Is that a $100 bill on the sidewalk?*

Philip Bond, University of Washington

July 2025

Abstract

I analyze a setting in which investors sequentially encounter a potentially profitable investment opportunity, and decide both whether to investigate its quality and whether to exploit the opportunity. Once exploited, the opportunity is unavailable to future investors. The key friction is that an investor observes only whether the opportunity remains available, but is ignorant about the number of investors who have already investigated the opportunity. The probability that the opportunity is exploited decreases in the number of investors. Safe opportunities are exploited more often than risky ones, even controlling for average profitability. Higher exploitation probabilities are generally associated with lower observed Sharpe ratios.

*I thank Zhiguo He and Haoxiang Zhu for helpful comments. Any errors are my own.

...an economics professor [is] walking with a keen-eyed student across the university quad. "Look", says the student, pointing at the ground, "a five-pound note". "It can't be", replies the rational professor. "If it was there, somebody would have picked it up by now."

From *The Economist*, October 20, 1984

1 Introduction

The joke in the epigraph is well-known and widely invoked, in large part because it accurately conveys the economic insight that profitable opportunities tend to be exploited. But the joke *also* conveys a potentially important countervailing force that limits such exploitation, namely that economic agents—henceforth, *investors*—will be unwilling to incur costs to examine a potentially attractive opportunity on the grounds that if the opportunity were indeed profitable then someone else would already have exploited it ("picked it up").

In this short paper I analyze this setting. Specifically, investors sequentially encounter a potentially profitable opportunity. Each investor first decides whether to pay an investigation cost to observe a noisy signal about the opportunity's profitability; and then decides whether or not to exploit the opportunity. Once exploited, the opportunity is unavailable to future investors. Crucially, each investor observes only whether or not the opportunity is available, but is ignorant about how many other investors have already examined the opportunity and decided not to exploit it.

In equilibrium, there is a strictly positive probability that *all* investors pass on the opportunity. This probability *increases* in the number of potential investors. Consequently, even as the investor pool grows arbitrarily large, the probability that the opportunity remains unexploited remains strictly positive.

Opportunities that generate a high payoff if they are good, but have small probabilities of being good, are especially affected. Consequently, the set of opportunities that are exploited is tilted towards "safe" prospects relative to the underlying pool of opportunities. After controlling for the return on bad opportunities, higher exploitation probabilities are associated with lower observed Sharpe ratios, provided that investigation produces relatively accurate information.

The analysis gives a parsimonious explanation for why profitable trading opportunities persist even in the presence of large quantities of arbitrage capital.[1] More generally, the analysis generates an endogenous limit to innovation, especially of high-risk high-reward projects.

[1] See the large literature in financial economics on limits to arbitrage, surveyed by, for example, Gromb and Vayanos (2010).

I am unaware of a previous analysis of this setting. Perhaps closest is Zhu (2012)'s analysis of quotes offered by buyers in an OTC market under the assumption that a seller contacts buyers in a random and unobserved order. If buyers share a common valuation and receive exogenous noisy signals about this common value then in equilibrium there is a positive probability of no trade even as the number of buyers grows arbitrarily large. Different from in Zhu's analysis, in my setting agents observe information only if they first pay a cost, and in equilibrium many agents remain uninformed; consequently, endogenous acquisition of information potentially changes the adverse selection problem faced by buyers.[2] Separately, and again different from Zhu, I derive comparative statics for how equilibrium exploitation probabilities vary with the number of traders and with project characteristics. Papers such as Sherman and Willett (1967) and Elberfeld and Wolfsetter (1999) study Bertrand competition with a pre-stage in which firms simultaneously decided whether or not to pay an entry cost, and show that entry is decreasing in the potential number of firms. Unlike in the current setting, there is no learning from the fact that the opportunity still exists. Herrera and Hörner (2013) and Monzón and Rapp (2014) study social learning settings in which agents sequentially make a binary decision, as in Banerjee (1992) and Bikhchandani et al (1992) and a large subsequent literature, but in which individual agents lack information about their place in the sequence. I share this last assumption, but the setting and results are significantly different. At a high level, the paradox underlying the sidewalk-dollar joke is related to the topic examined in Grossman and Stiglitz (1980), viz., if market prices convey accurate information then no investor acquires information, but if no investor acquires information then market prices don't convey information.

2 Model

An investment opportunity has quality $\omega \in \{g(ood), b(ad)\}$ with probability p_ω. A total of $n \geq 2$ risk-neutral investors sequentially encounter the opportunity, in random order. When an investor i encounters the opportunity, it is either unexploited, or already-exploited. If it is already-exploited then investor i can't do anything. If the opportunity is unexploited, investor i chooses (I) whether or not to pay an investigation cost $\kappa > 0$ to privately observe a signal σ_i of the opportunity's quality, and (II) whether to exploit the opportunity, yielding a payoff of π_ω, where $\pi_g > \pi_b$.

The critical assumption is that when an investor i encounters the opportunity he/she doesn't know how many other investors have already encountered the opportunity (and by extension, doesn't know whether other investors have already investigated). Instead, the

[2] In subsection 4.3 I analyze the effects of exogenous signal acquisition in my setting.

only information available to the investor is whether the opportunity remains unexploited.[3]

Conditional on the opportunity's quality ω, individual investors' signals σ_i are independent. The signal is binary, with $\sigma_i \in \{g, b\}$ and

$$\begin{aligned} \Pr(\sigma_i = g | \omega = g) &= 1 - \epsilon_g \\ \Pr(\sigma_i = b | \omega = b) &= 1 - \epsilon_b. \end{aligned}$$

That is, ϵ_ω is the signal's error rate given project quality ω.

Throughout, I assume that unconditionally exploiting the project is unprofitable,

$$p_g \pi_g + p_b \pi_b < 0, \tag{1}$$

but that the signal is accurate enough that, for a single investor acting in isolation, the strategy of exploiting following a good signal yields a payoff exceeding the investigation cost,

$$p_g (1 - \epsilon_g) \pi_g + p_b \epsilon_b \pi_b > \kappa. \tag{2}$$

The combination of (1) and (2) implies

$$\epsilon_g + \epsilon_b < 1, \tag{3}$$

which implies that an investor's posterior assessment that the opportunity is good is increased by observing a good signal. For use throughout, define

$$\mathcal{E} \equiv \frac{\epsilon_b}{1 - \epsilon_g} < 1.$$

In the main text I generally assume strictly positive error rates $\epsilon_\omega > 0$. But the setting remains well-behaved if $\epsilon_g = 0$ and/or $\epsilon_b = 0$; see Appendix B.

[3] In particular, I assume that calendar time doesn't provide an individual investor with useful information about the number of other investors who have already encountered the opportunity. Formally, one can think of the opportunity as arriving at Poisson rate λ, and then the n investors sequentially encountering the probability in random order. As $\lambda \to 0$, calendar time grows arbitrarily uninformative.

3 Analysis

3.1 Equilibrium

I characterize the symmetric equilibrium. Let α be the probability with which each investor investigates the opportunity if it remains unexploited. (As standard: Similar results emerge in a related setting in which each investor's investigation cost is independently drawn from some common distribution, and each investor follows a cutoff strategy in this cost.)

As a preliminary step: assumptions (1) and (2) imply that, in equilibrium, investors exploit an (unexploited) opportunity if and only they investigate and see a good signal.

Lemma 1 *In any equilibrium, the investigation probability α is strictly positive and an investor exploits the opportunity after receiving a good signal and does not exploit after either receiving a bad signal, or absent investigation.*

Given Lemma 4, an investor's expected payoff from investigating (gross of investigation cost κ) is

$$v(\alpha) \equiv \sum_{\omega=g,b} \pi_\omega \Pr(\omega|\sigma_i = g, \text{opportunity remains}) \Pr(\sigma_i = g|\text{opportunity remains}). \quad (4)$$

That is: An investor observes that the opportunity remains available, and consequently, that an uncertain number of previous investors having potentially investigated and drawn bad signals. Based on this, the investor assesses the probability of observing a signal $\sigma_i = g$; and then further assesses the probability that the opportunity is good ($\omega = g$) conditional on the signal σ_i being good *and* an uncertain number of previous investors having potentially investigated and drawn signals b.

The payoff $v(\alpha)$ can be explicitly calculated via repeated application of Bayes' rule (see Appendix C). But it is considerably easier to denote by Q_ω the joint probability that one of the n investors exploits the probability if the opportunity's quality is ω,

$$Q_\omega(\alpha) \equiv \Pr(\text{opportunity exploited}|\omega)$$

and then note that, by symmetry,

$$n \Pr(\text{opportunity remains})\, \alpha v(\alpha) = \sum_{\omega=g,b} Q_\omega(\alpha)\, p_\omega \pi_\omega. \quad (5)$$

That is: the expected payoff of each investor is the probability of encountering an unexploited opportunity times the probability of investigating the opportunity (α) times the expected payoff from doing so (v). The sum of these expected payoffs across investors must equal the aggregate payoff across all investors, which is given by the right hand side (RHS) of (5).

To evaluate (5), note that the opportunity is unexploited only if every investor in the sequence of n investors doesn't exploit, which occurs with probability

$$1 - Q_\omega(\alpha) = \Pr(\text{opportunity unexploited}|\omega) = (1 - \Pr(\sigma_i = g|\omega)\alpha)^n . \tag{6}$$

Moreover, and similarly,

$$\begin{aligned} & \Pr(\text{opportunity remains}) \\ = \; & \frac{1}{n}\sum_{k=1}^{n}\sum_{\omega=g,b} p_\omega \Pr\left(\text{opportunity remains}|\text{investor } k^{th} \text{ in line}, \omega\right) \\ = \; & \frac{1}{n}\sum_{\omega=g,b}\sum_{k=1}^{n} p_\omega (1 - \Pr(\sigma_i = g|\omega)\alpha)^{k-1} \\ = \; & \frac{1}{n}\sum_{\omega=g,b} p_\omega \frac{1 - (1 - \Pr(\sigma_i = g|\omega)\alpha)^n}{\Pr(\sigma_i = g|\omega)\alpha} \\ = \; & \sum_{\omega=g,b} \frac{Q_\omega(\alpha) p_\omega}{\Pr(\sigma_i = g|\omega) n\alpha}. \end{aligned} \tag{7}$$

Hence for $\alpha \in (0, 1]$,

$$v(\alpha) = \frac{\sum_{\omega=g,b} Q_\omega(\alpha) p_\omega \pi_\omega}{\sum_{\omega=g,b} \frac{Q_\omega(\alpha) p_\omega}{\Pr(\sigma_i = g|\omega)}} = \frac{\frac{Q_g(\alpha)}{Q_b(\alpha)} p_g \pi_g + p_b \pi_b}{\frac{Q_g(\alpha)}{Q_b(\alpha)} \frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}}. \tag{8}$$

As (8) makes clear, they key term is the ratio of exploitation probabilities $\frac{Q_g}{Q_b}$. Indeed, this ratio is a sufficient statistic for equilibrium characterization.

For a low individual investigation probability α the most likely case is a single investigation, and so the ratio $\frac{Q_g}{Q_b}$ simply coincides with the ratio of the conditional probabilities of seeing a good signal, namely $\frac{1-\epsilon_g}{\epsilon_b} > 1$ (by (3)). In contrast, if the individual investigation probability α is high then it is likely that some investor investigates and draws a good signal, even in the state $\omega = b$, and so $Q_g \approx Q_b \approx 1$. The following result formalizes and generalizes these observations:

Lemma 2 *For $n \geq 2$, the ratio $\frac{Q_g}{Q_b}$ approaches $\frac{1-\epsilon_g}{\epsilon_b} = \frac{1}{\mathcal{E}}$ as $\alpha \to 0$, equals $\frac{1-\epsilon_g^n}{1-(1-\epsilon_b)^n}$ at $\alpha = 1$, and is monotonically decreasing in $\alpha \in (0, 1)$.*

Since $\pi_g > \pi_b$ the RHS of (8) is increasing in the ratio $\frac{Q_g}{Q_b}$,[4] and hence (by Lemma 2)

[4]The derivative of the RHS of with respect to $\frac{Q_g}{Q_b}$ has the same sign as $\pi_g - \mathcal{E}\pi_b$.

$v(\alpha)$ is decreasing in α. In words: As others' investigation probability α increases, the observation that the opportunity remains available becomes an increasingly negative signal of the opportunity's quality ω, so that the expected payoff from investigation v decreases.

By Lemma 2 and assumption (2)

$$v(0) = \lim_{\alpha \to 0} v(\alpha) = (1 - \epsilon_g) p_g \pi_g + \epsilon_b p_b \pi_b > \kappa.$$

In words: if no-one else investigates ($\alpha = 0$) then it is strictly profitable to investigate.

Consequently:

Lemma 3 *There is a unique (symmetric) equilibrium given either by the solution to*

$$v(\alpha) = \kappa,$$

or by $\alpha = 1$ if $v(1) \geq \kappa$. The equilibrium is interior for all n sufficiently large.[5]

3.2 Comparative statics in the number of investors n

As the number of investors n increases, the signal that the opportunity is still available conveys increasingly negative information. Consequently, as n increases, the probability that any individual investor investigates falls.

The effect of the number of investors on the equilibrium exploitation probabilities, henceforth Q_g^* and Q_b^*, thus depends on the balance between more potential investors, and each investor investigating less. As an easy example: As n increases from 1 to 2, the investigation probability generally falls from $\alpha_1 = 1$ to $\alpha_2 < 1$; consequently, if $\epsilon_g = 0$, the exploitation probability Q_g^* falls from 1 to $\alpha_2 + (1 - \alpha_2)\alpha_2$, even though the number of investors has increased. This observation generalizes to cover Q_b^* as well as Q_g^*, and arbitrary error rates ϵ_ω and investor population n:

Proposition 1 *The equilibrium exploitation probabilities Q_g^* and Q_b^* are decreasing in the number of investors n.*

An immediate consequence of Proposition 1 is:

Corollary 1 *Even as the number of investors n grows arbitrarily large the equilibrium exploitation probabilities Q_g^* and Q_b^* remain bounded below* 1.

[5] This follows from Lemma 2 and assumption (1).

Indeed, the limiting equilibrium values of Q_g^* and Q_b^* as n grows large can be straightforwardly characterized using the equilibrium condition

$$\frac{\frac{Q_g}{Q_b} p_g \pi_g + p_b \pi_b}{\frac{Q_g}{Q_b} \frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}} = \kappa \tag{9}$$

and the fact that for any fixed value of $Q_g \in (0,1)$,[6]

$$1 - \lim_{n\to\infty} Q_b = (1 - Q_g)^{\mathcal{E}}. \tag{10}$$

3.3 Comparative statics in the ratio of good to bad opportunities

As the ratio of good to bad opportunities $\frac{p_g}{p_b}$ increases, an investor who observes that an opportunity remains unexploited attaches a higher probability to the explanation that no other investor has investigated the opportunity, and attaches a lower probability to the opportunity remaining available because it is bad, and other investors having observed bad signals. Consequently, an investor updates less from the observation that the opportunity remains available. This increases an investor's willingness to investigate, thereby increasing the equilibrium probability of exploitation.

An increase in $\frac{p_g}{p_b}$ also has the direct effect of increasing expected project profitability, pushing in the same direction. But even if π_g and π_b are adjusted so as to leave expected project profitability unchanged it remains the case that equilibrium exploitation probabilities rise in $\frac{p_g}{p_b}$.

Proposition 2 *If the ratio of good to bad opportunities $\frac{p_g}{p_b}$ weakly increases and expected profitability $Q_g^* p_g \pi_g + Q_b^* p_b \pi_b$ of the opportunity weakly increases (holding Q_g^* and Q_b^* fixed), with at least one increase strict, then the equilibrium exploitation probabilities Q_g^* and Q_b^* strictly increase.*

In words: Low-risk opportunities are more likely to be exploited than high-risk ones, even after controlling for expected profitability.

3.4 The Sharpe ratio of exploited opportunities

The opportunity is exploited with probability $Q_g^* p_g + Q_b^* p_b$, and conditional on exploitation yields π_g with probability $\frac{Q_g^* p_g}{Q_g^* p_g + Q_b p_b}$ and π_b with probability $\frac{Q_b^* p_b}{Q_g^* p_g + Q_b^* p_b}$. Hence the expected

[6]See appendix for derivation of (10).

return and standard deviation of returns are, respectively

$$\frac{Q_g^* p_g \pi_g + Q_b^* p_b \pi_b}{Q_g^* p_g + Q_b^* p_b}$$

and

$$\frac{\sqrt{Q_g^* p_g Q_b^* p_b}}{Q_g^* p_g + Q_b^* p_b} \left(\pi_g - \pi_b\right),$$

implying the associated Sharpe ratio is

$$\left(\left(\frac{Q_g^*}{Q_b^*} \frac{p_g}{p_b} \right)^{\frac{1}{2}} \pi_g + \left(\frac{Q_g^*}{Q_b^*} \frac{p_g}{p_b} \right)^{-\frac{1}{2}} \pi_b \right) \left(\pi_g - \pi_b\right)^{-1}. \tag{11}$$

Holding exploitation probabilities Q_ω^* fixed, the Sharpe ratio (11) is increasing in each of p_g and π_g,[7] as one would expect. However, increases in p_g and π_g also raise exploitation probabilities (Proposition 2) and hence decrease the ratio $\frac{Q_g^*}{Q_b^*}$ (Lemma 2), pushing the Sharpe ratio down. I next show that in a wide range of cases the latter effect is at least as strong as the former, so that the net effect is that better opportunities are associated with (weakly) lower equilibrium Sharpe ratios.

From the equilibrium condition (9) it is immediate that the equilibrium value of $\frac{Q_g^*}{Q_b^*} \frac{p_g}{p_b}$ is invariant to changes in the ratio $\frac{p_g}{p_b}$; consequently, the Sharpe ratio is likewise invariant to changes in $\frac{p_g}{p_b}$. Increases to $\pi_g - \pi_b$ (holding π_b fixed) and to π_b (holding $\pi_g - \pi_b$) generate conflicting effects. In both cases $\frac{Q_g^*}{Q_b^*}$ falls (Proposition 2), pushing the Sharpe ratio down; but there are also direct effects that potentially offset this effect. The following result gives simple sufficient conditions for the overall sign:

Proposition 3 *The Sharpe ratio (11) is invariant to changes in $\frac{p_g}{p_b}$; is increasing in $\pi_g - \pi_b$ (holding π_b constant) if $\mathcal{E} < \frac{1}{9}$;[8] and is decreasing in π_b (holding $\pi_g - \pi_b$ fixed) if $\frac{p_g}{p_b} \leq 1$.*

In many cases, one can think of exploitation of the opportunity as entailing some investment k. Bad opportunities yield nothing, while good opportunities yield a gross expected return R; hence $\pi_b = -k$ and $\pi_g = (R-1)k$. In this case, the combination of Propositions 2 and 3 delivers:

[7] For any positive constant a, the derivative of $\frac{a\pi_g + a^{-1}\pi_b}{\pi_g - \pi_b}$ with respect to π_g has the same sign as

$$a\left(\pi_g - \pi_b\right) - \left(a\pi_g + a^{-1}\pi_b\right) = -\left(a + a^{-1}\right)\pi_b,$$

which is positive by (1).

[8] In the case of state-invariant error rates $\epsilon_b = \epsilon_g$, the condition $\mathcal{E} < \frac{1}{9}$ corresponds to $\epsilon < 1/10$.

Corollary 2 *Controlling for the required investment k, if $\mathcal{E} < \frac{1}{9}$ then the equilibrium probability that an opportunity is exploited is negatively correlated with its equilibrium Sharpe ratio.*

4 Comparisons to benchmark cases

4.1 Investors know their place in the sequence

A first benchmark is the case in which investors know their place in the sequence. The equilibrium outcome is almost immediate: By assumption (2) the first investor investigates the opportunity, and exploits if a good signal is observed. If the opportunity remains available then investor $i > 1$ similarly investigates if the expected payoff form doing so exceeds the investigation cost κ,

$$(1-\epsilon_g)\Pr(\omega = g|\sigma_1 = \ldots = \sigma_{i-1} = b)\,\pi_g + \epsilon_b \Pr(\omega = b|\sigma_1 = \ldots = \sigma_{i-1} = b)\,\pi_b \geq \kappa.$$

Consequently, the first $m \geq 1$ investors investigate if the opportunity is available, while no agent $i > m$ investigates. Investigations cease because the information-cascade effect (Banerjee 1992 and Bikhchandani et al 1992) means that after a sequence of bad signals, which can be inferred from the opportunity remaining available, an investor would ignore even good signal, and so prefers to simply the save the cost of an investigation and do nothing.

Evaluating,

$$\Pr(\omega = g|\sigma_1 = \ldots = \sigma_{i-1} = b) = \frac{p_g \epsilon_g^{i-1}}{p_g \epsilon_g^{i-1} + p_b (1-\epsilon_b)^{i-1}}$$

and so the equilibrium m is the largest m such that

$$\frac{(1-\epsilon_g)\,\epsilon_g^{m-1} p_g \pi_g + \epsilon_b (1-\epsilon_b)^{m-1} p_b \pi_b}{(1-\epsilon_g)\,\epsilon_g^{m-1} \frac{p_g}{1-\epsilon_g} + \epsilon_b (1-\epsilon_b)^{m-1} \frac{p_b}{\epsilon_b}} \geq \kappa.$$

Equilibrium exploitation probabilities are then

$$\begin{aligned} \tilde{Q}_g &= 1 - \epsilon_g^m \\ \tilde{Q}_b &= 1 - (1-\epsilon_b)^m . \end{aligned}$$

The comparison of investor welfare relative to the case in which investors don't know their place in the sequence is immediate from the fact that, in this case, each investor has a zero expected payoff once investigation costs are accounted for:

Corollary 3 *Let n be sufficiently large that the equilibrium in the unknown-place-in-sequence is interior. Investor welfare is strictly raised if investors observe their place in the sequence.*

While the welfare comparison is straightforward, the comparison of exploitation probabilities is ambiguous.

For conciseness, for the remainder of this subsection I focus on the leading case in which the signal error rate is independent of the opportunity's quality ω, i.e.,

$$\epsilon_g = \epsilon_b = \epsilon. \tag{12}$$

In this case, only the first investor investigates ($m = 1$); and either that investor observes a positive signal and exploits the opportunity, or observes a negative signal and no future investor finds it worthwhile to investigate. Exploitation probabilities are simply

$$\tilde{Q}_g = 1 - \epsilon \text{ and } \tilde{Q}_b = \epsilon.$$

The reason is that, under (12), observing a negative signal and then a positive signal leaves an investor with posterior beliefs that match the prior belief p_g, which by assumption (1) leads to no-investment.

On the one hand, if the common error rate ϵ is sufficiently small, the probability that a good project is exploited is raised if investors know their place in the sequence:

Lemma 4 *If $\epsilon_g = \epsilon_b = \epsilon$ is sufficiently small and n is sufficiently large then*

$$\tilde{Q}_b < Q_g^* < \tilde{Q}_g.$$

Moreover, $\tilde{Q}_b < Q_b^*$ if and only if

$$\frac{(1 - \exp(-1))\, p_g}{(1 - \exp(-1))\, p_g + p_b} \pi_g > \kappa, \tag{13}$$

which holds if a single investor's return to investigation, $\frac{p_g \pi_g}{\kappa}$, is sufficiently large.

The ordering of exploitation probabilities in Lemma 4 isn't universal, as illustrated in the following example:

Example: Let $p_g = .2$, $\pi_g = 1,000$, $\pi_b = -300$, $\kappa = 6$, $\epsilon_g = \epsilon_b = .4$. If investors know their place in the sequence then $\tilde{Q}_g = .6$ and $\tilde{Q}_b = .4$. But if investors don't know their place in the sequence then the equilibrium as $n \to \infty$ is $Q_g^* \to .691 > \tilde{Q}_g$ and $Q_b^* \to .543 > \tilde{Q}_b$.[9]

In the example, if investors know their place in the sequence then, as discussed, only the first investor investigates; and because the error rate is high ($\epsilon_g = .4$) the exploitation probability is low. In contrast, if investors don't know their place in the sequence then the expected number of investigations rises, raising the exploitation probability.

Lemma 4 identifies conditions under which investors knowing their place in the sequence leads to better exploitation outcomes, in the sense that good opportunities are *more* likely to be exploited and worse projects are *less* likely to be exploited, relative to the case of unknown-place-in-sequence. This begs the question of whether the reverse is possible, viz., are there circumstances under which investors knowing their place in the sequence leads to unambiguously *worse* exploitation outcomes? The answer is negative:

Corollary 4 *If $Q_g^* > \tilde{Q}_g$ then $Q_b^* > \tilde{Q}_b$.*

Corollary 4 is immediate from the fact that (Lemma 2)

$$\frac{Q_g^*}{Q_b^*} < \frac{1-\epsilon}{\epsilon} = \frac{\tilde{Q}_g}{\tilde{Q}_b}.$$

4.2 Investors make investigation decisions simultaneously

A second benchmark is that in which all investors make investigation decisions simultaneously, without the opportunity to observe whether or not the opportunity still exists. Equivalently, this benchmark corresponds to investors committing to an investigation strategy before the game begins. If multiple investors investigate and then attempt to exploit the opportunity is randomly allocated to one of them.Note that this benchmark is closely related to previous analysis of Bertrand competition preceded by a costly entry decision (see references in introduction).

Let $\hat{v}(\alpha)$ denote an investor's expected payoff (gross of investigation costs) from investigation if all investors investigate with probability α. By the analogue of (5),

$$n\alpha\hat{v}(\alpha) = \sum_{\omega=g,b} Q_\omega(\alpha)\, p_\omega \pi_\omega, \tag{14}$$

where the exploitation probabilities $Q_g(\omega)$ and $Q_b(\omega)$ are again given by (6).

[9] Moreover, by Proposition 1 the equilibrium exploitation probabilities Q_g^* and Q_b^* are larger for smaller values of n.

When investors move sequentially they draw negative inferences from seeing that the opportunity remains available. This observation might suggest that exploitation probabilities in the simultaneous-move game are higher. In fact, the reverse is the case, and exploitation probabilities are *lower*. Formally, denote by $\hat{Q}_g$ and $\hat{Q}_b$ the equilibrium exploitation probabilities in the simultaneous-move game:

Lemma 5 *If n is sufficiently large that an interior equilibrium exists in the simultaneous-move game then $\hat{Q}_g < Q_g^*$ and $\hat{Q}_b < Q_b^*$.*

The economic force behind Lemma 5 is as follows. Consider the commitment interpretation of the simultaneous-move game: each investor decides on an investigation probability before the game begins, and then the investors investigate and choose whether to exploit in a randomly determined order. When it is an individual investor's turn to move, there are two possibilities: either (i) the opportunity remains available, or (ii) it is already exploited. Case (i) is exactly the situation that arises in the main sequential-move analysis; but case (ii) is strictly worse for an investor, since in this case the investigation cost κ yields zero benefit, but the investor is committed to investigate with probability α. Consequently,

$$\hat{v}(\alpha) < v(\alpha), \tag{15}$$

which indeed can be seen formally from the comparison of (5) and (14). It follows from (15) that the equilibrium investigation probability is lower in the simultaneous-move setting, yielding Lemma 5 (see appendix for details).

While exploitation probabilities are lower in the simultaneous-move game than in the main sequential-move analysis, investor welfare is exactly the same (and equals zero) in both cases, provided only that n is large enough to deliver an interior equilibrium in both cases.

It is worth highlighting that the ratio of good to bad opportunities, $\frac{p_g}{p_b}$, plays a different role in the sequential- and simultaneous-move games. This is easiest seen by considering a reduction in $\frac{p_g}{p_b}$ that is accompanied by changes to π_g and π_b that leave $p_g\pi_g$ and $p_b\pi_b$ unchanged. In the sequential-move game this change lowers equilibrium exploitation probabilities (Proposition 2). Economically, the smaller ratio of good to bad opportunities leads to more negative investor inferences from the observation that the opportunity remains available. In contrast, in the simultaneous-move game this change leaves exploitation probabilities *unaffected*.

4.3 Exogenous signals

A third benchmark is the case in which acquisition of signals σ_i is exogenous. Specifically: Exactly as in the baseline analysis, investors sequentially encounter the opportunity in random order. Different from the baseline: upon encountering the opportunity, an investor i immediately observes a signal σ_i of the opportunity's quality. The investor then decides whether to pay a cost $\check{\kappa}$ in order to exploit the opportunity. Exactly as in the benchmark case of endogenous signal acquisition, in equilibrium an investor only exploits the opportunity following a good signal. The fact that the cost $\check{\kappa}$ is paid only after observing the signal generates a mechanical advantage to an investor in the exogenous-signal case relative to the endogenous-signal case. In order to avoid this mechanical difference between the cases I assume

$$\check{\kappa} \sum_{\omega=g,b} p_\omega \Pr\left(\sigma_i = g|\omega\right) = \kappa, \tag{16}$$

i.e., the expected cost of exploitation for a single investor is equal across the exogenous- and endogenous-signal settings.

An important first step is to note that if the number of investors n is sufficiently large then it cannot be an equilibrium for an investor to exploit with probability 1 after observing a good signal. The reason is that, in this case, the observation that the opportunity remains available would indicate that all previous investors have observed a bad signal, and this information would overwhelm an investor's single good signal. Accordingly, the equilibrium must take the form of: each investor who observes a good signal exploits the opportunity with probability $\gamma \in (0, 1)$. I focus on the case of n sufficiently large for the remainder of the discussion.

Analogous to (4), define $\check{v}(\gamma)$ as the expected payoff of an investor who observes a good signal, gross of exploitation cost $\check{\kappa}$. Note that the probability that an opportunity of quality ω is exploited by some investor is

$$1 - \left(1 - \Pr\left(\sigma_i = g|\omega\right)\gamma\right)^n = Q_\omega\left(\gamma\right).$$

Consequently, the expected payoff $\check{v}(\gamma)$ is given by an analogous formula to (5):

$$n\left(\sum_{\omega=g,b} p_\omega \Pr\left(\text{opportunity remains}|\omega\right)\Pr\left(\sigma_i = g|\omega\right)\right)\gamma\check{v}\left(\gamma\right) = \sum_{\omega=g,b} Q_\omega\left(\gamma\right)p_\omega\pi_\omega.$$

The same steps as in (7) yield

$$\Pr(\text{opportunity remains}|\omega) = \frac{Q_\omega(\gamma)}{\Pr(\sigma_i = g|\omega)\, n\gamma},$$

hence

$$\check{v}(\gamma) = \frac{\frac{Q_g(\gamma)}{Q_b(\gamma)} p_g \pi_g + p_b \pi_b}{\frac{Q_g(\gamma)}{Q_b(\gamma)} p_g + p_b} = \frac{\frac{Q_g(\gamma)}{Q_b(\gamma)} \frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}}{\frac{Q_g(\gamma)}{Q_b(\gamma)} p_g + p_b} v(\gamma) > v(\gamma). \tag{17}$$

As in baseline analysis, Lemma 2 implies that $\check{v}$ is decreasing in γ, generalizing the above observation that if projects with good signals are exploited with high probability then the observation that the opportunity remains available conveys negative information. The inequality in (17) reflects the advantage that an investor enjoys in the exogenous-signal case of choosing whether to pay the exploitation cost only after seeing the signal realization.

More interestingly, the advantage that an investor derives from paying the exploitation cost only after seeing the signal exceeds the increase in investigation costs embodied in (16). Consequently, the equilibrium exploitation probability is higher in the case of exogenous signals than in the endogenous signal baseline:

Lemma 6 *If n is sufficiently large that the equilibrium of the exogenous-signal game is interior then $\check{Q}_g > Q_g^*$ and $\check{Q}_b > Q_b^*$.*

The intuition for Lemma 6 is that (16) scales up the cost κ using the prior distribution of project quality (p_g, p_b). In contrast, in equilibrium an investor faces an adversely selected pool of projects, making the ability to defer paying the exploitation cost more valuable than implied under the prior distribution.

5 Summary

I analyze a setting in which investors sequentially encounter a potentially profitable investment opportunity, and decide both whether to investigate its quality and whether to exploit the opportunity. Once exploited, the opportunity is unavailable to future investors. The key friction is that an investor observes only whether the opportunity remains available, but is ignorant about the number of investors who have already investigated the opportunity. The probability that the opportunity is exploited decreases in the number of investors. Safe opportunities are exploited more often than risky ones, even controlling for average profitability. Higher exploitation probabilities are generally associated with lower observed Sharpe ratios.

References

Banerjee, Abhijit V., 1992, "A Simple Model of Herd Behavior." Quarterly Journal of Economics 107, pp. 797-817.

Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch, 1992, "A theory of fads, fashion, custom and cultural change as informational cascades." Journal of Political Economy 100, pp. 992–1026.

Elberfeld Walter, Elmar Wolfstetter, 1999, "A dynamic model of Bertrand competition with entry." International Journal of Industrial Organization 17, pp. 513–525.

Gromb, Denis and Dimitri Vayanos, 2010. "Limits of arbitrage." Annual Review of Financial Economics 2, pp. 251–275.

Grossman, Sanford J. and Joseph E. Stiglitz, 1980, "On the Impossibility of Informationally Efficient Markets." American Economic Review 70, pp. 393-408.

Herrera, Helios and Johannes Hörner, 2013, "Biased social learning." Games and Economic Behavior 80, pp. 131-146.

Monzón, Ignacio and Michael Rapp, 2014, "Observational learning with position uncertainty." Journal of Economic Theory 154, pp. 375–402.

Sherman, Roger and Thomas D. Willett, 1967, "Potential Entrants Discourage Entry." Journal of Political Economy 75, pp. 400-403.

Zhu, Haoxiang, 2012, "Finding a good price in opaque over-the-counter markets." Review of Financial Studies 25, pp. 1255-1285.

A Proofs

Proof of Lemma 1: First, $\alpha = 0$ cannot be an equilibrium, as follows. In this case, no investigation occurs, and the observation that the opportunity remains available conveys no information, so every investor who encounters the opportunity believes the probability that the opportunity is good is p_g. But then assumption (2) implies that an individual investor would gain by deviating and investigating.

For the remainder of the proof, denote the exploitation probabilities after a good signal, no signal, and a bad signal by β_g, β_0, β_b. Note that $\beta_g \geq \beta_0 \geq \beta_b$, with $\beta_g = \beta_0 = 1$ if $\beta_b \in (0, 1)$ and $\beta_g = 1$ and $\beta_b = 0$ if $\beta_0 \in (0, 1)$ and $\beta_0 = \beta_b = 0$ if $\beta_g < 1$. Note first that $\alpha > 0$ and $\beta_g = 0$ cannot be an equilibrium, since in this case an investor would do better by avoiding the investigation cost κ.

Second, $\alpha > 0$ and $\beta_b = 1$ cannot be an equilibrium, as follows. In this case, no updating occurs, so every investor who encounters the opportunity believes the probability that the opportunity is good is p_g. But then assumption (1) implies that $\beta_0 = 0$, in turn implying $\beta_b = 0$, a contradiction.

Third, $\alpha > 0$ and $\beta_g < 1$ cannot be an equilibrium, since in this case an investor is at best indifferent between exploiting and doing nothing after observing a signal g, and so would do better by deviating to no-investigation and avoiding the investigation cost κ.

Fourth, $\alpha > 0$ and $\beta_b \in (0, 1)$ cannot be an equilibrium, as follows. From above, $\beta_g = 1$. So observing that the opportunity remains available leads an investor to revise beliefs about project quality downwards; and seeing a bad signal leads to a further downwards revision. Then (1) implies that $\beta_b = 0$.

Fifth, and finally, a similar argument implies that $\alpha > 0$ and $\beta_0 > 0$ cannot be an equilibrium.

Combined, these observations imply that in any equilibrium $\alpha > 0$, $\beta_g = 1$ and $\beta_0 = \beta_b = 0$, completing the proof.

Proof of Lemma 2: By l'Hopital's rule, as $\alpha \to 0$

$$\frac{Q_g}{Q_b} \to \frac{1-\epsilon_g}{\epsilon_b} = \frac{1}{\mathcal{E}} > 1.$$

Evaluation of $\frac{Q_g}{Q_b}$ at $\alpha = 1$ is immediate. The remainder of the proof establishes that $\frac{Q_g}{Q_b}$ is monotonically decreasing in $\alpha \in (0, 1)$.Write $A = 1 - \epsilon_g$ and $B = \epsilon_b$, and note that $B < A$. I establish that

$$\frac{1-(1-A\alpha)^n}{1-(1-B\alpha)^n}$$

is decreasing in $\alpha \in (0, 1)$, i.e., that

$$A(1-A\alpha)^{n-1}(1-(1-B\alpha)^n) < B(1-B\alpha)^{n-1}(1-(1-A\alpha)^n),$$

or equivalently,

$$\frac{A}{B}\frac{(1-A\alpha)^{n-1}}{(1-B\alpha)^{n-1}} < \frac{1-(1-A\alpha)^n}{1-(1-B\alpha)^n}.$$

This inequality holds with equality at $n = 1$. Consequently, it suffices to show that for any $n \geq 1$,

$$\frac{1-A\alpha}{1-B\alpha} < \frac{\frac{1-(1-A\alpha)^{n+1}}{1-(1-A\alpha)^n}}{\frac{1-(1-B\alpha)^{n+1}}{1-(1-B\alpha)^n}}.$$

Write $x_A = 1 - A\alpha$ and $x_B = 1 - B\alpha$, and note $0 < x_A < x_B < 1$. The above inequality is equivalent to

$$\frac{x_B^{-1}-1}{1-x_B^n} < \frac{x_A^{-1}-1}{1-x_A^n}.$$

Hence it suffices to show that

$$\frac{x^{-1}-1}{1-x^n}$$

is decreasing in $x \in (0,1)$, i.e., that

$$-x^{-2}(1-x^n) + nx^{n-1}\left(x^{-1}-1\right) < 0,$$

or equivalently,

$$-1 + (n+1)\,x^n - nx^{n+1} < 0.$$

The LHS equals zero at $x = 1$, and the derivative of LHS with respect to x is

$$n(n+1)\,x^{n-1}(1-x) > 0,$$

thereby establishing the required inequality and completing the proof.

Lemma A-1 *The relation between Q_b and Q_g is given by*

$$Q_b = 1-\left(1-\mathcal{E}\left(1-(1-Q_g)^{\frac{1}{n}}\right)\right)^n \tag{A-1}$$
$$Q_g = 1-\left(1-\mathcal{E}^{-1}\left(1-(1-Q_b)^{\frac{1}{n}}\right)\right).^n \tag{A-2}$$

For $n \geq 2$: Holding Q_g fixed, Q_b is increasing in n; and holding Q_b fixed, Q_g is decreasing in n.

Proof of Lemma A-1: Equation (A-1) follows straightforwardly from $1-(1-\epsilon_g)\,\alpha = (1-Q_g)^{\frac{1}{n}}$ and $\alpha = \frac{1-(1-Q_g)^{\frac{1}{n}}}{1-\epsilon_g}$. Similarly, (A-2) follows straightforwardly from $1-\epsilon_b\alpha = (1-Q_b)^{\frac{1}{n}}$ and $\alpha = \frac{1-(1-Q_b)^{\frac{1}{n}}}{\epsilon_b}$.

Rearranging (A-1) gives

$$\ln(1-Q_b) = n\ln\left(1-\mathcal{E}\left(1-(1-Q_g)^{\frac{1}{n}}\right)\right). \tag{A-3}$$

I show that the RHS of (A-3) is decreasing in n, i.e.,

$$\ln\left(1-\mathcal{E}\left(1-(1-Q_g)^{\frac{1}{n}}\right)\right) + \left(-\frac{1}{n^2}\right)n\frac{\mathcal{E}\cdot\ln(1-Q_g)\cdot(1-Q_g)^{\frac{1}{n}}}{1-\mathcal{E}\left(1-(1-Q_g)^{\frac{1}{n}}\right)} < 0, \tag{A-4}$$

which is equivalent to

$$\begin{aligned}
&\left(1-\mathcal{E}\left(1-(1-Q_g)^{\frac{1}{n}}\right)\right)\ln\left(1-\mathcal{E}\left(1-(1-Q_g)^{\frac{1}{n}}\right)\right)\\
<\;&\mathcal{E}\cdot(1-Q_g)^{\frac{1}{n}}\cdot\ln(1-Q_g)^{\frac{1}{n}}.
\end{aligned}$$

Hence it suffices to show that, for any $x\in(0,1)$,

$$(1-\mathcal{E}(1-x))\ln(1-\mathcal{E}(1-x))<\mathcal{E}x\ln x.$$

This inequality holds with equality at $\mathcal{E}=0,1$. The RHS is linear in $\mathcal{E}$. Hence it suffices to show that the LHS is convex in $\mathcal{E}$, which is indeed the case since its derivative with respect to $\mathcal{E}$ is

$$-(1-x)\ln(1-\mathcal{E}(1-x))-(1-x),$$

which is increasing in $\mathcal{E}$. This establishes (A-4).

Rearranging (A-2) gives

$$\ln(1-Q_g)=n\ln\left(1-\mathcal{E}^{-1}\left(1-(1-Q_b)^{\frac{1}{n}}\right)\right). \tag{A-5}$$

I show that the RHS is increasing in n, i.e.,

$$\ln\left(1-\mathcal{E}^{-1}\left(1-(1-Q_b)^{\frac{1}{n}}\right)\right)+\left(-\frac{1}{n^2}\right)n\frac{\mathcal{E}^{-1}\cdot\ln(1-Q_b)\cdot(1-Q_b)^{\frac{1}{n}}}{1-\mathcal{E}^{-1}\left(1-(1-Q_b)^{\frac{1}{n}}\right)}>0, \tag{A-6}$$

which is equivalent to

$$\begin{aligned}
&\left(1-\mathcal{E}^{-1}\left(1-(1-Q_b)^{\frac{1}{n}}\right)\right)\ln\left(1-\mathcal{E}^{-1}\left(1-(1-Q_b)^{\frac{1}{n}}\right)\right)\\
>\;&\mathcal{E}^{-1}\cdot(1-Q_b)^{\frac{1}{n}}\cdot\ln(1-Q_b)^{\frac{1}{n}}.
\end{aligned}$$

Hence it suffices to show that, for any $x\in(0,1)$,

$$\left(1-\mathcal{E}^{-1}(1-x)\right)\ln\left(1-\mathcal{E}^{-1}(1-x)\right)>\mathcal{E}^{-1}x\ln x.$$

This inequality holds with equality at $\mathcal{E}^{-1}=0,1$. The RHS is linear in $\mathcal{E}^{-1}$. Hence it suffices to show that the LHS is convex in $\mathcal{E}^{-1}$, which is established in the first part of the proof. This establishes (A-4) and completes the proof.

Proof of Proposition 1: The equilibrium value of $\frac{Q_g}{Q_b}$ is independent of n. Consider the ratio $\frac{Q_g}{Q_b}$ as a function of Q_g. By Lemma 2, it is a decreasing function; and by Lemma A-1 the function decreases as n increases. It follows that the equilibrium value of Q_g falls. Similarly, consider the ratio $\frac{Q_g}{Q_b}$ as a function of Q_b. Again by Lemma 2, it is a decreasing function; and by Lemma A-1 the function decreases as n increases. It follows that the equilibrium value of Q_b falls, completing the proof.

Derivation of (10): Fix Q_g. From (A-1),

$$1 - Q_b = \left(1 - \mathcal{E}\left(1 - (1 - Q_g)^{\frac{1}{n}}\right)\right)^n = \exp\left(\frac{\ln\left(1 - \mathcal{E}\left(1 - (1 - Q_g)^{\frac{1}{n}}\right)\right)}{\frac{1}{n}}\right).$$

Note that

$$\frac{\partial \ln\left(1 - \mathcal{E}\left(1 - (1 - Q_g)^{\frac{1}{n}}\right)\right)}{\partial\left(\frac{1}{n}\right)} = \frac{\mathcal{E}\ln(1 - Q_g)\cdot(1 - Q_g)^{\frac{1}{n}}}{\left(1 - \mathcal{E}\left(1 - (1 - Q_g)^{\frac{1}{n}}\right)\right)} \to \mathcal{E}\ln(1 - Q_g) \text{ as } n \to \infty.$$

So by l'Hopital's rule, as $n \to \infty$,

$$1 - Q_b \to (1 - Q_g)^{\mathcal{E}}.$$

Proof of Proposition 2: Denote $\psi = \frac{Q_g}{Q_b}$, and denote by ψ_0 the value of ψ prior to any parameter changes. The equilibrium condition is

$$\epsilon_b \frac{\psi p_g \pi_g + p_b \pi_b}{\psi p_g \mathcal{E} + p_b} = \kappa. \tag{A-7}$$

If p_g remains unchanged and π_g and/or π_b increase then it is immediate that ψ decreases and hence (by 2) that α, Q_g and Q_b all increase.

Next, consider the case in which p_g increases. From (A-7), and using $p_b = 1 - p_b$,

$$\left(\frac{\partial\psi}{\partial p_g} p_g \pi_g + \frac{\partial(\psi_0 p_g \pi_g + p_b \pi_b)}{\partial p_g}\right)(\psi_0 \mathcal{E} p_g + p_b) - \left(\frac{\partial\psi}{\partial p_g}\mathcal{E} p_g + \psi\mathcal{E} - 1\right)(\psi_0 p_g \pi_g + p_b \pi_b) = 0,$$

i.e.,

$$\begin{aligned}\frac{\partial\psi}{\partial p_g} &= \frac{(\psi\mathcal{E} - 1)(\psi_0 p_g \pi_g + p_b \pi_b) - \frac{\partial(\psi_0 p_g \pi_g + p_b \pi_b)}{\partial p_g}(\psi_0 \mathcal{E} p_g + p_b)}{p_g \pi_g (\psi_0 \mathcal{E} p_g + p_b) - \mathcal{E} p_g (\psi_0 p_g \pi_g + p_b \pi_b)} \\ &= \frac{(\psi\mathcal{E} - 1)(\psi_0 p_g \pi_g + p_b \pi_b) - \frac{\partial(\psi_0 p_g \pi_g + p_b \pi_b)}{\partial p_g}(\psi_0 \mathcal{E} p_g + p_b)}{p_g p_b \pi_g - \mathcal{E} p_g p_b \pi_b}.\end{aligned}$$

Note that $\psi\mathcal{E} < 1$ by Lemma 2. Moreover, the equilibrium condition implies $\psi_0 p_g \pi_g + p_b \pi_b >$

0. Hence if $\frac{\partial(\psi_0 p_g \pi_g + p_b \pi_b)}{\partial p_g} \geq 0$ then $\frac{\partial \psi}{\partial p_g} < 0$, implying (by Lemma 2) that α, Q_g and Q_b are all higher, completing the proof.

For reference, note that if both π_g and π_b are constant then.

$$\begin{aligned}
\frac{\partial \psi}{\partial p_g} &= \frac{(\psi\mathcal{E} - 1)(\psi p_g \pi_g + p_b \pi_b) - (\psi \pi_g - \pi_b)(\psi \mathcal{E} p_g + p_b)}{p_g p_b \pi_g - \mathcal{E} p_g p_b \pi_b} \\
&= \frac{\psi \mathcal{E} p_b \pi_b - \psi p_g \pi_g - (\psi p_b \pi_g - \psi \mathcal{E} p_g \pi_b)}{p_g p_b \pi_g - \mathcal{E} p_g p_b \pi_b} \\
&= \frac{\psi(\mathcal{E}\pi_b - \pi_g)}{p_g p_b \pi_g - \mathcal{E} p_g p_b \pi_b} = -\frac{\psi}{p_g p_b}.
\end{aligned}$$

Proof of Proposition 3: Denote $\phi = \frac{Q_g p_g}{Q_b p_b}$ and $\pi_g = \Delta + \pi_b$. The equilibrium condition is

$$\epsilon_b \frac{\phi \pi_g + \pi_b}{\mathcal{E}\phi + 1} = \epsilon_b \frac{\phi(\Delta + \pi_b) + \pi_b}{\mathcal{E}\phi + 1} = \kappa.$$

For use below, note that certainly

$$\phi \pi_g + \pi_b > 0. \tag{A-8}$$

Differentiating with respect π_g yields

$$\left(\frac{\partial \phi}{\partial \pi_g}\pi_g + \phi\right)(\mathcal{E}\phi + 1) - (\phi \pi_g + \pi_b)\mathcal{E}\frac{\partial \phi}{\partial \pi_g} = 0,$$

and hence

$$\frac{\partial \phi}{\partial \pi_g} = -\frac{\phi(\mathcal{E}\phi + 1)}{\pi_g(\mathcal{E}\phi + 1) - (\phi\pi_g + \pi_b)\mathcal{E}} = -\frac{\phi(\mathcal{E}\phi + 1)}{\pi_g - \mathcal{E}\pi_b}.$$

Similarly, differentiating with respect to π_b (holding $\Delta = \pi_g - \pi_b$ fixed) yields

$$\left(\frac{\partial \phi}{\partial \pi_b}\pi_g + \phi + 1\right)(\mathcal{E}\phi + 1) - (\phi \pi_g + \pi_b)\mathcal{E}\frac{\partial \phi}{\partial \pi_b} = 0$$

and hence

$$\frac{\partial \phi}{\partial \pi_b} = -\frac{(\phi + 1)(\mathcal{E}\phi + 1)}{\pi_g(\mathcal{E}\phi + 1) - (\phi\pi_g + \pi_b)\mathcal{E}} = -\frac{(\phi + 1)(\mathcal{E}\phi + 1)}{\pi_g - \mathcal{E}\pi_b}.$$

The derivative of the Sharpe ratio (11) with respect to π_g is

$$\phi^{\frac{1}{2}}(\pi_g - \pi_b)^{-1} + \frac{\partial \phi}{\partial \pi_g}\frac{1}{2}\left(\phi^{-\frac{1}{2}}\pi_g - \phi^{-\frac{3}{2}}\pi_b\right)(\pi_g - \pi_b)^{-1} - \left(\phi^{\frac{1}{2}}\pi_g + \phi^{-\frac{1}{2}}\pi_b\right)(\pi_g - \pi_b)^{-2} \tag{A-9}$$

which has the same sign as (multiplying by $\phi^{\frac{3}{2}}(\pi_g - \pi_b)^2$)

$$\phi^2(\pi_g - \pi_b) - \left(\phi^2\pi_g + \phi\pi_b\right) - \frac{\phi(\mathcal{E}\phi + 1)}{\pi_g - \mathcal{E}\pi_b}\frac{1}{2}(\phi\pi_g - \pi_b)(\pi_g - \pi_b)$$

and hence the same sign as (dividing by $-\pi_b$)

$$\phi(\phi + 1) - \frac{(\mathcal{E}\phi + 1)}{-\frac{\pi_g}{\pi_b} + \mathcal{E}}\frac{1}{2}\left(-\frac{\phi\pi_g}{\pi_b} + 1\right)\left(-\frac{\phi\pi_g}{\pi_b} + \phi\right)$$

and hence the same sign as (multiplying by $\frac{2}{\phi}\left(-\frac{\phi\pi_g}{\pi_b} + \mathcal{E}\phi\right)$)

$$2(\phi + 1)\left(-\frac{\phi\pi_g}{\pi_b} + \mathcal{E}\phi\right) - (\mathcal{E}\phi + 1)\left(-\frac{\phi\pi_g}{\pi_b} + 1\right)\left(-\frac{\phi\pi_g}{\pi_b} + \phi\right). \tag{A-10}$$

From (A-8), $-\frac{\phi\pi_g}{\pi_b} > 1$. To establish the comparative static in π_g I show that (A-10) is negative if $-\frac{\phi\pi_g}{\pi_b} > 1$ and $\mathcal{E} > \frac{1}{9}$. Evaluated at $-\frac{\phi\pi_g}{\pi_b} = 1$, expression (A-10) equals 0. Consider

$$2(\phi + 1)(z + \mathcal{E}\phi) - (\mathcal{E}\phi + 1)(z + 1)(z + \phi). \tag{A-11}$$

This is a concave quadratic in z. So to establish that (A-10) is negative it suffices to show that the derivative of (A-11) with respect to z is negative when evaluated at $z = 1$, i.e.,

$$2(\phi + 1) - (\mathcal{E}\phi + 1)(z + \phi) - (\mathcal{E}\phi + 1)(z + 1)\big|_{z=1} < 0,$$

i.e.,

$$2(\phi + 1) - (\phi\mathcal{E} + 1)(1 + \phi) - 2(\phi\mathcal{E} + 1) < 0,$$

i.e.,

$$-\mathcal{E}\phi^2 - \phi(3\mathcal{E} - 1) - 1 < 0. \tag{A-12}$$

If $\mathcal{E} \geq \frac{1}{3}$ then the proof is complete. If instead $\mathcal{E} \in \left(0, \frac{1}{3}\right)$, any roots of the quadratic in (A-12) are positive. The determinant of this quadratic is

$$(3\mathcal{E} - 1)^2 - 4\mathcal{E} = 1 - 10\mathcal{E} + 9\mathcal{E}^2 = (1 - 9\mathcal{E})(1 - \mathcal{E})$$

which is negative for $\mathcal{E} \in \left(\frac{1}{9}, 1\right)$. Hence for $\mathcal{E} \in \left(\frac{1}{9}, \frac{1}{3}\right)$ the quadratic in (A-12) is negative for all ϕ, completing the proof of the comparative static in π_g.

Finally, the derivative of the Sharpe ratio (11) with respect to π_b, holding $\Delta = \pi_g - \pi_b$

fixed, has the same sign as

$$\phi^{\frac{1}{2}} + \phi^{-\frac{1}{2}} + \frac{1}{2}\frac{\partial \phi}{\partial \pi_b}\left(\phi^{-\frac{1}{2}}\pi_g - \phi^{-\frac{3}{2}}\pi_b\right)$$

which has the same sign as (multiplying by $2\phi^{\frac{3}{2}}$)

$$2\phi\left(\phi+1\right) - \frac{\left(\phi+1\right)\left(\mathcal{E}\phi+1\right)}{\pi_g - \mathcal{E}\pi_b}\left(\phi\pi_g - \pi_b\right)$$

which has the same sign as (multiplying by $\frac{\pi_g - \mathcal{E}\pi_b}{\phi+1}$)

$$2\left(\phi\pi_g - \mathcal{E}\phi\pi_b\right) - \left(\mathcal{E}\phi+1\right)\left(\phi\pi_g - \pi_b\right) = \left(1-\mathcal{E}\phi\right)\left(\phi\pi_g + \pi_b\right). \tag{A-13}$$

By Lemma (2), $\mathcal{E}\phi = \mathcal{E}\frac{Q_g}{Q_b}\frac{p_g}{p_b} < \frac{p_g}{p_b}$. Combined with (A-8) this establishes that (A-13) is positive if $p_g \leq \frac{1}{2}$, thereby completing the proof.

Proof of Lemma 4: From (A-18) and (A-19), as $\epsilon \to 0$ and $n \to \infty$, Q_g approaches the solution to $Q_g^{0,\infty}$

$$\frac{Q_g^{0,\infty} p_g \pi_g}{Q_g^{0,\infty} p_g - p_b \ln\left(1 - Q_g^{0,\infty}\right)} = \kappa.$$

Note that, by assumption ((2)), $Q_g^{0,\infty} \in (0,1)$. Since $\tilde{Q}_g \to 1$ and $\tilde{Q}_b \to 0$ as $\epsilon \to 0$, this establishes that $\tilde{Q}_b < Q_g < \tilde{Q}_g$. It remains to order $\tilde{Q}_b$ relative to Q_b.

To do so, use (10) to write the equilibrium condition (for $n \to \infty$) in terms of Q_b, i.e.,

$$\frac{\left(1 - \left(1-Q_b\right)^{\frac{1-\epsilon}{\epsilon}}\right) p_g \pi_g + Q_b p_b \pi_b}{\left(1 - \left(1-Q_b\right)^{\frac{1-\epsilon}{\epsilon}}\right)\frac{p_g}{1-\epsilon} + Q_b \frac{p_b}{\epsilon}} = \kappa.$$

The LHS is decreasing in Q_b (Lemma (2)). Hence $Q_b > \tilde{Q}_b$ if and only if the LHS evaluated at $Q_b = \epsilon$ exceeds κ. Using

$$\lim_{\epsilon \to 0}\left(1-\epsilon\right)^{\frac{1-\epsilon}{\epsilon}} = e^{-1},$$

as $\epsilon \to 0$ the LHS evaluated at $Q_b = \epsilon$ approaches

$$\frac{\left(1-e^{-1}\right)p_g}{\left(1-e^{-1}\right)p_g + p_b}\pi_g,$$

completing the proof.

Proof of Lemma 5: Note that

$$\lim_{\alpha \to 0} \frac{Q_\omega(\alpha)}{n\alpha} = \Pr(\sigma_i = g|\omega).$$

Hence by assumption(2),

$$\kappa < \lim_{\alpha \to 0} \hat{v}(\alpha). \tag{A-14}$$

Moreover, $Q_\omega(1) \to 1$ as $n \to \infty$. By assumption (1), let n be large enough that

$$\hat{v}(1) < 0. \tag{A-15}$$

Let $\hat{\alpha}$ be an equilibrium investigation probability of the simultaneous move game. By (A-14) and (A-15) it follows that $\hat{\alpha} \in (0,1)$, with

$$\hat{v}(\hat{\alpha}) = \kappa.$$

From (15) it follows that

$$v(\alpha) > \kappa.$$

The result follows from Lemma (2), completing the proof.

Proof of Lemma 6: The equilibrium condition determining γ in the exogenous-signal game is $\check{v}(\gamma) = \check{\kappa}$, which by (16) and (17) can be written as

$$((1-\epsilon_g)p_g + \epsilon_b p_b) \frac{\frac{Q_g(\gamma)}{Q_b(\gamma)} \frac{p_g}{1-\epsilon_g} + \frac{p_b}{\epsilon_b}}{\frac{Q_g(\gamma)}{Q_b(\gamma)} p_g + p_b} v(\gamma) = \kappa,$$

or equivalently as

$$\frac{(\mathcal{E}^{-1}p_g + p_b)\left(\frac{Q_g(\gamma)}{Q_b(\gamma)}\mathcal{E}p_g + p_b\right)}{\frac{Q_g(\gamma)}{Q_b(\gamma)}p_g + p_b} v(\gamma) = \kappa.$$

Since $\frac{Q_g(\gamma)}{Q_b(\gamma)}$ is less than $\mathcal{E}^{-1}$ but exceeds 1, the expression multiplying $v(\gamma)$ lies between 1 and

$$\left(\mathcal{E}^{-1}p_g + p_b\right)(\mathcal{E}p_g + p_b) > 1.$$

It follows that the equilibrium value of γ in the exogenous-signal game exceeds the equilibrium value of α in the endogenous-signal game, completing the proof.

B Additional Appendix: Case, $\epsilon_b = 0$

In this case, $Q_b \equiv 0$,

$$\Pr\left(\text{opportunity available}\right) = \frac{Q_g\left(\alpha\right)p_g}{\left(1-\epsilon_g\right)n\alpha} + p_b,$$

and

$$v\left(\alpha\right) = \frac{Q_g\left(\alpha\right)p_g\pi_g}{\frac{Q_g(\alpha)p_g}{1-\epsilon_g} + p_b n\alpha}.$$

Recall

$$Q_g\left(\alpha\right) = 1 - \left(1 - \left(1-\epsilon_g\right)\alpha\right)^n,$$

and hence

$$1 - \left(1-\epsilon_g\right)\alpha = \left(1-Q_g\right)^{\frac{1}{n}}.$$

So v can be written directly in terms of Q_g as

$$v_0\left(Q_g\right) \equiv \frac{\left(1-\epsilon_g\right)Q_g p_g \pi_g}{Q_g p_g + p_b n\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right)}. \tag{A-16}$$

For $\epsilon_b > 0$, from (A-1),

$$v = \frac{\left(1-\epsilon_g\right)Q_g p_g \pi_g + \left(1-\epsilon_g\right)\left(1-\left(1-\mathcal{E}\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right)\right)^n\right)p_b\pi_b}{Q_g p_g + \left(1-\left(1-\mathcal{E}\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right)\right)^n\right)\mathcal{E}^{-1}p_b}.$$

By l'Hopital's rule,

$$\lim_{\mathcal{E}\to 0}\left(1-\left(1-\mathcal{E}\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right)\right)^n\right)\mathcal{E}^{-1} = n\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right),$$

and so

$$\lim_{\epsilon_b\to 0} v = \frac{\left(1-\epsilon_g\right)Q_g p_g \pi_g}{Q_g p_g + p_b n\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right)}, \tag{A-17}$$

which coincides with (A-16). By the same arguments,

$$\lim_{\epsilon_g=\epsilon_b\to 0} v = \frac{Q_g p_g \pi_g}{Q_g p_g + p_b n\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right)}. \tag{A-18}$$

Note, moreover, that

$$\lim_{n\to\infty} n\left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right) = -\ln\left(1-Q_n\right). \tag{A-19}$$

Note that

$$\lim_{Q_g \to 0} v_0\left(Q_g\right) = \left(1-\epsilon_g\right) p_g \pi_g,$$

which exceeds κ by assumption (2). Moreover, v_0 is decreasing in Q_g by the following result.

Lemma A-2 *The ratio is* $\frac{1-(1-Q_g)^{\frac{1}{n}}}{Q_g}$ *is increasing in* $Q_g \in (0,1)$ *for any* $n \geq 2$*.*

Proof of Lemma (A-2): I establish that

$$\frac{1}{n}\left(1-Q_g\right)^{\frac{1}{n}-1} Q_g - \left(1-\left(1-Q_g\right)^{\frac{1}{n}}\right) > 0.$$

Define

$$f\left(x\right) = \alpha x^{\alpha}\left(1-x\right) - \left(1-x^{\alpha}\right)x.$$

It suffices to show that $f\left(x\right) > 0$ for all $x \in (0,1)$ and $\alpha < 1$. Differentiating,

$$\begin{aligned} f'\left(x\right) &= \alpha^2 x^{\alpha-1}\left(1-x\right) - \alpha x^{\alpha} + \alpha x^{\alpha-1} \cdot x - \left(1-x^{\alpha}\right) \\ &= \alpha^2 x^{\alpha-1} + \left(1-\alpha^2\right)x^{\alpha} - 1 \\ f''\left(x\right) &= \left(\left(\alpha-1\right)\alpha^2 + \alpha\left(1-\alpha^2\right)x\right)x^{\alpha-2}. \end{aligned}$$

Note that $f\left(0\right) = f\left(1\right) = 0$; that $f'\left(x\right) > 0$ for all x sufficiently small while $f'\left(1\right) = 0$; and that f is concave if $x < \frac{\alpha}{1+\alpha}$ and convex if $x > \frac{\alpha}{1+\alpha}$. It follows first that $f\left(x\right) > 0$ for all $x \in [\frac{\alpha}{1+\alpha}, 1)$ and then that $f\left(x\right) > 0$ for all $x \in (0, \frac{\alpha}{1+\alpha}]$ also, completing the proof.

C Additional appendix: Explicit evaluation of (4)

$$
\begin{aligned}
& \Pr\left(\omega|\sigma_i = g, \text{opportunity available}\right) \\
= & \sum_{k=1}^{n} (\Pr\left(\omega|\sigma_i = g, \text{opportunity available, agent } i \text{ is } k^{th} \text{ in line}\right) \\
& \times \Pr\left(\sigma_i = g, \text{opportunity available, agent } i \text{ is } k^{th} \text{ in line}|\sigma_i = g, \text{opportunity available}\right)) \\
= & \sum_{k=1}^{n} (\frac{\Pr\left(\sigma_i = g, \text{opportunity available, agent } i \text{ is } k^{th} \text{ in line}|\omega\right) p_\omega}{\Pr\left(\sigma_i = g, \text{opportunity available, agent } i \text{ is } k^{th} \text{ in line}\right)} \\
& \times \frac{\Pr\left(\sigma_i = g, \text{opportunity available, agent } i \text{ is } k^{th} \text{ in line}\right)}{\Pr\left(\sigma_i = g, \text{opportunity available}\right)}) \\
= & \sum_{k=1}^{n} \frac{\Pr\left(\sigma_i = g|\omega\right) \frac{1}{n} \left(1 - \alpha \Pr\left(\sigma_i = g|\omega\right)\right)^{k-1} p_\omega}{\Pr\left(\sigma_i = g, \text{opportunity available}\right)} \\
= & \frac{\Pr\left(\sigma_i = g|\omega\right) p_\omega}{\Pr\left(\sigma_i = g, \text{opportunity available}\right)} \frac{1 - \left(1 - \alpha \Pr\left(\sigma_i = g|\omega\right)\right)^n}{n\alpha \Pr\left(\sigma_i = g|\omega\right)}.
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
& \Pr\left(\sigma_i = g, \text{opportunity available}|\text{opportunity available}\right) \\
= & \frac{\Pr\left(\sigma_i = g, \text{opportunity available}\right)}{\sum_{\tilde{\omega}} \Pr\left(\text{opportunity available}|\tilde{\omega}\right) p_{\tilde{\omega}}}
\end{aligned}
$$

and

$$
\begin{aligned}
& \Pr\left(\text{opportunity available}|\omega\right) \\
= & \sum_{k=1}^{n} \Pr\left(\text{opportunity available}|\omega, \text{agent } i \text{ is } k^{th} \text{ in line}\right) \Pr\left(\text{agent } i \text{ is } k^{th} \text{ in line}|\omega\right) \\
= & \frac{1}{n} \sum_{k=1}^{n} \left(1 - \alpha \Pr\left(\sigma_i = g|\omega\right)\right)^{k-1} \\
= & \frac{1 - \left(1 - \alpha \Pr\left(\sigma_i = g|\omega\right)\right)^n}{n\alpha \Pr\left(\sigma_i = g|\omega\right)}.
\end{aligned}
$$

Hence expected payoff from investigation is

$$
\begin{aligned}
& \sum_{\omega = g,b} \pi_\omega \Pr\left(\omega|\sigma_i = g, \text{opportunity available}\right) \Pr\left(\sigma_i = g, \text{opportunity available}|\text{opportunity available}\right) \\
= & \sum_{\omega = g,b} \pi_\omega \frac{\Pr\left(\sigma_i = g|\omega\right) p_\omega}{\Pr\left(\sigma_i = g, \text{opportunity available}\right)} \frac{1 - \left(1 - \alpha \Pr\left(\sigma_i = g|\omega\right)\right)^n}{n\alpha \Pr\left(\sigma_i = g|\omega\right)} \frac{\Pr\left(\sigma_i = g, \text{opportunity available}\right.}{\sum_{\tilde{\omega}} \Pr\left(\text{opportunity available}|\tilde{\omega}\right) p}
\end{aligned}
$$

$$= \frac{1}{n\alpha} \frac{\sum_{\omega=g,b} \left(1 - \left(1 - \alpha \Pr\left(\sigma_i = g|\omega\right)\right)^n\right) p_\omega \pi_\omega}{\sum_{\omega=g,b} \frac{1-(1-\alpha\Pr(\sigma_i=g|\omega))^n p_\omega}{n\alpha\Pr(\sigma_i=g|\omega)}}.$$