Supplementary problems: Sec. 1.1 # 11, 14, 15; Sec. 1.2 # 1, 6; Sec. 1.3 # 1, 9

Compulsory problems:

(1) **[5 pts.]** Notice that in Sec. 1.1 # 14, $f(E \cap F) \subseteq f(E) \cap f(F)$, but not necessarily equal. For $E, F \subseteq \mathbb{R}$, give an example when $f(E \cap F) \neq f(E) \cap f(F)$; i.e. $f(E) \cap f(F) \nsubseteq f(E \cap F)$.

(2) **[5 pts.]** Identify the incorrect implication in the following false "proof" and explain why this is incorrect.

$$x = y \Rightarrow x^2 = xy \Rightarrow x^2 - y^2 = xy - y^2 \Rightarrow (x + y)(x - y) = y(x - y) \Rightarrow x + y = y \Rightarrow 2y = y \Rightarrow 2 = 1.$$

(3) **[10 pts.]** Consider a sequence $a_n \in \mathbb{R}$ defined by the recursion $(2 - a_n)a_{n+1} = 1$. Prove that if $0 < a_1 < 1$, then $0 < a_n < a_{n+1} < 1$; i.e. it is a increasing sequence.

(4) **[10 pts.]** Prove that every natural number is either even or odd.

(5) **[10 pts.]** Prove that if there are injections $f : X \mapsto Y$ and $g : Y \mapsto X$, where X and Y are finite, then there is also a bijection from X to Y such that $\text{Card}(X) = \text{Card}(Y)$. (Hint: think about the cardinalities of finite sets) (Note: this is a special case of a famous theorem. Do not use the statement of that theorem to prove this!).

(6) **[10 pts.]** Prove that $\sqrt{6}$ is an irrational number.

Your homework raw score is: $\frac{n}{2m} \cdot M + \left(1 - \frac{n}{2m}\right) \cdot N = N + \frac{n}{2m}(M - N)$. For this homework, $M = 50$, $m = 7$, N is the number of compulsory problems you get correct, and n is the number of supplementary problems you complete. It should be noted that for the supplementary problems I will be looking for **full completion**, but I won't take off points for mistakes.