Cosigned vs. group loans

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Abstract

We analyze lending contracts when social sanctions are used to enforce repayments and borrowers differ in their abilities to impose sanctions. Symmetric group loans are preferred to cosigned loans when sanctioning abilities are similar; that is, when the power relation between borrowers is relatively equal. Conversely, cosigned loans are preferred when the power relation is unequal. We analyze why group lending arrangements offering different loan terms to members of the same group are difficult to implement. We also show that our comparison between symmetric group loans and cosigned loans is robust to endogenous matching.

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1. Introduction

The Grameen Bank and its group lending contract have received substantial academic attention in recent years. Grameen makes symmetric group loans: identical loans to a group of borrowers where all are punished if one does not repay. A large theoretical literature has established conditions under which symmetric group loans do better than individual loans (Ghatak and Guinnane, 1999). But symmetric group loans are just one way of lending to individuals who have insufficient collateral of their own. Another form of lending is through a
cosigned loan, in which a borrower provides a cosigner who does not receive a loan but is punished if the borrower does not repay. Such arrangements are also ubiquitous.¹

In this paper, we first compare these two commonly observed loan contracts. Under what circumstances will we observe symmetric group loans? And conversely, when will we observe cosigned loans?

Symmetric group loans and cosigned loans are extremes on the continuum of joint liability lending. Asymmetric group loans, in which group members are given different loan terms, are also an intermediate possibility. But leading microlenders such as the Grameen Bank and its replications, BancoSol in Bolivia, the Bank for Agriculture and Agricultural Co-operatives in Thailand and the Kenya Rural Enterprise Program all make group loans that are symmetric. In this paper, we identify one reason for why such lending schemes to small groups are generally symmetric. (At the end of the paper, we discuss how our analysis might change if applied to large groups.)

Our paper builds on Besley and Coate’s (1995) influential model of how socially sanctioned punishments can be used to enforce repayment. These sanctions include social ostracism, shame and exclusion from informal insurance networks, and are widespread in villages and other close-knit communities.² Nonetheless, for any pair of individuals, we would expect that one is more able to sanction the other, or equivalently, that one is more susceptible to sanctions than the other. Throughout the paper, we will speak of the power relation between any pair of individuals; we refer to an individual in a pair as being stronger or weaker, according to his/her sanctioning ability. A key implication is that weaker borrowers have a higher willingness to repay, since they are threatened with tougher sanctions ex post. In practice, however, the power relation of a pair is difficult for an outsider to observe. A loan officer in a Bangladeshi village can hardly be expected to know if Rashida can punish Farzana or vice versa. It is in such a private information environment that we analyze cosigned and group loans.

When would we expect cosigned loans to be used instead of group loans? If one of the individuals does not have an investment opportunity, then there is no point in lending to both and so cosigned loans are trivially the best option. But we show that, even when both individuals have investment opportunities, cosigned loans are preferred to group loans if the power relation is sufficiently unequal. Conversely, if the power relation is relatively equal, then symmetric group loans are preferred.

As discussed above, microlenders that lend to small groups rely heavily on symmetric loan contracts. Why? After all, in making a symmetric group loan, a bank is forced to “level down” loan size to a point at which even the member who faces the least significant social sanction still repays. In contrast, with an asymmetric group loan, the bank could provide more funds to borrowers who face larger sanctions in the event of default. While one possibility is that lenders make symmetric group loans out of a sense of fairness, we show here that the lender’s inability to

¹ The distinction between cosigned loans and group loans has been noted in the literature (see Ghatak and Guinnane, 1999; Banerjee et al., 1994), but has received little theoretical attention. A recent exception is Gangopadhyay and Lensink (2005).

² Social sanctions form the basis of informal contract enforcement (see Greif, 1996 for a review). Recent evidence suggests social sanctions can help explain the high repayment rates of microlenders. Karlan (2004) finds that Peruvian groups with social ties are more likely to repay than groups without social ties. Ahlin and Townsend (2003) find that Thai villages where borrowers report that they will be excluded from informal village credit markets if they do not repay the microlender have higher repayment rates than villages where borrowers do not report these socially sanctioned punishments.
accurately observe power relations makes asymmetric group loans hard to successfully implement. Ideally, the bank would like to expand its delivery of credit by offering a menu of loan contracts that induces groups to reveal their internal power relations. We show, however, that unless borrowers are very productive no such menu is possible. So conditional on making a group loan, the bank will make a symmetric group loan.

While the bank’s ignorance of power relations makes it difficult to make an effective group loan, this lack of information does not in any way hamper the bank’s ability to make an effective cosigned loan. Given its ignorance, a key concern for the bank in making a cosigned loan is that the cosigner is the weaker of the borrower–cosigner pair and so is unable to sanction the borrower in the event of default. However, since the cosigner has collateral at stake, he will never agree to cosign a loan for somebody he cannot sanction.

In addition to analyzing cosigned and group loans where the power relations are the same for all pairs of borrowers, we also consider an extended economy in which agents match endogenously into equal or unequal pairs in response to the lending contract. Our basic comparison is robust to such endogenous matching. Further, we show that if the bank can target a community of borrowers with relatively equal power relations, as many microlenders do in practice, then it can improve welfare by making symmetric group loans.

In contrast to the adverse selection or moral hazard problems that have been the focus of the microcredit literature, our paper deals with limited enforcement. Even though many believe that enforcement difficulties are a crucial reason for financial constraints in developing countries, there have only been a few papers on this topic. Besley and Coate (1995) is the closest study to ours. They study lending contracts with identical borrowers, and so neither asymmetric group loans nor cosigned loans arise in their model. In contrast, the potential borrowers in our model have unequal and unobserved sanctioning abilities. This allows us to study a richer set of contracts for which symmetric group loans and cosigned loans are special cases.

We proceed as follows. In Section 2, we describe the basic model and show that under the benchmark of full information, either cosigned loans or asymmetric group loans are efficient. In Sections 3 and 4, we assume that the bank cannot observe power relations. In Section 3, we compare two simple and commonly observed loan contracts – cosigned loans and symmetric group loans – and establish that cosigned loans are preferred and achieve the first best when power relations are sufficiently unequal. In Section 4, we establish circumstances under which more general loan contracts are ineffective in preventing stronger borrowers from pretending to be weaker and defaulting on the bank. In Section 5, we analyze an extended economy in which agents can match into equal or unequal pairs. We discuss how our results might apply to large groups in Section 6. We conclude in Section 7.

3 This literature on adverse selection and moral hazard includes Armendariz de Aghion and Gollier (2000), Banerjee et al. (1994), Ghatak (2000), Laffont (2003), Rai and Sjöström (2004) and Stiglitz (1990), among others. In all of these papers, borrower returns are contractible, i.e., borrowers will repay as long as they have enough funds to do so. In our paper, by contrast, borrowers must be induced to repay by threatening punishment, e.g., the seizure of collateral by the bank or social sanctions imposed by other villagers. (Note also that the private information in these papers is on the riskiness of borrower projects, effort levels or ability to repay, while the private information in our paper is on the borrower’s willingness to repay.)

4 Besley and Coate (1995) and Laffont and N’Guessan (2001) study limited enforcement in microcredit contracts. Ligon et al. (2002) provide evidence for how limited enforcement constrains insurance in south Indian villages.
2. The economy

There are two agents, \( i \in \{1, 2\} \), and a bank.\(^5\) As in much of the literature, we will assume a non-convex production possibility set to motivate credit constraints. Let \( \alpha \) be the minimum investment level. Each agent can invest \( x_i \geq \alpha \) in a project with certain rate of return \( \rho > 1 \). If an agent has \( x_i < \alpha \), he must use a costless storage technology. Let \( f(x_i) \) denote output from input \( x_i \):

\[
f(x_i) = \begin{cases} 
\rho x_i & \text{if } x_i \geq \alpha \\
 x_i & \text{if } x_i < \alpha 
\end{cases}
\]

Aside from their unequal abilities to impose social sanctions on each other, which we discuss in detail below, agents are \textit{ex ante} identical. They have no funds of their own to invest. Each agent has collateral \( c \). The bank can threaten to seize this collateral if the borrower does not repay; in such cases, the bank can sell the collateral for an amount \( c \). To make the problem of interest, we assume throughout that

\[
c < \alpha
\]  

(1)

That is, borrowers do not possess enough collateral to raise \( \alpha \) directly. So lending is impossible with individual loans.

Agents can also impose sanctions on each other. We model these sanctions in the same fashion as Besley and Coate (1995) as an exogenous social norm. Specially, it is socially acceptable for one agent to sanction another if the other agent’s action causes harm, and not otherwise. In the credit context, this implies that if one agent’s default causes the other to lose collateral, then the latter can sanction the former for imposing harm on him.

In general, for any pair of agents, we would expect that one is more able to sanction the other, or equivalently, that one is more susceptible to sanctions than the other. For example, a villager may have a plot of land that is upstream from another and so can sanction the downstream villager by restricting irrigation water. In contrast, the downstream villager cannot sanction the upstream villager. As another example, imagine agent 1 is a shopkeeper and agent 2 is customer. The sanction agent 1 can impose on agent 2 is to refuse to sell his goods. The sanction agent 2 can impose on agent 1 is to stop patronizing his shop. Typically, these sanctions will differ in their monetary equivalence. Throughout the paper, we will speak of the \textit{power relation} between any pair of individuals; we refer to an agent in a pair as being \textit{stronger} or \textit{weaker}, according to their sanctioning ability. We denote the power relation between agents 1 and 2 as either SW or WS.

Much of our analysis relates to the consequences of the relative power disparity between the two agents in our economy. To this end, we define a parameter \( \mu \) to index the extent to which sanctioning abilities are equal (or unequal) in the pair. The parameter \( \mu \) ranges from 0 to 1/2 where higher values denote more equal power relations. We write the sanctioning ability of the stronger

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\(^5\) For most of the paper, we restrict attention to lending schemes involving at most two borrowers. In Section 5, we extend our model to an economy with many potential borrowers, but again look only at lending contracts that involve one or two agents. Finally, in Section 6, we discuss how our results might change if the lending contract instead involved a much larger number of individuals.
agent as \((1 - \mu)s\) and the sanctioning ability of the weaker agent as \(\mu s\).\(^6\)\(^7\) For the remainder of this section, we assume that the power relation is observed and that agent 1 is the weaker agent.

We turn now to lending arrangements. We denote a loan contract for agent \(i\) by \((x_i, R_i)\), where \(x_i\) is the loan size and \(R_i\) is the required repayment. Regardless of the lending contract, the largest total punishments that can be imposed on agents 1 and 2 are \(c + (1 - \mu)s\) and \(c + \mu s\). As such, necessary conditions for payments \(R_1\) and \(R_2\) to be incentive compatible are

\[
R_1 \leq c + (1 - \mu)s \quad \text{and} \quad R_2 \leq c + \mu s. \tag{2}
\]

Below, we show that these conditions are sufficient as well as necessary.

We will refer to \(c + (1 - \mu)s\) as the willingness to repay of the weaker borrower and \(c + \mu s\) as the willingness to repay of the stronger borrower. The timing is as shown in Fig. 1.

We distinguish between two types of joint liability lending: cosigned loans where only one agent receives a loan and the other is a cosigner, and group loans where both agents receive a cosigned loan (and consequently both are cosigners).

In a cosigned loan contract, only one agent receives a loan, while collateral is seized from both agents if the loan is not repaid. Suppose that agent 2 is the loan recipient, so only agent 2 has a repayment decision to make. If agent 2 repays \(t_2 < R_2\), the bank seizes collateral \(c\) from both agents. Since in this case agent 1 also loses his own collateral claim, he imposes a social sanction of \(\mu s\) on agent 2. Thus, agent 2 can be induced to repay any amount \(R_2 \leq c + \mu s\) by the cosigned loan contract. Consider agent 2’s decision of whether to repay or default:\(^8\):

<table>
<thead>
<tr>
<th>Agent 2 repays 0</th>
<th>Agent 2 repays (R_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank action</td>
<td>Bank seizes (c) from both agents</td>
</tr>
<tr>
<td>Agent 1’s action</td>
<td>Agent 1 imposes sanction</td>
</tr>
<tr>
<td>Agent 2’s utility</td>
<td>(-c - \mu s)</td>
</tr>
</tbody>
</table>

Clearly, if \(R_2 < c + \mu s\), then agent 2 is best off paying \(R_2\), while if \(R_2 = c + \mu s\), he is indifferent between paying \(R_2\) and paying 0.

\(^6\) Notice that both agents have the same endowment of collateral \(c\). Provided collateral endowments are observable by the bank, this assumption could be straightforwardly relaxed without qualitatively changing our results.

\(^7\) An alternative interpretation of \(\mu\) is as different bargaining powers. Suppose agents have the same sanctioning ability \(s\) but different skills in renegotiating the imposition of social sanctions. Suppose agent 2 fails to repay and the bank seizes \(c\) from agent 1 as a consequence. Agent 1 is now in a position to impose a sanction \(s\) on agent 2. Then, the agents have the incentive to renegotiate: they would be collectively better off if no sanction were imposed. This is a standard split-the-surplus game. If \(\mu\) denotes agent 1’s bargaining power, then the outcome is for agent 2 to pay (“bribe”) agent 1 an amount \(\mu s\) in return for not imposing the sanction. Conversely, if agent 1’s bargaining power is \(1 - \mu\), then agent 2 pays a bribe of \((1 - \mu)s\). The net effect is that a weaker agent faces a welfare loss of \((1 - \mu)s\) and the stronger agent faces a welfare loss of \(\mu s\) if he causes harm to the other agent.

\(^8\) In what follows, for expositional ease, we shall only consider cases in which agents repay the loan entirely or repay nothing. This is without loss since agents will never choose to make partial repayments or to pay more than what is required.
In a group loan, both agents receive loans and are expected to make strictly positive payments to the bank. Again, provided that condition (2) is satisfied, there is a loan contract under which it is an equilibrium for agents 1 and 2 to repay \( R_1 \) and \( R_2 \), respectively. If one is not concerned about equilibrium uniqueness, this is straightforward. Consider the contract in which the bank threatens to seize collateral \( c \) from both agents if either agent 1 fails to repay \( R_1 \) or agent 2 fails to repay \( R_2 \). An agent \( i \) is socially sanctioned if he defaults on \( R_i \), while his partner \( j \) repays \( R_j \) but not otherwise. As such, the agents play the following game:

\[
\begin{array}{ccc}
 & 0 & R_2 \\
0 & -c, -c & -c-(1-\mu)s, -R_2-c \\
R_1 & -R_1-c, -c-\mu s & -R_1, -R_2 \\
\end{array}
\]

Provided that (2) holds, there is an equilibrium in which agents 1 and 2 pay \( R_1 \) and \( R_2 \), respectively. Under the above contract, there is also an equilibrium in which both agents simply pay nothing, i.e., default. However, as we show below, more complicated loan contracts can avoid this problem of multiplicity of equilibria (see Ma, 1988 and Ma et al., 1988).

Specifically, the bank can augment the group loan contract to ensure that the repayment equilibrium is unique. One such example is as follows. Let \( A_i \) be any repayment that is higher than required, i.e., \( A_i > R_i \). The bank seizes collateral \( c \) from borrower \( i \) if he does not repay \( R_i \), and seizes collateral from both borrowers if borrower \( i \) repays strictly less than \( R_i \), while borrower \( j \) does not pay strictly more than \( R_j \). In addition to giving each borrower \( i \) the option to make his own repayment \( R_i \), the bank also allows each borrower to repay extra, i.e., repay \( A_i \).9 Whenever one or more borrowers chooses this second option, the bank issues “refunds”, as follows: (a) if both borrowers repay \( A_i(i=1, 2) \), the bank returns \( A_i-R_i \) to borrower \( i \) \((i=1, 2)\); (b) if borrower \( i \) repays \( A_i \) and borrower \( j \neq i \) repays \( R_j \), the bank returns \( R_j \) to borrower \( j \); and (c) if borrower \( i \) repays \( A_i \) and borrower \( j \) repays 0, the bank returns \( A_i \) to borrower \( i \).

This augmented group loan contract gives the repayment subgame shown below in which each agent chooses either to default, repay \( R_i \) or repay \( A_i \). As before, borrower \( i \) sanctions borrower \( j \) whenever borrower \( j \)’s default causes borrower \( i \) to lose his collateral — i.e., social sanctions are imposed for repayments \((R_1, 0)\) and \((0, R_2)\).10

\[
\begin{array}{ccc}
 & 0 & A_2 \\
0 & -c, -c & -c-(1-\mu)s, -R_2-c & -c, 0 \\
R_1 & -R_1-c, -c-\mu s & -R_1, -R_2 & 0, -A_2 \\
A_1 & 0, -c & -A_1, 0 & -R_1, -R_2 \\
\end{array}
\]

Provided that (2) holds, there is a unique equilibrium in which agents 1 and 2 pay \( R_1 \) and \( R_2 \), respectively. There is no longer an equilibrium in which both agents default, since each agent prefers to repay \( A_i \).

Many contracts of this form are possible. The key is to make sure that each borrower has an incentive to deviate from playing “default” when the other borrower also plays default. The

9 The repayments must be feasible, i.e., less than \( f(x_i) \). Since \( A_i \) can be made arbitrarily close to \( R_i \), this implies that the repayment \( A_i \) can be selected to be feasible whenever \( R_i < f(x_i) \). For the case in which the borrower derives zero surplus from the loan, \( R_i = f(x_i) \), the repayment \( A_i \) can be selected to be feasible provided that the borrower receives (an arbitrarily small) exogenous endowment at the repayment date.

10 Other social sanctioning rules are conceivable. For example, one might speculate that the social sanctioning norm is for borrower \( i \) to sanction borrower \( j \) whenever borrower \( i \) pays at least \( R_i \) and borrower \( j \) pays less than \( R_j \). For all alternate social sanctioning norms that we have explored, we have been able to construct a group loan contract similar to the one above which implements full repayment as the only equilibrium.
example we give above is reasonably realistic. The bank never collects total repayments in excess of what it is owed. The bank interprets repayments \((A_1, R_2)\) as evidence that borrower 1 is better able to repay than borrower 2 and so returns \(R_2\) to borrower 2. Further the bank rewards borrower 1 for his “good faith” gesture if repayment is \((A_1, 0)\). We should point out that the refund payments do not have to be explicitly labelled as such and can take the form of future loans.11

So far we have derived the repayment constraints that the bank faces. Next we turn to the bank’s contract design problem. We take the bank’s objective to be the maximization of the aggregate welfare of the agents, subject to the constraint that it makes non-negative profits. Both altruistic lenders subject to a tight funding constraint (such as development banks) and competitive profit-maximizing banks can be expected to behave broadly in this manner. Formally, the bank chooses a loan and repayment pair \((x_i, R_i)\) for each \(i\) to maximize aggregate welfare

\[
\sum_{i=1,2} (f(x_i) - R_i)
\]

subject to the repayment constraints (2),12 the break even constraint

\[
\sum_{i=1,2} (x_i - R_i) \leq 0
\]

and the individual rationality constraints

\[
f(x_i) - R_i \geq 0
\]

The solution to this problem is given in Proposition 1. Even though individual lending is impossible, lending is feasible with group loans or cosigned loans. Given full observability of social sanctions, the efficient group loan will generally be asymmetric.

Proposition 1. (Benchmark: bank observes borrower types)

(i) The bank lends to both agents (asymmetric group loans) if

\[
\alpha \leq \min \left\{ c + \frac{s}{2}, 2c + s - \frac{c + (1-\mu)s}{\rho} \right\}
\]

(ii) The bank lends only to the weaker agent (cosigned loan) if inequality (5) does not hold, but \(\alpha \leq c + (1-\mu)s\).

(iii) If \(\alpha > c + (1-\mu)s\) no surplus-producing loan is possible.

The proof is in the appendix. Proposition 1 establishes that the form of lending depends on relationship between the minimum investment \(\alpha\) and the collateral endowments \(c, \mu s\) and \((1-\mu) s\). First consider the case in which the minimum investment \(\alpha\) is smaller than the stronger borrower’s willingness to repay, i.e., \(\alpha < c + \mu s\). It is easily shown that this is a subcase of

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11 Further, the bank could achieve the same outcome (a unique repayment equilibrium) with smaller refunds, as follows. If repayments are \((A_i, R_i)\), then the bank makes a refund of max\(\{0, R_i - c\} + \epsilon\) to borrower \(j\). If repayments are \((A_i, 0)\), then the bank makes a refund of max\(\{0, A_i - c\} + \epsilon\) to borrower \(i\).

12 Notice that cosigned loans are just an especially asymmetric group loan where \(R_1 = 0\) or \(R_2 = 0\).
condition (5). In this case, it is feasible to recover $\alpha$ from both agents and so group loans are efficient. The stronger agent 2 is asked for a smaller repayment than the weaker agent 1; i.e., $R_1 = c + (1 - \mu)s$ and $R_2 = c + \mu s$. There is some freedom in how to set the loan sizes: depending on the project return $\rho$, it may be possible to give equal loans and still satisfy the individual rationality constraint of both borrowers. In general, however, the loan granted to the weaker borrower will be larger, to reflect his larger repayment.

The more interesting case is when $c + \mu s < \alpha < c + (1 - \mu)s$. Now the bank can recover $\alpha$ from the weaker agent but not from the stronger agent. The bank would like to lend to both using group loans. But that would mean losing money on the stronger agent since his loan of $\alpha$ exceeds his willingness to repay. Therefore, group loans are feasible only if the bank can make enough money on the larger loan to the weaker agent without violating the weaker agent’s individual rationality constraint. Note that the smaller $\mu$ is, the greater the discrepancy in the repayment sizes. Consequently, when $\mu$ is sufficiently small, the weaker agent is no longer willing to cross-subsidize the stronger agent and so it becomes impossible to lend at least $\alpha$ to each. In such a situation (case (ii)), the bank will only lend to the weaker agent using the stronger agent as cosigner. Finally, if it is impossible to recover $\alpha$ from even the weaker agent, i.e., if $\alpha > c + (1 - \mu)s$, then lending is clearly infeasible.

Group loans generate a higher surplus than cosigned loans: the total surplus using group loans is $(\rho - 1)(2c + s)$, while the total surplus with a cosigned loan is only $(\rho - 1)(c + (1 - \mu)s)$. But group loans are only feasible if the minimum investment size $\alpha$ is sufficiently low relative to the collateral the agents possess.

To illustrate our results throughout the paper, we make use of the following two example economies:

**Example 1 (Full information benchmark).** Suppose that collateral $c = 80$, minimum investment size $\alpha = 100$ and rate of return $\rho = 1.14$. Further, suppose the weaker agent can sanction the stronger agent in the amount $\mu s = (1/4)80 = 20$ and the stronger agent can sanction the weaker agent up to $(1 - \mu)s = (3/4)80 = 60$. Therefore, the weaker agent’s willingness to repay is $140$, while the stronger agent’s willingness to repay is $100$. In this case, condition (5) is satisfied and the bank makes an asymmetric group loan to the pair. The bank offers a loan contract $(100, 100)$ to the stronger agent and $(140, 140)$ to the weaker agent.

**Example 2 (Full information benchmark).** Suppose $c$, $\alpha$ and $\rho$ are as in Example 1. However, the power relation between the two agents is more unequal: specifically, the weaker agent cannot sanction the stronger agent (i.e., $\mu s = 0$), while the stronger agent can sanction the weaker agent up to $(1 - \mu)s = 80$. The willingness to repay of the weaker agent is $160$, while the willingness to repay of the stronger agent is just $80$. Even though the total willingness to repay is the same as in Example 1, condition (5) is not satisfied and so the bank cannot lend to both agents and still break even. Instead, the bank makes a cosigned loan of $(160, 160)$ to the weaker agent.

So far we have established that asymmetric group loans with the weaker borrower receiving a larger loan than the stronger borrower are efficient if feasible. However, in practice, we observe a large number of microlenders offering only symmetric group loans. Even though villagers surely differ in their sanctioning abilities, as we argued above, lenders often do not appear to take these differences into account. We shall discuss their reasons for doing so in Section 4.
3. Symmetric group loans vs. cosigned loans

In practice, banks are outsiders with limited information on the power relations between villagers. The rest of the paper deals with consequences of this observation.

To motivate the way in which we model the specifics of the bank’s information, consider a bank seeking to lend to a pair agents in a village. Although the bank is unlikely to know the power relation of any specific pair of individuals, it does presumably have some information about the range of possibilities that the power relation may take.

For example, an external lender may know that the only feasible sanction within a particular village is social ostracism. It also knows that even this sanction cannot always be imposed: under the prevailing norms of village society, an individual can only be ostracized by someone who is more “senior” in the village’s informal hierarchy. What the lender cannot observe, however, is the internal hierarchy itself. If social ostracism hurts all individuals an equal amount \( s \), then the bank’s information boils down to this: it knows that, for any pair of villagers 1 and 2, either villager 1 can impose sanction \( s \) on villager 2, while villager 2 cannot impose any sanction on villager 1 or vice versa. In terms of our notation, the bank knows the magnitude of the sanctions \( \mu s \) and \((1 - \mu)s\) (here \( \mu = 0 \)), but does not know whether the power relation between villagers 1 and 2 is SW or WS.

In this section and next, we study lending arrangements under exactly the informational assumptions of the example above. There are just two possibilities, SW and WS, for the power relation between the pair of potential borrowers 1 and 2, and while the bank cannot observe the power relation, it does know the values \( \mu s \) and \((1 - \mu)s\). In other words, the willingness to repay of each potential borrower is unobserved. Subsequently, in Section 5, we extend our analysis to an environment in which there is a broader range of possible power relations. This will enrich our analysis by introducing an additional consideration: how does the contract proposed by the bank affect how borrowers match with one another? In contrast, when the only possible power relations between a given pair of agents are SW and WS, the matching problem is degenerate.

In general, the bank’s contract design problem is to offer a menu of loan contracts that induces agents to reveal their power relation (and hence reveal how much each is willing to repay). Since there are two possible power relations, WS and SW, the menu will include two possible contracts:

\[
\{(x_1^{\text{WS}}, x_2^{\text{WS}}, R_1^{\text{WS}}, R_2^{\text{WS}}), (x_1^{\text{SW}}, x_2^{\text{SW}}, R_1^{\text{SW}}, R_2^{\text{SW}})\}
\]

Note that, since individual lending is impossible under assumption (1), without loss all loan contracts are assumed to be group or cosigned loans.

For now, we restrict attention to anonymous menus — that is, those in which if the names of the two borrowers were interchanged, the menu of possible contracts would be unchanged. Formally, an anonymous menu is one that satisfies \( x_1^{\text{WS}} = x_2^{\text{SW}}, x_1^{\text{SW}} = x_2^{\text{WS}}, R_1^{\text{WS}} = R_2^{\text{SW}} \) and \( R_1^{\text{SW}} = R_2^{\text{WS}} \).

When the menu of contracts offered is non-degenerate – that is, the contracts are not identical – how do the two borrowers decide which contract to accept? It is natural to suppose that the stronger agent will have more of a say in selecting a menu option than the weaker agent. We model this by assuming that it is the stronger agent who chooses which contract to accept. However, once the contract has been selected, the non-selecting (weaker) agent has

\[\text{See Section 4.2 for non-anonymous loan contracts.}\]
veto power and can decline the selected contract. When this happens, no loan is made at all. Consequently, the stronger agent will only choose a contract that satisfies the weaker agent’s individual rationality constraint.

In practice, the most common loan contracts are the symmetric group loan and a cosigned loan. In this section, we analyze the performance of these two contracts. In Section 4, we then consider whether the bank could offer an alternate loan contract that outperforms symmetric group loans and cosigned loans. As we will see in Section 4, under a wide range of parameter values, symmetric group loans and cosigned loans are in fact the most efficient lending contracts available to the bank.

3.1. The self-selection property of cosigned loans

Formally, a cosigned lending policy under asymmetric information is a particularly simple menu in which there are two menu items: one in which agent 1 takes a loan cosigned by agent 2, and the other in which agent 2 takes a loan cosigned by agent 1. In terms of the notation defined in (6), \( x_1^{SW} = x_2^{WS} = R_1^{SW} = R_2^{WS} = 0 \).

As we saw in the Section 2, when the bank’s objective is to maximize aggregate borrower welfare, then conditional on making a cosigned loan he prefers to lend to the weaker borrower. The reason is simple — the weaker borrower can be called upon to repay \( c + (1 - \mu) s \), while the stronger borrower can only be induced to repay \( c + \mu s \). Consequently, a larger loan can be made to the weaker borrower.

A striking property of cosigned loans is that the bank’s ignorance of the relative sanctioning abilities of the two borrowers does not impede this targeting of the weaker borrower. This can be seen as follows. The bank would ideally like to make a cosigned loan of \( x = c + (1 - \mu) s \) to the weaker borrower, with a repayment of \( R = x \). Consider what happens if it offers the cosigned loan menu defined above, with the loan sizes \( x_1^{WS} = x_2^{SW} \) and repayments \( R_1^{WS} = R_2^{SW} \) both set to the preferred level \( c + (1 - \mu) s \).

Without loss, suppose that agent 1 is the weaker agent — i.e., the state is WS. Under this menu, agent 2, the stronger agent, is happy to select the WS contract: agent 1 gets the loan of \( c + (1 - \mu) s \), while agent 2 is the cosigner. The reason is that, under this selection, agent 1 will indeed repay and so agent 2 does not lose his collateral. In contrast, if agent 2 selects the contract SW, then agent 1 foresees that agent 2 will default on the repayment \( c + (1 - \mu) s \) — and so agent 1 vetoes the selection, since it leaves him with negative welfare.

To summarize, the bank is able to make a cosigned loan of \( c + (1 - \mu) s \) to any agent who can find a cosigner, and be sure that only a weaker agent will take such a loan — and the weaker agent will not default. Recall, moreover, that from Proposition 1 cosigned loans achieve the constrained first best when the minimum loan size \( \alpha \) is relatively large compared to the collateral endowments \( c \) and \( s \). Consequently, under these same conditions, the bank’s ignorance of agents’ sanctioning abilities does not reduce social welfare:

**Corollary 1 (Self-selection of cosigned loans).** If (5) does not hold, then self selection using cosigned loans allows an uninformed bank to lend as much as if it were fully informed.

As we will see in Section 4, when the solution to the full information problem is for the bank to employ an asymmetric group loan (i.e., when (5) holds), the situation is very different: the bank’s lack of knowledge of the power relation constrains its ability to lend efficiently. Specifically, unless the agents’ project return \( \rho \) is very high, the bank is unable to effectively induce agents to
reveal their relative willingness to repay. Instead, the bank is forced to use either a cosigned loan or a symmetric group loan, even though neither is efficient under full information.

3.2. The choice between group loans and cosigned loans

So far we have discussed one commonly observed contract, namely cosigned loans. We now turn to a second commonly observed contract, group loans in which loans and repayments are identical across members. We have termed such contracts symmetric group loans. Formally, symmetric group loans are a degenerate menu of loan contracts in which both borrowers are offered identical loans: $x_{1}^{WS} = x_{2}^{SW} = x_{1}^{SW} = x_{2}^{WS}$ and $R_{1}^{WS} = R_{2}^{SW} = R_{1}^{SW} = R_{2}^{WS}$.

Our next result characterizes the bank’s choice between cosigned and symmetric group loans as a function of the minimum project size $\alpha$ and the inequality parameter $\mu$. It is graphically depicted in Fig. 2.

**Proposition 2 (Symmetric group loans vs. cosigned loans).**

(i) The bank lends to both agents (symmetric group loans) if

$$c + \mu s \geq \max \{(1-2\mu)s, \alpha\}$$  \hspace{1cm} (7)

(ii) The bank lends only to the weaker agent (cosigned loan) if (7) does not hold, but $\alpha \leq c + (1-\mu)s$.

(iii) No surplus-producing loan is possible if $\alpha > c + (1-\mu)s$.

Fig. 2. Private information.
The proof of Proposition 2 is straightforward. As in the full-information problem (see Proposition 1), the bank’s choice of contract depends on the relative size of the minimum investment $\alpha$ and the collateral endowments $c$, $\mu s$ and $(1-\mu)s$. Since the strong borrower is only willing to repay $c+\mu s$, the bank can recover at most $c+\mu s$ from each borrower with symmetric group loans. Symmetric loans are feasible if $\alpha \leq c+\mu s$. Symmetric group loans are preferred to cosigned loans if $2c+2\mu s$, the total lending using symmetric group loans is higher than $c+(1-\mu)s$, the total lending with cosigned loans, or equivalently if $c+\mu s \geq (1-2\mu)s$ holds. When group loans are infeasible, i.e., when power relations are sufficiently unequal or the minimum investment size is sufficiently large, then the bank will just give a cosigned loan to the weaker agent. Even though the bank cannot tell the agents apart, by offering a cosigned loan that the stronger agent is unwilling to repay (and consequently the weaker agent is unwilling to cosign), the bank effectively selects the weaker agent. This comparison is illustrated in our examples as:

**Example 1 (Comparison).** The bank can make a symmetric group loan since condition (7) is satisfied. Each agent receives a loan of $(100, 100)$.

**Example 2 (Comparison).** This economy is relatively unequal (compared with Example 1): Since condition (7) is not satisfied, the bank cannot give a symmetric group loan and still ensure that each agent receives a minimum loan of 100. So the bank offers a cosigned loan of $(160, 160)$. The bank lends just as much as in the benchmark case of full information, illustrating Corollary 1.

Restating Proposition 2 slightly gives:

**Corollary 2 (Inequality).** If the power relation is sufficiently unequal, the bank uses cosigned loans in preference to symmetric group loans. Conversely, if $\alpha \leq c+s/2$ and the power relation is sufficiently equal, the bank uses symmetric group loans in preference to cosigned loans.\(^{14}\)

Notice that the bank always prefers to give cosigned loans for $\mu$ sufficiently small. This is unlike the full information case: from Proposition 1, absent private information when the minimum project $\alpha$ is small\(^{15}\) the bank prefers asymmetric group loans to cosigned loans even when the power relation is very unequal.

4. Alternate contracts

In the previous section, we compared the efficacy of cosigned loans and symmetric group loans. We have focused on this pair of contracts because of their empirical prevalence. In particular, as we have discussed above, group lending schemes typically do not differentiate between group members, at least when the group size is small.

Nonetheless, there remains the theoretical question of why symmetric contract terms are as prevalent as they are. One possible reason is, of course, that microlenders treat group members equally out of a sense of fairness. In terms of our model, the main advantage of departing from

\(^{14}\) If $\alpha > c+s/2$, then symmetric group loans are never feasible.

\(^{15}\) Specifically, below $\min\{c+(s/2), (2-(1/\rho))c+(1-(1/\rho)s)\}$. 

symmetric terms is that larger loans could be given to group members who are more susceptible to social sanctions. However, such an increase in loan size is predicated on the ability of other villagers who are already more powerful within village society to actually implement these sanctions. It is easy to see why a microlender might feel uncomfortable about mobilizing existing social inequalities to increase loan size.\textsuperscript{16}

Aside from fairness issues, though, our model also directly suggests an important impediment to the use of asymmetric loan terms. As we have discussed, an outside lender will generally be uninformed about the specifics of power relations between group members. The consequence of this is that an outside lender has at most limited ability to tailor its loan contracts to take into account the differences in willingness to pay that arise from agents’ different susceptibilities to sanctions.

The basic intuition of this argument is clear. However, one still might wonder whether the bank would be able to use a suitably designed menu of loan contracts to induce agents to reveal the power relation between them. This is the main issue we explore in the current section (see Section 4.1).

A second and related possibility, which we also consider (see Section 4.2), is whether it is really so bad if a bank sometimes misjudges the power relation between the agents it lends to. That is, suppose the bank offers agents a single loan contract which violates our anonymity restriction: it offers agent 1 a higher loan than agent 2 and demands a higher repayment than the stronger agent’s willingness to repay. Such a contract will work well if the bank correctly guesses that agent 1 occupies the weaker position in the power relation between the two, but will lead to default if the bank guesses wrongly.

The results of this section are easily summarized: there are only limited circumstances in which either of the above strategies can be effectively implemented. In particular, unless the project return $\rho$ is high, symmetric group loans and cosigned loans are the best contracts at the bank’s disposal.

### 4.1. Alternate menus

Let us start with the question of whether the bank can do better by offering non-degenerate menus of loan contracts (other than cosigned loans). We show that such menus can be useful, but only in very limited circumstances. In particular, unless projects are very productive then restricting attention to simple contracts, i.e., symmetric group loans or cosigned loans, is without any loss of surplus.

To understand why menus are of limited use in making group loans, it is useful to start by asking if menus can indeed play a useful role in the economy of Example 1. Recall that in this example, total surplus is lower under asymmetric information when the bank uses only cosigned and symmetric group loans. Specifically, total lending was 200 using a symmetric group loan, which is lower than the full information benchmark lending level of 240. Can the uninformed bank improve welfare in this example by offering an appropriate menu in which the borrower pair reveals its type via the contract selected?

Consider the menu in which repayments are set equal to the willingness to pay of the two borrowers, i.e., $R_1^{WS} = R_2^{SW} = 140$ and $R_2^{WS} = R_1^{SW} = 100$. That is, if the power relation is

\textsuperscript{16} Related, many microlenders stress solidarity within the group. This can (somewhat speculatively) be interpreted as an attempt to reduce the power disparity within the group, and hence increase the willingness to repay of the strongest group member.
WS and the borrowers choose the WS contract from the menu offered, then borrower 1 (the weaker agent) is asked to repay 140, while borrower 2 (the stronger agent) is asked to repay 100. Recall that we assume that the stronger borrower chooses the contract and the weaker agent has veto power. Consequently, the menu of contracts must be such that, when the power relation is WS, the stronger borrower prefers choosing the contract WS to contract SW.

If in this case the stronger borrower chooses the contract SW, then both borrowers will default. The stronger borrower then loses his collateral, worth 80, which is better than repaying 100. The only way to induce the stronger borrower to choose the WS contract is if the loan he receives is much higher in the WS than SW contract. For example, suppose that the menu of contracts gives the stronger borrower a loan of 140 and the weaker borrower a loan of 100 (i.e., \( x^{WS}_{1} = x^{SW}_{2} = 100 \) and \( x^{WS}_{2} = x^{SW}_{1} = 140 \)). The stronger borrower will choose the contract WS when the power relation is WS, since his utility from doing so is \( \rho 140 - 100 \), whereas his utility from choosing contract SW is \( \rho 100 - 80 \).\( \rho 140 - 100 \).

The potential problem, of course, is that, since the stronger borrower is receiving such favorable loan terms (a large loan and small repayment), the weaker borrower must be receiving correspondingly unfavorable terms. Specifically, if he does not veto the stronger agent’s contract choice, his equilibrium utility is \( \rho 100 - 140 \). If the project return is high enough (\( \rho > 1.4 \)), then this is positive and the proposed menu succeeds in delivering the first-best surplus even under asymmetric information. However, if the project return is below 1.4, as it is in our example (where \( \rho = 1.14 \)), then the menu fails. The above discussion generalizes to:

**Proposition 3 (Limits to menus).** Fix \( \mu \in [0, 1/2] \). Then there exists a \( \bar{\rho}(\mu) > 1 \) such that, if \( \rho \leq \bar{\rho}(\mu) \), there exists no menu in which the bank is able to lend more than is possible using cosigned loans or a symmetric group loan, while itself breaking even.

The proof is in the appendix. This negative result contrasts starkly with Corollary 1. Unlike menus of asymmetric group loans, cosigned loans are a useful selection device because they are so extreme. The cosigner receives nothing and so if a weaker agent anticipates default, then he has no incentive to cosign a loan for a stronger agent since he will certainly lose \( c \). In contrast, if the bank offers a menu of asymmetric group loans, both borrowers receive at least \( \alpha \). So even if the stronger borrower selects a loan on which both borrowers will default, the weaker agent will not veto this choice — his payoff is at least \( \rho \alpha - c > 0 \). For this reason, separating borrower types is much more difficult if the bank is trying to lend to both borrowers than if the bank is only trying to lend to one borrower.

### 4.2. Non-anonymous contracts

The main disadvantage of the symmetric group loans that we have considered in Section 3 is that both agents are asked to repay \( c + \mu s \). This is lower than the weaker agent’s willingness to repay. If the bank were to offer a symmetric group loan with higher repayments, the borrowers would default with probability 1. But suppose the bank takes a stab in the dark about the power relation between the pair of borrowers and offers a contract which agent 1 would only repay if he is the weaker of the two agents. Such a loan is repaid only half the time, i.e., when the power relation is WS. Of course, such a contract violates the anonymity assumption that we have maintained in the rest of the paper. Nonetheless, in this section, we briefly consider whether such
non-anonymous contracts can generate more surplus than either symmetric group loans or cosigned loans.

Again, it is useful to consider Example 1 to motivate non-anonymous contracts. In that example, the weaker agent had a willingness to repay of 140 and the stronger agent had a willingness to repay of 100. Suppose the bank offered non-anonymous loans of \((x_1, 140)\) to agent 1 and \((x_2, 100)\) to agent 2. If accepted by both agents, such loans are repaid in the state WS but are defaulted on in the state SW. The bank collects 240 in the state WS but only 160 (in collateral seized) in the state SW. So the bank can only make loans of \(x_1 + x_2 \leq 200\). Since each loan must be exceed the minimum investment size \(\alpha = 100\), the only possibility is to set \(x_1 = x_2 = 100\). Clearly, agent 1 will never accept such a loan unless he is very productive (\(\rho \geq 1.4\)). So there is limited scope for such non-anonymous loan contracts.

This type of argument generalizes to:

**Proposition 4 (Limits to non-anonymous loan contracts).** No non-anonymous contract delivers higher surplus than is possible using either a cosigned loan or a symmetric group loan if either \(\mu \geq 1/4\), or if \(\rho \leq \hat{\rho}(\mu) = [c + \mu s]/[c + \mu s - (\alpha - c)]\).

The proof is in the appendix.

5. Targeting and endogenous matching

Up to now, we have assumed that there are just two possibilities for the power relation between any pair of agents \(i\) and \(j\): either \(i\) is stronger than \(j\), or \(j\) is stronger than \(i\). Equivalently, in terms of punishment abilities, either \(i\) can impose a sanction \((1-\mu)s\) on \(j\), while \(j\) can impose a sanction \(\mu s\) on \(i\), or vice versa. Formally, we say that the power relation is either SW or WS.

In this section, we extend our comparison between symmetric group and cosigned loans to the case in which there are instead three possible power relationships between a pair of agents. In addition to SW and WS, we introduce the possibility that the two agents are equal in their sanctioning abilities: each of \(i\) and \(j\) can impose a sanction of \(s/2\) on the other. We write \(E\) for this additional power relation.\(^{17}\)

Broadly speaking, there are two possible informational assumptions that one can make in this extended model. We analyze their implications in turn.

5.1. Targeting

First, suppose that the bank can observe whether the power relation between a pair of agents lies in \{SW, WS\}, or whether the power relation is \(E\). How will the bank use this information? From Section 3, it is easier for the bank to make a loan to a pair of agents when the power relation between them is relatively equal. In our extended model, this can be seen starkly as follows: if \(c + \mu s < \alpha < c + (1/2)s\), then the bank will only make a group loan to a pair of agents with power relation \(E\).

Lending only to groups with relatively equal internal power relations resembles the observed microfinance practice of targeting. For example, the Grameen Bank restricts its group loans to

\(^{17}\) Note that additional power relation is one in which the two agents are exactly equal. More generally, it would be straightforward to extend our analysis by instead introducing a pair of additional power relations SW' and WS', in which \(\mu' > \mu\), corresponding to less disparity than SW and WS.
landless women. To reiterate, we are not assuming here that the bank knows the exact internal power relations between borrowers, but only that it knows that there are not great disparities within the group.

5.2. Endogenous group formation

At the opposite extreme, the bank may be completely unable to distinguish whether the power relation between a pair of agents lies in \{SW, WS\}, or whether the power relation is \(E\). In terms of our previous notation, it is as if the parameter \(\mu\) can now take two possible values, one of which is \(1/2\). The bank no longer knows the magnitude of sanctions a pair of agents can impose on one another.

When the “equal” power relation cannot be distinguished from the unequal power relations \{SW, WS\}, the bank faces a non-trivial matching problem. Previously, when the bank designed a lending contract it knew that in all pairs, one borrower would be stronger than the other. Now, it is possible that the paired agents are equal. To the extent that agents can choose their partners, the bank must now take into account the effect of its proposed lending contract on how agents match.

Formally, a matching for this economy is a partition of the set of agents into pairs. We refer to any matching in which all pairs are unequal as an unequal matching, and to a matching in which all pairs are equal as an equal matching. Notice that in between these two extremes are other matching outcomes, in which some pairs are equal and other pairs are unequal. To ease the presentation of our analysis, we assume (I) that the number of agents in the economy is divisible by 4, and (II) that for any four randomly selected agents there is one way of pairing the four agents such that both pairs are equal, and another way of pairing the four agents such that both pairs are unequal.\(^{18}\) Consequently, there exists at least one unequal matching and at least one equal matching for the economy.

As a benchmark, we start by characterizing socially optimal matchings conditional on the bank’s choice of loan contract (i.e., cosigned loans or symmetric group loans). Suppose first that the bank makes cosigned loans. The bank would ideally like the repayment on these cosigned loans to be \(c + (1 - \mu)s\), the highest possible willingness to repay. The stronger agent in an unequal pair would accept such a cosigned loan (and the loan would be repaid) but neither agent in equal pairs would agree to cosign for the other. So in order to maximize welfare with cosigned loans, the bank can do no better than an unequal matching. For any other matching, there would be one or more equal pairs, thereby reducing total lending. Next imagine that the bank makes symmetric group loans. The bank can recover \(c + (1/2)s\) from each agent in an equal pair, but can only recover a smaller amount, \(c + \mu s\), from each agent in an unequal pair. So the bank can do no better than with an equal matching and would be able to lend more with symmetric group loans than previously (Proposition 2). To summarize, when the bank makes cosigned loans unequal matching is socially optimal, while if the bank makes symmetric group loans an equal matching is socially optimal.

5.2.1. Cosigned loans

We now turn to an analysis of how agents match in response to the lending contract offered. Following Roth and Sotomayor (1990), we define a stable matching as one in which no two

\(^{18}\) As should be clear below, our conclusions do not depend critically on these assumptions.
agents who are not paired would both strictly\textsuperscript{19,20} prefer each other to their actual partners. Our first result is that, if the bank offers cosigned loans, the unequal matching is stable. As discussed above, when the bank uses cosigned loans, social surplus is maximized by the unequal matching. Consequently:

**Proposition 5 (Cosigned loans with matching).** If the bank offers a cosigned loan of $c + (1 - \mu)s$, then the unequal matching is stable, the bank lends $c + (1 - \mu)s$ to each pair and the loans are repaid. This is the most the bank can lend using cosigned loans.

The proof of Proposition 5 is straightforward. Observe that, when the bank offers cosigned loans, every matching is stable, as follows. Recall that a matching is unstable if and only if there exists an alternate matching that two unmatched agents both strictly prefer. No matter how the agents match, no-one will agree to cosign a loan that he knows will end in default. Given this, in every matching at least one agent in each pair has zero utility. But then every matching must be stable. Consequently, if the bank offers the cosigned loan $c + (1 - \mu)s$, then the unequal matching is certainly stable, each pair takes the loan, and repays. In contrast, any higher cosigned loan will not be accepted.

### 5.2.2. Symmetric group loans

As discussed, if borrowers match into equal pairs the bank would be able to make a symmetric group loan of $c + (1/2)s$. Unfortunately, however, if the bank offers this loan contract, an equal matching is not stable. As a result, the bank is constrained to offer group loan contracts in which the repayment is less that $c + \mu s$.

**Proposition 6 (Symmetric group loans with matching).** If the bank offers a symmetric group loan of $c + \mu s$, then any matching is stable, the bank lends $c + \mu s$ to every borrower, and loans are repaid. This is the most the bank can lend using symmetric group loans.

On the one hand, if the bank offers symmetric group loan with repayment $R_1 = R_2 = R$ such that $R \leq c + \mu s$, then under any matching all groups will repay their loans.

On the other hand, consider a symmetric group loan with repayments $R_1 = R_2 = R$ such that $c + \mu s < R \leq c + (1/2)s$. In this case, any matching in which there is more than one equal pair is unstable. To see this, suppose to the contrary that a matching with two equal pairs is stable. But any equal pair repays its loan, while any unequal pair defaults. Since borrower welfare is higher under default, all four borrowers in the two equal pairs would be strictly better off under the alternate matching in which they are all in unequal pairs.

Given that the only stable matchings are those in which at most one pair is equal, all but one pair default on the loan and the bank will lose money whenever the total number of agents is large enough.

Finally, if the bank offers a symmetric group loan with repayment $R_1 = R_2 = R$ such that $R > c + (1/2)s$, then in all matchings all pairs default on their loans.

\textsuperscript{19} See their definition in Section 4 and also their example 2.15 in Appendix A.

\textsuperscript{20} In many settings, Propositions 5 and 6 would hold even under the following more demanding alternate definition of a stable matching: a matching is said to be unstable if there exists an alternate matching in which two unmatched agents are both weakly better off and at least one is strictly better off. A proof is available from the authors.
5.2.3. The choice between group loans and cosigned loans, revisited

In Section 3, we compared cosigned loans and symmetric group loans in an economy where all pairs were unequal. In this section, we have analyzed an extended economy in which agents match endogenously into equal or unequal pairs in response to the lending contract. On the one hand, we would expect that the bank could do better with symmetric group loans in this extended economy because agents in equal pairs have higher willingness to repay such loans (see Section 5.1 on loan targeting). On the other hand, since agents are free to choose their partners, this imposes additional constraints on what the bank can recover. Our analysis shows that the net effect leaves our previous conclusions unchanged. In particular, Proposition 2’s comparison of cosigned and symmetric group loans applies to the extended economy in which agents match endogenously.

6. Discussion: large groups

Formally, our model has dealt only with borrowing groups of size two; more generally, it is intended to apply to the large number of group lending schemes that make use of small groups. Although the best known group lending schemes do indeed use small groups, there also exist lending programs in which the group size is large (e.g., village banking by FINCA). For that reason, we briefly discuss the likely applicability of our analysis to large groups.21

Symmetric group loans arise in our model because for any pair of potential borrowers the lender is unable to tell which of the pair is more susceptible to social sanctions. This forces the lender to level down all repayments to the point at which even the least susceptible borrower is willing to repay. This same effect is clearly present for large groups as well. However, at least under some circumstances the lender’s ignorance about the power relations pertaining between pairs of borrowers may matter less in large groups. Loosely speaking, while a lender may not know whether borrower 1 can sanction borrower 2 or borrower 2 can sanction borrower 1, he may know that on average borrower 1 is more susceptible to social sanctions than is borrower 2. Put slightly differently, “diversification” of social sanctions across a large group reduces the importance of the lender’s limited information.

To formalize this observation, consider the following extension of our basic model. Suppose now that agents differ in some characteristic that is readily observed by the lender. Gender, race, religious affiliation and, in some cases, class or caste are all obvious examples. For specificity, we discuss an example in which there are two types of people: blues and greens.

Assume that, on average, blues are more likely to be able to impose sanctions on greens than vice versa. That is, if individual 1 is blue and individual 2 is green, then the probability that the power relation between the pair is SW is \( q > 1/2 \). (We assume that if two individuals are the same color then this conveys no information.) Under this extension, then, the lender possesses a noisy signal of the power relation of at least some pairs of individuals.

It is readily verified that if \( q \) is not much greater than 1/2 then the introduction of this noisy signal has no effect on our existing results concerning lending to small groups (specifically, groups of size 2). In contrast, however, it has a very great effect on the lender’s information about the susceptibility to sanctions of an individual in a large group. The reason is as follows. Information about whether individual 1 is blue or green tells the lender little about that...

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21 It is beyond the scope of the paper to account for why group size varies across lending schemes. The discussion below might suggest that large groups are better. However, large groups also have disadvantages, such as free rider problems, which are not modelled here.
individual’s susceptibility to punishment by a particular individual 2. By the law of large numbers, however, it is very informative about individual 1’s average susceptibility to punishment by a large group.

Consider the average sanction that members of a large group can impose on a blue member. Some of the other blue members can impose a sanction $\mu s$; others can impose a sanction $(1-\mu)s$. Since the two possibilities are equally likely, the average sanctioning ability of a blue member on a fellow blue member is $(1/2)s$.

Turning to green members, again some can impose a sanction $\mu s$, while others can impose a sanction $(1-\mu)s$. However, since the probability that a green member can impose the higher sanction on a blue member is $(1-q)<(1/2)$, the average sanctioning ability of a green member on a blue member is $(q\mu+(1-q)(1-\mu))s<(1/2)s$.

If the proportion of blues in the population is $\beta$, then by the law of large numbers the average ability of group members to punish a blue individual converges to

$$\left(\frac{\beta}{2} + (1-\beta)(q\mu + (1-q)(1-\mu))\right)s < \frac{s}{2},$$

while the average ability of group members to punish a green individual converges to

$$\left(\frac{1-\beta}{2} + \beta(q(1-\mu) + (1-q)(1-\mu))\right)s > \frac{s}{2}.$$ 

Consequently, while the lender knows little about the extent to which particular blue individuals can be sanctioned by particular green individuals, when a group is large the lender knows that with very high probability a green individual can be sanctioned more effectively than a blue individual. Armed with this information it is then able to extend larger loans to green group members, secure in the knowledge that green members do indeed have a higher willingness to repay.22

7. Conclusions

We have analyzed lending contracts in a model where borrowers have unobserved power relations (and, consequently, unobserved willingness to repay). We have shown that simple and commonly observed loan contracts are constrained efficient unless projects are very productive. Symmetric group loans are constrained efficient when power relations between individuals are relatively equal. When power relations are relatively unequal, cosigned loans are efficient. We have also shown that this comparison remains true even if borrowers can choose to match into equal or unequal pairings.

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22 This discussion has focused on the effect of group size on the symmetry prediction of our main model. Group age may also impact the extent to which lending terms differ across group members. For example, if the lender learns about group members’ susceptibilities to punishment over time, then he may be able to shift to treating different borrowers differently. Or if there are vacancies in groups created by dropouts, then the bank may fill those with new members who have smaller initial loan sizes. We leave a fuller exploration of this effect for future research.
Appendix A. Proofs

Proof of Proposition 1. First, consider loans in which both borrowers receive at least \( \alpha \). Suppose there exists such a loan satisfying constraints (2), (3) and (4). It is easily seen that there exists an alternative loan satisfying the same constraints, with the repayment constraint (2) and bank break-even constraint (3) both binding. As such, there exists a loan in which both agents receive at least \( \alpha \) if and only if there exists a loan size \( x_1 \) for agent 1 such that \( x_1 \geq \alpha, 2c+s-x_1 \geq \alpha, \rho x_1 \geq c+(1-\mu)s \) and \( \rho(2c+s-x_1) \geq c+\mu s \). In other words, such a loan exists if and only if the intervals \([\alpha, 2c+s-\alpha]\) and \(\left[\frac{c+(1-\mu)s}{\rho}, 2c+s-\frac{c+\mu s}{\rho}\right]\) have a non-empty intersection. These intervals intersect if and only if \( \alpha \leq 2c+s-\alpha, \frac{c+(1-\mu)s}{\rho} \leq 2c+s \) and \( 2c+s-\frac{c+\mu s}{\rho} \geq \alpha \). Since \( 1-\mu \geq \mu \), these conditions are in turn equivalent to condition (5).

From above, when it is possible to lend \( \alpha \) to both agents, it is possible to do so in such a way that total lending equals \( 2c+s \), the maximum that the bank can ever lend while breaking even. As such, whenever a loan of this sort exists, it is preferred to all alternatives. If instead condition (5) is not satisfied, the bank can lend more than \( \alpha \) to at most one of the agents. Since the other agent receives no surplus, no cross-subsidy is possible. It follows that the best the bank can do is to make a cosigned loan to the weaker of the two agents. Finally, if \( \alpha > c+(1-\mu)s \), then even this option is not available. \( \square \)

Proof of Proposition 3. Suppose the menu induces borrowers to always default. Then the bank can lend at most \( 2c \) in total. Since \( 2c < 2\alpha \), the bank can only lend to one of the agents, and we have established that using cosigned loans to lend to the weaker agent is efficient in such a case. So this menu can do no better than cosigned lending. For a menu to perform better, then in at least one of states SW and WS the borrowers must select a contract and then repay. By the anonymity assumption, and since the stronger borrower makes the contract selection in both states, the situation is exactly symmetric across states. Without loss, we focus on state SW. Without loss, write the menu as \( \{x^S, x^W, R^S, R^W\}, (x^W, x^S, R^W, R^S) \), and assume that the menu is designed so that the stronger borrower selects the contract which gives him a loan of \( x^S \) and repayment \( R^S \). We refer to this contract as the intended contract.

We claim first that, when the unintended contract is selected, the borrowers default. To see this, note first that the menu can only dominate a symmetric group loan if one of the repayments \( R^S \) and \( R^W \) exceeds \( c+\mu s \). Since repayment occurs when the intended contract is selected, \( R^S \leq c+\mu s \). But then \( R^W > c+\mu s \), which implies that if the unintended contract is selected default occurs.

Observe that the stronger borrower chooses the intended contract if and only if

\[
\rho x^S - R^S \geq \rho x^W - c.
\]  

(8)

Given this preliminary observation, in order for the menu to deliver at least \( \alpha \) to each borrower, and for the bank to break-even, the loan parameters \( x^S, x^W, R^S, R^W \) must satisfy – in addition to constraint (8) – the following seven inequalities: \( x^S \geq \alpha \) and \( x^W \geq \alpha \) (both borrowers receive at least \( \alpha \)); \( R^S \leq c+\mu s \) and \( R^W \leq c+(1-\mu)s \) (both borrowers repay); \( \rho x^S - R^S \geq 0 \) and \( \rho x^W - R^W \geq 0 \) (both borrowers have positive utility); and \( R^S + R^W \geq x^S + x^W \) (the bank breaks even). These inequalities define a constraint set, \( X \) say. We will show that the set \( X \) is empty if

\[
\rho(2-\rho)\alpha > \rho c + (\rho-1)\mu s
\]

(9)

and one of
\[ x \leq \frac{c + (1-\mu)s}{\rho} \]  
(10)

\[ x > c + \frac{1}{2} \left( 1 - \frac{\mu}{\rho} \right) s \]  
(11)

Suppose to the contrary that \( X \) is non-empty. So it contains some element \((x^S, x^W, R^S, R^W)\). It is easily seen that it must then contain an element in which either (a) the weaker borrower’s IR constraint binds, \( \rho x^W = R^W \), or (b) the weaker agent receives the minimum feasible loan, \( \rho x^W = R^W \). (If a contract satisfies neither condition, we can always reduce \( x^W \) and increase \( x^S \), while preserving the stronger borrower’s incentive to choose the right contract.)

First, suppose a menu exists that satisfies the stated constraints and in which the weaker borrower receives zero utility, i.e., \( \rho x^W = R^W \). For the bank to break even, \( R^S + (\rho - 1) x^W \geq x^S \), and so the stronger borrower’s utility \( \rho x^S - R^S \) from choosing the intended contract is certainly less than \( (\rho - 1)R^S + \rho(\rho - 1)x^W \). A necessary condition for the stronger borrower to choose the intended contract is thus \( (\rho - 1)R^S + \rho(\rho - 1)x^W \). The repayment \( R^S \) must be less than \( c + \mu s \), otherwise the borrower will not repay. So our contract must satisfy \( (\rho - 1)(c + \mu s) + \rho(\rho - 1)x^W \geq \alpha \), i.e., \( \rho c + (\rho - 1)\mu s \geq \rho(2 - \rho)x^W \). Since \( x^W \geq \alpha \), this contradicts inequality (9).

Second, suppose a menu exists that satisfies the stated constraints and in which the weaker borrower receives the lowest feasible loan, \( x^W = \alpha \). By assumption, the weaker borrower’s IR constraint is satisfied, \( \rho x = R^W \geq 0 \). So if the repayment that can be extracted from the weaker borrower is high enough, i.e., \( c + 1 - \mu s \geq \rho \alpha \), then we can always raise the repayment owed by the weaker borrower so that his IR constraint binds, \( \rho x = R^W \) — but we have just ruled out this case. So if inequality (10) holds, the proof is complete.

The remainder of the proof deals with the case in which the weaker borrower’s maximum repayment is lower, i.e., \( c + (1 - \mu s) < \rho \alpha \). Note that we can assume without loss that the weaker borrower is being asked to repay the maximum amount, \( R^W = c + (1 - \mu s) \), since this leaves him with positive utility. So for the bank to break even, the loan to the stronger borrower must satisfy \( x^S \leq R^S + c + (1 - \mu) s - \alpha \). The stronger borrower’s utility is thus less than \( (\rho - 1)R^S + \rho(\alpha + (1 - \mu) s - \alpha) \). A necessary condition for the stronger borrower to choose the intended contract is thus \( (\rho - 1)R^S + \rho(\alpha + (1 - \mu) s - \alpha) \geq \rho x - c \). As before, the stronger borrower’s repayment \( R^S \) must be less than \( c + \mu s \). So our contract must satisfy \( (\rho - 1)(c + \mu s) + \rho(\rho - 1)(\alpha + (1 - \mu) s - \alpha) \geq \rho x - c \), or equivalently \( c + \frac{1}{2} \left( 1 - \frac{\mu}{\rho} \right) s \geq \alpha \). This contradicts assumption (11).

To complete the proof, we explicitly construct a value \( \bar{\rho}(\mu) \) such that if \( \rho \leq \bar{\rho}(\mu) \) then inequality (9) holds, along with either inequality (10) or (11). Inequality (9) is equivalent to \( \rho(2 - \rho) \alpha - \rho c - (\rho - 1)\mu s > 0 \). This is a concave quadratic in \( \rho \) and is equal to \( \alpha - c > 0 \) at \( \rho = 1 \). Its larger root is at

\[ \rho = 1 + \sqrt{\frac{(c + \mu s)^2 + 4\alpha(\alpha - c - c(c + \mu s))}{2\alpha}}. \]
At least one of inequalities (10) and (11) holds whenever
\[
\frac{c + (1-\mu)s}{\rho} \geq c + \frac{1}{2}\left(1 - \frac{\mu}{\rho}\right)s,
\]
or equivalently whenever
\[
\rho \leq \frac{c + (1-\mu)s}{c + \frac{\mu}{2}}.
\]
So one (not necessarily tight) possibility for \(\bar{\rho}(\mu)\) is
\[
\bar{\rho}(\mu) = 1 + \min\left\{ \frac{(1-\mu)s}{c + \frac{\mu}{2}}, \sqrt{\frac{(c + \mu)^2 + 4\alpha(x-c)-(c + \mu s)}{2\alpha}} \right\}.
\]
This completes the proof. □

**Proof of Proposition 4.** Suppose to the contrary that such a contract exists. For it to improve on a cosigned loan, it must grant a loan of at least \(\alpha\) to both agents. For it to improve on a symmetric group loan, it must entail total lending of more than \(2(c + \mu s)\). For the bank to breakeven, it then follows that at least one of the requested repayments must exceed \(c + \mu s\). So without loss assume that \(R_1 > c + \mu s\).

First, observe that since \(R_1 > c + \mu s\), if the power relation between agents 1 and 2 is SW and the agents accept the loan, they will subsequently default. Since by assumption \(x_1, x_2 \geq \alpha\), it then follows that the agents will always accept the loan when their type is SW.

Since the bank loses money when the agents’ type is SW, the contract must be accepted – and subsequently repaid – when their type is WS. Taking into account that half the time the agents’ type is SW, the bank’s breakeven constraint is
\[
x_1 + x_2 \leq \frac{1}{2} (R_1 + R_2) + \frac{1}{2}(c + c).
\]
Since by agent 1’s IR constraint \(x_1 \geq R_1/\rho\), this implies
\[
x_2 \leq \frac{1}{2} R_2 + c - \left(\frac{1}{\rho} - \frac{1}{2}\right)R_1.
\]
Since the contract is repaid, \(R_2 \leq c + \mu s\). So provided \(\rho < 2\),
\[
x_2 \leq \frac{1}{2}(c + \mu s) + c - \left(\frac{1}{\rho} - \frac{1}{2}\right)(c + \mu s) = c + \left(1 - \frac{1}{\rho}\right)(c + \mu s).
\]
If \(\rho \leq \frac{c + \mu s}{c + \mu s - (x-c)}\) this implies that \(x_2\) is less than \(\alpha\), giving a contradiction.

Finally, note that, even if \(\rho > \frac{c + \mu s}{c + \mu s - (x-c)}\), the most the bank can recover (and thus lend) is
\[
\frac{1}{2}(c + \mu s + c + (1-\mu)s) + \frac{1}{2}(c + c) = 2(c + \frac{s}{2}).
\]
In contrast, the bank can easily lend \(2c + 2\mu s\) using the symmetric loan contract. So if \(\frac{1}{2} \leq 2\mu\), we again have a contradiction. □
References