1, \pm 2, \ldots, \pm n, \ldots\}$$
$$\mathbb{Q} := \left\{ \frac{n}{m} : m, n \in \mathbb{Z}, n \neq 0 \right\}$$
$$\mathbb{Q}^c := \mathbb{R} \setminus \mathbb{Q}$$

Now lets go over some set operations and other important definitions.

Definition 2.
(1) The union of sets A and B is $A \cup B := \{x : x \in A \text{ or } B\}$.
(2) The intersection of sets A and B is $A \cap B := \{x : x \in A \text{ and } B\}$.
(3) The complement (set minus) of B relative to A is $A \setminus B := \{x : x \in A \text{ and } x \in B\}$

Definition 3. The empty set, denoted as $\emptyset$, is a set with no elements.

Definition 4. Two sets A and B are said to be disjoint if $A \cap B = \emptyset$

We went over examples of these definitions in class.

Now lets look at a theorem on set identities.

Theorem 1. *Consider the sets A, B, C. The following identities hold,*
(1) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

(2) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

Before we prove the theorem, lets first think of what strategy we are going to use. Recall that in order to prove equality we have to do two proofs each:

(1) (a) $A \setminus (B \cup C) \subseteq (A \setminus B) \cap (A \setminus C)$
   (b) $(A \setminus B) \cap (A \setminus C) \subseteq A \setminus (B \cup C)$

(2) (a) $A \setminus (B \cap C) \subseteq (A \setminus B) \cup (A \setminus C)$
   (b) $(A \setminus B) \cup (A \setminus C) \subseteq A \setminus (B \cap C)$

Proof. (1) (a) Suppose $x \in A \setminus (B \cup C)$, then $x \in A$, but $x \notin B \cup C \Rightarrow x \notin B, x \notin C \Rightarrow x \in A \setminus B$ and $x \in A \setminus C \Rightarrow x \in (A \setminus B) \cap (A \setminus C)$.
(b) Suppose $x \in (A \setminus B) \cap (A \setminus C)$, then $x \in A \setminus B$ and $A \setminus C \Rightarrow x \in A$, but $x \notin B, x \notin C \Rightarrow x \notin (B \cup C) \Rightarrow x \in A \setminus (B \cup C)$.
Since (a) and (b) hold, $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

(2) (a) Suppose $x \in A \setminus (B \cap C)$, then $x \in A$, but $x \notin (B \cap C) \Rightarrow x \notin B$ or $x \notin C$. If $x \notin B$, then $x \in (A \setminus B)$. if $x \in C$, then $x \in (A \setminus C) \Rightarrow x \in (A \setminus B) \cup (A \setminus C)$.
(b) Suppose $x \in (A \setminus B) \cup (A \setminus C)$, then $x \in (A \setminus B)$ or $x \in (A \setminus C)$. If $x \in (A \setminus B)$, $x \in A$ but $x \notin B$. If $x \in (A \setminus C)$, $x \in A$ but $x \notin C \Rightarrow x \notin (B \cup C) \Rightarrow x \in A \setminus (B \cap C)$.
Since (a) and (b) hold, $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

Thereby completing the proof. □