

- (1) Sketch the direction field for $y' = y(y^2 - 4)$ and state what happens as $t \rightarrow \infty$.
- (2) Sketch the direction field for $y' = y(y - 2)^2$.
- (3) Consider the ODE $2t^2y'' + 3ty' - y = 0$.
 - (a) What is the order of the equation? Is it linear or nonlinear?
 - (b) Is $y_1 = t^{1/2}$ a solution?
 - (c) Is $y_2 = t^{-1}$ a solution?
 - (d) If they are both solutions find the Wronskian of those solutions.
- (4) What is the order of the ODE $\frac{d}{dx} \left(x \frac{dy}{dx} \right) = \frac{\ln x}{xy}$. Is it linear or nonlinear?
- (5) Solve the IVP $\frac{dy}{dx} = \frac{x}{y(1+x^2)}$, $y(0) = -2$.
- (6) Solve the IVP $y' = y^2 - 1$, $y(0) = -2$.
- (7) Solve the IVP $y' + y = e^{-t}$, $y(0) = y_0$. Find the value of y_0 such that the solution $y(t)$ reaches its maximum at $t = 4$.
- (8) Solve the IVP $ty' + 2y = 4t^2$, $y(1) = 4$.
- (9) Find a linear homogeneous constant coefficient ODE that has the roots of its characteristic equation as $r = -2, 3$.
- (10) Solve the IVP $y'' - y' - 2y = 0$; $y(0) = \alpha$, $y'(0) = \beta$. What relation between α and β will give us a bounded (for all time) solution?
- (11) Consider the IVP: $t(t - 4)y'' - 3ty' + 4y = 2$; $y(3) = 0$, $y'(3) = -1$. Determine the longest interval for which the IVP is guaranteed to have a unique solution.
- (12) Consider the IVP: $4y'' + 12y' + 9y = 0$; $y(0) = -1$, $y'(0) = \alpha$.
 - (a) For what α does the solution change signs at $t = 1/2$?
 - (b) How many times does this solution (for the α above) change signs for $t > 0$?
- (13) A tank with a capacity of 6 L initially contains 10 g of salt and 1 L of water. A mixture containing 1 g/L of salt enters the tank at a rate of 3 L/hr. The mixture leaves the tank at a rate of $\frac{1}{2}V(t)$ L/hr, where $V(t)$ is the volume of fluid in the tank (which may be less than the volume of the tank itself).
 - (a) Formulate the IVP for the volume of fluid in the tank then solve that IVP.
 - (b) Formulate an ODE for the amount of salt in the tank. Show that this ODE is the same as the ODE for the volume above.
- (14) A tank initially contains 120 L of fresh water. A mixture containing a concentration of γ g/L of salt enters the tank at a rate of 2 L/min, and the well-stirred mixture leaves the tank at a rate of 3 L/min.
 - (a) When will the tank be empty (i.e. come up with an IVP for the volume).
 - (b) Formulate and then solve an IVP for the amount of salt in the tank.