Silence is safest: non-disclosure when the audience's preferences are uncertain*

Philip Bond† Yao Zeng‡

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Abstract

We examine voluntary disclosure when the firm ("sender") is risk-averse and uncertain about audience preferences. We show that some firms stay silent in equilibrium, in contrast to classic "unravelling" results. Silence reduces the sensitivity of a firm's payoff to audiences' preferences, which is attractive to risk-averse firms, i.e., "silence is safest." Increases in firm risk-aversion reduce disclosure by firm-types who bear a higher risk under disclosure. In contrast, silence imposes risk on the audience, and consequently, increases in audience risk-aversion increase disclosure. We discuss applications to corporate non-disclosure, and to regulatory rules mandating that disclosure be entirely public.

Keywords: information disclosure, risk-aversion, uncertainty, preferences.

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†Foster School of Business, University of Washington, USA. Email: apbond@uw.edu

‡Foster School of Business, University of Washington, USA. Email: yaozeng@uw.edu

1 Introduction

An important and long-standing question in the economics of information is whether voluntary disclosure leads to full disclosure. A compelling and intuitive argument, often described as the "unravelling" argument, suggests that it does.[1] In brief, the argument is that the firm, or more generally the "sender," with the most favorable information will voluntarily disclose. So the audience for the disclosure will interpret silence as indicating that the firm does not have the most favorable information. But given this, the firm with the second most favorable piece of information will disclose, and so on. All the firms thus disclose in the end.

Despite the force of the unravelling argument, the prediction of full disclosure appears too strong. There are many cases in which valuable information that is potentially disclosable is not disclosed, that is, the would-be sender is "silent." Many private firms do not voluntarily reveal their cash flows. Similarly, firms are frequently reluctant to disclose important details of their operations, such as CEO pay, the fraction of profits stemming from specific geographic or product markets (e.g., Apple Watch sales, Amazon Prime membership), or inventory levels. In such cases, potential disclosers believe that silence is in their best interests, even though audiences may interpret silence with skepticism. Moreover, the unravelling argument has the strong implication that disclosure laws and regulations are unnecessary, which is inconsistent with the vigorous arguments associated with the introduction of such rules.

In this paper, we give a new yet simple explanation for silence. Our explanation captures the idea that some firms fear disclosure because, if they disclose, they face the risk of "losing" with respect to some audience; and consequently, that staying silent and not disclosing is the safest option. Moreover, our explanation has the advantage of applying even in cases where disclosure has no direct cost, and in which there is no uncertainty that the firm possesses information to disclose, which are arguably the leading existing explanations of silence (see Grossman and Hart, 1980; Jovanovic, 1982; Dye, 1985). Note that the examples above—firm cash flow, CEO pay, profit decomposition, inventory levels—are all instances in which the firm certainly has information to disclose, and the direct costs of doing so are likely to be negligible.

Our explanation has two key ingredients, both of which we show to be necessary for

[1] See Viscusi (1978), Grossman and Hart (1980), Milgrom (1981), Grossman (1981), and Milgrom and Roberts (1986). Dranove and Jin (2010) survey the literature.

silence to occur in equilibrium. The first ingredient is that the firm faces uncertainty about the audience's collective preferences. For example, firms often disclose to a mix of investors, who wish to see high cash flows; and other parties, such as regulators, labor unions, tax authorities, and competitors, who they would like to convince that cash flows are low. Whether a firm benefits from convincing its audience that its cash flow is high or low depends on the relative strength of the preferences of different members of the firm's audience. Alternatively, even if all members of the audience have the same preferences for high cash flows, if the firm can only disclose a signal that is imperfectly correlated with cash flows, the firm may still not know the audience's beliefs concerning the correlation between signals and cash flows. For example, a firm that is considering disclosing CEO pay may be unsure whether investors regard high CEO pay as indicating that future cash flows will be high, because the CEO is good; or as low, because of poor governance. Many applications indeed feature audience members with intrinsically different preferences, or different beliefs, as we discuss in Section 3.

The second key ingredient in our analysis is firm risk-aversion. Absent firm risk-aversion, firm uncertainty about audience preferences is not enough to generate silence. The reason is that the expected payoff from disclosure can still be ordered, so that one can still identify firms with the highest incentive to disclose, and the unravelling argument still applies. Under firm risk-aversion, silence potentially delivers an additional benefit of making the firm's payoffs safer, thereby breaking the unravelling argument. We show that silence arises precisely when it is safer than disclosure. Firm risk-aversion can arise in a number of ways: for example, if the firm is closely held, as many private firms are; or if the firm's managers are risk-averse and are exposed to firm outcomes through their incentive contracts; or if financing frictions lead to a concave firm value function.[2]

In a little more detail, consider, for example, a firm with private information about the level of its cash flow, which it can voluntarily disclose. Along the lines above, the firm discloses to an audience composed of investors and a regulator. While investors always reward the firm for high cash flows, the regulator may treat the firm more harshly if it believes cash flows are higher. However, the firm is uncertain about

[2]Conversely, a firm is effectively risk neutral in the widely-studied case in which it is owned by well-diversified shareholders, its decisions have no impact on systematic risk, there is no agency problem between shareholders and management, and no financing frictions.

whether the regulator treats firms *uniformly*, i.e., acts independently of perceived firm cash flow; or is instead *discriminatory*, i.e., treats firms with higher perceived cash flows more harshly. If the regulator acts uniformly, then the firm wants to convince its audience that its cash flows are high. The reverse is true if the regulator is sufficiently discriminatory.

A disclosing firm faces a lottery over different outcomes, where the lottery realization depends on whether the regulator is uniform or discriminatory. Firms disclosing extreme cash flows—i.e., either very high, or very low—face particularly high-risk lotteries, because they receive very different treatment from uniform and discriminatory regulators. In contrast, a firm disclosing moderate cash flows is treated in a similar way by the two types of regulator, and hence faces a much safer lottery.

In a typical equilibrium that we study, firms with extreme information stay silent and do not disclose, while firms with intermediate information disclose. Audiences correctly interpret silence as indicating extreme information—in the example above, either very low or very high cash flows. The audience's response to silence is thus based on the average of these extremes, i.e., a belief that the firm has moderate cash flows. In particular, this means that uniform and discriminatory regulators treat the firm in similar ways. So silence generates a lower-risk lottery for firms with extreme information, relative to the alternative of disclosing.

The discussion above highlights the firm's uncertainty about the regulator's type. But investors' presence in the audience is important, because it means the firm is unsure about the ordinal preferences of its combined audience. If instead investors were absent, a firm would know that it is best off when its audience believes cash flows are low, and standard unravelling forces would lead to full disclosure.

Although we described this example in terms of investors and a regulator, the same forces operate in many other settings. For instance, instead of a being concerned about how it will be treated by a regulator, a firm may be concerned about how a potential competitor will behave; and instead of different regulator types, the firm's uncertainty is instead over whether the probability of competitor entry is largely independent of its cash flows, or is instead sharply increasing in its cash flows.

Given the economic forces underlying equilibrium silence and the failure of unravelling, it is natural to conjecture that silence becomes more likely as firm risk-aversion increases. Similarly, silence exposes audiences to risk by reducing their ability to differentiate between different firm-types. Consequently, silence becomes less likely as the audience become more risk-averse. Section 6 formalizes these comparative statics.

Finally, in an extension we explore the impact of rules that mandate that any disclosure should be entirely public, such as Regulation Fair Disclosure in the US. We show that such rules may reduce total disclosure, because they remove firms' ability to shield themselves from uncertainty about audience preferences by selectively targeting audience segments with known preferences.

Although our formal model is couched in terms of the firm being a seller and an audience composed of buyers, with different audiences corresponding to different buyer preferences, this formal framework covers a wide range of applications, including the regulator and competitor examples discussed above. We also discuss a number of more involved but natural applications in Section 3, including the disclosure of corporate news in cases where investors differ in their beliefs about the correlation between news and cash flows.

Previous research has identified other possible reasons for why full unravelling may not occur, and some firms choose to remain silent instead of disclosing. As noted above, the most widely applicable existing explanations are that full unravelling does not occur if disclosure is costly (Grossman and Hart, 1980; Jovanovic, 1982); and that full unravelling does not occur if there is some probability that the firm is unable to disclose (Dye, 1985).

While the assumptions of costly disclosure and unobservably impossible disclosure are certainly satisfied in some settings, there are also many settings in which disclosure is costless, and there is no uncertainty as to whether the firm is able to disclose, but voluntary disclosure does not generate full disclosure. As we noted earlier, a firm certainly knows the amount it is paying its executives, its profits from different geographic and product markets, and its inventory levels, and the direct costs of disclosing these pieces of information are likely negligible. Moreover, accounting scholars have suggested that "big data"—i.e., the improvement of information technology and the resulting mass production of information—will likely reduce accounting and reporting costs, which implies lower disclosure costs and less uncertainty as to whether firms have information in the first place.[3] Our paper can explain silence in these settings where previous explanations cannot. Moreover, it captures precisely the idea that staying silent and not disclosing is the "safest" course of action.

Unravelling results have been generalized to wider classes of economies by papers

[3]See Warren et al (2015) for a survey.

such as Okuno-Fujiwara et al (1990) and Seidmann and Winter (1997).[4] Okuno-Fujiwara et al (1990) stress the importance of the monotonicity of the sender's expected utility in the receiver's beliefs, and include examples in which a failure of this property leads to silence. Our paper can be viewed as identifying a set of natural economic conditions that generate non-monotonicity of the sender's expected utility in receiver beliefs. In doing so, we characterize the extent of silence—typically, partial rather than full—along with comparative statics with respect to sender and receiver risk aversion.

The literature on disclosure is large, and has suggested a number of further alternative explanations of silence, as surveyed in Dranove and Jin (2010). Among them, some share our focus on audience heterogeneity, though rely on very different economic forces. For example, Fishman and Hagerty (2003) show that silence arises if some audience members are unable to process the information content of disclosure. Harbaugh and To (2017) consider a setting in which the sender's type is drawn from the interval $[0, 1]$, but disclosures are restricted to specifying which element of a finite partition of $[0, 1]$ the type belongs to. Moreover, the audience is endowed with a private signal about the sender's type. Consequently, the best senders in a partition element may prefer to remain silent in order to avoid mixing with mediocre senders in the same partition element, and thus the unraveling argument breaks down. Similarly, Quigley and Walther (2018) show that when disclosing is costly while the audience observes a separate noisy signal about the sender, the best sender may remain silent, rely on the audience's signal, and thus save the disclosure cost. This then generates "reverse unraveling" in which other sender-types also remain silent in order to pool with higher sender-types.

Dutta and Trueman (2002), Suijs (2007), and Celik (2014) all analyze relatively special situations in which the sender is unsure how the audience will respond to a disclosure. However, Dutta and Trueman (2002) assume that there is a strictly positive probability that the sender has nothing to disclose, and state that this is

[4] Giovannoni and Seidmann (2007) study a setting similar to Seidmann and Winter (1997), and characterize conditions under which no disclosure occurs. Differently from our paper, the sender knows the audience's preferences. Instead silence arises because different sender types desire different audience responses, as in the following simple example (which is closely related to examples in these two papers). The sender's type x is uniform over $[-1, 1]$, and the audience takes an action a equal to its posterior estimate of x. If the sender's payoff is given by $-ax$, there is an equilibrium with no disclosure, since no disclosure yields a payoff of 0 for all sender types, while disclosure by type x yields a payoff of $-x^2$.

critical for their results. In Suijs (2007)'s environment (unlike ours), there is a direct benefit to silence.[5] In Celik (2014), a seller chooses whether to disclose a location on a Hotelling line, and *also* makes a take-it-or-leave-it price offer to a buyer whose location on the Hotelling line is assumed to follow a uniform distribution.[6] The details of price formation are important: if instead there were several buyers in competition, the only equilibrium would be full disclosure.

2 Model

We consider a sender—henceforth, the *firm*—that has an attribute or *type* x, and that interacts with an *audience* that is composed of one or more members, as detailed below. The firm's type x is drawn from a set X, where X is a compact interval of the real line.[7] The prior distribution of x has full support over X, and admits a density function f. We normalize the endpoints of X so that $X = [0, 1]$.

The firm is privately informed about its type x. The firm can, at zero cost, credibly *disclose* its type x to an audience; or stay *silent* and not disclose any information. The firm is uncertain about the composition of the audience to which it discloses. The firm's utility is determined by the composition of the audience and the audience's information about the firm's type.

Looking ahead, the firm's uncertainty about the audience's collective ordinal preferences plays a central role in our analysis. By modeling firm uncertainty about the audience's composition, as we do next, we are able to capture both (i) cases in which the firm is unsure of the ordinal preferences of individual audience members, for example, the firm is unsure whether investors or a regulator is listening, as well as (ii) cases in which the firm knows the ordinal preferences of each member, but remains unsure how they aggregate. For example, we can formalize the example discussed in the introduction, in which the firm discloses to investors and a regulator but is unsure

[5] To be specific, in Suijs (2007)'s model, disclosure gives a payoff of either $U(0)$ or $U(1)$, with probabilities $1 - p(\phi)$ and $p(\phi)$ respectively, where ϕ is the sender's type. Silence gives payoffs of $U\left(\frac{1}{2}\right)$ and something at least $U(0)$, with corresponding probabilities, and *regardless* of audience inferences about what silence means. So if the type space is such that $1 - p(\phi)$ is sufficiently high for all types, silence is an equilibrium.

[6] These assumptions imply that disclosing sellers at the ends of the line face a severe trade off between proposing a higher price and achieving a reasonable sale probability.

[7] The assumption that X is compact ensures that there exists an equilibrium of the disclosure game we describe. If instead X is non-compact, it is straightforward to give examples in which no equilibrium exists.

whether the regulator treats firms *uniformly* or *discriminatory.*

Formally, an audience is composed of one or more *receivers*. The set of possible receivers is $\{1, 2, \ldots, n\}$, where $n \geq 1$. Let $\mathcal{P}(n)$ denote the power set of $\{1, 2, \ldots, n\}$. The set of possible audiences is $\mathcal{N} \subset \mathcal{P}(n)$, and we write $N \in \mathcal{N}$ to denote a representative audience. The firm does not know what audience it faces when making disclosure decisions; let $\Pr(N)$ be the probability it assigns to facing audience $N \in \mathcal{N}$. Given a realized audience N, any disclosure is observed by all its members $i \in N$.

The firm's utility is determined by the combination of disclosure decisions and the identity of receivers in the audience it faces. We denote the firm's payoff from an individual receiver $i \in N$ by p_i. The firm's payoff from audience N is then $p_N \equiv \sum_{i \in N} p_i$. The firm's risk preferences are determined by v, a differentiable and strictly increasing function. Hence the firm's expected utility is

$$E_N [v(p_N)] = \sum_{N \in \mathcal{N}} \Pr(N) v(p_N).$$

Note that, for clarity, we typically write E_N when the expectation is being taken over audiences $N \in \mathcal{N}$.

The firm's payoff p_i from an individual receiver $i \in N$ is determined by

$$E_x [u_i (g_i(x) - p_i) | \mathcal{I}] = u_i(0), \tag{1}$$

where $\mathcal{I}$ is the receiver's information (i.e., either the particular x the firm discloses, or nothing), u_i is continuous, strictly increasing and weakly concave, and g_i is differentiable. The form of (1) is motivated by a firm of type x selling an item to a set of competing buyers, each of whom has risk preferences given by u_i and a valuation of the item of $g_i(x)$, so that p_i is the competitive price. For other applications, it is frequently useful to set u_i to be linear, so that (1) simplifies to

$$p_i = E_x [g_i(x) | \mathcal{I}].$$

Note that we impose no assumption on the relationship between different g_i's or as to whether g_i is increasing, decreasing, or non-monotone.

The variation in g_i across receivers may arise from variation in preferences over x, as in the investor-regulator example from the introduction, and as we discuss below in subsections 3.1 and 3.2. Alternatively, variation in g_i may arise from variation in beliefs about the correlation between the disclosure x and some latent payoff-relevant fundamental y, even if all receivers have the same preferences over the fundamental y, as we discuss in subsections 3.3 and 3.4.

For use throughout, we denote the firm's expected utility from disclosing x by $V^D(x)$. This quantity is straightforward to calculate, since in this case the firm's payoff p_i from receiver i is simply $p_i = g_i(x)$, and so

$$V^D(x) \equiv E_N\left[v\left(\sum_{i \in N} p_i\right)\right] = E_N\left[v\left(\sum_{i \in N} g_i(x)\right)\right].$$

It is also convenient to define the aggregation of audience N's preferences, $g_N(x)$:

$$g_N(x) \equiv \sum_{i \in N} g_i(x).$$

We say an equilibrium features *full disclosure* if the probability that the firm discloses is 1, otherwise it features *silence*. More specifically, we say an equilibrium features *partial silence* if the probability that the firm discloses is strictly between 0 and 1, and that it features *full silence* if the probability that the firm discloses is 0.

Throughout, we write $\left(p_N^S\right)_{N \in \mathcal{N}}$ for the "prices" received from the different audiences following silence. Note that these prices are endogenously determined in equilibrium.

We make the following mild regularity assumptions, which rule out economically uninteresting outcomes in which unravelling does not occur because an interval of firm-types all derive exactly the same utility from disclosure. First, no audience has flat preferences over the firm's type:

Assumption 1 *For any $N \in \mathcal{N}$ and any subset $\tilde{X} \subset X$ with positive measure, there exists $\tilde{x} \in \tilde{X}$ such that $g_N(\tilde{x}) > E_x\left[g_N(x)\,|\tilde{X}\right]$.*

Second, the expected price (as opposed to utility) received after disclosure is not flat in the firm's type:

Assumption 2 *Either: For any subset $\tilde{X} \subset X$ with positive measure, there exists $\tilde{x} \in \tilde{X}$ such that $E_N[g_N(\tilde{x})] > E_x\left[E_N[g_N(x)]\,|\tilde{X}\right]$; or else the firm is strictly risk-averse.*

Note that Assumption 2 holds generically in the space of probability distributions over the audience's type (as a consequence of Assumption 1). Moreover, Assumption 2 allows for the non-generic case of a flat expected price if the firm is strictly risk-averse. This is useful primarily because it enables us to use a very simple example in Section 4 to illustrate our results.

Before proceeding, we note the following straightforward result, which is directly implied by receivers' (weak) risk-aversion, and which we use repeatedly:

Lemma 1 *For any audience $N \in \mathcal{N}$,*

$$p_N \leq E_x \left[g_N(x) | \mathcal{I} \right], \tag{2}$$

where the inequality is strict if u_i is strictly concave for any $i \in N$ and the posterior of x given information $\mathcal{I}$ is non-degenerate.

3 Model applications

Our model is general enough to accommodate many economically relevant applications in which disclosing is costless but the firm faces uncertainty about audience preferences. We have described the baseline model in terms of a firm that sells an item with attribute x to buyers (audiences). The seller chooses whether or not to disclose the attribute x. Importantly, different buyers have different preferences over the attribute x. For example, a firm may be unsure whether consumers prefer an innovative or a conventional product; a financial advisor may be unsure about clients' risk-return preferences; and in a mergers and acquisitions setting, a target firm may be unsure as to whether the bidding firms' technology is a complement or a substitute to its own technology.

Below, we expand on several applications for which the mapping from our model to the application is more involved. Although we believe our model is relevant for both corporate and non-corporate settings, we emphasize corporate applications, and discuss non-corporate applications more briefly, with details relegated to a supplementary online appendix.

3.1 Conflict between debt and equity

A leading case of distinct investor preferences is that between equity- and debt-holders, where different preferences stem from the different structure of these securities.

Specifically, consider a firm that anticipates that it will issue new securities in the future. With some probability q it will issue a quantity κ_1 of equity, while with probability $1-q$ it will issue κ_2 of debt. For simplicity we take the firm's issuance decision as exogenous, and uncorrelated with factors determining the firm's cash flow. The firm's future cash flow y is a random variable. The firm does not know its future cash flow realization, but it does know its type, x, which determines the distribution of y. For example, x may represent the firm's choice of projects, which affect both the mean and variance of cash flows. The firm can disclose x.

Let $g_{\{1\}}(x)$ and $g_{\{2\}}(x)$ respectively be an investor's valuation of κ_1 units of equity and the κ_2 units of debt if the investor knows the firm's type is x. Notably, the ordinal properties of $g_{\{1\}}$ and $g_{\{2\}}$ may differ. In particular, whenever x conveys information about the volatility of future cash flows, standard arguments imply that equity and debt values move in opposite directions.

3.2 Conflict between investors and regulators (or labor union, tax authority, or competitor)

In the introduction we discussed the case of a firm choosing whether to disclose its expected cash flow x to an audience composed of investors and a regulator, with the firm uncertain about whether the regulator acts uniformly, and treats all firms the same; or instead is discriminatory, and treats more harshly firms that it believes have higher cash flows.

We formalize this case as follows. Let $n=3$ (three receivers); g_1 a strictly increasing function, $g_2(x) \equiv -\kappa$, some constant κ, and g_3 some strictly decreasing function, with u_2 and u_3 both linear (receiver 1 represents investors, receiver 2 is the uniform regulator, and receiver 3 is the discriminatory regulator); and $\mathcal{N} = \{\{1,2\},\{1,3\}\}$ (the audience either consists of investors and a uniform regulator, or investors and a discriminatory regulator).

By relabeling, our model also covers similar applications in which the regulator is replaced by a labor union, a tax authority, a competitor, or some combination of these entities.

3.3 Disclosure of imperfect signals of the underlying attribute

So far, we have considered cases in which the firm discloses information that investors and other receivers directly care about, such as cash flows or product characteristics. But in many cases, the information that a firm considers disclosing is instead valuable simply because it is correlated with what investors and other receivers ultimately care about. For example, and as discussed in the introduction, investors are likely to be interested in CEO compensation primarily because it represents a signal about, among other things, the likely quality of the CEO, and the corporate governance of the firm, both of which in turn affect future cash flows. Similarly, investors are likely to care about inventory levels primarily because of what they reveal about future sales and hence cash flows.

Importantly, in these cases investors may disagree about the correlation between the object being disclosed, and the future cash flows. For example, some investors may believe the correlation between CEO pay and future cash flows is positive, while others may believe just the opposite. The same is true for the correlation between inventory levels and future cash flows.

Formally, let y be future cash flow (or more generally, some other underlying attribute that receivers care about). The firm cannot disclose y, but can disclose some other quantity x (e.g., CEO pay, or inventory), which is potentially correlated with y. Audiences care about cash flows y, but do not have any direct preference over x. For simplicity, we assume audiences are risk neutral over y.

Although all audiences thus have the same preferences, they differ in what they believe x reveals about y. Specifically, all audiences have the same prior of the distribution of y, with support $[0, 1]$. However, they differ in their assessment of the distribution of the signal x conditional on y. For simplicity, we focus on a stark case to illustrate our results. Each audience believes that x is either perfectly correlated with y, and specifically equals y; or that x is perfectly negatively correlated with y, and specifically equals $1 - y$. Audience N attaches probabilities λ_N and $1 - \lambda_N$ to these two possibilities.

Consequently, audience N's conditional expectation of y after observing x is[8]

$$E^N[y|x] = \lambda_N x + (1 - \lambda_N)(1 - x). \tag{3}$$

From (3), one can see that if an audience N believes that the signal is sufficiently likely to be positively (negatively) correlated with the underlying attribute, that is, $\lambda_N > (<) 1/2$, the conditional expectation is increasing (decreasing) in x. This setting thus falls within our general framework where $g_N(x) = E^N[y|x]$.

Importantly, in this setting differences between audiences arise even though all audiences have the same preferences over the underlying attribute (e.g., they all prefer higher cash flows to lower cash flows), but differ in other information, which leads them to form different beliefs after disclosure.[9]

3.4 Disclosure of ratings

Closely related to the disclosure of imperfect signals discussed immediately above is the disclosure of a rating issued by a third party. To give a few examples: firms can decide whether to disclose credit or other ratings; students decide whether to disclose test scores; schools decide whether to disclose test scores; and restaurants decide whether to disclose quality ratings. In all these cases, receivers are likely to have the same intrinsic preferences over the underlying characteristics; for example, almost all restaurant patrons prefer better food safety. Nonetheless, and as surveyed by Dranove and Jin (2010), there are a number of settings in which firms do not disclose ratings, even when disclosure of the rating is costless.

The chief difference between the disclosure of ratings, and the disclosure of imperfect signals discussed in subsection 3.3, is that while an item such as CEO pay may

[8]In expression (3), an audience does not update its beliefs about whether x and y are positively or negatively correlated based on the observation of x. One interpretation is simply that different audiences have heterogenous prior beliefs about these possibilities. Alternatively, if y is symmetrically distributed over $[0, 1]$, then the observation of x does not generate any updating; in this case, (3) is consistent with audiences starting from a common prior, but different audiences subsequently observing different pieces of information that lead to different posteriors on whether x and y are positively or negatively correlated.

[9]Note that the heterogeneity in audience information is independent of the information the firm is disclosing, in contrast to Harbaugh and To (2017) and Quigley and Walther (2018). Related, the forces behind silence in our paper are very different from in these papers, as evidenced by the fact that firm risk-aversion plays a critical role in our results (see Proposition 2), while coarse disclosure and disclosure costs respectively play a critical role in Harbaugh and To (2017) and Quigley and Walther (2018).

plausibly be either positively or negatively correlated with future cash flows, even the most hardened skeptics of the information quality of ratings rarely argue that ratings are negatively correlated with the characteristic they purport to measure.

Nonetheless, and just as in subsection 3.3, differences in audience beliefs about signal structure can easily lead to different audiences to having different ordinal rankings of ratings. The intuition is follows. In many settings, the audience is concerned that a rating is only a very noisy signal of the truth. For many statistical distributions, this concern implies that if the firm discloses a very high rating, the audience attaches more probability to the rating being largely noise. Consequently, the posterior expectation $E[y|x]$ may decrease in the rating x.[10] In words: an audience may well regard a very favorable rating as being "too good to be true," and respond by largely ignoring the rating. Finally, the range of ratings values over which $E[y|x]$ declines in x naturally depends on audience beliefs about the statistical properties of the rating, and hence different audiences have different ordinal rankings over ratings.

Although these forces could in principle apply to the disclosure of credit and other ratings in corporate settings, casual empiricism suggests that the "too good to be true" response to very favorable ratings is of greater relevance in settings such as student test scores, school rankings, and food safety ratings (see Dranove and Jin, 2010).[11] For this reason, we relegate the formal version of the above arguments to an online appendix.

3.5 Disclosure prior to elections

The central forces in our model are plausibly very relevant in elections. In particular, electoral candidates are typically uncertain about the electorate's composition, and its preferences over both proposed policies and candidate attributes. As such, our framework provides a natural explanation for why candidates gain from remaining vague (i.e., staying silent) about both their stances on policy questions, and on their past behavior, even in spite of electorate suspicion about such equivocation.

[10] Dawid (1973) gives conditions under which $E[y|x] \to E[y]$ as x approaches its supremum.

[11] Relatedly, non-disclosure of credit ratings is an instance where the explanation proposed by Dye (1985) is straightforwardly applicable: Investors do not know whether or not a firm has obtained a rating from a ratings agency, and so do not know whether the firm has a rating to disclose. Moreover, the cost-based explanation of Grossman and Hart (1980) and Jovanovic (1982) would apply in cases where the firm's payment to the rating agency is partially contingent on whether the rating is actually released.

Although elections also occur in corporate settings—perhaps most notably, elections for board directors—the above issues seem considerably more important in political elections. Accordingly, we again relegate a formal treatment of disclosure prior to elections to an online appendix. However, one point that emerges from the formal analysis that is worth stressing here is that it illustrates that concavity in the sender's "utility" function v can arise from concavity in how policies and candidate attributes map to the probability of electoral success, as opposed to being determined by the sender's utility function.

4 Necessary conditions for silence

In this section we show how equilibria with silence can emerge in our setting, and derive a pair of necessary conditions. We start with the following simple example:

Example 1: Consider the investor-regulator application of the introduction, which we formalized in subsection 3.2. Recall that receiver 1 corresponds to investors, receiver 2 corresponds to a uniform regulator, and receiver 3 corresponds to a discriminatory regulator; and the audience is either $\{1, 2\}$ or $\{1, 3\}$. We adopt the following specific parameterization: $g_1(x) = x$, $g_2(x) = -1$, $g_3(x) = -2x$, u_i is linear for all receivers i, the firm is strictly risk-averse (v is strictly concave), the unconditional mean $E[x]$ of the firm's type is $\frac{1}{2}$, and the two audiences are equally probable.

Consequently, the aggregated preferences of audience $\{1, 2\}$ (investors and uniform regulator) are given by $g_{\{1,2\}}(x) = x - 1$, while the aggregated preferences of audience $\{1, 3\}$ are given by $g_{\{1,3\}}(x) = x - 2x = -x$.

There is an equilibrium in which all firm types stay silent, as follows. The firm's disclosure payoff function is $V^D(x) = \frac{1}{2}v(x-1) + \frac{1}{2}v(-x)$. In the claimed equilibrium, a firm's payoff from remaining silent is $\frac{1}{2}v(E[x]-1)+\frac{1}{2}v(E[x]-2E[x])$, which coincides with $V^D(E[x])$. The payoff to any firm $x \neq \frac{1}{2}$ from silence is hence strictly higher than the payoff from disclosure because V^D is a strictly concave function that is symmetric over $[0, 1]$, and hence obtains it maximum at $\frac{1}{2} = E[x]$.

In words: In the equilibrium described, disclosure induces a lottery over outcomes $x - 1$ and $-x$, depending on the regulator's type. The expectation of these lottery payoffs is $-\frac{1}{2}$. In contrast, silence induces a degenerate lottery with a certain outcome $-\frac{1}{2}$, obtained regardless of whether the regulator is uniform or discriminatory. The firm is risk-averse, and so strictly prefers silence, because it is safer.

As Example 1 makes clear, the two key properties driving equilibrium silence are (I) receivers differ in their preference orderings over some firm-types, which gives rise to a risky lottery; and (II) firm risk-aversion. We next establish the necessity of these two properties.

First, silence can only arise if at least some audiences differ in their preference orderings:

Proposition 1 *If there is no uncertainty over audience preference orderings, i.e., g_{N_1} is ordinally equivalent to g_{N_2} in the sense that $g_{N_1}(x) < (\leq)\, g_{N_1}(\tilde{x})$ if and only if $g_{N_2}(x) < (\leq)\, g_{N_2}(\tilde{x})$ for any $N_1, N_2 \in \mathcal{N}$, then disclosure occurs with probability* 1.[12]

By Proposition 1, uncertainty over only the *strength* of audience preferences for a higher value of x is insufficient to generate silence, since in this case all the audiences have ordinally equivalent preferences, and a version of the standard unravelling proof applies. In contrast, silence requires the firm to be unsure about whether an audience values higher or lower values of x, at least over some range.

To highlight this point, consider the following perturbation of Example 1:

Example 2: Identical to Example 1, except that $\mathcal{N} = \{\{2\}, \{3\}\}$, i.e., the firm discloses only to a regulator, though is still uncertain whether the regulator is uniform or discriminatory. Hence the disclosure payoff function is simply $V^D(x) = \frac{1}{2}v(-1) + \frac{1}{2}v(-2x)$, and in particular, is monotone decreasing. The standard unravelling argument applies, and the only equilibrium entails full disclosure.

Example 2 highlights the role of the investors in Example 1. Without investors (Example 2), the firm only faces uncertainty about the cardinal strength of the audience's preferences, and full unravelling occurs. With investors (Example 1), the firm faces uncertainty about the ordinal properties of the audience's preferences, and there is an equilibrium with full silence.

We also highlight that Proposition 1 is true even if g_N is non-monotone, illustrating that non-monotone audience preferences (and hence non-monotone firm payoffs) alone

[12]Note that Proposition 1 can be also stated with respect to receivers: silence can only arise if at least some receivers differ in their preference orderings. The logic is straightforward: if g_i is ordinally equivalent to g_j for any receivers i, j, it must be that g_{N_1} is ordinally equivalent to g_{N_2} for any audiences $N_1, N_2 \in \mathcal{N}$.

are not sufficient to generate silence in equilibrium. Roughly speaking, if g_N is non-monotone, but all audiences have ordinally equivalent preferences, the unravelling argument still applies after a change in variables from x to $g_N(x)$.

We turn now to our second necessary condition, firm risk-aversion. Recall that firm risk-aversion naturally arises from any of: concentrated ownership; managerial risk-aversion coupled with internal agency fictions; external financing frictions. If the firm is either risk-neutral or risk-loving, then unravelling occurs, and all firms disclose:

Proposition 2 *If the firm's utility function v is linear or strictly convex then disclosure occurs with probability* 1.

In particular, if the firm and receiver utility functions v and u_i are all linear, then one can simply switch variables from x to $E_N[g_N(x)]$, and apply the standard unravelling argument with respect to $E_N[g_N(x)]$. The proof of Proposition 2 extends this argument to cover convex v functions and concave u_i functions.

Remark: A separate point that Example 1 illustrates is that our setting regularly has multiple equilibria. Full-disclosure can always be supported as an equilibrium, simply by assigning off-equilibrium beliefs on silence that load on the type with the lowest utility from disclosure. Accordingly, our main results are concerned with characterizing silence equilibria when they exist, and with comparative statics on silence equilibria.

5 Silence is safest: Characteristics of silence equilibria

We next characterize silence equilibria. In light of Proposition 2, for the remainder of the paper we impose:

Assumption 3 *The firm's utility function v is strictly concave.*

In addition, we further assume that the receiver payoff functions g_i are concave. The reason for this assumption is that if instead the payoff functions are convex, silence creates a direct benefit to the firm via standard Jensen's inequality effects, which makes the economics underlying a silence equilibrium less interesting. We elaborate on this point in more detail in subsection 8.1.

Assumption 4 *For any $i \in \{1, 2, ..., n\}$, the receiver payoff function g_i is weakly concave.*

In many cases, Assumption 4 has a very natural economic interpretation. For example, in the regulator and tax authority applications of Section 2 and subsection 3.2, concavity corresponds to progressive "taxation" by the regulator or tax authority. In the debt-equity application of subsection 3.1, concavity (indeed linearity) arises if the attribute x corresponds to risky investments in market securities, so that overall firm value is constant in x (see online appendix for details). Moreover, concavity is also satisfied in the imperfect signal disclosure application in subsection 3.3 (see (3)) and in the rating disclosure application in subsection 3.4 (see online appendix for details).

Because Assumption 4 rules out a direct benefit to silence it strengthens Lemma 1 to

$$p_N \leq E_x [g_N(x)|\mathcal{I}] \leq g_N (E_x [x|\mathcal{I}]) . \tag{4}$$

Note, moreover, that Assumptions 3 and 4 imply that the disclosure utility V^D is strictly concave in the firm's type (see Figure 1).

5.1 Silence by firms with extreme types

Example 1 of Section 4 has no disclosure at all. However, this is an unusual case, in the sense that it can arise only if

$$\max_{\tilde{x}} E_N [v (g_N (\tilde{x}))] \leq E_N \left[v \left(p_N^S\right)\right], \tag{5}$$

which by (4) implies

$$\max_{\tilde{x}} E_N [v (g_N (\tilde{x}))] \leq E_N [v (g_N (E_x [x]))], \tag{6}$$

which requires the knife-edge condition $\arg\max_{\tilde{x}} E_N [v (g_N (\tilde{x}))] = E_x [x]$.

More generally, partial silence equilibria entail some firm-types disclosing and other types not disclosing. Specifically, any partial silence equilibrium has silence by extreme firm-types, and disclosure by intermediate firm-types, as illustrated in Figure 1:

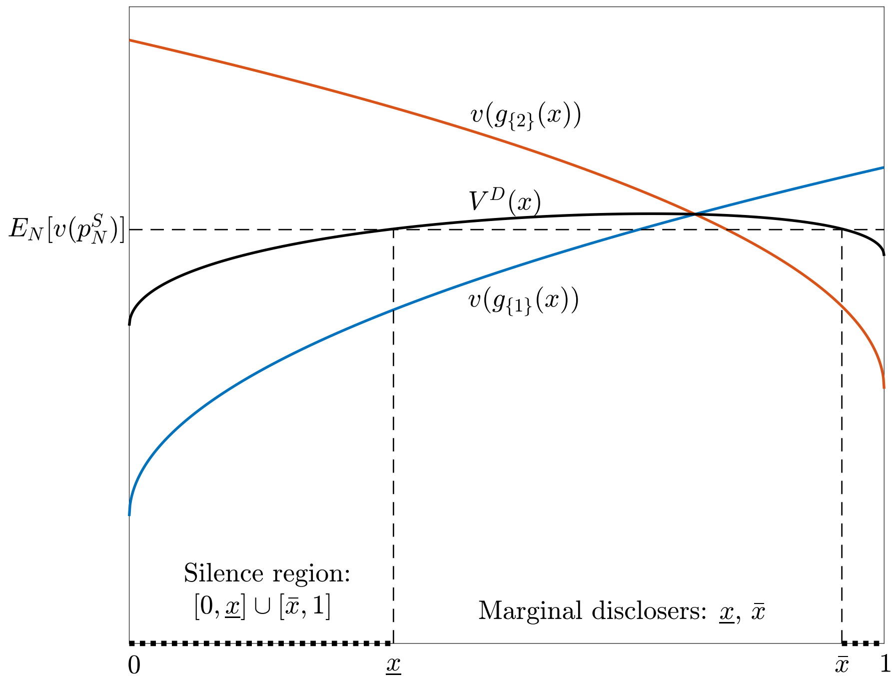

Figure 1: Illustration of a generic partial silence equilibrium

Proposition 3 *In any equilibrium with silence there exist $\underline{x}, \bar{x} \in (0,1)$ with $\underline{x} \leq \bar{x}$ such that all firms $x \in (\underline{x}, \bar{x})$ strictly prefer to disclose and all firms $x < \underline{x}$ and $x > \bar{x}$ strictly prefer silence. Moreover, $V^D(\underline{x}) = V^D(\bar{x}) = E_N\left[v\left(p_N^S\right)\right]$.*

If $\underline{x} = \bar{x}$ in Proposition 3, the equilibrium features *full* silence, as in Example 1 of Section 4. If instead $\underline{x} < \bar{x}$, the equilibrium features *partial* silence.

The proof of Proposition 3 is intuitive. Suppose first that firms sufficiently close to the extremes $0, 1$ do not disclose. If the equilibrium features partial silence, the continuity of V^D implies that there exist firms $\underline{x}$ and $\bar{x} > \underline{x}$ that are indifferent between disclosure and silence. Since silence delivers the same expected utility to all firm-types, the fact that both $\underline{x}$ and $\bar{x}$ are indifferent between disclosure and silence also implies $V^D(\underline{x}) = V^D(\bar{x})$. So if $x \in (\underline{x}, \bar{x})$ then, by the strict concavity of V^D,

$$V^D(x) > V^D(\underline{x}) = V^D(\bar{x}), \tag{7}$$

i.e., all firms in $(\underline{x}, \bar{x})$ strictly prefer disclosure to silence. Similarly, any firm with type below $\underline{x}$ or above $\bar{x}$ strictly prefers silence.

If instead the equilibrium features full silence, simply set $\underline{x} = \bar{x} = E_x[x]$. As noted above, full silence implies $\max_{\tilde{x}} E_N[v(g_N(\tilde{x}))] = E_N[v(g_N(E_x[x]))]$. Moreover, by (4) and (5), $\max_{\tilde{x}} E_N[v(g_N(\tilde{x}))] \leq E_N\left[v\left(p_N^S\right)\right] \leq E_N[v(g_N(E_x[x]))]$. It immediately follows that $E_N\left[v\left(p_N^S\right)\right] = E_N[v(g_N(E_x[x]))] = V^D(E_x[x])$.

Finally, what if at least one of firms $x = 0, 1$ discloses? If both firms $x = 0, 1$ disclose, then an analogue of (7) implies that all firms disclose. If instead just one of firms $x = 0, 1$ discloses, the concavity of V^D implies that the silence set is either a lower or upper interval of X. Lemma A-1 in the appendix formally rules out this possibility. The intuition is as follows. Economically, silence is attractive for extreme firm-types only if receivers interpret silence as meaning that the firm either has a very low or very high type, and so on average is of an intermediate type. In this case, silence allows an extreme type agent to replace a very risky lottery over prices $(g_N(x))_{N\in\mathcal{N}}$ with a safer lottery over more similar prices $\left(p_N^S\right)_{N\in\mathcal{N}}$.

In light of Proposition 3, we define a marginal discloser x_m as follows:

Definition 1 *In an equilibrium with silence, a firm-type x_m is a marginal discloser if $V^D(x_m) = E_N\left[v\left(p_N^S\right)\right]$.*

As we remarked earlier, whenever a silence equilibrium exists, there also exists an equilibrium with full disclosure. An additional form of multiplicity arises if the equilibrium condition $V^D(\underline{x}) = V^D(\bar{x}) = E_N\left[v\left(p_N^S\right)\right]$ has multiple solutions (recall that p_N^S is a function of $\underline{x}$ and $\bar{x}$). Whether such multiplicity arises is determined by the density f of the firm's type, on which we have imposed no assumptions. We phrase all results below in a way that allows for the existence of multiple silence equilibria. It is also worth noting that, given the concavity of V^D, an immediate corollary of Proposition 3 is that equilibria are straightforwardly ranked in terms of the sets of firm-types who disclose:

Corollary 1 *Suppose that multiple silence equilibria exist, and let $\{\underline{x}, \bar{x}\}$ and $\{\underline{x}', \bar{x}'\}$ be the marginal disclosers in two such equilibria. Then either $(\underline{x}, \bar{x}) \subset (\underline{x}', \bar{x}')$ or $(\underline{x}', \bar{x}') \subset (\underline{x}, \bar{x})$.*

5.2 Silence is safest

Our next result formalizes the idea that the lottery over $\left(p_N^S\right)_{N\in\mathcal{N}}$ is safer. That is, silence is safest. For use both here and below, we state the following mild condition, which guarantees strictness of some key inequalities:

Condition 1 *There is at least one receiver i for which either u_i or g_i is strictly concave.*

In particular, in any silence equilibrium Condition 1 strengthens inequality (4) to the strict inequality $p_N^S < g_N\left(E_x\left[x|\text{silence}\right]\right)$ for any audience containing i.

Proposition 4 *Consider an equilibrium with silence, and marginal disclosers $\underline{x}$ and $\bar{x}$, where $\underline{x} \leq \bar{x}$. Then*

$$\underline{x} \leq E_x\left[x|\textit{silence}\right] \leq \bar{x}, \tag{8}$$

and moreover, there is at least one marginal discloser $x_m \in \{\underline{x}, \bar{x}\}$ for which

$$E_N\left[p_N^S\right] \leq E_N\left[g_N\left(x_m\right)\right]. \tag{9}$$

All three inequalities are strict if the equilibrium has partial silence (i.e., $\underline{x} < \bar{x}$) and Condition 1 holds.

Equation (8) in Proposition 4 formalizes the idea that silence is attractive because receivers' equilibrium expectation of the firm's type given silence lies between the marginal discloser types $\underline{x}$ and $\bar{x}$. Inequality (9) says that the silence lottery is safer than the disclosure lottery of at least one of the marginal disclosers, in the following sense: since the lotteries provide the same expected utility to the firm (this is the definition of a marginal discloser), a lower expected payment implies that the lottery must be safer. In words, "silence is safest."

5.3 Existence of silence equilibria

Propositions 3 and 4 characterize silence equilibria, conditional on such equilibria existing. In general, an equilibrium with silence indeed exists provided that (I) at least some audiences have different preference orderings over extreme firm-types; (II) the probability of different audiences is such that extreme firm-types dislike disclosure sufficiently equally; and (III) receivers are not too risk-averse. Proposition 5 establishes existence of silence equilibria under these conditions.

The result requires some mild regularity conditions on audience preferences over extreme firm-types, and on the prior density f of extreme firm-types. For clarity, we state these regularity assumptions separately.

Assumption 5 *For all audiences $N \in \mathcal{N}$, the derivative $\frac{\partial v(g_N(x))}{\partial x}$ remains bounded as $x \to 0, 1$.*

Assumption 6 *For any constant $\kappa > 0$, $\lim_{x\to 0} \frac{f(x)}{f(1-\kappa x)}$ exists and is strictly positive.*

In addition, recall that at this point in the paper we have imposed Assumption 3, which states that the firm is strictly risk-averse.

Proposition 5 *Suppose that there are audiences $N_1, N_2 \in \mathcal{N}$ such that $g_{N_1}(0) < g_{N_1}(1)$ and $g_{N_2}(0) > g_{N_2}(1)$. Then an equilibrium with silence exists if the distribution of audiences $\{Pr(N)\}$ is such that $\left|V^D(0) - V^D(1)\right|$ is sufficiently small, and all receivers are sufficiently close to risk-neutral.*

The proof of Proposition 5 is based on standard fixed-point arguments, and we sketch a special case here to illustrate how it works. Let everything be the same as in the above Example 1, with the exception that now $E_x[x] \neq \frac{1}{2}$. A specific property of the example, which considerably simplifies the argument below, is that $p^S_{N_1} + p^S_{N_2} = -1$, so that the expected utility from silence is simply $\Pr(N_1)v\left(p^S_{N_1}\right) + \Pr(N_2)v\left(-1 - p^S_{N_1}\right) = V^D\left(p^S_{N_1} + 1\right)$.[13] Note that the condition that $\left|V^D(1) - V^D(0)\right|$ is sufficiently small is certainly satisfied, since in the example $V^D(0) = V^D(1)$.

To show that an equilibrium exists, we look for a candidate equilibrium in which types $X \backslash [\underline{x}, \bar{x}]$ stay silent and do not disclose, while types $[\underline{x}, \bar{x}]$ disclose. From Proposition 3, we know that any silence equilibrium has this structure. To this end, we vary the candidate value of $\underline{x}$ continuously from $\arg\max_{\tilde{x}} V^D(\tilde{x}) = \frac{1}{2}$ down to 0. The corresponding candidate value of $\bar{x} > \frac{1}{2}$ is determined by the equilibrium condition $V^D(\underline{x}) = V^D(\bar{x})$. Given candidate values of $\underline{x}$, $\bar{x}$, the corresponding payoffs associated with silence are $p^S_{N_1} = E_x[x|X \backslash [\underline{x}, \bar{x}]] - 1$ and $p^S_{N_2} = -E_x[x|X \backslash [\underline{x}, \bar{x}]]$.

On the one hand, at $\underline{x} = \bar{x} = \frac{1}{2}$, we know $p^S_{N_1} + 1 = E_x[x] \neq \frac{1}{2}$, so that $V^D(\underline{x}) > V^D\left(p^S_{N_1} + 1\right)$. That is, the firm $\underline{x}$ strictly prefers disclosure to silence, implying that full silence is not an equilibrium.

On the other hand, as $\underline{x}$ approaches 0, $\bar{x}$ approaches 1. Under the regularity conditions on the tails of the density function of Assumption 6, it follows that $p^S_{N_1} + 1$ is bounded away from both 0 and 1. Consequently, for all $\underline{x}$ sufficiently close to 0, we

[13]The proof in the appendix is general and does not rely on this property.

know $V^D(\underline{x}) < V^D\left(p^S_{N_1}+1\right)$, since V^D obtains its minimum value at the extremes $x = 0, 1$. In words, the firm $\underline{x}$ strictly prefers silence to disclosure as $\underline{x}$ approaches 0.

By continuity, it follows that there is at least one candidate equilibrium $\underline{x} \in \left(0, \frac{1}{2}\right)$ that satisfies the equilibrium condition $V^D(\underline{x}) = V^D(\bar{x}) = V^D\left(p^S_{N_1}+1\right)$.

Among other things, the above argument highlights the role of the condition in Proposition 5 that $\left|V^D(0) - V^D(1)\right|$ needs to be sufficiently small. This condition ensures that for any candidate specification of a marginal discloser with low type (i.e., a small $\underline{x}$), it remains possible to find a corresponding marginal discloser with high type (i.e., a large $\bar{x}$).

At the same time, it is worth emphasizing that Proposition 5 states just one set of sufficient conditions for silence. Silence equilibria can certainly exist even when $V^D(0)$ and $V^D(1)$ are very different.

6 Comparative statics

Given that a key economic force driving equilibrium silence is that silence reduces risk faced by firms, especially for firms with extreme types, it is natural to conjecture that silence is increasing in firm risk-aversion. Propositions 6 and 7 make this intuition precise. It is also natural to expect that disclosure is increasing in receiver risk-aversion because silence exposes receivers to risk by reducing their ability to differentiate between different firm-types, and thus a more risk-averse receiver is less willing to pay a high price to a non-disclosing firm. This is formalized in Proposition 8.

6.1 Increasing firm risk-aversion

Proposition 4 says that in a partial silence equilibrium, silence reduces risk for at least one of the marginal disclosers $\underline{x}$ and $\bar{x}$. Given this, a natural conjecture is that as firm risk-aversion increases, firms close to this marginal discloser are less likely to disclose, and more likely to remain silent. Concretely, variations in firm risk-aversion correspond to variation in ownership concentration, managerial risk-version, internal agency frictions, or external financing frictions.

For the case of two audiences ($|\mathcal{N}| = 2$), we can establish this result using Pratt's (1964) general ordering of risk preferences.

Proposition 6 *Suppose that $|\mathcal{N}| = 2$, Condition 1 holds, and that an equilibrium with partial silence exists when the firm's preferences are given by v. Suppose that the firm's preferences change to $\tilde{v} = \phi \circ v$ for some increasing and strictly concave ϕ, corresponding to greater risk-aversion. Then there is a marginal discloser x_m for whom silence is safer than disclosure in the original equilibrium, i.e., $E_N \left[p_N^S\right] < E_N \left[g_N \left(x_m\right)\right]$, and a new silence equilibrium under preferences $\tilde{v}$, such that silence strictly increases in the neighborhood of x_m.*

The restriction to two audiences in Proposition 6 is needed because, as is widely appreciated, it is hard to produce general comparative statics on choices between risky lotteries with respect to risk preferences (see, e.g., Ross (1981) for a discussion of this point), without imposing significant structure on either the utility function or on the distribution of payoffs. Specifically, with just two audiences we are able to show that, for at least one of the marginal disclosers $x_m \in \{\underline{x}, \bar{x}\}$, the prices associated with silence, i.e., $p_{N_1}^S, p_{N_2}^S$, lie within the range of possible prices associated with disclosure, i.e., lie in the interval $[\min \{g_{N_1}(x_m), g_{N_2}(x_m)\}, \max \{g_{N_1}(x_m), g_{N_2}(x_m)\}]$. This property allows us to apply results based on Pratt's ordering of risk preferences (specifically, Hammond (1974)).

For more than two audiences, we are unable to guarantee this property. Since we then lack structure on the distribution of payoffs, we must instead impose more structure on the set of utility functions to produce similar comparative statics with respect to firm risk-aversion. We have the following result:

Proposition 7 *Suppose that Condition 1 holds, and that an equilibrium with partial silence exists when the firm's preferences are given by v. Suppose that the firm's preferences change to $\tilde{v}$, where $\alpha \tilde{v}(x) + x = v(x)$ for some constant $\alpha > 0$, corresponding to greater risk-aversion. Then there is a marginal discloser x_m for whom silence is safer than disclosure in the original equilibrium, i.e., $E_N \left[p_N^S\right] < E_N \left[g_N \left(x_m\right)\right]$, and a new silence equilibrium under preferences $\tilde{v}$, such that silence strictly increases in the neighborhood of x_m.*

In words, the comparison of risk preferences used in Proposition 7 amounts to saying: preferences represented by $\tilde{v}$ are more risk-averse than preferences represented by v if v corresponds to a mixture of $\tilde{v}$ and risk neutral preferences. This ordering is closely related to Ross's (1981) notion of preferences becoming "strongly more risk averse." Note that in the specific case of mean variance preferences, this comparison corresponds to a greater dislike of variance.

6.2 Increasing receiver risk-aversion

Another interesting question is how equilibrium disclosure would change when the audience becomes more risk averse. To address this question formally, we consider an increase in receiver risk-aversion, also in the sense of Pratt. Intuitively, while silence helps risk-averse firms by delivering a safer lottery, it hurts risk-averse receivers, because it means that they buy an item of uncertain quality. Consequently, an increase in receiver risk-aversion reduces the prices paid to a non-disclosing firm. Hence higher risk-aversion of receivers makes silence less attractive for firms. Consequently, when the receivers, and thus the audience, become more risk averse, silence is reduced:

Proposition 8 *Suppose that Condition 1 holds and an equilibrium with silence exists when receivers' preferences are given by $\{u_i\}$. Suppose that some receiver j's preferences change to $\tilde{u}_j = \phi \circ u_j$ for some increasing and strictly concave ϕ, corresponding to greater risk-aversion of j. Then all equilibria feature strictly more disclosure than the equilibrium with the least amount of disclosure under $\{u_i\}$.*

Note that, in our setting, disclosure by a firm eliminates all risk for the audience. However, the economic force in Proposition 8 continues to hold even in situations where disclosure reduces the risk faced by the audience, instead of completely eliminating it.

7 Targeted disclosure and Regulation Fair Disclosure

So far, we have assumed that all disclosure is fully public, in the sense that it is received by all members of the audience. Here, we briefly explore the implications of instead allowing the firm to exclude some subset of receivers from receiving the disclosure. By considering this extension of targeted disclosure, we are then able to analyze the consequences of rules such as the U.S. Regulation Fair Disclosure (Reg FD) that mandate that any disclosure by a public firm must be fully public, and so inhibit a firm's ability to exclude particular receivers. In particular, we show how rules mandating public disclosure may end up reducing total disclosure. This is consistent with the empirical evidence documented in Bailey et al (2002) that firms disclose less payoff-relevant information overall after the introduction of Reg FD.

A first and almost immediate prediction of our model is that if a firm could target disclosure to a subset of receivers, so as to remove uncertainty about audience preferences, then it would do so.

To fix ideas, consider the following perturbation of Example 1:

Example 3: The set of possible receivers is $\{1, 1', 2, 3\}$, where receivers 1 and $1'$ are different classes of investors, receiver 2 is the uniform regulator, and receiver 3 is the discriminatory regulator. Investors 1 and $1'$ have aligned preferences, i.e., for some $\lambda \in (0,1)$, $g_1(x) = \lambda x$ and $g_{1'}(x) = (1-\lambda)x$. Under the public disclosure benchmark (i.e., the case we have thus far focused on), the audience is either $\{1, 1', 2\}$ or $\{1, 1', 3\}$. All other elements are the same as Example 1.

We extend our model to allow for targeted disclosure by allowing the firm to exclude some receivers. Formally, we allow the firm to exclude some exogenously fixed subset of receivers $M \subset \{1, \ldots, n\}$. Hence firm-type x now chooses between three actions, namely: (I) disclose to whoever the audience is; (II) stay silent; or (III) disclose to whoever the audience is, while excluding receivers M.[14] A firm's choice of whether to exclude receivers M is not publicly observed, though clearly any receiver $i \in M$ who receives a disclosure can infer that the firm has not excluded M. The firm's payoff is determined exactly as before; however, if a firm discloses and excludes M, then receivers $i \in M$ have coarser information to form expectations of the firm's type x than receivers in $N \backslash M$.

As an application, in Example 3 we set $M = \{1', 2, 3\}$. That is: the firm could disclose to a subset of targeted investors, corresponding to receiver 1; or disclose publicly, in which case the regulator also observes the disclosure; or stay silent completely. We emphasize that in order to fit the applications we have in mind we have given the firm only limited ability to selectively disclose.[15]

The targeted disclosure prediction of our model is that in equilibrium there is full disclosure to receiver 1. This is simply the standard unravelling argument, which

[14] The first two actions are exactly as before; it is the third option that is new to this extension.

[15] We have also remained consistent with out main motivation, and continued to assume that the firm is unable to distinguish between the uniform and discriminatory regulator, i.e., between receivers 2 and 3.

applies since the firm has the option of disclosing just to receiver 1; and that when disclosing to receiver 1, the firm is completely certain about the preferences of the audience observing the disclosure (i.e., receiver 1).

Moreover, for λ sufficiently small (i.e., a small number of targeted investors), an equilibrium outcome of allowing this form of targeted disclosure in Example 3 is as follows. As just noted, all firms disclose to receiver 1. In addition, firms with cash flows close to $\frac{1}{2}$ also disclose publicly, while most firms exclude M, i.e., disclose only to the targeted investors (receiver 1).[16]

Now consider the effect of introducing a rule along the lines of Reg FD that prevents a firm from selectively disclosing to just the investor subset 1. In line with our discussion above, we assume that once the firm discloses to both investor subsets 1 and $1'$, the disclosure is public, and so is also observed by the regulator. That is, we interpret Reg FD as blocking the firm's ability to exclude the receiver subset $M = \{1', 2, 3\}$. Consequently, Reg FD returns the model to the case we have focused on, in which all disclosures are public. In this case, we are back to Example 1, and it is an equilibrium for all firms to remain silent, and disclose nothing. Hence Reg FD can end up reducing disclosure, instead of expanding it, in the sense that originally all firms disclose to at least a subset of investors, while after Reg FD is imposed all firms switch to complete silence.

The intuition of Reg FD discouraging disclosure is that a mandate that any disclosure be fully public reduces the ability of firms to control the amount of uncertainty about audience preferences that they face.[17]

[16]The argument that it is an equilibrium outcome for most firms to exclude M is as follows. Exactly the same argument as used in Example 1 establishes that if $\lambda = 0$ then it is an equilibrium for all firms to exclude M. By continuity, for a sufficiently small λ it is an equilibrium outcome for most firms to exclude M.

[17]Related but different from us, Guembel and Rossetto (2009) also argue that Reg FD may lead to less disclosure. In their model, unsophisticated receivers may misunderstand complex messages, and thus the firm prefer to disclose to sophisticated receivers only. Under Reg FD, therefore, the firm may prefer not to say anything rather than risk being misunderstood.

8 Discussion and extensions

8.1 Direct benefits to silence

The analysis of Sections 5 and 6 is all conducted under Assumption 4, which states that the payoff functions g_i are weakly concave. In this subsection, we briefly relax this assumption and explore the opposite case in which the payoff functions are strictly convex. As we noted when introducing Assumption 4, convexity of g_i (and hence the resulting convexity of g_N) introduces a direct gain to silence. Here we illustrate this point in more detail. Although this is not uninteresting, this force is separate from the effects due to firm uncertainty about the receiver's type, and firm risk-aversion, both of which are necessary for silence, and so are central effects we wish to study.

We focus on the specific case in which all receivers have linear preferences u_j, and for all audiences N, there is a constant α_N such that $g_N(x) = v^{-1}(\alpha_N x)$. Since v is strictly concave, this implies that g_N is strictly convex. In this analytically very tractable case we show how the convexity of g_N generates a direct gain to silence, and in turn leads to an equilibrium with full silence. (In contrast, recall that, under Assumption 4, full silence is non-generic in the space of probability distributions over audiences.)

In this case, the firm's expected utility from disclosure, $V^D(x)$, is clearly linear. Assuming that α_N does not have the same sign for all audiences (see Proposition 1), we can choose probabilities $\{\Pr(N)\}$ such that V^D has a slope arbitrarily close to 0. And whenever the slope is sufficiently close 0, there is an equilibrium in which no one discloses, as we next show.

If all firms are silent, the expected utility from silence is

$$E_N\left[v\left(E_x\left[g_N(x)\right]\right)\right],$$

and so the expected utility gain from silence relative to disclosure for firm $\hat{x}$ is

$$E_N\left[v\left(E_x\left[g_N(x)\right]\right)\right] - V^D(\hat{x}) = E_N\left[v\left(E_x\left[g_N(x)\right]\right)\right] - E_N\left[v\left(g_N\left(E_x[x]\right)\right)\right] + V^D\left(E_x[x]\right) - V^D(\hat{x}). \tag{10}$$

The sense in which convexity of g_N generates a direct benefit to silence is then that, since g_N is strictly convex, for any audience,

$$E_x [g_N (x)] - g_N (E_x [x]) > 0.$$

Thus, the first difference in (10) is the direct benefit to silence induced by the convexity of g_N, which is bounded away from 0. The second term in (10) approaches 0 as the slope of V^D approaches 0. So provided probabilities $\{\Pr(N)\}$ are chosen so that V^D has a slope sufficiently close to 0, there is indeed an equilibrium in which no one discloses. As discussed, this equilibrium outcome is driven by the fact that silence generates a direct benefit.

8.2 Welfare consequences of mandated disclosure

In many circumstances, regulations and laws mandate disclosure. In cases where the standard unravelling argument applies, such regulations should have little effect on equilibrium outcomes and utilities. In contrast, in the cases we have characterized where the equilibrium outcome is less than full disclosure, such regulations clearly increase disclosure. This affects welfare differently for firms and receivers.

For firms, mandated disclosure can only lower welfare, since an unregulated firm always has the option of disclosing.

Under the competitive condition (1), receiver utility is always simply $u_i (0)$, so that receiver utility is unaffected by mandated disclosure. But more generally, one could imagine replacing (1) with alternative assumptions that leave receivers some surplus. (Such a change would not affect the key economic forces in our analysis.) In this case, mandated disclosure has the potential to increase receiver welfare, by reducing the risk to which they are exposed.

8.3 Generalized disclosure

Thus far, we have considered the case in which the firm either discloses that its type is in the singleton set $\{x\}$, or else discloses nothing. Here we consider instead the case in which the firm can disclose any member A of some family of sets $\mathcal{X}$, provided that $x \in A$. We assume that, at a minimum, $\mathcal{X}$ contains all singletons, all closed subintervals of the interval X, and all binary unions of closed subintervals of X. To avoid economically uninteresting mathematical complications, we assume that all members of $\mathcal{X}$ are closed. Note that silence simply corresponds to disclosing X.

This enlarged set of disclosure possibilities is most likely to be relevant if disclosure takes the form of a trustworthy auditor reporting a firm's type x to receivers; or alternatively, if severe ex-post penalties can be inflicted on firms who are found to have lied (see discussion in Glode et al (2018)). If instead disclosure takes the form of simply displaying some attribute to receivers, then our benchmark analysis so far covers the relevant case.[18]

Note that this expansion of the firm's disclosure possibilities does not affect standard unravelling results. Indeed, it is straightforward to adapt the proofs of Propositions 1 and 2 to show that, under the conditions stated in these results, in any equilibrium a firm discloses $\{x\}$ with probability one.

Our main result in this section is that, given the expanded set of disclosure possibilities, an equilibrium with less than full disclosure—"silence" in the sense that the firm does not fully disclose its type—exists under a very wide range of circumstances. This is true if the key conditions we identify in this paper are satisfied, namely, firm risk-aversion, differences in audience preferences, and receivers who are not too risk-averse. In particular, we are able to establish existence of an equilibrium with less than full disclosure without imposing the sufficient condition that $V^D(0)$ is sufficiently close to $V^D(1)$, which we used to establish Proposition 5.

Proposition 9 *If (A) there exist $\underline{\xi}, \bar{\xi} \in (0,1)$ and a pair of some audiences N_1, N_2 such that $\underline{\xi} \neq \bar{\xi}$, $V^D\left(\underline{\xi}\right) = V^D\left(\bar{\xi}\right)$, and $g_{N_1}(x) \neq g_{N_2}(x)$ for $x = \underline{\xi}, \bar{\xi}$, and (B) all receivers are sufficiently close to risk neutral, then there is an equilibrium with less than full disclosure, i.e., there is a positive probability of a firm disclosing a signal other than $\{x\}$.*

It is worth stressing that the condition (A) is satisfied whenever audiences have different preferences (g_{N_1} differs from g_{N_2} for at least some audiences N_1, N_2), and

[18]Specifically, Glode et al (2018) analyze a setting in which the sender can disclose any subset of the type space that includes its own type. Their analysis also differs from ours in two other important respects. First, the receiver has all the bargaining power, which implies that any sender obtains zero surplus if it fully discloses its type. Second, their paper is primarily concerned with the case in which the sender can commit to a disclosure rule before seeing its type. As an extension, they also consider the non-commitment case, and show that partial disclosure survives as an equilibrium, since given the bargaining power assumption the sender prefers to preserve some uncertainty about its type in order to obtain at least some informational rent.

these different preferences generate non-monotonicity of the expected utility from disclosing $\{x\}$, as given by the function V^D.

The proof of Proposition 9 is very close to previous analysis, and we give it here. We establish the existence of an equilibrium characterized by $\underline{x}, \bar{x} \in \left(\underline{\xi}, \bar{\xi}\right)$, in which firms with $x \in (\underline{x}, \bar{x})$ and $x \in X \backslash \left[\underline{\xi}, \bar{\xi}\right]$ disclose their exact type $\{x\}$; while the remaining firms with $x \in \left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$ disclose simply $\left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$.

The proof of Proposition 9 builds on the proof of Proposition 5. First, if one restricts firms to disclose either $\{x\}$ or $\left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$, the proof is the same as that of Proposition 5.[19]

It then remains to ensure that firms do not deviate to other disclosures. The equilibrium is supported by the following off-equilibrium beliefs: If the firm discloses $A \in \mathcal{X}$, and $A \neq \left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$, off-equilibrium beliefs place full mass on the firm's type being in $\arg\min_{\tilde{x} \in A} V^D(\tilde{x})$. These off-equilibrium beliefs immediately imply that firms with $x \in X \backslash \left(\left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]\right)$ do not have a profitable deviation. For firms with $x \in \left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$, note that these off-equilibrium beliefs ensure that any deviation is at least weakly worse than the deviation of disclosing $\{x\}$—which has already been established to be an unprofitable deviation, by the first step of the proof.

8.4 Possible empirical tests

The main empirical prediction of our model is that non-disclosure can arise even in cases in which receivers know that the firm has information to disclose, and in which there is no direct cost of disclosure (or, more generally, the direct costs of disclosure are very small). Moreover, our mechanism is based on very parsimonious assumptions, which are likely to hold in a wide array of circumstances.

As we have stressed, our proposed mechanism relies on two ingredients: uncertainty over audience preferences, and firm risk aversion. While it is likely to be challenging for an econometrician to isolate significant shocks to risk-aversion, legal and regulatory changes may lead to changes in the firm's uncertainty over audience preferences. In particular, anything that changes the feasibility of targeting disclosure to just a subset of receivers would allow one to test the key economic channel in our mechanism. The closest paper we are aware of that studies such an event is Bailey

[19] Indeed, the fact that $\underline{\xi}, \bar{\xi} \in (0,1)$ means that the proof avoids the complications of what happens to utility and density functions as $x \to 0, 1$, which is what allows use to dispense with the regularity conditions contained in Assumptions 5 and 6.

et al (2002)'s empirical study of Reg FD (see discussion in Section 7).

Another specific prediction of our model is that it is firms with extreme information who are likely to stay silent. A significant challenge in testing this prediction is that the econometrician does not observe the information possessed by firms that stay silent. One potential approach to overcoming this challenge would be to examine settings in which a new mandatory disclosure requirement is introduced, and in which the information being disclosed is persistent over time. In such cases, the econometrician is effectively able to observe the information of firms who stayed silent in the voluntary disclosure regime.

9 Conclusion

There are many settings in which voluntary disclosure is possible, but in which disclosure occurs with probabilities below 1, despite classic unravelling arguments. In this paper we explore a possible explanation, which is new to the literature, namely that potential disclosers do not know the preference ordering of the audience to whom they are disclosing, and because of risk-aversion they dislike the risk that this imposes. We show how these two features together naturally deliver equilibrium silence.

In contrast to existing leading explanations of silence, our explanation does not require disclosure to be either costly, or impossible for some (unobservable) subset of would-be disclosers. As such, our paper can explain silence even in settings where disclosure is costless, and there is no uncertainty about whether disclosure is possible.

Our explanation captures the intuitive notion that a firm may prefer to stay silent because anything that it says will make some audiences very unhappy, while staying silent avoids this extreme outcome. That is, silence is safest. Specifically, silence reduces the risk borne by potential disclosers with extreme information. Consequently, disclosure decreases when potential disclosers grow more risk-averse, in a sense we make precise. On the other hand, silence reduces the information available to the audience for disclosures, thereby increasing the risk borne by the audience. Because of this, potential disclosers benefit more from disclosing when audiences grow more risk-averse, leading to increased equilibrium disclosure.

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Appendix

Throughout the appendix, we denote by S the set of firm-types that stay silent in equilibrium.

Results omitted from main text

Lemma A-1 *Let Assumptions 1, 3 and 4 hold. Let* $\underline{x}, \bar{x}$ *be such that* $0 \leq \underline{x} < \bar{x} \leq 1$*; all firms in* $(\underline{x}, \bar{x})$ *disclose; and all firms* $x < \underline{x}$ *and* $x > \bar{x}$ *stay silent. Then* $E_x[x|S] \in [\underline{x}, \bar{x}]$*. Under Condition 1, moreover,* $E_x[x|S] \in (\underline{x}, \bar{x})$*.*

Proof of Lemma A-1: For any audience N, (4) implies

$$p_N^S \leq g_N\left(E_x[x|S]\right), \tag{A-1}$$

and hence

$$E_N\left[v\left(p_N^S\right)\right] \leq E_N\left[v\left(g_N\left(E_x[x|S]\right)\right)\right] = V^D\left(E_x[x|S]\right),$$

i.e., firm $E_x[x|S]$ weakly prefers disclosure to silence. By the strict concavity of v it follows that $E_x[x|S] \in [\underline{x}, \bar{x}]$. Finally, under Condition 1 inequality (A-1) holds strictly, implying that firm $E_x[x|S]$ strictly prefers to disclosure to silence, and so $E_x[x|S] \in (\underline{x}, \bar{x})$, completing the proof.

Proofs of results stated in main text

Proof of Lemma 1: Concavity of u_i and Jensen's inequality imply

$$u_i\left(E_x\left[g_i(x) - p_i|\mathcal{I}\right]\right) \quad \geq \quad E_x\left[u_i\left(g_i(x) - p_i\right)|\mathcal{I}\right] = u_i(0),$$

which in turn implies $p_i \leq E_x[g_i(x)|\mathcal{I}]$ and then immediately $p_N \leq E_x[g_N(x)|\mathcal{I}]$.

Proof of Proposition 1: Suppose to the contrary that the probability of silence is strictly positive. So there exists some non-zero-measure subset $S \subset [0,1]$ of firm-types who do not disclose.

Write $\mathcal{N} = \left\{N_1, N_2, \ldots, N_{|\mathcal{N}|}\right\}$. We recursively define $x_1, \ldots, x_{|\mathcal{N}|} \in S$ as follows. First, by Assumption 1, define $x_1 \in S$ such that $g_{N_1}(x_1) > E_x\left[g_{N_1}(x)|S\right]$. Next, suppose that $x_1, \ldots, x_{k-1}$ are defined, with the properties that $x_{k-1} \in S$, and $g_N(x_{k-1}) > E_x[g_N(x)|S]$ for all audiences $N = N_1, \ldots, N_{k-1}$. Then, define $x_k \in S$ such that $g_{N_k}(x_k) \geq g_{N_k}(x_{k-1})$ and $g_{N_k}(x_k) > E_x[g_{N_k}(x)|S]$. To see that such a choice is possible, note that if $g_{N_k}(x_{k-1}) > E_x[g_{N_k}(x)|S]$ then one can simply set $x_k = x_{k-1}$; while if instead $E_x[g_{N_k}(x)|S] \geq g_{N_k}(x_{k-1})$, by Assumption 1 there must exist $x_k \in S$ with $g_{N_k}(x_k) > E_x[g_{N_k}(x)|S] \geq g_{N_k}(x_{k-1})$. Since $g_{N_k}(x_k) \geq g_{N_k}(x_{k-1})$, by ordinal equivalence $g_N(x_k) \geq g_N(x_{k-1})$ for any audience N, and hence $g_N(x_k) > E_x[g_N(x)|S]$ for all audiences $N = N_1, \ldots, N_k$, establishing the recursive step.

So in particular, $v\left(g_N\left(x_{|\mathcal{N}|}\right)\right) > v(E_x[g_N(x)|S])$ for all audiences $N \in \mathcal{N}$. By Lemma 1, $E_x[g_N(x)|S] \geq p_N^S$. Hence $v\left(g_N\left(x_{|\mathcal{N}|}\right)\right) > v\left(p_N^S\right)$ for all audiences $N \in \mathcal{N}$. But this implies that firm $x_{|\mathcal{N}|} \in S$ would strictly gain by deviating and disclosing. The contradiction completes the proof.

Proof of Proposition 2: Suppose to the contrary that the probability of silence is strictly positive. So there exists some non-zero-measure subset $S \subset [0,1]$ of firm-types who disclose with probability below 1. Since any firm-type $x' \in S$ prefers silence to disclosure, Lemma 1 implies

$$E_N[v(g_N(x'))] \leq E_N\left[v\left(p_N^S\right)\right] \leq E_N[v(E_x[g_N(x)|S])].$$

Since v is weakly convex,

$$E_N[v(E_x[g_N(x)|S])] \leq E_N[E_x[v(g_N(x))|S]] = E_x[E_N[v(g_N(x))]|S].$$

Combining these two inequalities implies that, for any $x' \in S$,

$$E_N[v(g_N(x'))] \leq E_x[E_N[v(g_N(x))]|S].$$

If v is strictly convex, the above inequality is strict, giving a contradiction. If instead v is linear, this inequality contradicts Assumption 2, completing the proof.

Proof of Proposition 4: First, consider the case of an equilibrum with partial silence, i.e., $0 < \underline{x} < \bar{x} < 1$. By Lemma A-1, $E_x[x|S] \in [\underline{x}, \bar{x}]$, i.e., (8) holds; and moreover, (4) implies

$$E_N\left[p_N^S\right] \leq E_N[g_N(E_x[x|S])]. \tag{A-2}$$

By Lemma 1,

$$E_N\left[p_N^S\right] \leq E_N\left[E_x\left[g_N(x)|S\right]\right] = E_x\left[E_N\left[g_N(x)|S\right]\right],$$

and so certainly there exists $\hat{x} \in X \backslash [\underline{x}, \bar{x}]$ such that

$$E_N\left[p_N^S\right] \leq E_N\left[g_N(\hat{x})\right]. \tag{A-3}$$

Since g_N is concave (Assumption 4), it follows from (A-2) and (A-3) that either $E_N\left[p_N^S\right] \leq E_N\left[g_N(\underline{x})\right]$ or $E_N\left[p_N^S\right] \leq E_N\left[g_N(\bar{x})\right]$, establishing (9). Finally, under Condition 1 inequality (A-2) holds strictly and $E_x[x|S] \in (\underline{x}, \bar{x})$, in turn implying that (9) holds strictly.

Second, consider the case of an equilibrium with full silence, i.e., $\underline{x} = \bar{x}$. Then (6) implies

$$E_N\left[v\left(g_N\left(E_x[x]\right)\right)\right] = \max_{\tilde{x}} E_N\left[v\left(g_N(\tilde{x})\right)\right] = E_N\left[v\left(p_N^S\right)\right]. \tag{A-4}$$

Since V^D is strictly concave, $E_x[x]$ is the unique maximizer of V^D, and hence $\bar{x} = E_x[x]$. Moreover, (A-4) combines with (4) to imply $p_N^S = g_N\left(E_x[x]\right)$ for all audiences N, completing the proof.

Proof of Proposition 5: Under the stated conditions, there exists some distribution of audiences $\{\Pr(N)\}_{N\in\mathcal{N}}$ such that $V^D(0) = V^D(1)$. We establish the existence of a silence equilibrium for this distribution, and for the case in which all receivers are risk neutral (u_i linear for all $i \in \{1, 2, ..., n\}$). The general result then follows by continuity.

Because receivers are risk neutral, silence prices are simply given by $p_i^S = E_x\left[g_i(x)|S\right]$ and $p_N^S = E_x\left[g_N(x)|S\right]$.

Note that Assumptions 1 and 3 imply that V^D is strictly concave. Define $x_{\max} = \arg\max_{\tilde{x}} V^D(\tilde{x})$.

If $V^D(x_{\max}) \leq E_N\left[v\left(E_x\left[g_N(x)\right]\right)\right]$ then there is an equilibrium in which no firm discloses, and the proof is complete. So for the remainder of the proof, we consider the case in which

$$V^D(x_{\max}) > E_N\left[v\left(E_x\left[g_N(x)\right]\right)\right]. \tag{A-5}$$

For any $\underline{x} \in (0, x_{\max})$, define $\eta(\underline{x}) \in (x_{\max}, 1)$ by $V^D(\eta(\underline{x})) = V^D(\underline{x})$. Note that $\eta(\underline{x})$ exists and is unique, since $V^D(0) = V^D(1)$ and V^D is strictly concave.

Moreover, η is continuous, with $\eta(\underline{x}) \to 1$ as $\underline{x} \to 0$, and

$$\frac{\partial}{\partial \underline{x}} \eta\left(\underline{x}\right) = \frac{\left.\frac{\partial}{\partial x} V^D\left(x\right)\right|_{x=\underline{x}}}{\left.\frac{\partial}{\partial x} V^D\left(x\right)\right|_{x=\eta(\underline{x})}}.$$

Since $V^D\left(0\right) = V^D\left(1\right)$, and V^D is strictly concave, $\frac{\partial}{\partial x} V^D\left(x\right)$ remains bounded away from 0 as $x \to 0, 1$. Assumption 5 then implies that $\frac{\partial}{\partial \underline{x}} \eta\left(\underline{x}\right)$ remains bounded away from both 0 and $-\infty$ as $\underline{x} \to 0$. Assumption 6 and l'Hôpital's rule then imply that the following limit exists, and is bounded away from 0:

$$\lim_{\underline{x} \to 0} \frac{\int_0^{\underline{x}} f\left(x\right) dx}{\int_{\eta(\underline{x})}^1 f\left(x\right) dx} = -\lim_{\underline{x} \to 0} \frac{f\left(\underline{x}\right)}{f\left(\eta\left(\underline{x}\right)\right) \frac{\partial}{\partial \underline{x}} \eta\left(\underline{x}\right)}.$$

Strict concavity of v (Assumption 3) and the condition that there are audiences $N_1, N_2 \in \mathcal{N}$ such that $g_{N_1}\left(0\right) < g_{N_1}\left(1\right)$ and $g_{N_2}\left(0\right) > g_{N_2}\left(1\right)$ then implies that

$$\lim_{\underline{x} \to 0} E_N\left[v\left(E_x\left[g_N\left(x\right) | X \backslash \left[\underline{x}, \eta\left(\underline{x}\right)\right]\right]\right)\right] - E_N\left[E_x\left[v\left(g_N\left(x\right)\right) | X \backslash \left[\underline{x}, \eta\left(\underline{x}\right)\right]\right]\right] > 0. \quad \text{(A-6)}$$

Also note that

$$E_N\left[E_x\left[v\left(g_N\left(x\right)\right) | X \backslash \left[\underline{x}, \eta\left(\underline{x}\right)\right]\right]\right] = E_x\left[E_N\left[v\left(g_N\left(x\right)\right)\right] | X \backslash \left[\underline{x}, \eta\left(\underline{x}\right)\right]\right] = E_x\left[V^D\left(x\right) | X \backslash \left[\underline{x}, \eta\left(\underline{x}\right)\right]\right].$$

Hence, and using $V^D\left(0\right) = V^D\left(1\right)$,

$$\lim_{\underline{x} \to 0} \left(E_N\left[E_x\left[v\left(g_N\left(x\right)\right) | X \backslash \left[\underline{x}, \eta\left(\underline{x}\right)\right]\right]\right] - V^D\left(\underline{x}\right)\right) = 0. \quad \text{(A-7)}$$

It follows by (A-6) that

$$V^D\left(\underline{x}\right) - E_N\left[v\left(E_x\left[g_N\left(x\right) | X \backslash \left[\underline{x}, \eta\left(\underline{x}\right)\right]\right]\right)\right] < 0$$

for all $\underline{x}$ sufficiently close to 0.

Combined with (A-5), continuity then implies that there exists some $\underline{x} \in (0, x_{\max})$ such that

$$V^D\left(\underline{x}\right) = V^D\left(\eta\left(\underline{x}\right)\right) = E_N\left[v\left(E_x\left[g_N\left(x\right) | X \backslash \left[\underline{x}, \eta\left(\underline{x}\right)\right]\right]\right)\right].$$

Hence there is an equilibrium in which firms $\left[\underline{x}, \eta\left(\underline{x}\right)\right]$ disclose, while firms $X \backslash \left[\underline{x}, \eta\left(\underline{x}\right)\right]$

remain silent and do not disclose, completing the proof.

Proof of Proposition 6: Consider any partial silence equilibrium, with a silence set $[0, \underline{x}) \cup (\bar{x}, 1]$.

Claim A: For each audience N, $p_N^S \leq \max\{g_N(\underline{x}), g_N(\bar{x})\}$.

Proof of claim: If g_N is monotone over $[\underline{x}, \bar{x}]$, then

$$p_N^S \leq E_x[g_N(x)|S] \leq g_N(E_x[x|S]) \leq \max\{g_N(\underline{x}), g_N(\bar{x})\},$$

where the first inequality follows from Lemma 1, the second inequality follows from Jensen's inequality and the concavity of g_N, and the last inequality follows from Proposition 4 and the monotonicity of g_N over $[\underline{x}, \bar{x}]$.

If instead g_N is non-monotone over $[\underline{x}, \bar{x}]$, then by concavity, it is strictly increasing over $[0, \underline{x})$ and strictly decreasing over $(\bar{x}, 1]$. Hence $g_N(x) < \max\{g_N(\underline{x}), g_N(\bar{x})\}$ for all $x \in [0, \underline{x}) \cup (\bar{x}, 1]$. So by Lemma 1,

$$p_N^S \leq E_x[g_N(x)|S] < \max\{g_N(\underline{x}), g_N(\bar{x})\}.$$

Claim B: For some $x \in \{\underline{x}, \bar{x}\}$, $p_{N_1}^S, p_{N_2}^S \in [\min\{g_{N_1}(x), g_{N_2}(x)\}, \max\{g_{N_1}(x), g_{N_2}(x)\}]$.

Proof of Claim: Now consider any silence equilibrium in which the silence set is $[0, \underline{x}) \cup (\bar{x}, 1]$. The equilibrium condition implies that $g_{N_1}(\bar{x}) - g_{N_1}(\underline{x})$ and $g_{N_2}(\bar{x}) - g_{N_2}(\underline{x})$ have opposite signs. Without loss, assume $g_{N_1}(\underline{x}) \leq g_{N_1}(\bar{x})$ and $g_{N_2}(\bar{x}) \leq g_{N_2}(\underline{x})$. So Claim A implies $p_{N_1}^S \leq g_{N_1}(\bar{x})$ and $p_{N_2}^S \leq g_{N_2}(\underline{x})$. The equilibrium condition then implies $p_{N_2}^S \geq g_{N_2}(\bar{x})$ and $p_{N_1}^S \geq g_{N_1}(\underline{x})$, and so $p_{N_1}^S \in [g_{N_1}(\underline{x}), g_{N_1}(\bar{x})]$ and $p_{N_2}^S \in [g_{N_2}(\bar{x}), g_{N_2}(\underline{x})]$.

If the sets $[g_{N_1}(\underline{x}), g_{N_1}(\bar{x})]$ and $[g_{N_2}(\bar{x}), g_{N_2}(\underline{x})]$ are ranked by the strong set order (Veinott, 1989) then the result is straightforward: If $[g_{N_1}(\underline{x}), g_{N_1}(\bar{x})] \preceq [g_{N_2}(\bar{x}), g_{N_2}(\underline{x})]$ under this order, then $p_{N_1}^S, p_{N_2}^S \in [g_{N_1}(\underline{x}), g_{N_2}(\underline{x})]$; while if instead $[g_{N_2}(\bar{x}), g_{N_2}(\underline{x})] \preceq [g_{N_1}(\underline{x}), g_{N_1}(\bar{x})]$, then $p_{N_1}^S, p_{N_2}^S \in [g_{N_2}(\bar{x}), g_{N_1}(\bar{x})]$.

Next, consider the cases where the two sets $[g_{N_1}(\underline{x}), g_{N_1}(\bar{x})]$ and $[g_{N_2}(\bar{x}), g_{N_2}(\underline{x})]$ are not ranked by the strong set order. There are two sub-cases. In the first sub-case, $[g_{N_1}(\underline{x}), g_{N_1}(\bar{x})] \subset [g_{N_2}(\bar{x}), g_{N_2}(\underline{x})]$, and so either $p_{N_2}^S \in [g_{N_2}(\bar{x}), g_{N_1}(\bar{x})]$ or $p_{N_2}^S \in [g_{N_1}(\underline{x}), g_{N_2}(\underline{x})]$ (or both), while both $p_{N_1}^S \in [g_{N_2}(\bar{x}), g_{N_1}(\bar{x})]$ and $p_{N_1}^S \in [g_{N_1}(\underline{x}), g_{N_2}(\underline{x})]$. In the second sub-case, $[g_{N_2}(\bar{x}), g_{N_2}(\underline{x})] \subset [g_{N_1}(\underline{x}), g_{N_1}(\bar{x})]$, and so either $p_{N_1}^S \in [g_{N_1}(\underline{x}), g_{N_2}(\underline{x})]$ or $p_{N_1}^S \in [g_{N_2}(\bar{x}), g_{N_1}(\bar{x})]$ (or both), while both

$p^S_{N_2} \in [g_{N_1}(\underline{x}), g_{N_2}(\underline{x})]$ and $p^S_{N_2} \in [g_{N_2}(\bar{x}), g_{N_1}(\bar{x})]$.

Claim C: If $x_m \in \{\underline{x}, \bar{x}\}$ and $p^S_{N_1}, p^S_{N_2} \in [\min\{g_{N_1}(x_m), g_{N_2}(x_m)\}, \max\{g_{N_1}(x_m), g_{N_2}(x_m)\}]$ then $E_N\left[p^S_N\right] \le E_N[g_N(x_m)]$.

Proof of Claim: If instead $E_N\left[p^S_N\right] > E_N[g_N(x_m)]$ then Theorem 3 of Hammond (1974) implies that $E_N\left[v\left(p^S_N\right)\right] > E_N[v(g_N(x_m))]$, contradicting the equilibrium condition.

Completing the proof: From above, for at least one $x_m \in \{\underline{x}, \bar{x}\}$, we know $p^S_{N_1}, p^S_{N_2} \in [\min\{g_{N_1}(x_m), g_{N_2}(x_m)\}, \max\{g_{N_1}(x_m), g_{N_2}(x_m)\}]$ and $E_N\left[p^S_N\right] \le E_N\left[g^S_N(x_m)\right]$, along with the equilibrium condition $E_N\left[v\left(p^S_N\right)\right] = E_N[v(g_N(x_m))]$. So for any increasing and strictly concave function ϕ, Theorem 3 of Hammond (1974) implies that

$$E_N\left[\phi\left(v\left(p^S_N\right)\right)\right] \ge E_N[\phi(v(g_N(x_m)))]. \tag{A-8}$$

Moreover, under Condition 1, Claim A holds strictly (by Proposition 4), and hence Claims B and C hold strictly also, and so (A-8) likewise holds strictly.

Given inequality (A-8), a straightforward modification of the argument in the proof of equilibrium existence in Proposition 5 implies that, for preferences $\tilde{v}$, there exists an equilibrium in which firms $[0, \underset{\sim}{x}) \cup (\tilde{x}, 1]$ do not disclose, where if $x_m = \underline{x}$ then $\underset{\sim}{x} > \underline{x}$, and if $x_m = \bar{x}$ then $\tilde{x} < \bar{x}$. This completes the proof.

Proof of Proposition 7: Given Proposition 3, when the firm's preferences are given by v, consider an equilibrium in which firms in $[0, \underline{x}) \cup (\bar{x}, 1]$ do not disclose. By Proposition 4, for some $x_m \in \{\underline{x}, \bar{x}\}$,

$$E_N\left[p^S_N\right] < E_N[g_N(x_m)]. \tag{A-9}$$

It follows that

$$E_N\left[\tilde{v}\left(p^S_N\right)\right] > E_N[\tilde{v}(g_N(x_m))], \tag{A-10}$$

since otherwise (A-9) and the definition that $v(x) = \alpha\tilde{v}(x) + x$ at all $x \in X$ implies that

$$E_N\left[v\left(p^S_N\right)\right] < E_N[v(g_N(x_m))],$$

contradicting the equilibrium condition when the firm's preferences are given by v. Given (A-10), the result follows as in the last step of the proof of Proposition 6.

Proof of Proposition 8: Consider the equilibrium with the least amount of disclosure. For any marginal discloser x_m the equilibrium condition $E_N\left[v\left(p_N^S\right)\right] = E_N\left[v\left(g_N\left(x_m\right)\right)\right]$ holds. Following the increase in receiver j's risk-aversion, if the silence set stays unchanged then p_j^S strictly decreases, and so does p_N^S for any audience N containing j. Hence, for both marginal disclosers $x_m \in \{\underline{x}, \bar{x}\}$ we have $E_N\left[v\left(p_N^S\right)\right] < E_N\left[v\left(g_N\left(x_m\right)\right)\right]$ for any audience N containing j. The result follows as in the last step of the proof of Proposition 6.

Online Appendix

Silence is safest: non-disclosure when
the audience's preferences are uncertain

Not for publication

Calculation for subsection 3.1

We show that Assumption 4 is satisfied in the debt/equity application of subsection 3.1 for the simple case noted following Assumption 4.

Let $E(x)$ and $D(x)$ be the value of debt and equity. Total firm value is $V(x) \equiv E(x) + D(x)$. In the case noted in the text following Assumption 4, x represents investments in market securities. In this case, V is simply a constant.

So provided that $E(x)$ is strictly monotone in x, one can change variables, and think of $\tilde{x} = E(x)$ as the firm's type. In this case, $g_{\{1\}}(\tilde{x}) = \tilde{x}$ corresponds to equity issuance, and $g_{\{2\}}(\tilde{x}) = V - \tilde{x}$ corresponds to debt issuance.

Proof of no updating in correlation discussion in subsection 3.3

Suppose x and y are both distributed over $[0,1]$; y has a fixed distribution; x is either perfectly correlated, and equals y, or is perfectly negatively correlated, and equals $1-y$.

By Bayes rule:

$$
\begin{aligned}
\Pr(+\text{ve corr}|x) &= \frac{\Pr(+\text{ve corr and } x)}{\Pr(+\text{ve corr and } x) + \Pr(-\text{ve corr and } x)} \\
&= \frac{\Pr(x|+\text{ve corr})\Pr(+\text{ve corr})}{\Pr(x|+\text{ve corr})\Pr(+\text{ve corr}) + \Pr(x|-\text{ve corr})\Pr(-\text{ve corr})}.
\end{aligned}
$$

Hence there is no updating after seeing x if

$$\Pr(x|+\text{ve corr}) = \Pr(x|-\text{ve corr}),$$

which is equivalent to

$$\Pr(y) = \Pr(1-y).$$

Rating disclosure application in subsection 3.4

Here we provide a formal mathematical treatment of the rating disclosure application in subsection 3.4. We highlight the idea of a firm refraining from disclosing a high rating due to the concern of the rating being perceived as "too good to be true," as discussed in subsection 3.4.

Suppose that the firm has a true underlying type or attribute, y, e.g., "quality". The members of any particular audience N are homogeneous. All audiences have the same preferences and are risk neutral. Neither the firm nor the audience knows y. Instead, the firm knows the realization of a signal x that is correlated with y, and is able to disclose x to audiences.

To capture ratings and potentially upward biases in rating, we assume that audiences first differ in their prior assessment of the distribution of the underlying attribute y;[20] we denote by $\psi_N(y)$ the density corresponding to the prior of audience N. Audiences also differ in their assessment of the distribution of the signal x conditional on the underlying attribute y, i.e., $H_N(x|y)$, the distribution of x conditional on y. Hence the conditional expectation of audience N, $E_N[y|x]$, potentially differs across types, both because of differences in priors about the underlying type, ψ_N, and differences in assessments of the process via which the signal is generated, $H_N(\cdot|\cdot)$.

As a simple example to illustrate how this can lead to different audience preference orderings over the disclosable signal x, consider the specific case in which the signal x is either perfectly correlated with the underlying attribute y, or is completely uncorrelated, with density $\phi(x)$. For example, a rating is either completely accurate, or is simply noise; or, in the inventory example, a firm's inventory is either completely driven by quality, or is unrelated to quality. A audience of type N attaches probabilities λ_N and $(1-\lambda_N)$ to these two possibilities. Without loss, if the signal y is perfectly correlated with x, it simply equals x.

In this case, upon observing signal x, audience N assesses the probability that it

[20]For transparency, in this application we directly consider aggregated audience beliefs, as opposed to receiver beliefs.

is perfectly correlated with the underlying attribute y as

$$\frac{\lambda_N \psi_N(x)}{\lambda_N \psi_N(x) + (1-\lambda_N)\phi(x)}. \tag{A-11}$$

Note that this expression depends both on audience N's prior assessment λ_N of how likely the signal is to be perfectly correlated, and on audience N's prior ψ_N of the distribution of the attribute.

The unconditional expectation of the attribute y is $E^N[y] = \int x\psi_N(x)\,dx$, where the superscript N denotes that the expectation is taken using audience N's priors. Since the signal x perfectly reveals the attribute if it is perfectly correlated, and provides no information if it is completely uncorrelated, audience N's conditional expectation of the attribute y after observing x is

$$E^N[y|x] = \frac{\lambda_N \psi_N(x)}{\lambda_N \psi_N(x) + (1-\lambda_N)\phi(x)}\left(x - E^N[y]\right) + E^N[y]. \tag{A-12}$$

As a simple parameterization, consider the case in which when the signal is uncorrelated with the attribute, it is drawn from an upper-triangular distribution over $[0,1]$, i.e.,

$$\phi(x) = 2x, \tag{A-13}$$

while audiences' priors follow a mixture of lower- and upper-triangular distributions, i.e., for audience N there is constant α_N such that

$$\psi_N(y) = 2(1-y)(1-\alpha_N) + 2y\alpha_N. \tag{A-14}$$

Among other interpretations, this parameterization captures in a simple way that ratings (i.e., the signal x) are upwards biased relative to the truth (i.e., the attribute y); in other words, ratings are "inflated".

In the following, we show that if an audience N has a sufficiently negative prior about the distribution of the attribute y (i.e., $\alpha_N < \hat{\alpha}(\lambda_N)$, for some $\hat{\alpha}(\lambda_N)$), the conditional expectation $E^N[y|x]$ is first increasing then decreasing in x, with the maximizing signal x itself increasing in the audience's assessment λ_N that the signal x is perfectly correlated with the attribute y. That is, higher signal realizations x reduce the audience's posterior of the correlation between the signal and the underlying attribute by enough that the audience's conditional expectation of the attribute declines towards its unconditional mean $E^N[y]$.[21] In contrast, if the audience has a more positive prior about the attribute (i.e., $\alpha_N \geq \hat{\alpha}(\lambda_N)$), the conditional expectation $E^N[y|x]$ is monotonically increasing in the signal x. Hence, this setting falls within our general framework, where $g_N(x) = E^N[y|x]$, and different audiences correspond to differences in priors of both the distribution of the underlying attribute, as parameterized by α_N, and of the correlation between the signal and the attribute, as parameterized by λ_N.

In detail, audience N's unconditional expectation of the attribute y is

$$E^N[y] = (1-\alpha_N)\frac{1}{3} + \alpha_N\frac{2}{3} = \frac{1+\alpha_N}{3}, \tag{A-15}$$

and the substituting (A-13) and (A-14) into (A-11) implies that, upon observing signal x, audience N assesses the probability that it is perfectly correlated with the underlying attribute as

$$\frac{\lambda_N(2(1-x)(1-\alpha_N)+2x\alpha_N)}{\lambda_N(2(1-x)(1-\alpha_N)+2x\alpha_N)+(1-p_N)2x} = \frac{\lambda_N(1-\alpha_N+x(2\alpha_N-1))}{\lambda_N(1-\alpha_N+x(2\alpha_N-1))+(1-\lambda_N)x}. \tag{A-16}$$

As one would expect, this probability is increasing in λ_N, the audience's prior that the signal x is perfectly correlated with the attribute y; and is also increasing in α_N for high signals $x > \frac{1}{2}$. By straightforward differentiation, it is decreasing in x (and strictly so if $\alpha_N < 1$).

Substituting (A-15) and (A-16) into (A-12) yields

$$\begin{aligned} E^N[y|x] &= \frac{\lambda_N(1-\alpha_N+x(2\alpha_N-1))}{\lambda_N(1-\alpha_N+x(2\alpha_N-1))+(1-\lambda_N)x}\left(x-\frac{1+\alpha_N}{3}\right)+\frac{1+\alpha_N}{3} \\ &= \lambda_N\frac{(2\alpha_N-1)x^2-\frac{2}{3}(\alpha_N^2+2\alpha_N-2)x-\frac{1}{3}(1-\alpha_N^2)}{x(1-2\lambda_N(1-\alpha_N))+\lambda_N(1-\alpha_N)}+\frac{1+\alpha_N}{3}. \end{aligned}$$

Differentiation yields[22]

[21] Although we demonstrate this in a highly parameterized setting, this property emerges much more widely, and Dawid (1973) gives conditions under which $E[y|x] \to E[y]$ as x approaches its supremum.

[22] To obtain the following expressions, note that for arbitrary constants a, b, c, d, e,

$$\frac{\partial}{\partial x}\frac{ax^2+bx+c}{dx+e} = \frac{adx^2+2aex+be-cd}{(dx+e)^2},$$

$$
\begin{aligned}
\frac{\partial}{\partial x} E^N[y|x] &= \lambda_N \frac{(2\alpha_N - 1)(1 - 2\lambda_N(1-\alpha_N))x^2 + 2(2\alpha_N - 1)\lambda_N(1-\alpha_N)x}{(\lambda_N(1-\alpha_N) + x(1 - 2\lambda_N(1-\alpha_N)))^2} \\
&+ \lambda_N \frac{\frac{1}{3}(1-\alpha_N^2)(1 - 2\lambda_N(1-\alpha_N)) - \frac{2}{3}(\alpha_N^2 + 2\alpha_N - 2)\lambda_N(1-\alpha_N)}{(\lambda_N(1-\alpha_N) + x(1 - 2\lambda_N(1-\alpha_N)))^2} \\
&= \lambda_N \frac{(2\alpha_N - 1)(1 - 2\lambda_N(1-\alpha_N))x^2 + 2(2\alpha_N - 1)\lambda_N(1-\alpha_N)x}{(\lambda_N(1-\alpha_N) + x(1 - 2\lambda_N(1-\alpha_N)))^2} \\
&+ \lambda_N \frac{\frac{1}{3}(1-\alpha_N)(1 + 2\lambda_N + \alpha_N(1 - 4\lambda_N))}{(\lambda_N(1-\alpha_N) + x(1 - 2\lambda_N(1-\alpha_N)))^2}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial^2}{\partial x^2} E^N[y|x] &= 2\lambda_N \frac{(2\alpha_N - 1)\lambda_N^2(1-\alpha_N)^2 - (1 - 2\lambda_N(1-\alpha_N))\frac{1}{3}(1-\alpha_N)(1 + 2\lambda_N + \alpha_N(1 - 4\lambda_N))}{(\lambda_N(1-\alpha_N) + x(1 - 2\lambda_N(1-\alpha_N)))^3} \\
&= 2\lambda_N(1-\alpha_N)\frac{(2\alpha_N - 1)\lambda_N^2(1-\alpha_N) - \frac{1}{3}(1 - 2\lambda_N(1-\alpha_N))(1 + 2\lambda_N + \alpha_N(1 - 4\lambda_N))}{(\lambda_N(1-\alpha_N) + x(1 - 2\lambda_N(1-\alpha_N)))^3}.
\end{aligned}
$$

First, we show that $\frac{\partial^2}{\partial x^2} E^N[y|x] < 0$. The denominator term $\lambda_N(1-\alpha_N) + x(1 - 2\lambda_N(1-\alpha_N))$ is positive, since it is just a rewriting of $\lambda_N \psi_N(x) + (1-\lambda_N)\phi(x)$. The numerator is negative, as follows. Note first that the numerator term is a quadratic in λ_N, which at $\lambda_N = 0$ evaluates as $-\frac{1}{3}(1+\alpha_N) < 0$ and at $\lambda_N = 1$ evaluates as $(1-\alpha_N)((2\alpha_N - 1) - (1 - 2(1-\alpha_N))) = 0$. So it is sufficient to show that the numerator is increasing in λ_N at $\lambda_N = 1$. The derivative of the numerator term with respect to λ_N is

$$
2\lambda_N(2\alpha_N - 1)(1-\alpha_N) + \frac{2}{3}(1-\alpha_N)(1 + 2\lambda_N + \alpha_N(1 - 4\lambda_N)) - \frac{1}{3}(1 - 2\lambda_N(1-\alpha_N))(2 - 4\alpha_N),
$$

which at $\lambda_N = 1$ evaluates as

$$
2(2\alpha_N - 1)(1-\alpha_N) + 2(1-\alpha_N)^2 - \frac{2}{3}(1 - 2(1-\alpha_N))(1 - 2\alpha_N) = \frac{2}{3}\left(a_N^2 + a_N + 1\right) > \frac{2}{3}\left(a_N + \frac{1}{2}\right)^2 \geq 0,
$$

completing the proof that $\frac{\partial^2}{\partial x^2} E^N[y|x] < 0$.

Next, we show that there exists some $\hat{\alpha} < \frac{1}{2}$ such that, if $\alpha_N < \hat{\alpha}$, $E^N[y|x]$ obtains

---

$$
\frac{\partial^2}{\partial x^2} \frac{ax^2 + bx + c}{dx + e} = 2\frac{ae^2 - d(be - cd)}{(dx + e)^3}.
$$

its maximum at a signal value strictly below 1. At $x = 0$,

$$\frac{\partial}{\partial x} E^N[y|x] = \lambda_N \frac{\frac{1}{3}(1-\alpha_N)(1+2\lambda_N+\alpha_N(1-4\lambda_N))}{(\lambda_N(1-\alpha_N)+x(1-2\lambda_N(1-\alpha_N)))^2} > 0,$$

where the inequality follows from the fact that $1+2\lambda_N+\alpha_N(1-4\lambda_N)$ is positive at both $\lambda_N = 0$ and $\lambda_N = 1$. At $x = 1$,

$$\frac{\partial}{\partial x} E^N[y|x] = \lambda_N \frac{(2\alpha_N - 1) + \frac{1}{3}(1-\alpha_N)(1+2\lambda_N+\alpha_N(1-4\lambda_N))}{(\lambda_N(1-\alpha_N)+x(1-2\lambda_N(1-\alpha_N)))^2}.$$

Note that if $\alpha_N = 0$ then this expression is strictly negative, while if $\alpha_N = \frac{1}{2}$ it is strictly positive. Hence there is some $\hat{\alpha} < \frac{1}{2}$ such that, if $\alpha_N < \hat{\alpha}$, $E^N[y|x]$ obtains its maximum at a signal value strictly below 1.

Finally, we show that for $\alpha_N < \hat{\alpha}$, $\arg\max E^N[y|x]$ is increasing in λ_N. To do so, it suffices to show that the denominator term of $E^N[y|x]$,

$$\begin{aligned} & (2\alpha_N - 1)(1-2\lambda_N(1-\alpha_N))x^2 + 2(2\alpha_N-1)\lambda_N(1-\alpha_N)x \\ + & \frac{1}{3}(1-\alpha_N)(1+2\lambda_N+\alpha_N(1-4\lambda_N)), \end{aligned}$$

is increasing in λ_N, i.e., that

$$-2(2\alpha_N-1)(1-\alpha_N)x^2 + 2(2\alpha_N-1)(1-\alpha_N)x + \frac{1}{3}(1-\alpha_N)(2-4\alpha_N) > 0,$$

i.e. (and recalling that $1 - 2\alpha_N > 0$),

$$x^2 - x + \frac{1}{3} > 0.$$

This is indeed true since

$$x^2 - x + \frac{1}{3} > x^2 - x + \frac{1}{4} = \left(x - \frac{1}{2}\right)^2 \geq 0.$$

Political disclosure prior to elections

Here we provide a stripped-down but formal treatment of the political disclosure application in subsection 3.5. In this case, the sender's payoffs do not stem from prices paid by buyers. This case also illustrates that the concavity of sender's preference function v need not stem from fundamental risk preferences.

Consider a political candidate facing a pool of voters. The candidate has an attribute (either innate, or a policy position) x. For example, x may represent the strength of a candidate's links to some industry; or his stance on trade agreements; or his personal income. The candidate does not know how voters respond to this attribute. In particular, with probability $\Pr(\{1\})$, voters are of type 1 in the sense that they like this attribute, and respond positively to higher values of x. In contrast, with probability $\Pr(\{2\})$, voters are of type 2 in the sense that they dislike this attribute, and respond negatively.

In addition, and regardless of whether the pool of voters is type 1 or 2, voters also weigh other factors when deciding whether to vote the candidate. These other factors are represented by δ, which is uniformly distributed over $[0, 1]$. Specifically, if the pool of voters is type i, the candidate wins the election if

$$\log\left(E_x\left[g_i(x)|\mathcal{I}\right] + \kappa_a\right) + \log\delta \geq \log\kappa_b,$$

so that voters' preferences over x are captured by the functions g_i, where g_1 is increasing and g_2 is decreasing; and κ_a and κ_b are parameters capturing details of the political process, and the characteristics of the candidate's opponent(s). Consequently, the candidate wins the election if $\delta \geq \frac{\kappa_b}{E[g_i(x)|\mathcal{I}]+\kappa_a}$, and so has a winning probability of

$$1 - \frac{\kappa_b}{E_x\left[g_i(x)|\mathcal{I}\right] + \kappa_a}.$$

Normalizing the candidate's winning payoff to 1, and defining $v(p) = 1 - \frac{\kappa_b}{p+\kappa_a}$, the candidate's expected utility is hence

$$\sum_{i=1,2} \Pr(\{i\}) v\left(E_x\left[g_i(x)|\mathcal{I}\right]\right),$$

which falls within our framework. Note that v is strictly increasing, and concave. Also note in this example an audience is equivelent to a receiver.