

Bailouts and the information content of investments*

Philip Bond, University of Washington
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Abstract

I analyze the effect of government bailouts on the information content of investments. I establish conditions under which expectations of future bailouts cause both investment magnitudes and asset prices to become more divorced from the fundamental quality of an investment. This in turn means that each investor extracts less useful information from the investment decisions of other investors, and makes worse investment decisions. In order to establish these results with minimal assumptions on payoff functions and distributions, I derive new results relating to Lehmann's (1988) measure of information content. These results are potentially useful in other applications.

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1 Introduction

A leading explanation for the origin of the 2007-2008 financial crisis is that market participants held incorrect beliefs, especially about the future evolution of housing prices (e.g., Cheng, Raina, Xiong (2014)). In turn, incorrect beliefs can be attributed to a number of causes, ranging from simple bad luck (Biais, Rochet, Woolley (2015)) to more behavioral explanations. However, all such explanations must confront the wisdom of the crowd. That is, a long tradition in economics and finance suggests that market mechanisms effectively aggregate the views of dispersed agents (e.g., Hayek (1945), Roll (1984)). Other things equal, the wisdom of the crowd reduces the likelihood of agents systematically holding incorrect beliefs.

In this paper, I explore the possibility that government bailout policy muted the wisdom of the crowd. Consider, for example, the failure of the prices of either bank bonds or mortgage-backed security (MBS) issues to predict the financial crisis. It is likely that investors viewed these bonds as partially insured by governments.¹ Such an expectation of government bailouts would change bond prices, and in particular, change the amount of information that bond prices contain about the fundamental value of houses. In this paper, I theoretically analyze whether expectations of a bailout increase or decrease the information content of prices and investment decisions.

I show that, in many circumstances, expectations of government bailouts reduce the information content of prices and investment decisions, thereby potentially contributing to mistaken beliefs. The main exception is that an expectation of government bailouts for one type of investment may raise rather than lower the information content of investments not directly affected by the bailout.

Although the role of bailouts as a contributing factor in the financial crisis has been much discussed, the idea that bailouts matter because of their effect on information aggregation has received little attention.² In contrast, much of the literature on bank bailouts has focused on the idea that bailouts engender moral hazard among bank managers.³ One reason to explore the effects of bailouts stemming from non-moral

¹See O'Hara and Shaw (1990) and Gandhi and Lustig (2015) for evidence of investor beliefs about bank bailouts.

²Bond, Goldstein and Prescott (2010) and Bond and Goldstein (2015) both show that if government intervention is responsive to market prices, then this responsiveness can reduce price informativeness. The driving force for these results is the responsiveness of intervention to prices, rather than intervention *per se*.

³A separate idea is that deposit insurance undercuts the disciplining role of demandable deposits.

hazard channels is that there has been at least some skepticism about the importance of moral hazard. For example, Fahlenbrach and Stulz (2011) “find some evidence that banks with CEOs whose incentives were better aligned with the interests of shareholders performed worse and no evidence that they performed better.” Writing in the Financial Times, Kay (2011) argues that “Banks do not think: ‘We can afford to take big risks because the government will help if things go wrong.’ The downside of failure for senior executives and boards is large even if it is not as large as it should be.” Writing about older financial sector crises, Akerlof and Romer (1993) argue against the importance of moral-hazard-induced excessive risk-taking.

The intuition for the effect of bailouts on the information content of investments is well-illustrated by the extreme case in which investors believe a bailout is certain. In this case, the expected return on an investment is unrelated to the fundamental (non-bailout) quality of the investment, and so investment decisions are likewise unrelated to investors’ perceptions of fundamental quality. Consequently, investment decisions cease to convey any information about fundamental quality. In contrast, if investors believe that investments in one type of investment (housing, for example) are certain to be bailed out, this effectively removes risk from their portfolios, and results in their investments in other types of projects becoming more tightly related to fundamental quality.

The extreme case of certain bailouts is very unrealistic, of course. The main results in the paper establish the impact of bailouts on informativeness for the empirically relevant case in which investors are unsure whether a bailout will occur.

To obtain these results, I derive new theoretical results on measuring information content. The best known measure of information content is due to Blackwell (1953). However, Blackwell’s ordering is very incomplete, in the sense of failing to order many situations.⁴ Consequently, I make use of an alternate measure of information content developed by the statistics literature, and due to Lehmann (1988). Whereas Blackwell’s ordering says that one signal is more informative than another if it leads to better decisions in an arbitrary decision problem, Lehmann’s ordering requires only

This point is made in passing by Calomiris and Kahn (1991), and more explicitly by Diamond and Rajan (2001).

⁴See discussion in Lehmann (1988). A well-known example of the weakness of Blackwell’s ordering is the following. Let ε be a normally distributed random variable, and let δ be a non-normal random variable. Then an immediate consequence of Cramer’s decomposition theorem is that the Blackwell ordering is unable to rank the informativeness of $X = \theta + \varepsilon$ and $Y = \theta + \delta$ with respect to θ , even as the variance ratio $\frac{\text{var}(\varepsilon)}{\text{var}(\delta)}$ grows either very large or very small.

better decisions in, roughly speaking, one-dimensional monotone decision problems, i.e., decision problems in which a signal contains information about a one-dimensional underlying state variable, and the optimal full-information decision is monotone in the underlying state variable. Although this class of decision problems encompasses many cases of economic interest, Lehmann’s ordering has received little attention in the economics literature. The main exceptions are Persico (2000), who uses Lehmann’s measure to analyze information acquisition prior to auctions, and Quah and Strulovici (2009), who expand the class of decision problems for which Lehmann’s ordering is useful.

Relative to these papers, I make two main contributions. First, these prior papers treat the signal as exogenous, whereas for many economic applications, including the ones in this paper, the signal arises endogenously from economic actions. But some of the assumptions imposed by prior papers (in particular, an invariant support) are commonly violated when the signal is generated endogenously. I extend prior results to allow for shifting and non-compact supports.

Second, I derive new results to show that Lehmann’s information ordering is equivalent to the condition that a particular order statistic satisfies the single crossing property (SCP, Milgrom and Shannon (1994)). This result then implies that Lehmann’s condition is equivalent to the Spence-Mirrlees single-crossing condition, which is based on the ratio of first-derivatives and can be checked relatively easily in applications.

In the current paper, I use these tools to analyze the effect of bailouts on the information content of economic outcomes. These same tools should prove useful in other contexts also.

2 Models

An uninformed investor is contemplating an investment, the expected payoff of which depends on a state variable $\theta \in \Theta \subset \mathfrak{R}$, where Θ is compact. Throughout, I assume that the expected return is increasing in θ . The investor cannot observe θ directly. Instead, the investor observes a , which is the outcome of economic decisions, as described below. Importantly, although I refer to this investor as “uninformed,” he may possess a high-quality signal of θ —this simply changes the uninformed investors prior before seeing a .

The outcome a also depends on a second random variable t , which is irrelevant to

the uninformed investor's decision, and moreover, is unobserved by the uninformed investor. The random variable t has a continuous distribution, which without loss can be assumed to be uniform over the interval between 0 and 1.

After observing a , the uninformed investor makes an investment decision $b \in B \subset \mathfrak{R}$. The uninformed investor's payoff from investment b in state θ is

$$V(b, \theta).$$

Assumption 1 *V is continuous in b , and satisfies the SCP (Milgrom and Shannon (1994)) in (b, θ) .*

Assumption 2 *There exist $\underline{\theta}, \bar{\theta} \geq \underline{\theta}, \underline{b}$ and \bar{b} such that if $\theta \leq \underline{\theta}$ then $V(\cdot, \theta)$ is weakly decreasing for $b \geq \bar{b}$, and if $\theta \geq \bar{\theta}$ then $V(\cdot, \theta)$ is weakly increasing for $b \leq \underline{b}$.*

Note that Assumption 2 is trivially satisfied if both A and Θ are compact.

Assumption 3 *For any $b'' > b'$, $V(b'', \theta) - V(b', \theta)$ is weakly quasi-concave as a function of θ .*

Assumption 3 is a mild regularity condition, and states that the marginal benefit of choosing action b'' instead of b' is either monotone increasing or decreasing, or else is an increasing then decreasing function of θ . This regularity condition is required to apply Theorem 3 of Athey (2002): see Section 5 below.

2.1 Model 1: Bailouts for similar investments

In Model 1, the uninformed investor observes the investment decision of a second informed investor in a similar investment. For simplicity, this informed investor directly observes θ .

The informed investor is the beneficiary of a government bailout. As an example, consider an investment in bonds issued by a financial institution. Various government actions may reduce the risk of these bonds: for example, the government may effectively insure or the bonds; or it may take steps to ensure that the financial institution repays its bondholders, even if the financial institution's shareholders suffer.

Formally, let r be the gross return on the investment when it succeeds, and assume that the gross return after failure is 0%. The probability of success is $q(\theta, \psi)$, where

ψ is a parameter summarizing bailout policy. Specifically,

$$q(\theta, \psi) = \theta + (1 - \theta)(1 - \psi), \quad (1)$$

that is, absent bailouts there is a probability θ that the investment succeeds, while in the $(1 - \theta)$ -probability event that the project fails, there is a bailout with probability $(1 - \psi)$. Assume that

$$\theta r \geq 1 \text{ for all } \theta \in \Theta, \quad (2)$$

i.e., the investment has a positive rate of return even absent government bailouts.

The informed investor chooses a to maximize utility

$$q(\theta, \psi) u(ar; t) + (1 - q(\theta, \psi)) u(-a; t), \quad (3)$$

where $t \in \mathfrak{R}$ is a characteristic of the informed investor that is both unobserved by the uninformed investor, and irrelevant to the uninformed investor's decision problem. The payoff $u(\cdot; t)$ is continuous with respect to t .

The utility function $u(c; t)$ is a strictly increasing and strictly concave, and satisfies

$$\frac{\partial}{\partial t} \left(\frac{u_{cc}}{u_c} \right) < 0 \quad (4)$$

Note that (4) is satisfied both if

$$u(c; t) = u(W(t) + c),$$

where $W'(t) < 0$ and u has non-increasing absolute risk aversion (NIARA), or if

$$u(c; t) = (1 - t) u(W + c) + t u(W - L + c), \quad (5)$$

where $L < W$ is a loss independent of the investment a , and u has decreasing absolute risk aversion (DARA).⁵

⁵To see this, note that

$$\frac{u_{cc}}{u_c} = \frac{(1 - t) u_{cc}(W + c) + t u_{cc}(W - L + c)}{(1 - t) u_c(W + c) + t u_c(W - L + c)}.$$

2.2 Model 2: Bailouts for unrelated investments

Model 2 is very similar to Model 1 with specification (5), except that now government bailouts affect the dimension of risk that the uninformed investor does not care about. That is, the informed investor has an investment unrelated to θ that results in wealth W if it succeeds and $W - L$ if it fails. The success probability of this investment is $q(t, \psi)$, given by

$$q(t, \psi) = 1 - p(t) + p(t)\psi,$$

that is, absent bailouts there is a probability $1 - p(t)$ that the investment succeeds, while in the $p(t)$ -probability event that the project fails, there is a bailout with probability ψ . The success probability of the informed investor's investment a that is related to θ is simply θ . Hence the informed investor chooses a to maximize

$$\begin{aligned} & \theta q u(W + ar) + \theta(1 - q) u(W - L + ar) \\ & + (1 - \theta) q u(W - a) + (1 - \theta)(1 - q) u(W - L - a), \end{aligned} \quad (6)$$

where the utility function u has DARA. Assume that (2) holds, so that the investment related to θ has a positive rate of return.

Hence the sign of $\frac{\partial}{\partial t} \left(\frac{u_{cc}}{u_c} \right)$ coincides with the sign of

$$\begin{aligned} & (u_{cc}(W - L + c) - u_{cc}(W + c))((1 - t)u_c(W + c) + tu_c(W - L + c)) \\ & - (u_c(W - L + c) - u_c(W + c))((1 - t)u_{cc}(W + c) + tu_{cc}(W - L + c)) \end{aligned}$$

which equals

$$\begin{aligned} & (1 - t)u_c(W + c)u_{cc}(W - L + c) + tu_c(W - L + c)u_{cc}(W - L + c) \\ & - (1 - t)u_c(W + c)u_{cc}(W + c) - tu_c(W - L + c)u_{cc}(W + c) \\ & - (1 - t)u_c(W - L + c)u_{cc}(W + c) - tu_c(W - L + c)u_{cc}(W - L + c) \\ & + (1 - t)u_c(W + c)u_{cc}(W + c) + tu_c(W + c)u_{cc}(W - L + c) \end{aligned}$$

which in turn equals

$$u_c(W + c)u_{cc}(W - L + c) - u_c(W - L + c)u_{cc}(W + c),$$

which has the same sign as

$$-\frac{u_{cc}(W + c)}{u_c(W + c)} - \left(-\frac{u_{cc}(W - L + c)}{u_c(W - L + c)} \right),$$

which is negative by DARA and $L > 0$.

2.3 Model 3: Bailouts for similar investments, uninformed investor observes prices not quantities

In Models 1 and 2, the uninformed investor observes the magnitude of the informed investor's investment. In Model 3, the uninformed investor instead observes the price informed investors pay to access the investment.

Specifically, suppose now that there are multiple informed investors. Each informed investor buys x units of a bond that has a price a . The bonds pay r if an underlying project succeeds, and 0 otherwise. As in Model 1, the success probability is $q(\theta, \psi)$, given by (1). Each informed investor acts as a price-taker, and so chooses x to maximize

$$qu(W + x(r - a)) + (1 - q)u(W - xa), \quad (7)$$

where u is a utility function exhibiting constant relative risk aversion (CRRA) of $\gamma > 0$. Let $x(a, \theta, \psi)$ be the demand of an informed investor when the price is a . There is a mass α of informed investors. The total quantity of bonds available to buy is $s(t)$, where $s(\cdot)$ is an increasing and differentiable function with $s(0) = 0$ and $s(1) = \bar{s}$. Hence, as in Grossman and Stiglitz (1980) and Hellwig (1980), the supply of the financial asset is random (and not directly observed by any party).⁶

The equilibrium price is hence a function $\eta(\theta, t; \psi)$ that solves the market-clearing condition

$$\alpha x(\eta(\theta, t; \psi), \theta, \psi) = s(t). \quad (8)$$

3 Lehmann informativeness

In all models described, the uninformed investor observes $a \in \mathfrak{R}$. The uninformed investor is interested in a because the observation of a provides information about the decision-relevant state variable θ .

Lehmann (1988) defines a partial ordering that captures the informativeness of a about θ . Lehmann's ordering is defined using the inverse of the distribution function, and so implicitly makes assumptions on the support of a conditional on θ . Quah and Strulovici (2009) are explicit in their assumptions, and in particular, assume that

⁶In contrast to these papers, and much of the subsequent literature, I do not assume that the asset supply is normally distributed. For recent work on relaxing normality assumptions in models of this type, see Breon-Drish (2015) and Albagli, Hellwig and Tsyvinski (2015).

the support of a is compact, and independent of both the state θ and the regime ψ . The statement of Lehmann's ordering below generalizes the assumptions of Quah and Strulovici.

Let $F(\cdot|\theta; \psi)$ be the distribution function for a in state θ in bailout regime $\psi \in \Psi \subset \mathfrak{R}$, i.e.,

$$F(z|\theta; \psi) = \Pr(a \leq z|\theta; \psi).$$

Let $A(\theta; \psi)$ be the support of a in state θ in regime ψ , and define $A(\psi) \equiv \bigcup_{\theta \in \Theta} A(\theta; \psi)$. For any $a_0 \in A(\psi)$, define $\Theta(a_0; \psi)$ as the set of states such that a_0 lies in the support of a_0 , i.e., $\Theta(a_0; \psi) = \{\tilde{\theta} : a_0 \in A(\tilde{\theta}; \psi)\}$.

Property 1 *For all states θ and regimes ψ , the distribution function $F(\cdot|\theta; \psi)$ is continuous and strictly increasing over $A(\theta; \psi)$, with $\inf_{a \in A(\theta; \psi)} F(a|\theta; \psi) = 0$ and $\sup_{a \in A(\theta; \psi)} F(a|\theta; \psi) = 1$.*

Property 2 *If for some θ the support $A(\theta; \psi)$ is unbounded above (respectively, below) for some ψ , the same is true for any $\tilde{\psi} \neq \psi$.*

For the remainder of this section assume that Properties 1 and 2 hold.

Consider any pair of regimes, ψ' and ψ'' , and any $\theta \in \Theta$. Given Properties 1 and 2, define $S(\cdot, \theta; \psi', \psi'') : A(\theta; \psi'') \rightarrow A(\theta; \psi')$ by

$$F(S(a, \theta; \psi', \psi'')|\theta; \psi') = F(a|\theta; \psi'').$$

Lehmann's ordering is defined by:

Definition 1 *a is a more Lehmann-informative in regime ψ'' than ψ' if for all $a'' \in A(\psi'')$, the function $S(a'', \theta; \psi', \psi'')$ is weakly decreasing in $\theta \in \Theta(a''; \psi'')$.*

The following result, which represents a generalization of results in Lehmann (1988) and Quah and Strulovici (2009), establishes that an increase in Lehmann-informativeness indeed leads to better decisions.

Proposition 1 *Suppose a is a more Lehmann-informative in regime ψ'' than ψ' , and $\zeta : A(\psi') \rightarrow B$ is a weakly increasing function. Let a' and a'' be the random variables arising in regimes ψ' and ψ'' respectively. Then there exists $\phi : A(\psi'') \rightarrow B$ such that, for all θ , $V(\phi(a''), \theta)$ first-order stochastically dominates $V(\zeta(a'), \theta)$.*

As Quah and Strulovici (2009) observe, Lehmann-informativeness implies an improvement in the uninformed investor's payoff in a very robust sense, in that Proposition 1 is completely independent of the uninformed investor's prior beliefs of θ .

Given Property 1, for all $t \in (0, 1)$ the inverse $F^{-1}(1 - t|\theta; \psi)$ is uniquely defined. In addition, define $F^{-1}(0|\theta; \psi) = \inf A(\theta; \psi)$ and $F^{-1}(1|\theta; \psi) = \sup A(\theta; \psi)$, with the understanding that if $A(\theta; \psi)$ is unbounded below (respectively, above) then $\inf A(\theta; \psi) = -\infty$ (respectively, $\sup A(\theta; \psi) = \infty$).

The next result relates Lehmann informativeness to whether $F^{-1}(1 - t|\theta; \psi)$ satisfies the SCP. Note that $F^{-1}(1 - t|\theta; \psi)$ is simply the $(1 - t)$ -percentile of the distribution of a given state θ and regime ψ .

Proposition 2 *Suppose that, for any regime ψ and $\theta'' > \theta'$, the distribution of a given θ'' first-order stochastically dominates the distribution of a given θ' . Then the Lehmann-informativeness of a is increasing in the regime ψ if and only if $F^{-1}(1 - t|\theta; \psi)$ satisfies the SCP in $((\theta, t); \psi)$, where $\Theta \times [0, 1]$ has the product ordering.*

The first-order stochastic dominance condition of Proposition 2 can be checked directly in terms $F^{-1}(1 - t|\theta; \psi)$:

Lemma 1 *Fix ψ , θ' and $\theta'' \geq \theta'$. Then the distribution of a given θ'' first-order stochastically dominates the distribution of a given θ' if and only if*

$$F^{-1}(1 - t|\theta''; \psi) \geq F^{-1}(1 - t|\theta'; \psi) \text{ for all } t. \quad (9)$$

To check whether $F^{-1}(1 - t|\theta; \psi)$ satisfies the SCP, it is useful to relate it to the Spence-Mirrlees single-crossing condition, which is expressed in terms of derivatives. Milgrom and Shannon's (1994) Theorem 3 establishes the equivalence (under certain conditions) between the Spence-Mirrlees condition and the SCP under the lexicographic ordering. Under Property 1, $F^{-1}(1 - t|\theta; \psi)$ is strictly decreasing in t , and under first-order stochastic dominance, $F^{-1}(1 - t|\theta; \psi)$ is weakly increasing in θ (see Lemma 1). Under these conditions, the SCP under the product ordering coincides with the SCP under the lexicographic ordering, which in turn coincides with the Spence-Mirrlees condition.

Proposition 3 *Suppose that, for any regime ψ and $\theta'' > \theta'$, the distribution of a given θ'' first-order stochastically dominates the distribution of a given θ' . Then:*

(I) $F^{-1}(1 - t|\theta; \psi)$ satisfies the SCP in $((\theta, t); \psi)$, where $\Theta \times [0, 1]$ has the product ordering, if and only if $F^{-1}(1 - t|\theta; \psi)$ satisfies the SCP in $((\theta, t); \psi)$, where $\Theta \times [0, 1]$ has the lexicographic ordering,

(II) If, moreover, $F^{-1}(1 - t|\theta; \psi)$ is differentiable with respect to θ and t , with derivatives continuous in (θ, t, ψ) , then the Lehmann-informativeness of a is increasing in the regime ψ if and only if $F^{-1}(1 - t|\theta; \psi)$ satisfies the Spence-Mirrlees single-crossing condition, i.e.,

$$\frac{\frac{\partial}{\partial \theta} F^{-1}(1 - t|\theta; \psi)}{\left| \frac{\partial}{\partial t} F^{-1}(1 - t|\theta; \psi) \right|} \text{ is increasing in } \psi. \quad (10)$$

4 The effect of bailouts on Lehmann-informativeness

I next use the tools developed in Section 3 to the three models of Section 2. In particular, I use Proposition 3 to assess the impact of bailouts on the information content of investments.

4.1 Model 1: Bailouts for similar investments reduce Lehmann-informativeness

The first-order condition (FOC) corresponding to the informed investor's maximization problem (3) is

$$qr u_c(ar; t) - (1 - q) u_c(-a; t) = 0. \quad (11)$$

Write $\eta(\theta, t; \psi)$ for the solution to (11), and note that, by (2), $\eta(\theta, t; \psi) \geq 0$. Hence

$$\eta_t(\theta, t; \psi) = -\frac{qr u_{ct}(\eta r; t) - (1 - q) u_{ct}(-\eta; t)}{qr^2 u_{cc}(\eta r; t) + (1 - q) u_{cc}(-\eta; t)} = -\frac{qr u_c(\eta r; t) \left(\frac{u_{ct}(\eta r; t)}{u_c(\eta r; t)} - \frac{u_{ct}(-\eta; t)}{u_c(-\eta; t)} \right)}{qr^2 u_{cc}(\eta r; t) + (1 - q) u_{cc}(-\eta; t)}. \quad (12)$$

Note that (4) is equivalent to $\frac{\partial^2}{\partial t \partial c} \ln u_c < 0$, and hence also equivalent to

$$\frac{\partial}{\partial c} \left(\frac{u_{ct}}{u_c} \right) < 0. \quad (13)$$

Hence (using $\eta \geq 0$)

$$\frac{u_{ct}(\eta r; t)}{u_c(\eta r; t)} < \frac{u_{ct}(-\eta; t)}{u_c(-\eta; t)}, \quad (14)$$

and so expression (12) is strictly negative, that is, the informed investor's investment a is decreasing in t . Accordingly, the $(1 - t)$ -percentile of the distribution of a , $F^{-1}(1 - t | \theta; \psi)$, coincides with $\eta(\theta, t; \psi)$. Evaluating the Spence-Mirrlees condition (10) and applying Proposition 3 gives:

Proposition 4 *In Model 1, bailouts reduce the Lehmann-informativeness of a .*

This is a case in which the support of the informed investor's action naturally varies with θ and ψ . In particular, consider specification (5). In this case, the support of the informed investor's investment a is the interval between

$$\arg \max_{\tilde{a}} q(\theta, \psi) u(W - L + \tilde{a}r) + (1 - q(\theta, \psi)) u(W - L - \tilde{a})$$

and

$$\arg \max_{\tilde{a}} q(\theta, \psi) u(W + \tilde{a}r) + (1 - q(\theta, \psi)) u(W - \tilde{a}),$$

and both quantities depend on θ and ψ .

4.2 Model 2: Bailouts for unrelated investments increase Lehmann-informativeness

The FOC corresponding to the informed investor's maximization problem (3) is

$$\frac{qu'(W - a) + (1 - q)u'(W - L - a)}{qu'(W + ar) + (1 - q)u'(W - L + ar)} = \frac{\theta r}{1 - \theta}. \quad (15)$$

Write $\eta(\theta, t; \psi)$ for the solution to (15), and note that, by (2), $\eta(\theta, t; \psi) \geq 0$.

I first show that the informed investor's investment η is decreasing in t , that is, decreasing in the investor's risk of failure on his other investment. Denote the LHS of (15) by $g(a, q)$. So $g_a \eta_t = -q_t g_q$. Since $g_a > 0$ and $q_t < 0$, η_t has the same sign as g_q , which in turn has the same sign as

$$\begin{aligned} & (u'(W - \eta) - u'(W - L - \eta))(qu'(W + \eta r) + (1 - q)u'(W - L + \eta r)) \\ & - (qu'(W - \eta) + (1 - q)u'(W - L - \eta))(u'(W + \eta r) - u'(W - L + \eta r)) \\ & = qu'(W - \eta)u'(W + \eta r) + (1 - q)u'(W - \eta)u'(W - L + \eta r) \\ & - qu'(W - L - \eta)u'(W + \eta r) - (1 - q)u'(W - L - \eta)u'(W - L + \eta r) \\ & - qu'(W - \eta)u'(W + \eta r) + qu'(W - \eta)u'(W - L + \eta r) \end{aligned}$$

$$\begin{aligned}
& - (1 - q) u'(W - L - \eta) u'(W + \eta r) + (1 - q) u'(W - L - \eta) u'(W - L + \eta r) \\
& = u'(W - \eta) u'(W - L + \eta r) - u'(W - L - \eta) u'(W + \eta r),
\end{aligned}$$

which in turn has the same sign as

$$\frac{u'(W - L + \eta r)}{u'(W - L - \eta)} - \frac{u'(W + \eta r)}{u'(W - \eta)}.$$

This is negative since $\eta \geq 0$ and $\frac{\partial}{\partial x} \frac{u'(x + \eta r)}{u'(x - \eta)} > 0$ (because, by DARA, $\frac{u''(x + \eta r)}{u'(x + \eta r)} - \frac{u''(x - \eta)}{u'(x - \eta)} > 0$).

Since the informed investor's investment a is decreasing in t , the $(1 - t)$ -percentile of the distribution of a , $F^{-1}(1 - t|\theta; \psi)$, coincides with $\eta(\theta, t; \psi)$. Evaluating the Spence-Mirrlees condition (10) and applying Proposition 3 gives:

Proposition 5 *In Model 2, bailouts increase the Lehmann-informativeness of a .*

Again, this is a case in which the support of the informed investor's action naturally varies with θ and ψ . In particular, consider specification (5). In this case, the support of the informed investor's investment a is the interval between

$$\begin{aligned}
& \arg \max_{\tilde{a}} \theta \psi u(W + \tilde{a}r) + \theta(1 - \psi)u(W - L + \tilde{a}r) \\
& + (1 - \theta)\psi u(W - \tilde{a}) + (1 - \theta)(1 - \psi)u(W - L - \tilde{a}),
\end{aligned}$$

and

$$\arg \max_{\tilde{a}} \theta u(W + \tilde{a}r) + (1 - \theta)u(W - \tilde{a}),$$

and both quantities depend on θ , while the first depends on ψ also.

4.3 Model 3: Bailouts for similar investments reduce the Lehmann-informativeness of prices

The FOC for the informed investor is

$$q(r - a)u'(W + x(r - a)) - (1 - q)au'(W - xa) = 0. \tag{16}$$

Since $q_\psi < 0$, it follows immediately that $x_\psi < 0$. Note that as $a \rightarrow 0$, $x(a, \theta, \psi) \rightarrow \infty$, and $x(a = qr, \theta, \psi) = 0$. So for any t , by continuity the market clearing condition

$$\alpha x(a, \theta, \psi) = s(t)$$

has a solution, denoted $\eta(\theta, t; \psi)$.

Moreover, x_a satisfies

$$\begin{aligned} & \left(q(r-a)^2 u''(W+x(r-a)) + (1-q)a^2 u''(W-xa) \right) x_a \\ &= qu'(W+x(r-a)) + (1-q)u'(W-xa) \\ &+ q(r-a)xu''(W+x(r-a)) - (1-q)axu''(W-xa). \end{aligned} \quad (17)$$

Note that $x_a < 0$ if $-(r-a)x \frac{u''(W+x(r-a))}{u'(W+x(r-a))} < 1$, i.e., $\gamma < \frac{W+x(r-a)}{x(r-a)}$, for which a sufficient condition is that $\gamma < \frac{W}{xr} + 1$.⁷ I focus on the case in which the maximum supply \bar{s} is sufficiently small that

$$\gamma < \frac{\alpha W}{\bar{s}r} + 1.$$

Since at the market clearing price $x = \frac{s(t)}{\alpha} \leq \frac{\bar{s}}{\alpha}$, it follows that

$$x_a(\eta(\theta, t; \psi), \theta, \psi) < 0.$$

From this, it follows that the price $\eta(\theta, t; \psi)$ is decreasing in the supply of bonds $s(t)$, i.e., decreasing in t . Hence the $(1-t)$ -percentile of the distribution of price a , $F^{-1}(1-t|\theta; \psi)$, coincides with $\eta(\theta, t; \psi)$.

From the market-clearing condition (8),

$$\begin{aligned} \eta_t x_a &= \alpha^{-1} s_t(t) \\ \eta_\theta x_a &= -x_\theta \\ \eta_\psi x_a &= -x_\psi. \end{aligned}$$

Hence

$$\frac{\eta_\theta}{|\eta_t|} = \alpha \frac{x_\theta}{s_t(t)},$$

⁷Note that this result holds independently of \bar{s} when relative risk aversion is $\gamma \leq 1$.

and so (using $x_\psi < 0$)

$$\begin{aligned}
\text{sign} \left(\frac{\partial}{\partial \psi} \left| \frac{\eta_\theta}{\eta_t} \right| \right) &= \text{sign} \left(\frac{\partial}{\partial \psi} x_\theta \right) = \text{sign} (\eta_\psi x_{\theta a} + x_{\theta \psi}) \\
&= \text{sign} \left(-\frac{x_\psi}{x_a} x_{\theta a} + x_{\theta \psi} \right) = \text{sign} \left(\frac{x_{\theta a}}{x_a} - \frac{x_{\theta \psi}}{x_\psi} \right). \\
&= \text{sign} \left(\frac{\partial}{\partial \theta} (\ln(-x_a) - \ln(-x_\psi)) \right) = \text{sign} \left(\frac{\partial}{\partial \theta} \left(\frac{x_a}{x_\psi} \right) \right).
\end{aligned}$$

Proposition 6 *In Model 3, bailouts reduce the Lehmann-informativeness of a .*

Again, the support of the price varies with θ and ψ ; in particular, the maximum price is $q(\theta, \psi)r$.

5 Checking the assumption of Proposition 1 that the decision function is increasing in a

Proposition 1 is predicated on the uninformed investor's decision ζ being (weakly) increasing in his observation of a . Lehmann (1988) and Quah and Strulovici (2009) both justify this condition by assuming that a satisfies the the monotone likelihood ratio (MLR) property, i.e., for any $\theta'' \geq \theta'$, the ratio is $\frac{\frac{\partial}{\partial a} F(a|\theta''); \psi}{\frac{\partial}{\partial a} F(a|\theta'); \psi}$ increasing in a . Note that in these papers the distribution of a is exogenous, and so the MLR property can be simply imposed as an assumption.

Despite its prevalence in economic analysis, verifying that an endogenous outcome a satisfies the MLR property is non-trivial. To circumvent this problem, I make use of Athey's (2002) Theorem 3, which establishes that the informed investor's decision ζ is increasing in a if a satisfies the monotone probability ratio (MPR) property, i.e., $F(\cdot|\cdot; \psi)$ is log-supermodular, i.e., for any $a'' \geq a'$ and $\theta'' \geq \theta'$, $F(a''|\theta''); \psi) F(a'|\theta'); \psi) \geq F(a'|\theta''); \psi) F(a''|\theta'); \psi)$.⁸ The MPR property is necessary but not sufficient for the MLR property (Eeckhoudt and Gollier (1995)).

Just as it is often most convenient in applications to check Lehmann informativeness using derivatives of the $(1-t)$ -percentile $F^{-1}(1-t|\theta; \psi)$, it is likewise convenient to relate the MPR property to these derivatives:

⁸Note that provided $F(a'|\theta'); \psi) > 0$, this inequality is equivalent to $\frac{F(a''|\theta''); \psi)}{F(a''|\theta'); \psi)} \geq \frac{F(a'|\theta''); \psi)}{F(a'|\theta'); \psi)}$.

Lemma 2 *Suppose that, for any regime ψ and $\theta'' > \theta'$, the distribution of a given θ'' first-order stochastically dominates the distribution of a given θ' . Then (assuming sufficient differentiability), the MPR property is equivalent to: for all $t \in [0, 1]$,*

$$\frac{\partial}{\partial \theta} F^{-1}(1 - t|\theta) = 0 \text{ or } (1 - t) \frac{\partial}{\partial t} \ln \left(\frac{\frac{\partial}{\partial \theta} F^{-1}(1 - t|\theta)}{\left| \frac{\partial}{\partial t} F^{-1}(1 - t|\theta) \right|} \right) + 1 \geq 0. \quad (18)$$

The cost of replacing the MPR property is that one needs more conditions on the payoff function V . This is the role of the regularity condition Assumption 3, which is imposed in Athey’s (2002) Theorem 3.

6 Conclusion

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A Appendix: Proofs (excluding Proposition 1)

Proof of Lemma 1:

Necessity of (9): Suppose to the contrary that for some t , $F^{-1}(1 - t|\theta'; \psi) > F^{-1}(1 - t|\theta''; \psi)$. If $1 - t \neq 1$, first-order stochastic dominance and Property 1 imply

$$F\left(F^{-1}(1 - t|\theta'; \psi) | \theta'; \psi\right) \geq F\left(F^{-1}(1 - t|\theta'; \psi) | \theta''; \psi\right) > F\left(F^{-1}(1 - t|\theta''; \psi) | \theta''; \psi\right),$$

i.e., $1 - t > 1 - t$, a contradiction. Similarly, if $1 - t \neq 0$, first-order stochastic dominance and Property 1 imply

$$F\left(F^{-1}(1 - t|\theta'; \psi) | \theta'; \psi\right) > F\left(F^{-1}(1 - t|\theta''; \psi) | \theta'; \psi\right) \geq F\left(F^{-1}(1 - t|\theta''; \psi) | \theta''; \psi\right),$$

which is again a contradiction.

Sufficiency of (9): Suppose to the contrary that there exists z such that $F(z|\theta''; \psi) > F(z|\theta'; \psi)$. By Property 1, $F^{-1}(1 - t|\theta''; \psi)$ is strictly decreasing in t , and combined with (9), this implies

$$\begin{aligned} F^{-1}(1 - (1 - F(z|\theta''; \psi)) | \theta''; \psi) &> F^{-1}(1 - (1 - F(z|\theta'; \psi)) | \theta''; \psi) \\ &\geq F^{-1}(1 - (1 - F(z|\theta'; \psi)) | \theta'; \psi). \end{aligned}$$

and hence $z > z$, a contradiction, completing the proof.

Proof of Proposition 2:

SCP implies Lehmann-informativeness: Fix $\psi', \psi'' > \psi'$, $a'' \in A(\psi'')$ and $\theta', \theta'' \in \Theta(a''; \psi'')$. Let t' and t'' be such that

$$F^{-1}(1 - t'|\theta'; \psi'') = F^{-1}(1 - t''|\theta''; \psi'') = a''. \quad (19)$$

Hence

$$\begin{aligned} F(a''|\theta'; \psi'') &= 1 - t' \\ F(a''|\theta''; \psi'') &= 1 - t''. \end{aligned}$$

From these equations it follows both that (using first-order stochastic dominance)

$t'' \geq t'$, and that

$$\begin{aligned} F\left(F^{-1}(1-t'|\theta';\psi')|\theta';\psi'\right) &= 1-t' = F(a''|\theta';\psi'') \\ F\left(F^{-1}(1-t''|\theta'';\psi')|\theta'';\psi'\right) &= 1-t'' = F(a''|\theta'';\psi''). \end{aligned}$$

Hence

$$\begin{aligned} S(a'',\theta';\psi',\psi'') &= F^{-1}(1-t'|\theta';\psi') \\ S(a'',\theta'';\psi',\psi'') &= F^{-1}(1-t''|\theta'';\psi'). \end{aligned}$$

Equality (19) and the SCP then imply

$$F^{-1}(1-t''|\theta'';\psi') \leq F^{-1}(1-t'|\theta';\psi'),$$

establishing the result.

Lehmann-informativeness implies SCP:

Suppose that, contrary to the claimed result, the SCP is violated, i.e., there exist $t', t'' \geq t', \theta', \theta'' \geq \theta', \psi'$ and $\psi'' \geq \psi'$ such that either

$$\begin{aligned} F^{-1}(1-t''|\theta'';\psi') &= F^{-1}(1-t'|\theta';\psi') \\ F^{-1}(1-t''|\theta'';\psi'') &< F^{-1}(1-t'|\theta';\psi''), \end{aligned}$$

or

$$\begin{aligned} F^{-1}(1-t''|\theta'';\psi') &> F^{-1}(1-t'|\theta';\psi') \\ F^{-1}(1-t''|\theta'';\psi'') &\leq F^{-1}(1-t'|\theta';\psi''). \end{aligned}$$

From Lemma 1,

$$F^{-1}(1-t'|\theta';\psi'') \leq F^{-1}(1-t'|\theta'';\psi'').$$

It follows from Property 1 that there exists $t''' \in [t', t'']$ such that

$$F^{-1}(1-t'''|\theta'';\psi') > F^{-1}(1-t'|\theta';\psi') \tag{20}$$

$$F^{-1}(1-t'''|\theta'';\psi'') = F^{-1}(1-t'|\theta';\psi''). \tag{21}$$

Let $a'' = F^{-1}(1-t'|\theta';\psi'')$. Note that, by Property 2, $a' \notin \{-\infty, \infty\}$, and hence

$a'' \in A(\theta'; \psi''), A(\theta''; \psi'')$. From (21) and the definition of a''

$$\begin{aligned} F(a''|\theta'; \psi'') &= 1 - t' \\ F(a''|\theta''; \psi'') &= 1 - t'''. \end{aligned}$$

By an identical argument to that used in the first half of the proof,

$$\begin{aligned} S(a'', \theta'; \psi', \psi'') &= F^{-1}(1 - t'|\theta'; \psi') \\ S(a'', \theta''; \psi', \psi'') &= F^{-1}(1 - t'''|\theta''; \psi'). \end{aligned}$$

So the Lehmann-informativeness condition implies

$$F^{-1}(1 - t'''|\theta''; \psi') \leq F^{-1}(1 - t'|\theta'; \psi'),$$

contradicting (20) and completing the proof.

Proof of Proposition 3:

Part (I): If (θ'', t'') exceeds (θ', t') under the product order, it does so under the lexicographic order also. As such, it is immediate that if $F^{-1}(1 - t|\theta; \psi)$ satisfies the SCP under the lexicographic order, it does so under the product order also. To establish the opposite implication, suppose that $F^{-1}(1 - t|\theta; \psi)$ satisfies the SCP under the product order, but that, contrary to the claimed result, there exists (θ', t') and (θ'', t'') with (θ'', t'') greater than (θ', t') under the lexicographic order, and such that either $F^{-1}(1 - t''|\theta''; \psi') \geq F^{-1}(1 - t'|\theta'; \psi')$ and $F^{-1}(1 - t''|\theta''; \psi'') < F^{-1}(1 - t'|\theta'; \psi'')$, or $F^{-1}(1 - t''|\theta''; \psi') > F^{-1}(1 - t'|\theta'; \psi')$ and $F^{-1}(1 - t''|\theta''; \psi'') \leq F^{-1}(1 - t'|\theta'; \psi'')$. If (θ'', t'') is greater than (θ', t') under the product order, there is an immediate contradiction. If instead $\theta'' > \theta'$ and $t'' < t'$ then since F^{-1} is increasing in θ (by first-order stochastic dominance and Lemma 1), it follows that $F^{-1}(1 - t''|\theta''; \psi') \geq F^{-1}(1 - t''|\theta'; \psi')$, while since $F^{-1}(1 - t|\cdot; \cdot)$ is strictly decreasing in t (Property 1), it follows that $F^{-1}(1 - t''|\theta''; \psi'') \leq F^{-1}(1 - t'|\theta'; \psi'') < F^{-1}(1 - t''|\theta'; \psi'')$. Since (θ'', t'') is greater than (θ', t') under the product ordering, this contradicts the SCP under the product ordering, and thereby completes the proof.

Part (II): As noted in the main text, Part (II) is an application of Milgrom and Shannon's (1994) Theorem 3. To apply this result it is necessary to verify the condition that $F^{-1}(1 - t|\theta; \psi)$ is completely regular, which, given that F^{-1} is weakly

increasing in θ , is equivalent to checking that if

$$F^{-1}(1 - t'|\theta'; \psi) = F^{-1}(1 - t''|\theta''; \psi)$$

for some $\theta'' > \theta'$, then for any $\theta \in (\theta', \theta'')$ there exists $t(\theta)$ continuous in θ such that

$$F^{-1}(1 - t(\theta)|\theta; \psi) = F^{-1}(1 - t'|\theta'; \psi). \quad (22)$$

This condition is indeed satisfied since

$$F^{-1}(1 - t'|\theta; \psi) \geq F^{-1}(1 - t'|\theta'; \psi) = F^{-1}(1 - t''|\theta''; \psi) \geq F^{-1}(1 - t''|\theta; \psi),$$

and hence (by continuity) there exists a unique $t(\theta)$ such (22) holds. Continuity follows since F^{-1} is continuous in (θ, t, ψ) .

Proof of Proposition 4: Differentiating (11) gives

$$\begin{aligned} \eta_\theta &= - \frac{q_\theta (ru_c(\eta r; t) + u_c(-\eta; t))}{qr^2u_{cc}(\eta r; t) + (1 - q)u_{cc}(-\eta; t)} \\ &= - \frac{q_\theta ru_c(\eta r; t) \left(1 + \frac{q}{1 - q}\right)}{qr^2u_{cc}(\eta r; t) + (1 - q)u_{cc}(-\eta; t)}, \end{aligned}$$

while η_t is given by (12). Recall $\eta_t < 0$. Hence

$$\frac{\eta_\theta}{|\eta_t|} = - \frac{\eta_\theta}{\eta_t} = \frac{q_\theta}{(1 - q)q} \frac{1}{\frac{u_{ct}(-\eta; t)}{u_c(-\eta; t)} - \frac{u_{ct}(\eta r; t)}{u_c(\eta r; t)}}.$$

Moreover, from $q_\theta > 0$ it follows that $\eta_\theta > 0$. Also, from $q_\psi < 0$ and (11) it follows that $\eta_\psi < 0$.

From $\eta_\psi < 0$ and (13), it follows that $\frac{\partial}{\partial \psi} \frac{u_{ct}(-\eta; t)}{u_c(-\eta; t)} < 0$ and $\frac{\partial}{\partial \psi} \frac{u_{ct}(\eta r; t)}{u_c(\eta r; t)} > 0$. Moreover,

$$\frac{\partial}{\partial \psi} \left(\frac{q_\theta}{(1 - q)q} \right) = \frac{\partial}{\partial \psi} \left(\frac{\psi}{(1 - \theta)\psi(1 - (1 - \theta)\psi)} \right) > 0 \quad (23)$$

Hence $\eta_\theta/|\eta_t|$ is increasing in ψ , which by Proposition 3 completes the proof.

Proof of Proposition 5: From (15), $g_\alpha \eta_\theta = \frac{\partial}{\partial \theta} \left(\frac{\theta r}{1 - \theta} \right)$ and $g_\alpha \eta_t = -q_t g_q$. Since

$\eta_t < 0$, $\eta_\theta/|\eta_t| = \frac{\partial}{\partial\theta} \left(\frac{\theta r}{1-\theta} \right) / q_t g_q$, and so $\frac{\partial}{\partial\psi} (\eta_\theta/|\eta_t|)$ has the same sign as $-\frac{\partial}{\partial\psi} (q_t g_q)$,

$$-(q_{t\psi} g_q + q_t (g_{qq} q_\psi + g_{qa} \eta_\psi)) = -q_{t\psi} g_q - q_t \left(g_{qq} q_\psi - g_{qa} \frac{q_\psi g_q}{g_a} \right),$$

where the equality follows from the implication of (15) that $g_a \eta_\psi = -q_\psi g_q$. Since $g_q < 0$, it follows that $\frac{\partial}{\partial\psi} (\eta_\theta/|\eta_t|)$ has the same sign as

$$q_{t\psi} + q_t q_\psi \left(\frac{g_{qq}}{g_q} - \frac{g_{qa}}{g_a} \right) = q_{t\psi} + q_t q_\psi \frac{\partial}{\partial q} \left(\ln \left(-\frac{g_q}{g_a} \right) \right).$$

Define

$$\begin{aligned} K &= -(qu''(W-a) + (1-q)u''(W-L-a))(qu'(W+ar) + (1-q)u'(W-L+ar)) \\ &\quad - (qu'(W-a) + (1-q)u'(W-L-a))(qru''(W+ar) + (1-q)ru''(W-L+ar)). \end{aligned}$$

Then

$$-\frac{g_q}{g_a} = \frac{-(u'(W-a_1)u'(W-L+a_1r) - u'(W-L-a_1)u'(W+a_1r))}{K}.$$

Hence

$$\frac{\partial}{\partial q} \left(\ln \left(-\frac{g_q}{g_a} \right) \right) = -\frac{\partial}{\partial q} (\ln K),$$

which converges to zero as $L \rightarrow 0$. Since $-\frac{q_{t\psi}}{q_t q_\psi} = \frac{1}{t(1-\psi)}$, it follows that $\frac{\partial}{\partial\psi} (\eta_\theta/|\eta_t|)$ is positive, completing the proof.

Proof of Proposition 6: As established in the main text, it is sufficient to show that $\frac{\partial}{\partial\theta} \left(\frac{x_a}{x_\psi} \right)$ is positive. From (17) and an analogous equation for x_ψ , the ratio x_a/x_ψ equals

$$\begin{aligned} &\frac{qu'(W+x(r-a)) + (1-q)u'(W-xa)}{-q_\psi((r-a)u'(W+x(r-a)) + au'(W-xa))} \\ &+ \frac{q(r-a)xu''(W+x(r-a)) - (1-q)axu''(W-xa)}{-q_\psi((r-a)u'(W+x(r-a)) + au'(W-xa))}. \end{aligned}$$

Hence $\frac{\partial}{\partial\theta} \left(\frac{x_a}{x_\psi} \right)$ has the same sign as

$$-q_\theta q_\psi (u'(W+x(r-a)) - u'(W-xa) + (r-a)xu''(W+x(r-a)) + axu''(W-xa))$$

$$\begin{aligned}
& + q_{\theta\psi} (qu' (W + x (r - a)) + (1 - q) u' (W - xa)) \\
& + q_{\theta\psi} (q (r - a) xu'' (W + x (r - a)) - (1 - q) axu'' (W - xa)).
\end{aligned}$$

From (1), $q_{\theta\psi} = 1$ and $q_{\theta}q_{\psi} = -(1 - q)$. Hence $\frac{\partial}{\partial\theta} \left(\frac{x_a}{x_\psi} \right)$ has the same sign as

$$\begin{aligned}
& (1 - q) (u' (W + x (r - a)) - u' (W - xa)) + (r - a) xu'' (W + x (r - a)) + axu'' (W - xa) \\
& + qu' (W + x (r - a)) + (1 - q) u' (W - xa) + q (r - a) xu'' (W + x (r - a)) - (1 - q) axu'' (W - xa) \\
& = u' (W + x (r - a)) + (r - a) xu'' (W + x (r - a)) \\
& = \frac{u' (W + x (r - a))}{W + x (r - a)} \left(W + x (r - a) + x (r - a) (W + x (r - a)) \frac{u'' (W + x (r - a))}{u' (W + x (r - a))} \right) \\
& = \frac{u' (W + x (r - a))}{W + x (r - a)} (W - (\gamma - 1) x (r - a)).
\end{aligned}$$

Explicitly evaluating the FOC (16) gives

$$(q (r - a))^{-\frac{1}{\gamma}} (W + x (r - a)) = ((1 - q) a)^{-\frac{1}{\gamma}} (W - xa),$$

and so solving for x gives

$$\begin{aligned}
x & = W \frac{((1 - q) a)^{-\frac{1}{\gamma}} - (q (r - a))^{-\frac{1}{\gamma}}}{(r - a) (q (r - a))^{-\frac{1}{\gamma}} + a ((1 - q) a)^{-\frac{1}{\gamma}}} \\
& = W \frac{(q (r - a))^{\frac{1}{\gamma}} - ((1 - q) a)^{\frac{1}{\gamma}}}{(r - a) ((1 - q) a)^{\frac{1}{\gamma}} + a (q (r - a))^{\frac{1}{\gamma}}}.
\end{aligned}$$

Hence

$$\begin{aligned}
\text{sign} \left(\frac{\partial}{\partial\theta} \left(\frac{x_a}{x_\psi} \right) \right) & = \text{sign} \left(1 - \frac{(\gamma - 1) (r - a) \left((q (r - a))^{\frac{1}{\gamma}} - ((1 - q) a)^{\frac{1}{\gamma}} \right)}{(r - a) ((1 - q) a)^{\frac{1}{\gamma}} + a (q (r - a))^{\frac{1}{\gamma}}} \right) \\
& = \text{sign} \left((r - a) ((1 - q) a)^{\frac{1}{\gamma}} + a (q (r - a))^{\frac{1}{\gamma}} \right. \\
& \quad \left. - (\gamma - 1) (r - a) (q (r - a))^{\frac{1}{\gamma}} + (\gamma - 1) (r - a) ((1 - q) a)^{\frac{1}{\gamma}} \right) \\
& = \text{sign} \left(\gamma (r - a) ((1 - q) a)^{\frac{1}{\gamma}} + (a - (\gamma - 1) (r - a)) (q (r - a))^{\frac{1}{\gamma}} \right) \\
& = \text{sign} \left(\gamma \left(\frac{1 - q}{q} \frac{a}{r - a} \right)^{\frac{1}{\gamma}} + \left(\frac{a}{r - a} - (\gamma - 1) \right) \right).
\end{aligned}$$

At $a = qr$, this expression equals

$$\gamma + \frac{q}{1-q} - (\gamma - 1) > 0.$$

Hence, provided that \bar{s} is small enough that a is close enough to qr , $\frac{\partial}{\partial \theta} \left(\frac{x_a}{x_\psi} \right)$ is positive, completely the proof.

Proof of Lemma 2: Throughout the proof I drop the regime parameter ψ , since it plays no role. Given Property 1, $\frac{\partial}{\partial t} F^{-1}(1-t|\theta) < 0$ for all $t \in [0, 1]$. Given differentiability, MPR is equivalent to the condition that, for all a ,

$$F_{a\theta}(a|\theta) F(a|\theta) - F_a(a|\theta) F_\theta(a|\theta) \geq 0. \quad (24)$$

Note that if $F_\theta(a|\theta) = 0$ then first-order stochastic dominance implies $F_{a\theta}(a|\theta) \geq 0$. So for the remainder of the proof, suppose that $F_\theta(a|\theta) < 0$, or equivalently (by Lemma 1) $\frac{\partial}{\partial \theta} F^{-1}(1-t|\theta) > 0$.

Differentiating

$$F(F^{-1}(1-t|\theta)|\theta) = 1-t$$

gives

$$\begin{aligned} F_\theta(F^{-1}(1-t|\theta)|\theta) &= -\frac{\partial}{\partial \theta} F^{-1}(1-t|\theta) F_a(F^{-1}(1-t|\theta)|\theta) \\ \frac{\partial}{\partial t} F^{-1}(1-t|\theta) F_a(F^{-1}(1-t|\theta)|\theta) &= -1 \\ \frac{\partial}{\partial t} F^{-1}(1-t|\theta) F_{a\theta}(F^{-1}(1-t|\theta)|\theta) &= -\frac{\partial}{\partial t} F^{-1}(1-t|\theta) \frac{\partial}{\partial \theta} F^{-1}(1-t|\theta) F_{aa}(F^{-1}(1-t|\theta)|\theta) \\ &\quad - \frac{\partial^2}{\partial t \partial \theta} F^{-1}(1-t|\theta) F_a(F^{-1}(1-t|\theta)|\theta) \\ \left(\frac{\partial}{\partial t} F^{-1}(1-t|\theta) \right)^2 F_{aa}(F^{-1}(1-t|\theta)|\theta) &= -\frac{\partial^2}{\partial t^2} F^{-1}(1-t|\theta) F_a(F^{-1}(1-t|\theta)|\theta). \end{aligned}$$

At $a = F^{-1}(1-t|\theta)$, the LHS of (24) has the same sign as

$$\begin{aligned} &(1-t) \frac{\partial}{\partial t} F^{-1}(1-t|\theta) \frac{\partial}{\partial \theta} F^{-1}(1-t|\theta) F_{aa}(F^{-1}(1-t|\theta)|\theta) \\ &+ (1-t) \frac{\partial^2}{\partial t \partial \theta} F^{-1}(1-t|\theta) F_a(F^{-1}(1-t|\theta)|\theta) \\ &+ \frac{\partial}{\partial t} F^{-1}(1-t|\theta) \left(\frac{-1}{\frac{\partial}{\partial t} F^{-1}(1-t|\theta)} \right) \left(-\frac{\partial}{\partial \theta} F^{-1}(1-t|\theta) F_a(F^{-1}(1-t|\theta)|\theta) \right), \end{aligned}$$

which has the same sign as

$$(1-t) \frac{\partial}{\partial t} F^{-1}(1-t|\theta) \frac{\partial}{\partial \theta} F^{-1}(1-t|\theta) \left(\frac{-\frac{\partial^2}{\partial t^2} F^{-1}(1-t|\theta)}{\left(\frac{\partial}{\partial t} F^{-1}(1-t|\theta)\right)^2} \right) \left(\frac{-1}{\frac{\partial}{\partial t} F^{-1}(1-t|\theta)} \right) \\ + (1-t) \frac{\partial^2}{\partial t \partial \theta} F^{-1}(1-t|\theta) \left(\frac{-1}{\frac{\partial}{\partial t} F^{-1}(1-t|\theta)} \right) + \frac{\partial}{\partial \theta} F^{-1}(1-t|\theta) \left(\frac{-1}{\frac{\partial}{\partial t} F^{-1}(1-t|\theta)} \right)$$

which has the same sign as

$$-(1-t) \frac{\frac{\partial^2}{\partial t^2} F^{-1}(1-t|\theta)}{\frac{\partial}{\partial t} F^{-1}(1-t|\theta)} + (1-t) \frac{\frac{\partial^2}{\partial t \partial \theta} F^{-1}(1-t|\theta)}{\frac{\partial}{\partial \theta} F^{-1}(1-t|\theta)} + 1 \\ = (1-t) \frac{\partial}{\partial t} \left(\ln \frac{\partial}{\partial \theta} F^{-1}(1-t|\theta) - \ln \left(-\frac{\partial}{\partial t} F^{-1}(1-t|\theta) \right) \right) + 1 \\ = (1-t) \frac{\partial}{\partial t} \ln \left(\frac{\frac{\partial}{\partial \theta} F^{-1}(1-t|\theta)}{\left| \frac{\partial}{\partial t} F^{-1}(1-t|\theta) \right|} \right) + 1,$$

thereby completing the proof.

B Appendix: Proof of Proposition 1

The heart of proof of Proposition 1 is the following result, which generalizes Step 2 of Lemma 3 in Quah and Strulovici (2009) to the case in which the action space B is non-compact.

Lemma 3 *If $b(\theta)$ is a weakly decreasing function then there exists b^* such that $V(b^*, \theta) \geq V(b(\theta), \theta)$ for all $\theta \in \Theta$.*

Proof of Lemma 3: Consider first the case in which $b(\cdot)$ takes only finitely many values. Hence there is finite partition $\{\Theta_k : k = 1, \dots, K\}$ of Θ such that $b(\cdot)$ is constant over each partition element Θ_k , and every member of Θ_{k+1} exceeds every member of Θ_k . The proof establishes the slightly stronger result that there exists $b^* \geq b(\Theta_K)$ such that $V(b^*, \theta) \geq V(b(\theta), \theta)$ for all $\theta \in \Theta$.

The proof is by induction. Suppose there exists $\tilde{b}_k \geq b(\Theta_k)$ such that $V(\tilde{b}_k, \theta) \geq V(b(\theta), \theta)$ for all $\theta \in \bigcup_{j \leq k} \Theta_j$. To establish the result, it is sufficient to establish the inductive step that there exists $\tilde{b}_{k+1} \geq b(\Theta_{k+1})$ such that $V(\tilde{b}_{k+1}, \theta) \geq V(b(\theta), \theta)$

for all $\theta \in \bigcup_{j \leq k+1} \Theta_j$. Define \tilde{b}_{k+1} as the largest element of

$$\arg \max_{b \in [b(\Theta_{k+1}), \tilde{b}_k]} V(b, \sup \Theta_k).$$

By SCP, $V(\tilde{b}_{k+1}, \theta) \geq V(b(\theta), \theta)$ for all $\theta \in \Theta_{k+1}$. Moreover, $V(\tilde{b}_{k+1}, \theta) \geq V(\tilde{b}_k, \theta)$ for all $\theta \in \bigcup_{j \leq k} \Theta_j$, since if instead $V(\tilde{b}_k, \theta) > V(\tilde{b}_{k+1}, \theta)$ for some $\theta \in \bigcup_{j \leq k} \Theta_j$, SCP implies that $V(\tilde{b}_k, \sup \Theta_k) > V(\tilde{b}_{k+1}, \sup \Theta_k)$, which contradicts the definition of \tilde{b}_{k+1} . By supposition, it then follows that $V(\tilde{b}_{k+1}, \theta) \geq V(b(\theta), \theta)$ for all $\theta \in \bigcup_{j \leq k+1} \Theta_j$, establishing the inductive step and hence completing the proof of this case.

Next, consider the case in which $b(\cdot)$ take infinitely many values. Recall that $\underline{\theta}$, $\bar{\theta}$, \underline{b} and \bar{b} are defined in Assumption 2. Define

$$\beta(\theta) = \begin{cases} \min \{b(\theta), \max \{b(\underline{\theta}), \bar{b}\}\} & \text{if } \theta \leq \underline{\theta} \\ b(\theta) & \text{if } \theta \in (\underline{\theta}, \bar{\theta}) \\ \max \{b(\theta), \min \{b(\bar{\theta}), \underline{b}\}\} & \text{if } \theta \geq \bar{\theta} \end{cases}.$$

Define $\bar{B} = [\min \{b(\bar{\theta}), \underline{b}\}, \max \{b(\underline{\theta}), \bar{b}\}]$. Observe that β is weakly decreasing and $\beta(\Theta) \subset \bar{B}$. Moreover, if $\beta(\theta) \neq b(\theta)$ then either $\theta \leq \underline{\theta}$ and $b(\theta) > \beta(\theta) \geq \bar{b}$, or $\theta \geq \bar{\theta}$ and $b(\theta) < \beta(\theta) \leq \underline{b}$. So by Assumption 2,

$$V(\beta(\theta), \theta) \geq V(b(\theta), \theta) \text{ for all } \theta \in \Theta. \quad (25)$$

Let $\{B_n\}$ be a sequence of finite subsets of \bar{B} such that $B_n \subset B_{n+1}$ and $\bigcup_n B_n$ is dense in \bar{B} . Define $\beta_n(\theta)$ as the largest member of B_n that is weakly less than $\beta(\theta)$. Hence for any $\theta \in \Theta$, $\beta_{n+1}(\theta) \geq \beta_n(\theta)$ and $\beta_n(\theta) \rightarrow \beta(\theta)$.

For any n , the first part of the proof implies that there exists b_n^* such that $V(b_n^*, \theta) \geq V(\beta_n(\theta), \theta)$ for all $\theta \in \Theta$. Moreover, $b_n^* \in \bar{B}$. Hence b_n^* has a convergent subsequence, with limit b^* . By the continuity of V in its first argument, it follows that $V(b^*, \theta) \geq V(\beta(\theta), \theta)$ for all $\theta \in \Theta$. The result then follows from (25), completing the proof.

Proof of Proposition 1: Under Property 1, for any θ , $S(\cdot, \theta)$ is strictly increasing. Let $T(\cdot, \theta) : A(\theta; \psi') \rightarrow A(\theta; \psi'')$ be the inverse of $S(\cdot, \theta)$ with respect to its first argument. By Properties 1 and 2, the function $T(\cdot; \theta)$ is well-defined, and is strictly

increasing.

Note first that a'' and $T(a', \theta)$ have the same distribution, since for any $z \in A(\theta; \psi'')$,

$$\begin{aligned}
\Pr(a'' \leq z|\theta) &= F(z|\theta; \psi'') \\
&= F(S(z, \theta) | \theta; \psi') \\
&= \Pr(a' \leq S(z, \theta) | \theta) \\
&= \Pr(T(a', \theta) \leq T(S(z, \theta), \theta) | \theta) \\
&= \Pr(T(a', \theta) \leq z | \theta).
\end{aligned}$$

By the Lehmann-informativeness property, for any $a''_0 \in A(\psi'')$ the function $\zeta(S(a''_0, \theta))$ is weakly decreasing in θ over $\Theta(a''_0; \psi'')$. So by Lemma 3, there exists a function $\phi : A(\psi'') \rightarrow B$ such that, for any $a''_0 \in A(\psi'')$,

$$V(\phi(a''_0), \theta) \geq V(\zeta(S(a''_0, \theta)), \theta) \text{ for all } \theta \in \Theta(a''_0; \psi'').$$

It follows that, for any θ and \bar{V} ,

$$\begin{aligned}
\Pr(V(\phi(a''), \theta) \leq \bar{V} | \theta) &= \Pr(V(\phi(T(a', \theta)), \theta) \leq \bar{V} | \theta) \\
&\leq \Pr(V(\zeta(S(T(a', \theta), \theta)), \theta) \leq \bar{V} | \theta) \\
&= \Pr(V(\zeta(a'), \theta) \leq \bar{V} | \theta),
\end{aligned}$$

where the inequality uses $T(a', \theta) \in A(\theta; \psi'')$, completing the proof.