# Is that a $100 bill on the sidewalk?*

Philip Bond, University of Washington

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**Abstract**

I analyze a setting in which investors sequentially encounter a potentially profitable investment opportunity, and decide both whether to investigate its quality and whether to exploit the opportunity. Once exploited, the opportunity is unavailable to future investors. The key friction is that an investor observes only whether the opportunity remains available, but is ignorant about the number of investors who have already investigated the opportunity. The probability that the opportunity is exploited decreases in the number of investors. Safe opportunities are exploited more often than risky ones, even controlling for average profitability. Higher exploitation probabilities are generally associated with lower observed Sharpe ratios.

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...an economics professor [is] walking with a keen-eyed student across the university quad. "Look", says the student, pointing at the ground, "a five-pound note". "It can't be", replies the rational professor. "If it was there, somebody would have picked it up by now."

From *The Economist*, October 20, 1984

# 1 Introduction

The joke in the epigraph is well-known and widely invoked, in large part because it accurately conveys the economic insight that profitable opportunities tend to be exploited. But the joke *also* conveys a potentially important countervailing force that limits such exploitation, namely that economic agents—henceforth, *investors*—will be unwilling to incur costs to examine a potentially attractive opportunity on the grounds that if the opportunity were indeed profitable then someone else would already have exploited it ("picked it up").

In this short paper I analyze this setting. Specifically, investors sequentially encounter a potentially profitable opportunity. Each investor decides both whether to pay an investigation cost to observe a noisy signal about the opportunity's profitability; and whether or not to exploit the opportunity. Once exploited, the opportunity is unavailable to future investors. Crucially, each investor observes only whether or not the opportunity is available, but is ignorant about how many other investors have already examined the opportunity and decided not to exploit it.

In this setting, the professor in the joke is asserting that the probability of the opportunity being good conditional on it remaining available approaches 0. Is the professor correct?

I derive a simple inequality under which the equilibrium probability that the opportunity is ever-exploited is bounded away from 1 even as the number of potential investors grows arbitrarily large. In this case, the conditional probability that the opportunity is good is bounded away from 0, and the supposedly rational professor is wrong. Moreover, the inequality is mild in the sense that it is satisfied whenever the probability that an investor mistakes a bad opportunity for a good one is sufficiently small.

Opportunities that generate a high payoff if they are good, but have small probabilities of being good, are especially affected by investors' fears that an unexploited opportunity is likely to be bad. Consequently, the set of opportunities that are exploited is tilted towards "safe" prospects relative to the underlying pool of opportunities. After controlling for the return on bad opportunities, higher exploitation probabilities are associated with lower observed Sharpe ratios, provided that investigation produces relatively accurate information.

The analysis gives a parsimonious explanation for why profitable trading opportunities persist even in the presence of large quantities of arbitrage capital.[1] More generally, the analysis generates an endogenous limit to innovation, especially of high-risk high-reward projects.

I am unaware of a previous analysis of this setting. Perhaps closest is Zhu (2012)'s analysis of quotes offered by buyers in an OTC market under the assumption that a seller contacts buyers in a random and unobserved order. If buyers share a common valuation and receive exogenous noisy signals about this common value then in equilibrium there is a positive probability of no trade even as the number of buyers grows arbitrarily large. Different from in Zhu's analysis, in my setting agents observe information only if they first pay a cost, and in equilibrium many agents remain uninformed; consequently, endogenous acquisition of information potentially changes the adverse selection problem faced by buyers.[2] Separately, and again different from Zhu, I derive comparative statics for how equilibrium exploitation probabilities vary with the number of traders and with project characteristics. Papers such as Sherman and Willett (1967) and Elberfeld and Wolfsetter (1999) study Bertrand competition with a pre-stage in which firms simultaneously decided whether or not to pay an entry cost, and show that entry is decreasing in the potential number of firms. Unlike in the current setting, there is no learning from the fact that the opportunity still exists. Herrera and Hörner (2013) and Monzón and Rapp (2014) study social learning settings in which agents sequentially make a binary decision, as in Banerjee (1992) and Bikhchandani et al (1992) and a large subsequent literature, but in which individual agents lack information about their place in the sequence. I share this last assumption, but the setting and results are significantly different. At a high level, the paradox underlying the sidewalk-dollar joke is related to the topic examined in Grossman and Stiglitz (1980), viz., if market prices convey accurate information then no investor acquires information, but if no investor acquires information then market prices don't convey information.

# 2 Model

An investment opportunity has quality $\omega \in \{g(ood), b(ad)\}$ with probability $p_\omega$. A total of $n \geq 2$ risk-neutral investors sequentially encounter the opportunity, in random order. When an investor $i$ encounters the opportunity, it is either unexploited, or already-exploited. If it is already-exploited then investor $i$ can't do anything. If the opportunity is unexploited, investor $i$ chooses (I) whether or not to pay an investigation cost $\kappa > 0$ to privately observe

[1] See the large literature in financial economics on limits to arbitrage, surveyed by, for example, Gromb and Vayanos (2010).

[2] Online Appendix B.3 analyzes the effects of exogenous signal acquisition in my setting.

a signal $\sigma_i$ of the opportunity's quality, and (II) whether to exploit the opportunity, yielding a payoff of $\pi_\omega$, where $\pi_g > \pi_b$.

The critical assumption is that when an investor $i$ encounters the opportunity he/she doesn't know how many other investors have already encountered the opportunity—and by extension, doesn't know whether other investors have already investigated. Instead, the only information available to the investor is whether the opportunity remains unexploited.[3]

Conditional on the opportunity's quality $\omega$, individual investors' signals $\sigma_i$ are independent. The signal is binary, with $\sigma_i \in \{g, b\}$ and

$$\begin{aligned} \Pr(\sigma_i = g|\omega = g) &= 1 - \epsilon_g \\ \Pr(\sigma_i = b|\omega = b) &= 1 - \epsilon_b. \end{aligned}$$

That is, $\epsilon_\omega$ is the signal's error rate given project quality $\omega$. Throughout, I assume that the signal is accurate enough that, for a single investor acting in isolation, exploitation following a bad signal is unprofitable,

$$\epsilon_g p_g \pi_g + (1 - \epsilon_b) p_b \pi_b < 0; \tag{1}$$

and similarly, that a good signal is informative enough to justify the cost of investigation,

$$(1 - \epsilon_g) p_g \pi_g + \epsilon_b p_b \pi_b - \kappa > \max\{0, p_g \pi_g + p_b \pi_b\}. \tag{2}$$

Assumptions (1) and (2) imply both

$$\pi_g > 0 > \pi_b \tag{3}$$

and $\frac{1-\epsilon_g}{\epsilon_b} > \frac{\epsilon_g}{1-\epsilon_b}$, which is in turn equivalent to

$$\epsilon_g + \epsilon_b < 1. \tag{4}$$

Note that (4) implies that a signal $\sigma_i = g$ indeed raises the posterior of state $\omega = g$, i.e., $\Pr(\omega = g|\sigma_i = g) > \Pr(\omega = g|\sigma_i = b)$. For use throughout, define

$$\mathcal{E} \equiv \frac{\epsilon_b}{1 - \epsilon_g} < 1.$$

In the main text I generally assume strictly positive error rates $\epsilon_\omega > 0$. But the setting

[3] In particular, I assume that calendar time doesn't provide an individual investor with useful information about the number of other investors who have already encountered the opportunity. Formally, one can think of the opportunity as arriving at Poisson rate $\lambda$, and then the $n$ investors sequentially encountering the probability in random order. As $\lambda \to 0$, calendar time grows arbitrarily uninformative.

remains well-behaved if $\epsilon_g = 0$ and/or $\epsilon_b = 0$; see Online Appendix C.

# 3 Analysis

## 3.1 Equilibrium

I characterize the symmetric equilibrium. Let $\alpha$ be the probability with which each investor investigates the opportunity if it remains unexploited. (Section 4.1 considers a perturbed version of the model in which investigation costs $\kappa$ are heterogeneous, leading to a pure-strategy equilibrium in which investors with investigation costs below a cutoff investigate.)

As a preliminary step: assumptions (1) and (2) imply that, in equilibrium, investors exploit an opportunity if and only if they investigate and see a good signal.

**Lemma 1** *The equilibrium investigation probability $\alpha$ is strictly positive and an investor exploits the opportunity after receiving a good signal and does not exploit after either receiving a bad signal, or absent investigation.*

Given Lemma 1, an investor's expected payoff from investigating (gross of investigation cost $\kappa$) is

$$v(\alpha) \equiv \sum_{\omega=g,b} \pi_\omega \Pr(\omega|\sigma_i = g, i \text{ observes opportunity}) \Pr(\sigma_i = g|i \text{ observes opportunity}). \tag{5}$$

That is: An investor observes that the opportunity remains available, and consequently, that an uncertain number of previous investors have potentially investigated and drawn bad signals. Based on this, the investor assesses the probability of observing a signal $\sigma_i = g$; and then further assesses the probability that the opportunity is good ($\omega = g$) conditional on the signal $\sigma_i$ being good *and* an uncertain number of previous investors having potentially investigated and drawn signals $b$.

The payoff $v(\alpha)$ can be explicitly calculated via repeated application of Bayes' rule. But it is considerably easier to denote by $Q_\omega$ the joint probability that one of the $n$ investors exploits the probability if the opportunity's quality is $\omega$,

$$Q_\omega(\alpha) \equiv \Pr(\text{opportunity exploited by end of game}|\omega)$$

and then note that, by symmetry,

$$n \Pr(i \text{ observes opportunity}) \alpha v(\alpha) = \sum_{\omega=g,b} Q_\omega(\alpha) p_\omega \pi_\omega. \tag{6}$$

That is: the expected payoff of each investor is the probability of encountering an unexploited opportunity times the probability of investigating the opportunity ($\alpha$) times the expected payoff from doing so ($v$). The sum of these expected payoffs across investors must equal the aggregate payoff across all investors, which is given by the right hand side (RHS) of (6).

To evaluate (6), note that the opportunity is never exploited if none of the $n$ investors exploit, which occurs with probability

$$1 - Q_\omega(\alpha) = \Pr(\text{never exploited}|\omega) = (1 - \Pr(\sigma_i = g|\omega)\,\alpha)^n. \tag{7}$$

Moreover, and similarly,

$$\begin{aligned}
&\Pr(i \text{ observes opportunity}) \\
= &\sum_{\omega=g,b} p_\omega \sum_{k=1}^{n} \frac{1}{n} \Pr\left(i \text{ observes opportunity}|\text{investor } i \text{ is } k^{th} \text{ in line}, \omega\right) \\
= &\frac{1}{n} \sum_{\omega=g,b} p_\omega \sum_{k=1}^{n} (1 - \Pr(\sigma_i = g|\omega)\,\alpha)^{k-1} \\
= &\frac{1}{n} \sum_{\omega=g,b} p_\omega \frac{1 - (1 - \Pr(\sigma_i = g|\omega)\,\alpha)^n}{\Pr(\sigma_i = g|\omega)\,\alpha} \\
= &\sum_{\omega=g,b} \frac{Q_\omega(\alpha)\,p_\omega}{\Pr(\sigma_i = g|\omega)\,n\alpha}.
\end{aligned} \tag{8}$$

Hence for $\alpha \in (0, 1]$,

$$v(\alpha) = \frac{\sum_{\omega=g,b} Q_\omega(\alpha)\,p_\omega \pi_\omega}{\sum_{\omega=g,b} \frac{Q_\omega(\alpha) p_\omega}{\Pr(\sigma_i = g|\omega)}} = \frac{\frac{Q_g(\alpha)}{Q_b(\alpha)} p_g \pi_g + p_b \pi_b}{\frac{Q_g(\alpha)}{Q_b(\alpha)} \frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}}. \tag{9}$$

As (9) makes clear, the ratio of exploitation probabilities $\frac{Q_g}{Q_b}$ is a sufficient statistic for equilibrium characterization. The analysis heavily exploits this fact.

For a low individual investigation probability $\alpha$ the most likely case is a single investigation, and so the ratio $\frac{Q_g}{Q_b}$ simply coincides with the ratio of the conditional probabilities of seeing a good signal, namely $\frac{1-\epsilon_g}{\epsilon_b} > 1$ (by (4)). In contrast, if the individual investigation probability $\alpha$ is high then it is likely that some investor investigates and draws a good signal, even in the state $\omega = b$, and so $Q_g \approx Q_b \approx 1$. The following result formalizes and generalizes these observations:

**Lemma 2** *The ratio* $\frac{Q_g}{Q_b}$ *approaches* $\frac{1-\epsilon_g}{\epsilon_b} = \frac{1}{\mathcal{E}}$ *as* $\alpha \to 0$ *and equals* $\frac{1-\epsilon_g^n}{1-(1-\epsilon_b)^n}$ *at* $\alpha = 1$. *Both*

$\frac{Q_g}{Q_b}$ *and* $v(\alpha)$ *are monotonically decreasing in* $\alpha \in (0,1)$. *For any* $\alpha \in (0,1)$,

$$\lim_{n\to\infty} \frac{Q_g(\alpha)}{Q_b(\alpha)} = \frac{Q_g(\alpha)}{1-(1-Q_g(\alpha))^{\varepsilon}}. \tag{10}$$

In words: As others' investigation probability $\alpha$ increases, the observation that the opportunity remains available becomes an increasingly negative signal of the opportunity's quality $\omega$, so that the expected payoff from investigation $v$ decreases.

By Lemma 2 and assumption (2)

$$v(0) = \lim_{\alpha\to 0} v(\alpha) = (1-\epsilon_g)\, p_g\pi_g + \epsilon_b p_b \pi_b > \kappa.$$

In words: if no-one else investigates ($\alpha = 0$) then it is strictly profitable to investigate.

Consequently:

**Lemma 3** *There is a unique (symmetric) equilibrium given either by the solution to*

$$v(\alpha) = \kappa, \tag{11}$$

*or by* $\alpha = 1$ *if* $v(1) > \kappa$.

## 3.2 Comparative statics in the number of investors $n$

As the number of investors $n$ increases, the signal from the opportunity being available conveys increasingly negative information. Consequently, as $n$ increases, the probability $\alpha$ that any individual investor investigates falls.

The effect of the number of investors on the equilibrium exploitation probabilities, henceforth $Q_g^*$ and $Q_b^*$, thus depends on the balance between more potential investors, and each investor investigating less. As an easy example: As $n$ increases from 1 to 2, the investigation probability generally falls from $\alpha_1 = 1$ to $\alpha_2 < 1$; consequently, if $\epsilon_g = 0$, the exploitation probability $Q_g^*$ falls from 1 to $\alpha_2 + (1-\alpha_2)\alpha_2$, even though the number of investors has increased. This observation generalizes to cover $Q_b^*$ as well as $Q_g^*$, and arbitrary error rates $\epsilon_\omega$ and investor population $n$. The key step in the proof is Lemma A-1 in the Appendix, which establishes that the ratio $\frac{Q_g(\alpha)}{Q_b(\alpha)}$ is decreasing in $n$.

**Proposition 1** *The equilibrium exploitation probabilities* $Q_g^*$ *and* $Q_b^*$ *are weakly decreasing in the number of investors* $n$*; and strictly decreasing if the equilibrium is interior (*$\alpha < 1$*).*

By itself, the effect of more investors is to increase the overall investigation probability. Proposition 1 establishes that this direct effect is dominated by the equilibrium effect that each individual investor's investigation probability $\alpha$ shrinks.

### 3.3 Is the professor right? Conditions for less-than-certain exploitation as $n \to \infty$

An immediate consequence of Lemma 2 is:

**Corollary 1** *The equilibrium exploitation probabilities $Q_g^*$ and $Q_b^*$ are bounded away from 1 as $n \to \infty$ if and only if*

$$\frac{p_g \pi_g + p_b \pi_b}{\frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}} < \kappa. \tag{12}$$

*If condition (12) is satisfied then $\Pr(\omega = g|\text{never exploited})$ remains bounded away from 0 as $n \to \infty$.*

Corollary 1 gives the inequality referenced in the introduction. When (12) holds, the professor in the epigraph's joke is wrong. Specifically, the probability

$$\Pr(\omega = g|\text{never exploited}) = \frac{\left(1 - Q_g^*\right) p_g}{\left(1 - Q_g^*\right) p_g + \left(1 - Q_b^*\right) p_b} \tag{13}$$

remains bounded away from 0 even as the number of investors grows arbitrarily large.

If instead (12) fails, then the professor is correct, in the sense that as $n \to \infty$, the probability $\Pr(\omega = g|\text{never exploited})$ approaches 0. To see this, simply note that if (12) fails then the equilibrium is $\alpha = 1$;[4] and hence $\frac{1-Q_g^*}{1-Q_b^*} = \left(\frac{\epsilon_g}{1-\epsilon_b}\right)^n \to 0$ by (4).

Corollary 1 has three significant implications.

First: If an individual investor finds unconditional exploitation unprofitable under the prior probability $p_g$, i.e.,

$$p_g \pi_g + p_b \pi_b \leq 0, \tag{14}$$

then the equilibrium exploitation probabilities $Q_g^*$ and $Q_b^*$ are bounded below 1 *regardless* of how low the investigation cost $\kappa$ is. This stems from the negative inference that each individual investor draws from seeing that a project remains available. If (by way of contradiction) individual investigation probabilities $\alpha$ remain sufficiently high to generate $Q_g^*, Q_b^* \to 1$, this negative inference is strong, and coupled with (14) imply sufficiently negative beliefs that investigation is never worthwhile, no matter how low its cost.

[4]From Lemma A-1, this is true for any $n$.

Second: In light of these observations, it is perhaps surprising that if (12) doesn't hold then the equilibrium is that everyone investigates, $\alpha = 1$, regardless of how large $n$ is. From (1), it remains the case that an individual investor $i$ exploits if and only if signal $\sigma_i = g$ is observed. Why doesn't the combination of $\alpha = 1$ and large $n$ imply that investor $i$'s observation of a project still being available is an extremely negative signal?

The reason is that observing that the project is still available *also* suggests that an investor is reasonably "early" in the line. It is true that investors who somehow knew themselves to be late in the line would draw very negative inferences from observing that the opportunity is still available. But the observation that the project is still available suggests that the observing investor is very unlikely to be late in the line.

*Remark:* The discussion above of whether the student or professor is correct is from the post-game vantage point. One can also evaluate who is correct from the within-game vantage point, viz., under the assumption that the student and professor are participants in the game. In this case, the *professor is always wrong*, because (see appendix for derivation)

$$\Pr(\omega = g | i \text{ observes opportunity}) = \frac{Q_g^* \mathcal{E} p_g}{Q_g^* \mathcal{E} p_g + Q_b^* p_b}, \tag{15}$$

which (by Lemma 2) is bounded below by $\frac{\mathcal{E} p_g}{\mathcal{E} p_g + p_b}$.[5] The reason is the same is just discussed: even if all investors investigate with probability $\alpha = 1$, the negative inference that an individual investor observes from seeing a project available is bounded by the fact that this observation also suggests an early place in the line.

Third, inspection of condition (12) implies that less-than-certain exploitation always arises if the error rate $\epsilon_b$ is sufficiently low:

**Corollary 2** *For any $\kappa > 0$, there exists an $\bar{\epsilon}_b > 0$ such that if $\epsilon_b \leq \bar{\epsilon}_b$ then $Q_g^*$ and $Q_b^*$ are bounded away from 1 as $n \to \infty$.*

At first sight, Corollary 2 is surprising. In particular, it implies that, under some circumstances, lower error rates reduce equilibrium exploitation probabilities. Indeed, Lemma 4 below shows that the function $v(\alpha)$, which gives the value of investigation conditional on the opportunity being available, is *increasing* in the error rate $\epsilon_b$ whenever $\epsilon_b$ is sufficiently small. The economic force that drives this is: as the error rate in state $\omega = b$ increases,

[5] At face value expression (15) suggests that $\Pr(\omega = g | i \text{ observes opportunity})$ approaches 0 as $\epsilon_b$ approaches 0. But this isn't the case, for two reasons. First, for any $\alpha$, $Q_b(\alpha) \to 0$ as $\epsilon_b \to 0$, and so the limiting behavior of (15) is non-obvious. Second, $\alpha$ is an equilibrium value. Online Appendix C.3 explicitly evaluates (15) for the case $\epsilon_b = 0$, and establishes that for this case $\Pr(\omega = g | i \text{ observes opportunity}) \geq \frac{\kappa}{(1-\epsilon_g)\pi_g}$.

more bad projects are accidentally exploited, and so the probability of a project being good conditional on it remaining available increases.

**Lemma 4** *For any $\alpha > 0$ and any $n$: $\frac{\partial v(\alpha)}{\partial \epsilon_b} > 0$ for all $\epsilon_b$ sufficiently small.*

Finally, note that if condition (12) holds then limiting equilibrium values of $Q_g^*$ and $Q_b^*$ as $n$ grows large can be straightforwardly characterized by substituting (10) into the equilibrium condition (11).

## 3.4 Prediction: Expected return is lower for late investors

By assumption, investors do not observe their place in line. But if an econometrican were able to observe a proxy for this after the fact, the clear prediction to emerge from the equilibrium analysis is that the expected return on the opportunity experienced by later investors is lower than that experienced by earlier investors. This is an immediate consequence of the fact that later investors only encounter the opportunity if all earlier investors who investigated the opportunity received a bad signal. The prediction accords with investor perceptions that entering a trade late is disadvantageous.

## 3.5 Comparative statics in the ratio of good to bad opportunities

As the ratio of good to bad opportunities $\frac{p_g}{p_b}$ increases, an investor who observes that an opportunity remains unexploited attaches a higher probability to the explanation that no other investor has investigated the opportunity, and attaches a lower probability to the opportunity remaining available because it is bad, and other investors having observed bad signals. Consequently, an investor updates less from the observation that the opportunity remains available. This increases an investor's willingness to investigate, thereby increasing the equilibrium probability of exploitation.

An increase in $\frac{p_g}{p_b}$ also has the direct effect of increasing expected project profitability, pushing in the same direction. But even if $\pi_g$ and $\pi_b$ are adjusted so as to leave expected project profitability unchanged it remains the case that equilibrium exploitation probabilities rise in $\frac{p_g}{p_b}$.

**Proposition 2** *Let (12) hold, and $n$ be large enough that the equilibrium is interior. If the ratio of good to bad opportunities $\frac{p_g}{p_b}$ increases and the expected profitability $Q_g^* p_g \pi_g + Q_b^* p_b \pi_b$ weakly increases (holding $Q_g^*$ and $Q_b^*$ fixed) then the equilibrium exploitation probabilities $Q_g^*$ and $Q_b^*$ increase and $\Pr(\omega = g | \text{never exploited})$ decreases.*

In words: Low-risk opportunities are more likely to be exploited than high-risk ones, even after controlling for expected profitability. And the professor is closer to being correct for low-risk opportunities, in the sense that $\Pr(\omega = g|\text{never exploited})$ is lower.

### 3.6 The Sharpe ratio of exploited opportunities

The opportunity is exploited with probability $Q_g^* p_g + Q_b^* p_b$, and conditional on exploitation yields $\pi_g$ with probability $\frac{Q_g^* p_g}{Q_g^* p_g + Q_b p_b}$ and $\pi_b$ with probability $\frac{Q_b^* p_b}{Q_g^* p_g + Q_b^* p_b}$. Hence the expected return and standard deviation of returns are, respectively

$$\frac{Q_g^* p_g \pi_g + Q_b^* p_b \pi_b}{Q_g^* p_g + Q_b^* p_b}$$

and

$$\frac{\sqrt{Q_g^* p_g Q_b^* p_b}}{Q_g^* p_g + Q_b^* p_b} (\pi_g - \pi_b),$$

implying the associated Sharpe ratio is

$$\left( \left( \frac{Q_g^*}{Q_b^*} \frac{p_g}{p_b} \right)^{\frac{1}{2}} \pi_g + \left( \frac{Q_g^*}{Q_b^*} \frac{p_g}{p_b} \right)^{-\frac{1}{2}} \pi_b \right) (\pi_g - \pi_b)^{-1}. \tag{16}$$

Holding exploitation probabilities $Q_\omega^*$ fixed, the Sharpe ratio (16) is increasing in each of $p_g$ and $\pi_g$,[6] as one would expect. However, increases in $p_g$ and $\pi_g$ also raise exploitation probabilities (Proposition 2) and hence decrease the ratio $\frac{Q_g^*}{Q_b^*}$ (Lemma 2), pushing the Sharpe ratio down. I next show that in a wide range of cases the latter effect is at least as strong as the former, so that the net effect is that better opportunities are associated with (weakly) lower equilibrium Sharpe ratios.

It is immediate from (9) that the equilibrium value of $\frac{Q_g^*}{Q_b^*} \frac{p_g}{p_b}$ is invariant to changes in the ratio $\frac{p_g}{p_b}$; consequently, the Sharpe ratio is likewise invariant to changes in $\frac{p_g}{p_b}$. Increases to $\pi_g - \pi_b$ (holding $\pi_b$ fixed) and to $\pi_b$ (holding $\pi_g - \pi_b$) generate conflicting effects. In both cases $\frac{Q_g^*}{Q_b^*}$ falls (Proposition 2), pushing the Sharpe ratio down; but there are also direct effects that potentially offset this effect. The following result gives simple sufficient conditions for the overall sign:

[6]For any positive constant $a$, the derivative of $\frac{a\pi_g + a^{-1}\pi_b}{\pi_g - \pi_b}$ with respect to $\pi_g$ has the same sign as

$$a(\pi_g - \pi_b) - (a\pi_g + a^{-1}\pi_b) = -(a + a^{-1})\pi_b,$$

which is positive by (3).

**Proposition 3** *The Sharpe ratio (16) is invariant to changes in* $\frac{p_g}{p_b}$*; is increasing in* $\pi_g - \pi_b$ *(holding* $\pi_b$ *constant) if* $\mathcal{E} < \frac{1}{9}$*;*[7] *and is decreasing in* $\pi_b$ *(holding* $\pi_g - \pi_b$ *fixed) if* $\frac{p_g}{p_b} \leq 1$*.*

In many cases, one can think of exploitation of the opportunity as entailing some investment $k$. Bad opportunities yield nothing, while good opportunities yield a gross expected return $R$; hence $\pi_b = -k$ and $\pi_g = (R-1)\,k$. In this case, the combination of Propositions 2 and 3 delivers:

**Corollary 3** *Controlling for the required investment* $k$*, if* $\mathcal{E} < \frac{1}{9}$ *then the equilibrium probability that an opportunity is exploited is negatively correlated with its equilibrium Sharpe ratio.*

# 4 Extensions and benchmarks

## 4.1 Heterogeneous investigation costs

Here, I briefly consider a perturbation of the model in which investors have heterogenous investigation costs. Specifically, let each investor's investigation cost be an i.i.d. draw from a continuous distribution. Hence an investor's realized investigation cost is uncorrelated with the investor's place in line. Let $\kappa(\alpha)$ denote the $\alpha$-percentile of the distribution. In assumption (2), the $\kappa$ is replaced with $\kappa(0)$.

In this perturbed version of the baseline model the equilibrium is a cutoff percentile $\alpha^*$ such that an investor investigates if and only if his/her realized investigation cost is below $\kappa(\alpha^*)$. The equilibrium value $\alpha^*$ is determined by the straightforward analogue to Lemma 3, viz., the unique solution to

$$v(\alpha) = \kappa(\alpha), \tag{17}$$

with $\alpha = 1$ if $v(1) > \kappa(1)$.

Just as in the baseline case of homogeneous investigation costs, adverse selection implies that the unconditional probability $\alpha$ that an investor investigates decreases as the total population $n$ of investors increases. With heterogenous costs, it follows that the investigation cost $\kappa(\alpha)$ of the marginal investor decreases as $n$ increases. Relative to the baseline case, this dampens the decrease in equilibrium exploitation probabilities $Q_g^*$ and $Q_b^*$.

The fact that the investigation cost of the marginal investor falls in $n$ delivers the following clean characterization of equilibrium outcomes as $n \to \infty$. Strikingly, it is only the lowest investigation cost in the population that matters in the determination of equilibrium

[7] In the case of state-invariant error rates $\epsilon_b = \epsilon_g$, the condition $\mathcal{E} < \frac{1}{9}$ corresponds to $\epsilon < 1/10$.

exploitation probabilities $Q_g^*$ and $Q_b^*$. The reason is that if condition (18) is satisfied then the unconditional probability $\alpha$ of each investor investigating approaches 0, corresponding to investigations only being conducted by those with the lowest investigation costs in the population.

**Proposition 4** *If*

$$\frac{p_g\pi_g + p_b\pi_b}{\frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}} < \kappa(0) \tag{18}$$

*then as* $n$ *grows large* $Q_g^*$ *approaches* $Q_g^{*,\infty} < 1$*, defined as the unique solution to*

$$\frac{\frac{Q_g}{1-(1-Q_g)^{\varepsilon}} p_g\pi_g + p_b\pi_b}{\frac{Q_g}{1-(1-Q_g)^{\varepsilon}}\frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}} = \kappa(0).$$

*If instead*

$$\frac{p_g\pi_g + p_b\pi_b}{\frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}} > \kappa(0) \tag{19}$$

*then as* $n$ *grows large* $Q_g^*$ *and* $Q_b^*$ *both approach* 1.

## 4.2 Benchmarks

An accompanying Online Appendix compares equilibrium outcomes in the baseline model (homogenous investigation cost $\kappa$) with three benchmark cases, namely (I) investors know their place in the line; (II) investors make investigation decisions simultaneously, without the opportunity to observe whether or not the opportunity still exists; (III) investors receive signals about project quality for free, but incur a cost to exploit the opportunity.

To briefly summarize:

Benchmark (I) is an information-cascade setting (Banerjee 1992 and Bikhchandani et al 1992). If signal-error rates $\epsilon_g$ and $\epsilon_b$ are sufficiently low then exploitation-outcomes in this benchmark are better than those in the baseline model, in the sense that good opportunities are more likely to be exploited and bad opportunities are less likely to be exploited. But more generally, the ordering of exploitation probabilities between this benchmark and the baseline model depends on parameter values, and there are cases in which good projects are more likely to be exploited if investors don't know their place in the line.

Benchmark (II) is closely related to previous analysis of Bertrand competition preceded by a costly entry decision (see references in introduction). Perhaps surprisingly, exploitation probabilities are *lower* in this benchmark than in the baseline sequential-decision case in which investors draw negative inferences from the fact that an opportunity remains available. To see the intuition for this ordering, note first that benchmark (II) is equivalent to investors committing to an investigation probability $\alpha$ *before* the baseline sequential-move game begins. In this commitment-interpretation of (II), there are two possibilities when it is an individual investor's turn to move: either (i) the opportunity remains available, or (ii) it is already exploited. Case (i) is exactly the situation that arises in the main sequential-move analysis; but case (ii) is strictly worse for an investor, since in this case the investigation cost $\kappa$ yields zero benefit, but the investor is committed to investigate with probability $\alpha$. Hence the gains from investigation are lower in (II) than in the baseline sequential-move game.

In benchmark (III), investors benefit from deferring the decision of whether to incur a cost until after observing the signal, leading to higher exploitation probabilities than in the baseline case of endogenous signal acquisition. Moreover, this ordering continues to hold even if costs are normalized so that they are equal across the two cases for a single investor acting in isolation.

# 5 Summary

I analyze a setting in which investors sequentially encounter a potentially profitable investment opportunity, and decide both whether to investigate its quality and whether to exploit the opportunity. Once exploited, the opportunity is unavailable to future investors. The key friction is that an investor observes only whether the opportunity remains available, but is ignorant about the number of investors who have already investigated the opportunity.

A simple and mild inequality gives conditions under which the professor of the epigraph's joke is wrong—that is, that there is a positive probability that an opportunity is good even conditional on it being available, and even as the number of investors grows arbitrarily large. Relatedly, the probability that the opportunity is exploited decreases in the number of investors, and bounded away from 1 even as the number of investors increases. Safe opportunities are exploited more often than risky ones, even controlling for average profitability. Higher exploitation probabilities are generally associated with lower observed Sharpe ratios.

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# A Proofs

## A.1 Omitted results

**Lemma A-1** *The relation between $Q_b$ and $Q_g$ is given by*

$$Q_b = 1 - \left(1 - \mathcal{E}\left(1 - (1 - Q_g)^{\frac{1}{n}}\right)\right)^n \quad \text{(A-1)}$$

$$Q_g = 1 - \left(1 - \mathcal{E}^{-1}\left(1 - (1 - Q_b)^{\frac{1}{n}}\right)\right)^n . \quad \text{(A-2)}$$

*For $n \geq 2$: Holding $Q_g$ fixed, $Q_b$ is increasing in $n$; and holding $Q_b$ fixed, $Q_g$ is decreasing in $n$. Moreover, for any $\alpha$,*

$$\lim_{n \to \infty} (1 - Q_b(\alpha)) = (1 - Q_g(\alpha))^{\mathcal{E}} . \quad \text{(A-3)}$$

**Proof of Lemma A-1:** Equation (A-1) follows from $1 - (1 - \epsilon_g)\alpha = (1 - Q_g)^{\frac{1}{n}}$ and

$$\alpha = \frac{1 - (1 - Q_g)^{\frac{1}{n}}}{1 - \epsilon_g}. \quad \text{(A-4)}$$

Similarly, (A-2) follows from $1 - \epsilon_b \alpha = (1 - Q_b)^{\frac{1}{n}}$ and

$$\alpha = \frac{1 - (1 - Q_b)^{\frac{1}{n}}}{\epsilon_b}. \quad \text{(A-5)}$$

Rearranging (A-1) gives

$$\ln(1 - Q_b) = n \ln\left(1 - \mathcal{E}\left(1 - (1 - Q_g)^{\frac{1}{n}}\right)\right). \quad \text{(A-6)}$$

I show that the RHS of (A-6) is decreasing in $n$, i.e.,

$$\ln\left(1 - \mathcal{E}\left(1 - (1 - Q_g)^{\frac{1}{n}}\right)\right) + \left(-\frac{1}{n^2}\right) n \frac{\mathcal{E} \cdot \ln(1 - Q_g) \cdot (1 - Q_g)^{\frac{1}{n}}}{1 - \mathcal{E}\left(1 - (1 - Q_g)^{\frac{1}{n}}\right)} < 0, \quad \text{(A-7)}$$

which is equivalent to

$$\begin{aligned} &\left(1 - \mathcal{E}\left(1 - (1 - Q_g)^{\frac{1}{n}}\right)\right) \ln\left(1 - \mathcal{E}\left(1 - (1 - Q_g)^{\frac{1}{n}}\right)\right) \\ < \;& \mathcal{E} \cdot (1 - Q_g)^{\frac{1}{n}} \cdot \ln(1 - Q_g)^{\frac{1}{n}} . \end{aligned}$$

Hence it suffices to show that, for any $x \in (0, 1)$,

$$(1 - \mathcal{E}(1 - x)) \ln(1 - \mathcal{E}(1 - x)) < \mathcal{E} x \ln x.$$

This inequality holds with equality at $\mathcal{E} = 0, 1$. The RHS is linear in $\mathcal{E}$. Hence it suffices to show that the LHS is convex in $\mathcal{E}$, which is indeed the case since its derivative with respect to $\mathcal{E}$ is

$$-\left(1-x\right)\ln\left(1-\mathcal{E}\left(1-x\right)\right)-\left(1-x\right),$$

which is increasing in $\mathcal{E}$. This establishes (A-7).

Rearranging (A-2) gives

$$\ln\left(1-Q_g\right) = n\ln\left(1-\mathcal{E}^{-1}\left(1-\left(1-Q_b\right)^{\frac{1}{n}}\right)\right). \tag{A-8}$$

I show that the RHS is increasing in $n$, i.e.,

$$\ln\left(1-\mathcal{E}^{-1}\left(1-\left(1-Q_b\right)^{\frac{1}{n}}\right)\right)+\left(-\frac{1}{n^2}\right)n\frac{\mathcal{E}^{-1}\cdot\ln\left(1-Q_b\right)\cdot\left(1-Q_b\right)^{\frac{1}{n}}}{1-\mathcal{E}^{-1}\left(1-\left(1-Q_b\right)^{\frac{1}{n}}\right)} > 0, \tag{A-9}$$

which is equivalent to

$$\begin{aligned} & \left(1-\mathcal{E}^{-1}\left(1-\left(1-Q_b\right)^{\frac{1}{n}}\right)\right)\ln\left(1-\mathcal{E}^{-1}\left(1-\left(1-Q_b\right)^{\frac{1}{n}}\right)\right) \\ > \ & \mathcal{E}^{-1}\cdot\left(1-Q_b\right)^{\frac{1}{n}}\cdot\ln\left(1-Q_b\right)^{\frac{1}{n}}. \end{aligned}$$

Hence it suffices to show that, for any $x \in (0,1)$,

$$\left(1-\mathcal{E}^{-1}\left(1-x\right)\right)\ln\left(1-\mathcal{E}^{-1}\left(1-x\right)\right) > \mathcal{E}^{-1}x\ln x.$$

This inequality holds with equality at $\mathcal{E}^{-1} = 0, 1$. The RHS is linear in $\mathcal{E}^{-1}$. Hence it suffices to show that the LHS is convex in $\mathcal{E}^{-1}$, which is established in the first part of the proof. This establishes (A-7).

Finally, I establish (A-3). Fix $Q_g$. From (A-1),

$$1-Q_b = \left(1-\mathcal{E}\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right)\right)^n = \exp\left(\frac{\ln\left(1-\mathcal{E}\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right)\right)}{\frac{1}{n}}\right).$$

Note that

$$\frac{\partial\ln\left(1-\mathcal{E}\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right)\right)}{\partial\left(\frac{1}{n}\right)} = \frac{\mathcal{E}\ln\left(1-Q_g\right)\cdot\left(1-Q_g\right)^{\frac{1}{n}}}{\left(1-\mathcal{E}\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right)\right)} \to \mathcal{E}\ln\left(1-Q_g\right) \text{ as } n\to\infty.$$

So by l'Hopital's rule, as $n \to \infty$,

$$1 - Q_b \to (1 - Q_g)^{\mathcal{E}},$$

completing the proof.

## A.2 Results stated in main text

**Proof of Lemma 1:** First, $\alpha = 0$ cannot be an equilibrium, as follows. In this case, no investigation occurs, and the observation that the opportunity remains available conveys no information, so every investor who encounters the opportunity believes the probability that the opportunity is good is $p_g$. But then assumption (2) implies that an individual investor would gain by deviating and investigating.

For the remainder of the proof, denote the exploitation probabilities after good and bad signals by $\beta_g$ and $\beta_b$. Note that $\beta_g \geq \beta_b$, with $\beta_g = 1$ if $\beta_b \in (0,1)$ and $\beta_b = 0$ if $\beta_g < 1$. Note first that $\alpha > 0$ and $\beta_g = 0$ cannot be an equilibrium, since in this case an investor would do better by avoiding the investigation cost $\kappa$.

Second, $\alpha > 0$ and $\beta_b = 1$ cannot be an equilibrium, as follows. In this case, no updating occurs, so every investor who encounters the opportunity believes the probability that the opportunity is good is $p_g$. But then assumption (1) directly implies that $\beta_b = 0$, a contradiction.

Third, $\alpha > 0$ and $\beta_g < 1$ cannot be an equilibrium, since in this case an investor is at best indifferent between exploiting and doing nothing after observing a signal $g$, and so would do better by deviating to no-investigation and avoiding the investigation cost $\kappa$.

Fourth, $\alpha > 0$ and $\beta_b \in (0,1)$ cannot be an equilibrium, as follows. From above, $\beta_g = 1$. So observing that the opportunity remains available leads an investor to revise beliefs about project quality downwards relative to the prior belief $p_g$. Hence assumption (1) implies that $\beta_b = 0$, a contradiction.

Combined, these observations imply that in any equilibrium $\alpha > 0$, $\beta_g = 1$ and $\beta_b = 0$.

Finally: Let $\tilde{p}$ be the probability that an investor attaches to $\omega = g$ when encountering an opportunity. Note that $\alpha > 0$, $\beta_g = 1$ and $\beta_b = 0$ together imply that the observation that the opportunity is available leads to negative updating relative to the prior $p_g$, i.e., $\tilde{p} \leq p_g$. The gain to paying the investigation cost over the alternative of exploiting without paying the investigation cost is

$$\begin{aligned} & (1 - \epsilon_g)\,\tilde{p}\pi_g + \epsilon_b\,(1 - \tilde{p})\,\pi_b - \kappa - (\tilde{p}\pi_g + (1 - \tilde{p})\,\pi_b) \\ = \; & -(\epsilon_g \tilde{p}\pi_g + (1 - \epsilon_b)\,(1 - \tilde{p})\,\pi_b) - \kappa. \end{aligned} \tag{A-10}$$

The derivative with respect to $\tilde{p}$ is $-\epsilon_g \pi_g + (1-\epsilon_b)\pi_b < 0$. By assumption (2) expression (A-10) is positive at $\tilde{p} = p_g$, and hence is positive for $\tilde{p} \leq p_g$. Hence no investor exploits without investigating, completing the proof.

**Proof of Lemma 2:** By l'Hopital's rule, as $\alpha \to 0$

$$\frac{Q_g}{Q_b} \to \frac{1-\epsilon_g}{\epsilon_b} = \frac{1}{\mathcal{E}} > 1.$$

Evaluation of $\frac{Q_g}{Q_b}$ at $\alpha = 1$ is immediate.

If $\frac{Q_g}{Q_b}$ is decreasing in $\alpha$ then it follows straightforwardly that $v$ is likewise decreasing in $\alpha$: simply note that the derivative of $\frac{\psi p_g \pi_g + p_b \pi_b}{\psi \mathcal{E} p_g + p_b}$ with respect to $\psi$ has the same sign as $\pi_g - \mathcal{E}\pi_b$, which is positive since $\pi_g > \pi_b$.

I next establish that $\frac{Q_g}{Q_b}$ is monotonically decreasing in $\alpha \in (0,1)$.Write $A = 1-\epsilon_g$ and $B = \epsilon_b$, and note that $B < A$. I establish that

$$\frac{1-(1-A\alpha)^n}{1-(1-B\alpha)^n}$$

is decreasing in $\alpha \in (0,1)$, i.e., that

$$A(1-A\alpha)^{n-1}(1-(1-B\alpha)^n) < B(1-B\alpha)^{n-1}(1-(1-A\alpha)^n),$$

or equivalently,

$$\frac{A}{B}\frac{(1-A\alpha)^{n-1}}{(1-B\alpha)^{n-1}} < \frac{1-(1-A\alpha)^n}{1-(1-B\alpha)^n}.$$

This inequality holds with equality at $n=1$. Consequently, it suffices to show that for any $n \geq 1$,

$$\frac{1-A\alpha}{1-B\alpha} < \frac{\frac{1-(1-A\alpha)^{n+1}}{1-(1-A\alpha)^n}}{\frac{1-(1-B\alpha)^{n+1}}{1-(1-B\alpha)^n}}.$$

Write $x_A = 1-A\alpha$ and $x_B = 1-B\alpha$, and note $0 < x_A < x_B < 1$. The above inequality is equivalent to

$$\frac{x_B^{-1}-1}{1-x_B^n} < \frac{x_A^{-1}-1}{1-x_A^n}.$$

Hence it suffices to show that

$$\frac{x^{-1}-1}{1-x^n}$$

is decreasing in $x \in (0,1)$, i.e., that

$$-x^{-2}\left(1-x^{n}\right)+nx^{n-1}\left(x^{-1}-1\right)<0,$$

or equivalently,

$$-1+\left(n+1\right)x^{n}-nx^{n+1}<0.$$

The LHS equals zero at $x=1$, and the derivative of LHS with respect to $x$ is

$$n\left(n+1\right)x^{n-1}\left(1-x\right)>0,$$

thereby establishing the required inequality and establishing that $\frac{Q_g}{Q_b}$ is decreasing in $\alpha$.

**Proof of Proposition 1:** By Lemma 2, $v(\alpha)$ is strictly decreasing in $n$ for all $\alpha \in (0,1]$. Hence if the equilibrium is interior for some $n$, it is interior for all higher $n$.

If the equilibrium is interior then $\frac{Q_g^*}{Q_b^*}$ is independent of $n$; write this value as $\psi^*$. The value $Q_g^*$ is determined by $\frac{Q_g^*}{Q_b\left(Q_g^*\right)}=\psi^*$, where $Q_b\left(Q_g^*\right)$ denotes the function (A-1). By Lemma 2, the function $\frac{Q_g}{Q_b(Q_g)}$ is decreasing in $Q_g$; and by Lemma A-1, it is decreasing in $n$. Hence $Q_g^*$ is decreasing in $n$.

The proof that $Q_b^*$ is decreasing in $n$ is parallel: let $Q_g\left(Q_b^*\right)$ denote the function (A-2), and note from Lemmas 2 and A-1 that $\frac{Q_g^*(Q_b)}{Q_b}$ is decreasing in $Q_b$ and in $n$. This completes the proof.

**Derivation of (15):**

$$\begin{aligned}
&\Pr\left(\omega|i \text{ observes opportunity}\right)\\
=&\sum_{k=1}^{n}\frac{\Pr\left(\text{investor } i \text{ is } k^{th} \text{ in line}, i \text{ observes opportunity}|\omega\right)p_\omega}{\Pr\left(i \text{ observes opportunity}\right)}\\
=&\sum_{k=1}^{n}\frac{\frac{1}{n}\left(1-\Pr\left(\sigma_i=g|\omega\right)\alpha\right)^{k-1}p_\omega}{\Pr\left(i \text{ observes opportunity}\right)}\\
=&\frac{1}{n}\frac{1-\left(1-\Pr\left(\sigma_i=g|\omega\right)\alpha\right)^{n}}{\Pr\left(\sigma_i=g|\omega\right)\alpha}\frac{p_\omega}{\Pr\left(i \text{ observes opportunity}\right)}\\
=&\frac{1}{n}\frac{Q_\omega}{\Pr\left(\sigma_i=g|\omega\right)\alpha}\frac{p_\omega}{\Pr\left(i \text{ observes opportunity}\right)}.
\end{aligned}$$

Similarly,

$$\Pr\left(i \text{ observes opportunity}|\omega\right)=\frac{1}{n}\frac{Q_\omega}{\Pr\left(\sigma_i=g|\omega\right)\alpha}.$$

Identity (15) follows.

**Proof of Corollary 2:** Fix $\kappa > 0$. By assumption (2), fix $\bar{\psi} \in (1, \mathcal{E}^{-1})$ such that $\bar{\psi} p_g \pi_g + p_b \pi_b > 0$. Define $\bar{\epsilon}_b > 0$ by

$$\frac{\bar{\psi} p_g \pi_g + p_b \pi_b}{\bar{\psi} \frac{p_g}{1-\epsilon_g} + \frac{p_b}{\bar{\epsilon}_b}} = \frac{\kappa}{2}.$$

Hence $v\left(\bar{\psi}\right) < \kappa$ for any $\epsilon_b \leq \bar{\epsilon}_b$. The result follows from Lemma A-1, completing the proof.

**Proof of Lemma 4:** Holding $\alpha$ fixed, the probability $Q_g$ is independent of the error rate $\epsilon_b$. Evaluating,

$$\begin{aligned} sign\left(\frac{\partial v(\alpha)}{\partial \epsilon_b}\right) &= sign\left(\frac{\partial Q_b}{\partial \epsilon_b} p_b \pi_b \left(Q_g \frac{p_g}{1-\epsilon_g} + Q_b \frac{p_b}{\epsilon_b}\right) - \left(\frac{\partial Q_b}{\partial \epsilon_b} - \frac{1}{\epsilon_b}\right) \frac{p_b}{\epsilon_b} \left(Q_g p_g \pi_g + Q_b p_b \pi_b\right)\right) \\ &= sign\left(\frac{\partial Q_b}{\partial \epsilon_b} \left(\frac{Q_g p_b p_g \pi_b}{1-\epsilon_g} - \frac{Q_g p_b p_g \pi_g}{\epsilon_b}\right) + \frac{p_b}{\epsilon_b^2} \left(Q_g p_g \pi_g + Q_b p_b \pi_b\right)\right). \end{aligned}$$

Recall $1 - Q_b = (1 - \epsilon_b \alpha)^n$, and so

$$\frac{\partial Q_b}{\partial \epsilon_b} = \frac{n\alpha}{1 - \epsilon_b \alpha} \left(1 - Q_b\right).$$

Hence

$$sign\left(\frac{\partial v(\alpha)}{\partial \epsilon_b}\right) = sign\left(\frac{n\alpha}{1-\epsilon_b \alpha} (1 - Q_b) \left(\frac{Q_g p_g \pi_b}{1-\epsilon_g} - \frac{Q_g p_g \pi_g}{\epsilon_b}\right) + \frac{1}{\epsilon_b^2} \left(Q_g p_g \pi_g + Q_b p_b \pi_b\right)\right).$$

As $\epsilon_b \to 0$, $Q_b \to 0$, and so

$$sign\left(\frac{\partial v(\alpha)}{\partial \epsilon_b}\right) = sign\left(Q_g p_g \pi_g\right)$$

for all $\epsilon_b$ sufficiently small, completing the proof.

**Proof of Proposition 2:** Denote $\psi = \frac{Q_g}{Q_b}$, and denote by $\psi_0$ the value of $\psi$ prior to any parameter changes. The equilibrium condition is

$$\epsilon_b \frac{\psi p_g \pi_g + p_b \pi_b}{\psi p_g \mathcal{E} + p_b} = \kappa. \qquad \text{(A-11)}$$

If $p_g$ remains unchanged and $\pi_g$ and/or $\pi_b$ increase then it is immediate that $\psi$ decreases and hence (by Lemma 2) that $\alpha$, $Q_g$ and $Q_b$ all increase.

Next, consider the case in which $p_g$ increases. From (A-11), and using $p_b = 1 - p_g$,

$$\left(\frac{\partial \psi}{\partial p_g} p_g \pi_g + \frac{\partial \left(\psi_0 p_g \pi_g + p_b \pi_b\right)}{\partial p_g}\right) \left(\psi_0 \mathcal{E} p_g + p_b\right) - \left(\frac{\partial \psi}{\partial p_g} \mathcal{E} p_g + \psi \mathcal{E} - 1\right) \left(\psi_0 p_g \pi_g + p_b \pi_b\right) = 0,$$

i.e.,

$$\begin{aligned}\frac{\partial\psi}{\partial p_g} &= \frac{(\psi\mathcal{E}-1)(\psi_0 p_g\pi_g + p_b\pi_b) - \frac{\partial(\psi_0 p_g\pi_g+p_b\pi_b)}{\partial p_g}(\psi_0\mathcal{E}p_g+p_b)}{p_g\pi_g(\psi_0\mathcal{E}p_g+p_b) - \mathcal{E}p_g(\psi_0 p_g\pi_g+p_b\pi_b)} \\ &= \frac{(\psi\mathcal{E}-1)(\psi_0 p_g\pi_g + p_b\pi_b) - \frac{\partial(\psi_0 p_g\pi_g+p_b\pi_b)}{\partial p_g}(\psi_0\mathcal{E}p_g+p_b)}{p_g p_b\pi_g - \mathcal{E}p_g p_b\pi_b}.\end{aligned}$$

Note that $\psi\mathcal{E} < 1$ by Lemma 2. Moreover, the equilibrium condition implies $\psi_0 p_g\pi_g + p_b\pi_b > 0$. Hence if $\frac{\partial(\psi_0 p_g\pi_g+p_b\pi_b)}{\partial p_g} \geq 0$ then $\frac{\partial\psi}{\partial p_g} < 0$, implying (by Lemma 2) that $\alpha$, $Q_g$ and $Q_b$ are all higher.

Finally, the statement that $\Pr(\omega = g|\text{never exploited})$ decreases follows from (13), the fact that $\alpha$ has increased, and the fact that

$$\frac{1-Q_g}{1-Q_b} = \left(\frac{1-(1-\epsilon_g)\alpha}{1-\epsilon_b\alpha}\right)^n$$

is decreasing in $\alpha$, completing the proof.

**Proof of Proposition 3:** Denote $\phi = \frac{Q_g p_g}{Q_b p_b}$ and $\pi_g = \Delta + \pi_b$. The equilibrium condition is

$$\epsilon_b\frac{\phi\pi_g+\pi_b}{\mathcal{E}\phi+1} = \epsilon_b\frac{\phi(\Delta+\pi_b)+\pi_b}{\mathcal{E}\phi+1} = \kappa.$$

For use below, note that certainly

$$\phi\pi_g + \pi_b > 0. \tag{A-12}$$

Differentiating with respect $\pi_g$ yields

$$\left(\frac{\partial\phi}{\partial\pi_g}\pi_g+\phi\right)(\mathcal{E}\phi+1) - (\phi\pi_g+\pi_b)\mathcal{E}\frac{\partial\phi}{\partial\pi_g} = 0,$$

and hence

$$\frac{\partial\phi}{\partial\pi_g} = -\frac{\phi(\mathcal{E}\phi+1)}{\pi_g(\mathcal{E}\phi+1)-(\phi\pi_g+\pi_b)\mathcal{E}} = -\frac{\phi(\mathcal{E}\phi+1)}{\pi_g-\mathcal{E}\pi_b}.$$

Similarly, differentiating with respect to $\pi_b$ (holding $\Delta = \pi_g - \pi_b$ fixed) yields

$$\left(\frac{\partial\phi}{\partial\pi_b}\pi_g+\phi+1\right)(\mathcal{E}\phi+1) - (\phi\pi_g+\pi_b)\mathcal{E}\frac{\partial\phi}{\partial\pi_b} = 0$$

and hence

$$\frac{\partial\phi}{\partial\pi_b} = -\frac{(\phi+1)(\mathcal{E}\phi+1)}{\pi_g(\mathcal{E}\phi+1)-(\phi\pi_g+\pi_b)\mathcal{E}} = -\frac{(\phi+1)(\mathcal{E}\phi+1)}{\pi_g-\mathcal{E}\pi_b}.$$

The derivative of the Sharpe ratio (16) with respect to $\pi_g$ is

$$\phi^{\frac{1}{2}} \left(\pi_g - \pi_b\right)^{-1} + \frac{\partial \phi}{\partial \pi_g} \frac{1}{2} \left(\phi^{-\frac{1}{2}} \pi_g - \phi^{-\frac{3}{2}} \pi_b\right) \left(\pi_g - \pi_b\right)^{-1} - \left(\phi^{\frac{1}{2}} \pi_g + \phi^{-\frac{1}{2}} \pi_b\right) \left(\pi_g - \pi_b\right)^{-2} \quad \text{(A-13)}$$

which has the same sign as (multiplying by $\phi^{\frac{3}{2}} \left(\pi_g - \pi_b\right)^2$)

$$\phi^2 \left(\pi_g - \pi_b\right) - \left(\phi^2 \pi_g + \phi \pi_b\right) - \frac{\phi \left(\mathcal{E} \phi + 1\right)}{\pi_g - \mathcal{E} \pi_b} \frac{1}{2} \left(\phi \pi_g - \pi_b\right) \left(\pi_g - \pi_b\right)$$

and hence the same sign as (dividing by $-\pi_b$)

$$\phi \left(\phi + 1\right) - \frac{\left(\mathcal{E} \phi + 1\right)}{-\frac{\pi_g}{\pi_b} + \mathcal{E}} \frac{1}{2} \left(-\frac{\phi \pi_g}{\pi_b} + 1\right) \left(-\frac{\phi \pi_g}{\pi_b} + \phi\right)$$

and hence the same sign as (multiplying by $\frac{2}{\phi} \left(-\frac{\phi \pi_g}{\pi_b} + \mathcal{E} \phi\right)$)

$$2 \left(\phi + 1\right) \left(-\frac{\phi \pi_g}{\pi_b} + \mathcal{E} \phi\right) - \left(\mathcal{E} \phi + 1\right) \left(-\frac{\phi \pi_g}{\pi_b} + 1\right) \left(-\frac{\phi \pi_g}{\pi_b} + \phi\right). \quad \text{(A-14)}$$

From (A-12), $-\frac{\phi \pi_g}{\pi_b} > 1$. To establish the comparative static in $\pi_g$ I show that (A-14) is negative if $-\frac{\phi \pi_g}{\pi_b} > 1$ and $\mathcal{E} > \frac{1}{9}$. Evaluated at $-\frac{\phi \pi_g}{\pi_b} = 1$, expression (A-14) equals 0. Consider

$$2 \left(\phi + 1\right) \left(z + \mathcal{E} \phi\right) - \left(\mathcal{E} \phi + 1\right) \left(z + 1\right) \left(z + \phi\right). \quad \text{(A-15)}$$

This is a concave quadratic in $z$. So to establish that (A-14) is negative it suffices to show that the derivative of (A-15) with respect to $z$ is negative when evaluated at $z = 1$, i.e.,

$$2 \left(\phi + 1\right) - \left(\mathcal{E} \phi + 1\right) \left(z + \phi\right) - \left(\mathcal{E} \phi + 1\right) \left(z + 1\right)\big|_{z=1} < 0,$$

i.e.,

$$2 \left(\phi + 1\right) - \left(\phi \mathcal{E} + 1\right) \left(1 + \phi\right) - 2 \left(\phi \mathcal{E} + 1\right) < 0,$$

i.e.,

$$-\mathcal{E} \phi^2 - \phi \left(3 \mathcal{E} - 1\right) - 1 < 0. \quad \text{(A-16)}$$

If $\mathcal{E} \geq \frac{1}{3}$ then the proof is complete. If instead $\mathcal{E} \in \left(0, \frac{1}{3}\right)$, any roots of the quadratic in (A-16) are positive. The determinant of this quadratic is

$$\left(3 \mathcal{E} - 1\right)^2 - 4 \mathcal{E} = 1 - 10 \mathcal{E} + 9 \mathcal{E}^2 = \left(1 - 9 \mathcal{E}\right) \left(1 - \mathcal{E}\right)$$

which is negative for $\mathcal{E} \in \left(\frac{1}{9}, 1\right)$. Hence for $\mathcal{E} \in \left(\frac{1}{9}, \frac{1}{3}\right)$ the quadratic in (A-16) is negative for all $\phi$, completing the proof of the comparative static in $\pi_g$.

Finally, the derivative of the Sharpe ratio (16) with respect to $\pi_b$, holding $\Delta = \pi_g - \pi_b$ fixed, has the same sign as

$$\phi^{\frac{1}{2}} + \phi^{-\frac{1}{2}} + \frac{1}{2}\frac{\partial \phi}{\partial \pi_b}\left(\phi^{-\frac{1}{2}}\pi_g - \phi^{-\frac{3}{2}}\pi_b\right)$$

which has the same sign as (multiplying by $2\phi^{\frac{3}{2}}$)

$$2\phi\left(\phi+1\right) - \frac{\left(\phi+1\right)\left(\mathcal{E}\phi+1\right)}{\pi_g - \mathcal{E}\pi_b}\left(\phi\pi_g - \pi_b\right)$$

which has the same sign as (multiplying by $\frac{\pi_g - \mathcal{E}\pi_b}{\phi+1}$)

$$2\left(\phi\pi_g - \mathcal{E}\phi\pi_b\right) - \left(\mathcal{E}\phi+1\right)\left(\phi\pi_g - \pi_b\right) = \left(1-\mathcal{E}\phi\right)\left(\phi\pi_g + \pi_b\right). \tag{A-17}$$

By Lemma 2, $\mathcal{E}\phi = \mathcal{E}\frac{Q_g}{Q_b}\frac{p_g}{p_b} < \frac{p_g}{p_b}$. Combined with (A-12) this establishes that (A-17) is positive if $p_g \leq \frac{1}{2}$, thereby completing the proof.

**Proof of Proposition 4:** For any $Q_g \in [0,1)$ define $\alpha\left(Q_g; n\right)$ by (A-4). Note that $\alpha\left(Q_g; n\right)$ is decreasing in $n$, and that $\alpha\left(Q_g; n\right) \to 0$ as $n \to \infty$. Define $Q_b\left(Q_g; n\right)$ by (A-1).

*Case: (18) holds:* The equilibrium $Q_g^{*,n}$ is given by the solution to

$$\frac{\frac{Q_g^{*,n}}{Q_b\left(Q_g^{*,n};n\right)}p_g\pi_g + p_b\pi_b}{\frac{Q_g^{*,n}}{Q_b\left(Q_g^{*,n};n\right)}\frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}} = \kappa\left(\alpha\left(Q_g^{*,n}; n\right)\right). \tag{A-18}$$

(Inequality (A-1) ensures that a solution to (A-18) exists: the LHS approaches $\frac{p_g\pi_g + p_b\pi_b}{\frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}}$ as $Q_g^{*,n} \to 1$ while the RHS certainly exceeds $\kappa(0)$.)

Recall that as $n \to \infty$ the relationship between $Q_b$ and $Q_g$ is given by (10). Hence Lemma A-1 implies

$$\frac{\frac{Q_g}{Q_b(Q_g;n)}p_g\pi_g + p_b\pi_b}{\frac{Q_g}{Q_b(Q_g;n)}\frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}} > \frac{\frac{Q_g}{1-(1-Q_g)^{\mathcal{E}}}p_g\pi_g + p_b\pi_b}{\frac{Q_g}{1-(1-Q_g)^{\mathcal{E}}}\frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}}.$$

Define $\bar{Q}_g^n$ as the solution to

$$\frac{\frac{\bar{Q}_g^n}{Q_b\left(\bar{Q}_g^n;n\right)}p_g\pi_g + p_b\pi_b}{\frac{\bar{Q}_g^n}{Q_b\left(\bar{Q}_g^n;n\right)}\frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}} = \kappa\left(0\right).$$

By Lemma A-1, $\bar{Q}_g^n$ is decreasing in $n$. Moreover,

$$\begin{aligned} Q_g^{*,n} &< \bar{Q}_g^n & \text{(A-19)} \\ \lim_{n\to\infty} \bar{Q}_g^n &= Q_g^{*,\infty}. & \text{(A-20)} \end{aligned}$$

Define $\underline{Q}_g^n$ by the solution to

$$\frac{\frac{\underline{Q}_g^n}{1-\left(1-\underline{Q}_g^n\right)^{\varepsilon}} p_g \pi_g + p_b \pi_b}{\frac{\underline{Q}_g^n}{1-\left(1-\underline{Q}_g^n\right)^{\varepsilon}} \frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}} = \kappa\left(\alpha\left(\underline{Q}_g^n; n\right)\right).$$

Note that $\underline{Q}_g^n$ is increasing in $n$ and

$$\underline{Q}_g^n < Q_g^{*,n}. \tag{A-21}$$

The combination of (A-19) and (A-21) implies that the sequence $\underline{Q}_g^n$ has an upper bound strictly less than 1. Hence $\alpha\left(\underline{Q}_g^n; n\right) \to 0$, and so $\underline{Q}_g^n \to Q_g^{*,\infty}$. Hence $Q_g^{*,n} \to Q_g^{*,\infty}$.

*Case: (19) holds:* Define $\underline{\alpha} > 0$ as the solution to

$$\frac{p_g \pi_g + p_b \pi_b}{\frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}} = \kappa\left(\underline{\alpha}\right).$$

Note that $\frac{Q_g\left(\frac{\underline{\alpha}}{2};n\right)}{Q_b\left(\frac{\underline{\alpha}}{2};n\right)} \to 1$ as $n \to \infty$. Hence for all $n$ sufficiently large,

$$\frac{\frac{Q_g\left(\frac{\underline{\alpha}}{2};n\right)}{Q_b\left(\frac{\underline{\alpha}}{2};n\right)} p_g \pi_g + p_b \pi_b}{\frac{Q_g\left(\frac{\underline{\alpha}}{2};n\right)}{Q_b\left(\frac{\underline{\alpha}}{2};n\right)} \frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}} > \kappa\left(\frac{\underline{\alpha}}{2}\right).$$

Hence for all $n$ sufficiently large, the equilibrium investigation probability must exceed $\frac{\underline{\alpha}}{2}$. Consequently both $Q_g^{*,n}$ and $Q_b^{*,n}$ must approach 1, completing the proof.