Supplementary problems: Sec. 2.1 # 26; 2.2 # 16; 2.3 # 4, 5, 7, 9, 11, 14; 2.4 # 13

Compulsory problems:

(1) **[5 pts.]** Prove that if $|x - x_0| < \epsilon/2$ and $|y - y_0| < \epsilon/2$, then $|(x + y) - (x_0 + y_0)| < \epsilon$ and $|(x - y) - (x_0 - y_0)| < \epsilon$.

(2) **[5 pts.]** Prove that $\mathbb{R}$ is unbounded.

(3) **[5 pts.]** Prove that for every $x \in \mathbb{R}$ there is an $n \in \mathbb{Z}^+$ such that $n > x$.

(4) **[5 pts. each]** Find the sup and inf of the following
- (a) $S = \{2^{-p} + 3^{-p} + 5^{-r} : p, q, r \in \mathbb{Z}^+\}$
- (b) $S = \{x : 3x^2 - 10x + 3 < 0\}$
- (c) $S = \{x : (x - a)(x - b)(x - c)(x - d) < 0\}$, where $a < b < c < d$.

(5) **[10 pts.]** Suppose $A, B \subseteq \mathbb{R}^+$ are bounded above, and define $C \subseteq \mathbb{R}$ as $C := \{xy : x \in A, y \in B\}$.
Prove that $(\sup A)(\sup B) = \sup C$.

(6) **[15 pts.]** Suppose $A, B \subseteq \mathbb{R}$ are nonempty with $A \subseteq B$ and B is bounded above. Prove that $\sup A \leq \sup B$. Further, prove if $\sup A < \sup B$ then $A \subset B$; i.e. $\sup A < \sup B \Rightarrow A \neq B$.

Your homework raw score is: $\frac{n}{2m} \cdot M + \left(1 - \frac{n}{2m}\right) \cdot N = N + \frac{n}{2m}(M - N)$. For this homework, $M = 50$, $m = 9$, N is the number of compulsory problems you get correct, and n is the number of supplementary problems you complete. It should be noted that for the supplementary problems I will be looking for **full completion**, but I won't take off points for mistakes.