(1) We substitute $x = 5\tan\theta \Rightarrow dx = 5\sec^2\theta d\theta$, then

$$I = \int \frac{\sqrt{25\sec^2\theta - 25}(\cancel{5\sec\theta}\tan\theta)}{\cancel{5\sec\theta}} d\theta = 5\int \tan^2\theta d\theta = 5\int(\sec^2\theta - 1)d\theta = 5\tan\theta - 5\theta = \boxed{\sqrt{x^2-25} - 5\sec^{-1}\left(\frac{x}{5}\right) + C}.$$

(2) Here we use by parts, $u = \tan^{-1}x \Rightarrow du = dx/(1+x^2)$ and $dv = dx \Rightarrow v = x$, then

$$I = x\tan^{-1}x - \int \frac{x}{x^2+1}dx = \boxed{x\tan^{-1}x - \frac{1}{2}\ln|x^2+1| + C}.$$

(3) Let $u = \tan(3x) \Rightarrow du = 3\sec^2(3x)dx$, then

$$I = \int \tan^4(3x)(\tan^2(3x)+1)\sec^2(3x)dx = \frac{1}{3}\int(u^6+u^4)du = \frac{1}{3}\left[\frac{1}{7}u^7 + \frac{1}{5}u^5\right] + C = \boxed{\frac{1}{3}\left[\frac{1}{7}\tan^7(3x) + \frac{1}{5}\tan^5(3x)\right] + C}.$$

(4) Here we use partial fractions,

$$\frac{3}{(2x+1)(x^2+1)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+1} \Rightarrow (A+2B)x^2 + (B+2C)x + (A+C) = 3 \Rightarrow A = -2B, B = -2C \Rightarrow C = \frac{3}{5}, B = -\frac{6}{5}, A = \frac{12}{5}$$

Then our integral becomes

$$I = \frac{12}{5}\int\frac{dx}{2x+1} - \frac{3}{5}\int\frac{2x-1}{x^2+1}dx = \frac{12}{10}\ln|2x+1| - \frac{3}{5}\int\frac{2x}{x^2+1}dx + \frac{3}{5}\int\frac{dx}{x^2+1} = \boxed{\frac{6}{5}\ln|2x+1| - \frac{3}{5}\ln|x^2+1| + \frac{3}{5}\tan^{-1}x + C}.$$

(5) Let $x^2 = 2\sin\theta \Rightarrow \cancel{2}xdx = \cancel{2}\cos\theta d\theta$, then

$$I = 2\int\cos^2\theta d\theta = \int(1+\cos 2\theta)d\theta = \theta + \frac{1}{2}\sin 2\theta + C = \theta + \sin\theta\cos\theta + C = \boxed{\sin^{-1}\left(\frac{x^2}{2}\right) + \frac{x^2}{2}\cdot\frac{\sqrt{4-x^4}}{2} + C}.$$

(6) We integrate this directly be noticing the $\cos x$ can be factored out,

$$I = \int_{\pi/3}^{\pi/2}\cos x(\cos^2 x + \sin^2 x)dx = \int_{\pi/3}^{\pi/2}\cos x dx = \sin x\Big|_{\pi/3}^{\pi/2} = \boxed{1 - \frac{\sqrt{3}}{2}}.$$

(7) Here we must do an improper integral,

$$I = \lim_{t\to\infty}\int_2^t \frac{dx}{x^{3/2}} = \lim_{t\to\infty} -2x^{-1/2}\Big|_2^t = \lim_{t\to\infty} -2t^{-1/2} + \sqrt{2} = \boxed{\sqrt{2}}$$

(8) a) $k = 3$, b) any $k < 3$, c) any $k > 3$.

(9) First lets take the antiderivative by letting $u = \ln x \Rightarrow du = dx/x$, then

$$\int\frac{\ln x}{x}dx = \int u du = \frac{1}{2}u^2 = \frac{1}{2}(\ln x)^2$$

Now we do the improper integral

$$I = \lim_{t\to 0}\int_t^1 \frac{\ln x}{x} = \lim_{t\to 0}\frac{1}{2}(\ln x)^2\Big|_t^1 = \lim_{t\to 0} -\frac{1}{2}(\ln t)^2 = -\infty$$

Therefore the integral <u>diverges</u>.

(10) (a) $\Delta x = k$, then $x_0 = -2k$, $x_1 = -k$, $x_2 = 0$, $x_3 = k$, $x_4 = 2k$, and $y_0 = 2^4k^4$, $y_1 = k^4$, $y_2 = 0$, $y_3 = k^4$, $y_4 = 2^4k^4$, then

$$S_4 = \frac{k}{3}\left[16k^4 + 4k^4 + 4k^4 + 16k^4\right] = \boxed{\frac{40}{3}k^5}.$$

(b)

$$f'(x) = 4x^3, f''(x) = 12x^2, f'''(x) = 24x, f''''(x) = 24 \Rightarrow |E_s| \le \frac{24\cdot 4^5k^5}{180\cdot 4^4} = \boxed{\frac{8}{15}k^5}.$$

(c)

$$\int_{-2k}^{2k} x^4 dx = \frac{1}{5}x^5\Big|_{-2k}^{2k} = \frac{2}{5}(2k)^5 = \frac{2^6}{5}k^5 \Rightarrow |\text{Error}| = \left|\frac{2^6}{5}k^5 - \frac{40}{3}k^5\right| = \boxed{\frac{8}{15}k^5}.$$

(11) Here we use partial fractions,

$$\frac{2x^2+2}{x(x-1)^2}dx = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \Rightarrow (A+B)x^2 - (B+2A-C)x + A = 2x^2 + 2 \Rightarrow A = 2, B = 0, C = 4.$$

Then

$$I = 2\int\frac{dx}{x} + 4\int\frac{dx}{(x-1)^2} = \boxed{2\ln|x| - 4(x-1)^{-1} + C}.$$