Silence is safest: non-disclosure when the audience's preferences are uncertain*

Philip Bond Yao Zeng
University of Washington

October 2018

Abstract

We examine voluntary disclosure in a setting where the would-be discloser ("sender") is risk-averse and uncertain about the audience's ("receiver's") preferences. We show that some senders stay silent in equilibrium, in contrast to classic "unravelling" results. Non-disclosure reduces the sensitivity of a sender's payoff to receivers' preferences, which is attractive to risk-averse senders, i.e., "silence is safest." Increases in sender risk-aversion reduce disclosure by sender-types who bear a higher risk under disclosure. In contrast, non-disclosure imposes risk on receivers, and consequently, increases in receiver risk-aversion increase disclosure. We discuss a variety of applications.

Keywords: information disclosure, risk-aversion, uncertainty, preferences.
JEL: D81, D82, D83, G14.

*We thank Bruce Carlin (discussant), Selman Erol, Robin Greenwood, Rick Harbaugh, Will Gornall, Steven Malliaris, Christina Zhu, Ben Zou, and seminar audiences at Columbia University, Johns Hopkins University, the University of Georgia, the University of Indiana, the University of Oxford, the University of Warwick, the University of Washington, the Federal Reserve Bank of Cleveland, the FTG-LSE summer conference and the UBC summer conference for useful comments. Any errors are our own.

1 Introduction

An important and long-standing question in the economics of information is whether voluntary disclosure leads to full disclosure. A compelling and intuitive argument, often described as the "unravelling" argument, suggests that it does.[1] In brief, the argument is that the firm, or more generally the "sender," with the most favorable information will voluntarily disclose. So the audience for the disclosure—the "receiver"—will interpret non-disclosure as indicating that the firm does not have the most favorable information. But given this, the firm with the second most favorable piece of information will disclose, and so on. All the firms thus disclose in the end.

Despite the force of the unravelling argument, the prediction of full disclosure appears too strong. There are many cases in which valuable information that is potentially disclosable is not disclosed. Firms do not voluntarily reveal all value-relevant information. Students do not always voluntarily disclose test scores. Politicians do not always voluntarily reveal past tax returns before elections. In such cases, potential disclosers believe that non-disclosure is in their best interests, even though audiences often interpret non-disclosure with skepticism. Moreover, the unravelling argument has the strong implication that disclosure laws and regulations are unnecessary, which is inconsistent with the vigorous arguments associated with the introduction of such rules.

In this paper, we give a new yet simple explanation for non-disclosure. Our explanation captures in a simple way the idea that some types of sender fear disclosure because, if they disclose, the information revealed will make someone unhappy; and consequently, that staying silent and not disclosing is the safest option. Moreover, our explanation has the advantage of applying even in cases where disclosure has no direct cost, and in which there is no uncertainty that the sender possesses information to disclose, which are arguably the leading existing explanations of non-disclosure (see Grossman and Hart, 1980; Jovanovic, 1982; Dye, 1985).

Our explanation has two key ingredients, both of which we show to be necessary for non-disclosure. The first ingredient is that the sender does not know the receiver's preferences. In particular, the sender does not know whether he would benefit from convincing the receiver that his type is "low," or "high." For example, a firm may be

[1] See Viscusi (1978), Grossman and Hart (1980), Milgrom (1981), Grossman (1981), and Milgrom and Roberts (1986). Dranove and Jin (2010) provide a recent survey of the literature.

unsure whether the primary recipient of disclosures is an investor, who it would like to convince that it has high cash flows; or another party, such as a regulator, labor union, tax authority, or competitor, who it would like to convince that it has low cash flows. To take another example, a politician who is considering disclosing past tax returns may be unsure whether voters wish to see high income (thereby indicating that he is rich and successful) or low income (thereby excusing the low taxes he is known to have paid). Many applications indeed feature receivers with different preferences, as we discuss in Section 3.

The second key ingredient in our analysis is sender risk-aversion.[2] Absent sender risk-aversion, sender uncertainty about receiver preferences is not enough to generate non-disclosure. The reason is that the expected payoff from disclosure can still be ordered, so that one can still identify senders with the highest incentive to disclose, and the unravelling argument still applies. Under risk-aversion, non-disclosure potentially delivers an additional benefit of making the sender's payoffs safer, thereby breaking the unravelling argument. We show that non-disclosure arises precisely when it is safer than disclosure.

In a little more detail, consider, for example, a firm with private information about the level of its cash flow, which it can voluntarily disclose. The firm is unsure whether it is disclosing to an investor, or to a regulator. If it discloses, the firm's payoff if the receiver is an investor is increasing in cash flow, whereas if the receiver is a regulator, the firm's payoff is decreasing in the cash flow it discloses. Thus, a disclosing firm faces a lottery over different outcomes, where the lottery realization depends on the receiver's type.[3] In many cases, firms with extreme cash flows—i.e., either very high, or very low—face a particularly high-risk lottery. For example, a firm with low cash flows experiences a very bad outcome if the receiver is an investor, but a very good outcome if the receiver is a regulator; while a firm with high cash flows experiences a very good outcome if the receiver is an investor, but a very bad outcome if the receiver is a regulator. In contrast, a firm with moderate cash flows experiences moderate outcomes regardless of the receiver's type.

[2]Note that even if the sender is a firm, risk-aversion is still a natural assumption if either the firm's managers are risk-averse and are exposed to firm outcomes, or if financing frictions lead to a firm value function that is concave.

[3]For readers concerned that, in reality, both the investor and regulator are always present, in subsection 3.2 below we present a variant of this setting where uncertainty stems from the harshness with which a firm expects to be treated by the regulator.

In a typical non-disclosure equilibrium in our setting, firms with extreme information stay silent and do not disclose, while firms with intermediate information disclose. Receivers correctly interpret non-disclosure as indicating extreme information—in the example above, either very low or very high cash flows. The receiver's response to non-disclosure is thus based on the average of these extremes. In particular, this means that, regardless of whether the receiver is an investor or a regulator, the receiver treats the firm as if its cash flow is moderate. As discussed above, firms with moderate cash flows face a lottery with low risk. So non-disclosure generates a lower-risk lottery for firms with extreme information, relative to the alternative of disclosing.

Given the economic forces underlying equilibrium non-disclosure and the failure of unravelling, it is natural to conjecture that non-disclosure becomes more likely as sender risk-aversion increases. Similarly, non-disclosure exposes receivers to risk by reducing their ability to differentiate between different sender-types. Consequently, non-disclosure becomes less likely as receiver risk-aversion increases. Section 6 formalizes these comparative statics.

Although our formal model is couched in terms of the sender being a seller and receivers being buyers, with different receiver-types corresponding to different buyer preferences, this formal framework covers a wide range of applications, including the regulator example discussed above. Other applications, which we detail in Section 3, include the disclosure of ratings (security ratings, student test scores, school or food safety ratings etc.); the disclosure of corporate news that is imperfectly correlated with firm type, such as inventory levels; and disclosure in political economy settings.

Previous research has identified other possible reasons for why full unravelling may not occur, and some senders choose to remain silent instead of disclosing. As noted above, the most widely applicable existing explanations are that full unravelling does not occur if disclosure is costly (Grossman and Hart, 1980; Jovanovic, 1982); and that full unravelling does not occur if there is some probability that the sender is unable to disclose (Dye, 1985).

While the assumptions of costly disclosure and unobservably impossible disclosure are certainly satisfied in some settings, there are also many settings in which disclosure is costless, and there is no uncertainty as to whether the sender is able to disclose, but voluntary disclosure does not generate full disclosure. For example, disclosure of tax returns by a politician is both costless, and known to be feasible with complete certainty. Moreover, accounting scholars have suggested that "big data"—i.e., the improvement of information technology and the resulting mass production of information—will likely reduce accounting and reporting costs, which implies lower disclosure costs and less uncertainty as to whether firms have information in the first place.[4] Our paper can explain non-disclosure in these settings where previous explanations cannot. Moreover, it captures precisely the idea that staying silent and not disclosing is the "safest" course of action.

Unravelling results have been generalized to wider classes of economies by papers such as Okuno-Fujiwara et al (1990) and Seidmann and Winter (1997).[5] Okuno-Fujiwara et al (1990) stress the importance of sender payoff monotonicity, and exhibit examples in which a failure of monotonicity blocks unravelling and leads to complete non-disclosure. However, we show that payoff non-monotonicity alone is not sufficient to block unravelling. Our paper can be viewed as identifying a set of economically relevant conditions under which partial non-disclosure emerges as an equilibrium outcome in a natural setting.

The literature on disclosure is large, and has suggested a number of further alternative explanations of non-disclosure as surveyed in Dranove and Jin (2010). Among them, some share our focus on receiver heterogeneity, though rely on very different economic forces. For example, Fishman and Hagerty (2003) show that non-disclosure arises if some receivers are unable to process the information content of disclosure. Harbaugh and To (2017) consider a setting in which the sender's type is drawn from the interval $[0, 1]$, but disclosures are restricted to specifying which element of a finite partition of $[0, 1]$ the type belongs to. Moreover, the receiver is endowed with a private signal about the sender's type. Consequently, the best senders in a partition element may prefer to remain silent in order to avoid mixing with mediocre senders in the same partition element, and thus the unraveling argument breaks down. Similarly, Quigley and Walther (2018) show that when disclosing is costly while the receiver

[4] See Warren et al (2015) for a survey.

[5] Giovannoni and Seidmann (2007) study a setting similar to Seidmann and Winter (1997), and characterize conditions under which no disclosure occurs. Differently from our paper, the sender knows the receiver's preferences. Instead non-disclosure arises because different sender types desire different receiver responses, as in the following simple example (which is closely related to examples in these two papers). The sender's type x is uniform over $[-1, 1]$, and the receiver takes an action a equal to his posterior estimate of x. If the sender's payoff is given by $-ax$, there is an equilibrium with no disclosure, since no disclosure yields a payoff of 0 for all sender types, while disclosure by type x yields a payoff of $-x^2$.

observes a separate noisy signal about the sender, the best sender may remain silent, rely on the receiver's signal, and thus save the disclosure cost. This then generates "reverse unraveling" in which other sender-types also remain silent in order to pool with higher sender-types.

Dutta and Trueman (2002), Suijs (2007), and Celik (2014) all analyze relatively special situations in which the sender is unsure how the receiver will respond to a disclosure. However, Dutta and Trueman (2002) assume that there is a strictly positive probability that the sender has nothing to disclose, and state that this is critical for their results. In Suijs (2007)'s environment (unlike ours), there is a direct benefit to non-disclosure.[6] In Celik (2014), a seller chooses whether to disclose a location on a Hotelling line, and *also* makes a take-it-ot-leave-it price offer to a buyer whose location on the Hotelling line is assumed to follow a uniform distribution.[7] The details of price formation are important: if instead there were several buyers in competition, the only equilibrium would be full disclosure.

2 Model

Consider a firm—henceforth, the sender—that sells an item with characteristic x to a buyer—henceforth, the receiver. The sender has type $x \in X$, where X is a compact interval of the real line.[8] The prior distribution of x has full support over X, and admits a density function f. We normalize the endpoints of X so that $X = [0, 1]$.

The sender's type is private information to the sender. The sender can, at zero cost, credibly disclose his type x to the receiver, or not disclose any information. The sender's utility is determined by the receiver's type, and the receiver's beliefs about the sender's type, as described below.

The receiver can be of $n \geq 2$ types, denoted by $i \in N \equiv \{1, 2, \ldots, n\}$. The probability of type i is q_i. The preferences of a receiver of type i are given by $u_i\left(g_i(x) - p_i\right)$,

[6]To be specific, in Suijs (2007)'s model, disclosure gives a payoff of either $U(0)$ or $U(1)$, with probabilities $1 - p(\phi)$ and $p(\phi)$ respectively, where ϕ is the sender's type. Non-disclosure gives payoffs of $U\left(\frac{1}{2}\right)$ and something at least $U(0)$, with corresponding probabilities, and *regardless* of receiver inferences about what non-disclosure means. So if the type space is such that $1 - p(\phi)$ is sufficiently high for all types, non-disclosure is an equilibrium.

[7]These assumptions imply that disclosing sellers at the ends of the line face a severe trade off between proposing a higher price and acheiving a reasonable sale probability.

[8]The assumption that X is compact ensures that there exists an equilibrium of the disclosure game we describe. If instead X is non-compact, it is straightforward to give examples in which no equilibrium exists.

where p_i is the endogenous price that receiver-type i pays to the sender, u_i is continuous, strictly increasing and weakly concave, and g_i is differentiable. The concavity of u_i captures the receiver's risk-aversion. The function g_i captures receiver i's valuation of a sender's type or attribute x, and hence determines receiver i's preference ordering over different sender-types. Note that we impose no assumption on the relationship between different g_i's or as to whether g_i is monotone or not.

The price p_i paid by receiver-type i is determined by the competitive condition

$$E_x \left[u_i \left(g_i(x) - p_i \right) | \mathcal{I} \right] = u_i \left(0 \right), \tag{1}$$

where $\mathcal{I}$ is the receiver's information (i.e., either the particular x the sender discloses, or nothing). Note that, for clarity, we typically write E_x to make clear the expectation is being taken over sender-types x, and correspondingly write E_i when the expectation is taken over receiver-types $i \in N$.

Consequently, any disclosure decision by the sender leads to a lottery over prices $(p_i)_{i \in N}$, where p_i is received with probability q_i. The sender's utility from receiving p_i is $v\left(p_i\right)$, where v is differentiable and strictly increasing. The sender's expected utility from the lottery $(p_i)_{i \in N}$ is thus

$$E_i \left[v \left(p_i \right) \right] = \sum_{i \in N} q_i v \left(p_i \right).$$

We postpone further assumptions on the shape of v (i.e., concavity versus convexity) until further below. As discussed in the introduction, the sender's risk preferences are an important determinant of equilibrium disclosure outcomes.

For use throughout, we denote the sender's expected utility from disclosing x by $V^D\left(x\right)$. This quantity is straightforward to calculate, since in this case the price the sender gets from a sender of type i is simply $p_i = g_i\left(x\right)$, and thus

$$V^D \left(x \right) \equiv E_i \left[v \left(g_i \left(x \right) \right) \right] = \sum_{i \in N} q_i v \left(g_i \left(x \right) \right).$$

We say an equilibrium features *full disclosure* if the probability that the sender discloses is 1. We say an equilibrium features *non-disclosure* if the probability that the sender discloses is strictly less than 1.

Throughout, we write $\left(p_i^{ND}\right)_{i \in N}$ for the prices received from the different receiver-types following non-disclosure. Note that these prices are endogenous, and are determined in equilibrium.

We make the following mild regularity assumptions, which rule out economically uninteresting outcomes in which unravelling does not occur because an interval of sender-types all derive exactly the same utility from disclosure. First, no receiver has flat preferences over the sender's type:

Assumption 1 *For any $i \in N$ and any subset $\tilde{X} \subset X$ with positive measure, there exists $\tilde{x} \in \tilde{X}$ such that $g_i(\tilde{x}) > E_x\left[g_i(x)\,|\tilde{X}\right]$.*

Second, the expected price (as opposed to utility) received after disclosure is not flat in the sender's type:

Assumption 2 *Either: For any subset $\tilde{X} \subset X$ with positive measure, there exists $\tilde{x} \in \tilde{X}$ such that $E_i[g_i(\tilde{x})] > E_x\left[E_i[g_i(x)]\,|\tilde{X}\right]$; or else the sender is strictly risk-averse.*

Note that Assumption 2 holds generically in the space of probability distributions over the receiver's type (as a consequence of Assumption 1). Moreover, Assumption 2 even allows the non-generic case of a flat expected price if the sender is strictly risk-averse. This is useful primarily because it enables us to use a very simple example in Section 4 to illustrate our results.

Before proceeding, we note the following straightforward result, which is directly implied by the receiver's (weak) risk-aversion, and which we use repeatedly:

Lemma 1 *For any receiver-type $i \in N$,*

$$p_i \leq E_x[g_i(x)|\mathcal{I}], \tag{2}$$

where the inequality is strict if u_i is strictly concave and the posterior of x given information $\mathcal{I}$ is non-degenerate.

3 Model applications

Our model is general enough to accommodate many economically relevant applications in which disclosing is costless. We have described the baseline model in terms of the sender being a firm that sells an item with characteristic x to buyers (the receivers). The seller chooses whether or not to disclose the characteristic x. Importantly, different buyers have different preferences over the characteristic x. To give a few examples: a firm may be unsure whether consumers prefer an innovative or a conventional product; a financial advisor may be unsure about clients' risk-return preferences; and in a mergers and acquisitions setting, a target firm may be unsure as to whether the bidding firms' technology is a complement or a substitute to its own technology.

Below, we expand on four applications for which the mapping from our model to the application is more involved.

3.1 Conflict between debt and equity

A leading case of distinct investor preferences in financial economics is that between equity- and debt-holders, where different preferences stem from the different structure of these securities.

A firm anticipates that it will need to raise funding in the future. With probability q_1 it will prefer to issue equity, but with probability q_2 it will prefer to issue debt, where for simplicity we take the firm's preference between debt and equity as exogenous.

The firm's future cash flow y is a random variable. The firm does not know its future cash flow realization, but it does know its type, x, which determines the distribution of y. For example, x may represent the firm's choice of projects, which affect both the mean and variance of cash flows. The firm can disclose x.

The firm has outstanding equity and debt, with values $E(x)$ and $D(x)$, and total firm value is $V(x) \equiv E(x) + D(x)$. For simplicity, we assume that the firm's future issue of equity and debt is sufficiently small that the new issue does not affect prices. Hence if κ_1 and κ_2 denote the small amount of equity and debt that the firm will issue, then $g_1(x) = \kappa_1 E(x)$ and $g_2(x) = \kappa_2 D(x)$.

3.2 Conflict between investors and regulators (or labor union, tax authority, or competitor)

A distinct application in corporate finance is that of a firm disclosing to both investors and a regulator (or a labor union, a tax authority, or a competitor) who have different preferences over the firm's cash flows, but the firm being unsure how harsh the regulator is. This application also illustrates that the sender's payoff does not necessarily stem from an explicit price paid by the receiver; and that sender uncertainity can arise even when the sender knows the identity of receivers, and their ordinal preferencs, but not the cardinal strength of these preferences.

To fix ideas, consider a firm choosing whether to disclose its expected cash flow, which we denote by x. For simplicity, we focus on an all-equity firm, and assume that the firm benefits from a higher share price (either because it intends to issue more equity, or because of managerial compensation contracts).

The disclosure will be received by both investors and a regulator. Also for simplicity, we assume that the investors of the firm are risk-neutral. The share price is thus $E[x|\mathcal{I}]$. In contrast, the regulator may take some regulatory action that negatively affects the firm, based on its belief about the firm's cash flow. However, the firm is unsure how harsh the regulator is. With probability q_i, the firm is facing a regulator whose negative impact on the firm is captured by a decreasing function $\tilde{g}_i$. Hence, by setting u_i to be linear, the firm's expected utility is

$$\sum_i q_i v(E[x + \tilde{g}(x)|\mathcal{I}]).$$

By setting $g_i(x) \equiv x + \tilde{g}_i(x)$, this specification falls within our framework, where the type-i "receiver" corresponds the composite of investors and the type-i regulator, whose harshness is captured by $\tilde{g}_i$. In particular, if $\tilde{g}_i$ has a gentle slope then g_i is increasing in cash flow x, while if $\tilde{g}_i$ is sharply decreasing, then g_i is likewise decreasing.

By relabeling, this structure also covers cases in which the regulator is replaced by a labor union, tax authority, or competitor, or indeed, some combination of these entities. We also note that, mathematically, this application is isomorphic to the application we discussed in the introduction, where the firm is unsure whether its disclosure will be received by investors, or by a regulator (see appendix for details).

3.3 Political elections

We next consider another important case in which the sender's payoffs do not stem from prices paid by buyers, that is, political elections. This case also illustrates that the concavity of sender's preference function v need not stem from fundamental risk preferences. We present a very stripped-down model of elections, though (as with elsewhere) it could be straightforwardly enriched.

Consider a political candidate facing a pool of voters. The candidate has an attribute (either innate, or a policy position) $x \in (0,1)$. For example, x may represent the strength of a candidate's links to some industry; or his stance on trade agreements; or his personal income. The candidate does not know how voters respond to this attribute. In particular, with probability q_1, voters are of type 1 in the sense that they like this attribute, and respond positively to higher values of x. In contrast, with probability q_2, voters are of type 2 in the sense that they dislike this attribute, and respond negatively.

In addition, and regardless of whether the pool of voters is type 1 or 2, voters also weight other factors when deciding whether to vote the candidate. These other factors are represented by δ, which is uniformly distributed over $[0,1]$. Specifically, if the pool of voters is type i, the candidate wins the election if

$$\log\left(E_x\left[g_i(x)|\mathcal{I}\right]+\kappa_a\right)+\log\delta\geq\log\kappa_b,$$

so that voters' preferences over x are captured by the functions g_i, where g_1 is increasing and g_2 is decreasing; and κ_a and κ_b are parameters capturing details of the political process, and the characteristics of the candidate's opponent(s). Consequently, the candidate wins the election if $\delta \geq \frac{\kappa_b}{E[g_i(x)|\mathcal{I}]+\kappa_a}$, and so has a winning probability of

$$1-\frac{\kappa_b}{E_x\left[g_i(x)|\mathcal{I}\right]+\kappa_a}.$$

Normalizing the candidate's winning payoff to 1, and defining $v\left(p\right)=1-\frac{\kappa_b}{p+\kappa_a}$, the candidate's expected utility is hence

$$\sum_{i=1,2} q_i v\left(E_x\left[g_i(x)|\mathcal{I}\right]\right),$$

which falls within our framework. Note that v is strictly increasing, and concave.

3.4 Disclosure of ratings and other signals of the underlying attribute

In many cases, the object the sender is able to verifiably disclose is distinct from the object that receivers care about. A leading example is that receivers care about the quality of the object the sender is selling, but the sender is only able to disclose something that is imperfectly correlated with quality, such as a rating issued by a third party (e.g., firms disclosing security ratings; students disclosing test scores; schools disclosing test scores; and restaurants disclosing quality ratings). An alternative example is a firm disclosing total sales, or inventory, or similar, which is correlated with quality (e.g., high sales might indicate high quality). Importantly, in this setting differences among receivers can arise even when all receivers have the same preferences over the underlying attribute (e.g., they all prefer higher quality to lower quality), but differ in other information, which leads them to form different posteriors after disclosure.[9]

Formally, suppose that the sender has a true underlying type or attribute, y, e.g., "quality." All receivers have the same preferences over quality: for simplicity, assume that if a receiver knew the seller's good were of quality y, he would value it at y. Neither the sender nor the receiver knows y, however. Instead, the sender knows the realization of a signal x that is correlated with y, and is able to disclose x to receivers.

Receivers potentially differ in their prior assessment of the distribution of the underlying attribute y; we denote by $\psi_i(y)$ the density corresponding to the prior of receiver-type i. Receivers also differ in their assessment of the distribution of the signal x conditional on the underlying attribute i, i.e., $H_i\left(x|y\right)$, the distribution of x conditional on y. Hence the conditional expectation of receiver-type i, $E_i\left[y|x\right]$, potentially differs across types, both because of differences in priors about the underlying type, $\psi_i(x)$, and differences in assessments of the process via which the signal is generated, $H_i\left(x|y\right)$.

As a simple example to illustrate how this can lead to different receiver preference orderings over the disclosable signal x, consider the specific case in which the signal

[9]Note that the heterogeneity in receiver information is independent of the information the sender is disclosing, in contrast to Harbaugh and To (2017) and Quigley and Walther (2018). Related, the forces behind non-disclosure in our paper are very different from in these papers, as evidenced by the fact that sender risk-aversion plays a critical role in our results (see Proposition 2), while coarse disclosure and disclosure costs respectively play a critical role in Harbaugh and To (2017) and Quigley and Walther (2018).

x is either perfectly correlated with the underlying attribute y, or is completely uncorrelated, with density $\phi(x)$. For example, a rating is either completely accurate, or is simply noise; or, in the inventory example, a firm's inventory is either completely driven by quality, or is unrelated to quality. A receiver of type i attaches probabilities λ_i and $(1-\lambda_i)$ to these two possibilities. Without loss, if the signal y is perfectly correlated with x, it simply equals x.

In this case, upon observing signal x, receiver i assesses the probability that it is perfectly correlated with the underlying attribute y as

$$\frac{\lambda_i \psi_i(x)}{\lambda_i \psi_i(x) + (1-\lambda_i)\phi(x)}. \tag{3}$$

Note that this expression depends both on receiver i's prior assessment λ_i of how likely the signal is to be perfectly correlated, and on receiver i's prior ψ_i of the distribution of the attribute.

The unconditional expectation of the attribute y is $E^i[y] = \int x\psi_i(x)\,dx$, where the superscript i denotes that the expectation is taken using receiver i's priors. Since the signal x perfectly reveals the attribute if it is perfectly correlated, and provides no information if it is completely uncorrelated, receiver i's conditional expectation of the attribute y after observing x is

$$E^i[y|x] = \frac{\lambda_i \psi_i(x)}{\lambda_i \psi_i(x) + (1-\lambda_i)\phi(x)}\left(x - E^i[y]\right) + E^i[y]. \tag{4}$$

As a simple parameterization, consider the case in which when the signal is uncorrelated with the attribute, it is drawn from an upper-triangular distribution over $[0,1]$, i.e.,

$$\phi(x) = 2x, \tag{5}$$

while receivers' priors follow a mixture of lower- and upper-triangular distributions, i.e., for receiver i there is constant α_i such that

$$\psi_i(x) = 2(1-x)(1-\alpha_i) + 2x\alpha_i. \tag{6}$$

Among other interpretations, this parameterization captures in a simple way that ratings (i.e., the signal x) are upwards biased relative to the truth (i.e., the attribute y).

In the appendix, we show that if a receiver i has a sufficiently negative prior about the distribution of the attribute y (i.e., $\alpha_i < \hat{\alpha}(\lambda_i)$, for some $\hat{\alpha}(\lambda_i)$), the conditional expectation $E^i[y|x]$ is first increasing then decreasing in x, with the maximizing signal x itself increasing in the receiver's assessment λ_i that the signal x is perfectly correlated with the attribute y. That is, higher signal realizations x reduce the receiver's posterior of the correlation between the signal and the underlying attribute by enough that the receiver's conditional expectation of the attribute declines towards his unconditional mean $E^i[y]$.[10] In contrast, if the receiver has a more positive prior about the attribute (i.e., $\alpha_i \geq \hat{\alpha}(\lambda_i)$), the conditional expectation $E^i[y|x]$ is monotonically increasing in the signal x. Hence, this setting falls within our general framework, where $g_i(x) = E^i[y|x]$, and different receiver-types correspond to differences in priors of both the distribution of the underlying attribute, as parameterized by α_i, and of the correlation between the signal and the attribute, as parameterized by λ_i.

4 Necessary conditions for non-disclosure

We start by showing how equilibria with non-disclosure can emerge in our setting, and deriving a pair of necessary conditions. The following simple example illustrates these necessary conditions:

Example: There are two receiver-types ($n = 2$), both of whom are risk-neutral (u_i is linear for $i = 1, 2$) and their preferences over sender-types are linear and symmetric ($g_1(x) = x$ and $g_2(x) = 1 - x$); the sender is strictly risk-averse (v is strictly concave); there is an equal probability of each receiver-type ($q_1 = q_2 = \frac{1}{2}$); and the unconditional mean $E[x]$ of the sender's type is $\frac{1}{2}$.

Under these conditions, there is an equilibrium with no disclosure at all, as follows. In such an equilibrium, non-disclosure results in prices

$$\begin{aligned} p_1^{ND} &= E[x] = \frac{1}{2} \\ p_2^{ND} &= E[1-x] = \frac{1}{2}, \end{aligned}$$

[10] Although we demonstrate this in a highly parameterized setting, this property emerges much more widely, and Dawid (1973) gives conditions under which $E[y|x] \to E[y]$ as x approaches its supremum.

and so the sender's expected utility from non-disclosure is simply $v\left(\frac{1}{2}\right)$.

On the other hand, if a sender of type x discloses, he faces a lottery over prices x and $1-x$, with a probability $\frac{1}{2}$ of each outcome. This lottery has an expected payoff of $\frac{1}{2}$. So, since the sender is risk-averse, he strictly prefers non-disclosure to disclosure.[11]

As the Example makes clear, the two key properties driving equilibrium non-disclosure are (I) receiver-types differ in their preferences over sender-types, at least over some range, which gives rise to a risky lottery over prices; and (II) sender risk-aversion. We next establish the necessity of these two properties.

First, non-disclosure can only arise if receiver-types differ in their preference orderings:

Proposition 1 *If there is no uncertainty over receiver preference orderings, i.e., g_i is ordinally equivalent to g_j in the sense that $g_i(x) < (\leq) g_i(\tilde{x})$ if and only if $g_j(x) < (\leq) g_j(\tilde{x})$ for all $x, \tilde{x} \in X$ and $i, j \in N$, then disclosure occurs with probability 1.*

By Proposition 1, non-disclosure requires the sender to be unsure about whether a receiver values higher or lower values of x, at least over some range. In contrast, uncertainty over the strength of receiver preferences for a higher value of x is not by itself enough to generate non-disclosure, since in this case a version of the standard unravelling proof applies.

We also highlight that Proposition 1 is true even if g_i is non-monotone, illustrating that non-monotone receiver preferences (and hence non-monotone sender payoffs) alone are not sufficient to generate non-disclosure in equilibrium. Roughly speaking, if g_i is non-monotone, but all receiver-types have ordinally equivalent preferences, the unravelling argument still applies after a change in variables from x to $g_i(x)$.

Proposition 1 highlights a feature of the investor-regulator example of subsection 3.2. If a sender knows the receiver's identity, and knows the receiver's ordinal preferences (e.g., the receiver is an investor, and investors pay more for shares in firms with high cash flows), then uncertainty about the cardinal strength of the receiver's preferences (e.g., how much more will investors pay for high cash flows?) is insufficient to escape the implication of full disclosure. However, the outcome is very

[11] A sender with type $x = \frac{1}{2}$ is indifferent.

different if the sender faces multiple classes of receiver, since in this case, uncertainty about the cardinal strength of the preferences of some class of receivers can lead to uncertainty about the *ordinal* properties of the "composite" receiver's preferences, potentially leading to equilibrium non-disclosure (see results below).

Second, if the sender is either risk-neutral or risk-loving, then unravelling occurs, and all senders disclose.

Proposition 2 *If the sender's utility function v is linear or strictly convex then disclosure occurs with probability* 1.

In particular, if the sender and receiver utility functions v and u_i are all linear, then one can simply switch variables from x to $E_i[g_i(x)]$, and apply the standard unravelling argument with respect to $E_i[g_i(x)]$. The proof of Proposition 2 extends this argument to cover convex v functions and concave u_i functions.

Remark: A separate point that the Example illustrates is that our setting regularly has multiple equilibria. Full-disclosure can always be supported as an equilibrium, simply by assigning off-equilibrium beliefs on non-disclosure that load on the type with the lowest utility from disclosure. Accordingly, our main results are concerned with characterizing non-disclosure equilibria when they exist, and with comparative statics on non-disclosure equilibria.

5 Silence in safest: Characteristics of non-disclosure equilibria

We next characterize non-disclosure equilibria. In light of Proposition 2, for the remainder of the paper we impose:

Assumption 3 *The sender's utility function v is strictly concave.*

In addition, we further assume that the receiver payoff functions g_i are concave. The reason for this assumption is that if instead the payoff functions are convex, non-disclosure creates a direct benefit to the sender via standard Jensen's inequality effects, which makes the economics underlying a non-disclosure equilibrium less interesting. We elaborate on this point in more detail in subsection 7.3.

Assumption 4 *For any $i \in N$, the receiver payoff function g_i is weakly concave.*

In many cases, Assumption 4 is easily satisfied and has a very natural economic interpretation. For example, in the debt-equity example of subsection 3.1, concavity (indeed linearity) arises if the attribute x corresponds to risky investments in market securities, so that overall firm value V is constant in x (see appendix for details). In the regulator and tax authority examples of subsection 3.2, concavity corresponds to progressive taxation by the regulator or tax authority. In the voting example of subsection 3.3, concavity roughly corresponds to type-1 voters having relatively flat preferences over ranges of the attribute x that type-2 voters feel very strongly about, and vice versa. Moreover, concavity is also satisfied in subsection 3.4 (see appendix for details).

Because Assumption 4 rules out a direct benefit to non-disclosure, it strengthens Lemma 1 to

$$p_i \quad \leq \quad E_x \left[g_i(x) | \mathcal{I} \right] \leq g_i \left(E_x \left[x | \mathcal{I} \right] \right). \tag{7}$$

Note, moreover, that Assumptions 3 and 4 imply that the disclosure utility V^D is strictly concave in the sender's type (see Figure 1).

5.1 Non-disclosure by senders with extreme types

The Example of Section 4 has no disclosure at all. However, this is an unusual case, in the sense that it can arise only if

$$\max_{\tilde{x}} E_i \left[v \left(g_i \left(\tilde{x} \right) \right) \right] \leq E_i \left[v \left(p_i^{ND} \right) \right], \tag{8}$$

which by (7) implies

$$\max_{\tilde{x}} E_i \left[v \left(g_i \left(\tilde{x} \right) \right) \right] \leq E_i \left[v \left(g_i \left(E_x \left[x \right] \right) \right) \right], \tag{9}$$

which requires the knife-edge condition $\arg\max_{\tilde{x}} E_i \left[v \left(g_i \left(\tilde{x} \right) \right) \right] = E_x \left[x \right]$.

More generally, non-disclosure equilibria entail some sender-types disclosing and other types not disclosing. Specifically, any non-disclosure equilibrium has non-disclosure by extreme sender-types, and disclosure by intermediate sender-types, as illustrated in Figure 1:

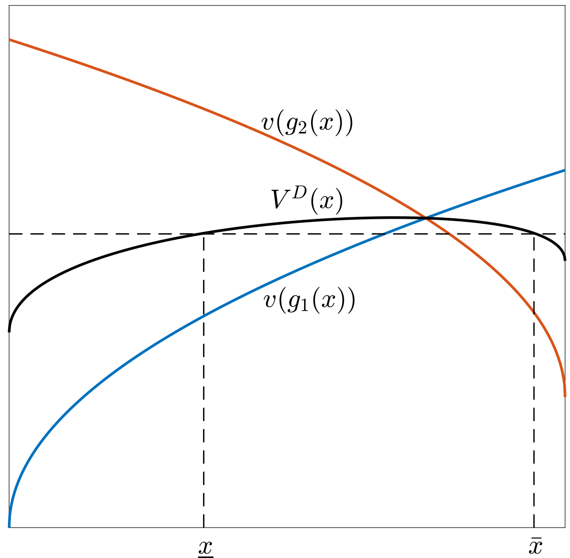

Figure 1: Illustration of a generic partial non-disclosure equilibrium

Proposition 3 *In any equilibrium with non-disclosure there exist* $\underline{x}, \bar{x} \in (0,1)$ *with* $\underline{x} \leq \bar{x}$ *such that all senders* $x \in (\underline{x}, \bar{x})$ *strictly prefer to disclose and all senders* $x < \underline{x}$ *any* $x > \bar{x}$ *strictly prefer non-disclosure. Moreover,* $V^D(\underline{x}) = V^D(\bar{x}) = E_i\left[v\left(p_i^{ND}\right)\right]$.

If $\underline{x} = \bar{x}$ in Proposition 3, the equilibrium features *full* non-disclosure, as in the Example of Section 4. If instead $\underline{x} < \bar{x}$, the equilibrium features *partial* non-disclosure.

The proof of Proposition 3 is intuitive. Suppose first that senders sufficiently close to the extremes $0, 1$ do not disclose. If the equilibrium features partial non-disclosure, the continuity of V^D implies that there exist senders $\underline{x}$ and $\bar{x} > \underline{x}$ who are indifferent between disclosure and non-disclosure. Since non-disclosure delivers the same expected utility to all sender-types, the fact that both $\underline{x}$ and $\bar{x}$ are indifferent between disclosure and non-disclosure also implies $V^D(\underline{x}) = V^D(\bar{x})$. So if $x \in (\underline{x}, \bar{x})$ then, by the strict concavity of V^D,

$$V^D(x) > V^D(\underline{x}) = V^D(\bar{x}), \tag{10}$$

i.e., all senders in $(\underline{x}, \bar{x})$ strictly prefer disclosure to non-disclosure. Similarly, any sender with type below $\underline{x}$ or above $\bar{x}$ strictly prefers non-disclosure.

If instead the equilibrium features full non-disclosure, simply set $\underline{x} = \bar{x} = E_x[x]$.

As noted above, full non-disclosure implies $\max_{\tilde{x}} E_i [v(g_i(\tilde{x}))] = E_i [v(g_i(E_x[x]))]$. Moreover, by (7) and (8), $\max_{\tilde{x}} E_i [v(g_i(\tilde{x}))] \leq E_i \left[v\left(p_i^{ND}\right)\right] \leq E_i [v(g_i(E_x[x]))]$. It immediately follows that $E_i \left[v\left(p_i^{ND}\right)\right] = E_i [v(g_i(E_x[x]))] = V^D(E_x[x])$.

Finally, what if senders sufficiently close to the extremes disclose? If this is true at both extremes $0, 1$, then an analogue of (10) implies that all senders disclose. If instead senders at only one extreme disclose, the concavity of V^D implies that the non-disclosure set is either a lower or upper interval of X. Lemma A-1 in the appendix formally rules out this possibility. The intuition is as follows. Economically, non-disclosure is attractive for extreme sender-types only if receivers interpret non-disclosure as meaning that the sender either has a very low or very high type, and so on average is of an intermediate type. In this case, non-disclosure allows an extreme type agent to replace a very risky lottery over prices $(g_i(x))_{i\in N}$ with a safer lottery over more similar prices $\left(p_i^{ND}\right)_{i\in N}$.

Consistent with the result that it is extreme types who do not disclose, Luca and Smith (2015) find that top business schools are least likely to disclose their rankings, whereas mid-ranked schools are most likely to disclose. Similarly, Bederson et al (2018) find that the highest-rated restaurants do not disclose their hygiene ratings, while relatively higher-rated restaurants disclose to stand out from other lower-rated ones.

In light of Proposition 3, we define a marginal discloser x_m as follows:

Definition 1 *In an equilibrium with non-disclosure, a sender-type x_m is a marginal discloser if $V^D(x_m) = E_i \left[v\left(p_i^{ND}\right)\right]$.*

As we remarked earlier, whenever a non-disclosure equilibrium exists, there also exists an equilibrium with full disclosure. An additional form of multiplicity arises if the equilibrium condition $V^D(\underline{x}) = V^D(\bar{x}) = E_i \left[v\left(p_i^{ND}\right)\right]$ has multiple solutions (recall that p_i^{ND} is a function of $\underline{x}$ and $\bar{x}$). Whether such multiplicity arises is determined by the density f of the sender's type, on which we have imposed no assumptions. We phrase all results below in a way that allows for the existence of multiple non-disclosure equilibria. It is also worth noting that, given the concavity of V^D, an immediate corollary of Proposition 3 is that equilibria are straightforwardly ranked in terms of the sets of sender-types who disclose:

Corollary 1 *Suppose that multiple non-disclosure equilibria exist, and let $\{\underline{x}, \bar{x}\}$ and*

$\{\underline{x}', \bar{x}'\}$ *be the marginal disclosers in two such equilibria. Then either* $(\underline{x}, \bar{x}) \subset (\underline{x}', \bar{x}')$ *or* $(\underline{x}', \bar{x}') \subset (\underline{x}, \bar{x})$.

5.2 Silence is safest

Our next result formalizes the idea that the lottery over $\left(p_i^{ND}\right)_{i \in N}$ is safer. That is, silence is safest. For use both here and below, we state the following mild condition, which guarantees strictness of some key inequalities:

Condition 1 *There is at least one receiver-type i for which either u_i or g_i is strictly concave.*

In particular, in any non-disclosure equilibrium Condition 1 strengthens inequality (7) to the strict inequality $p_i^{ND} < g_i\left(E_x\left[x|\text{non-disclosure}\right]\right)$ for at least some receiver-type i.

Proposition 4 *Consider an equilibrium with non-disclosure, and marginal disclosers $\underline{x}$ and $\bar{x}$, where $\underline{x} \le \bar{x}$. Then*

$$\underline{x} \le E_x\left[x|\textit{non-disclosure}\right] \le \bar{x}, \tag{11}$$

and moreover, there is at least one marginal discloser $x_m \in \{\underline{x}, \bar{x}\}$ for which

$$E_i\left[p_i^{ND}\right] \le E_i\left[g_i\left(x_m\right)\right]. \tag{12}$$

All three inequalities are strict if the equilibrium has partial non-disclosure (i.e., $\underline{x} < \bar{x}$) and Condition 1 holds.

Equation (11) in Proposition 4 formalizes the idea that non-disclosure is attractive because receivers' equilibrium expectation of the sender's type given non-disclosure lies between the marginal discloser types $\underline{x}$ and $\bar{x}$. Inequality (12) says that the non-disclosure lottery is safer than the disclosure lottery of at least one of the marginal disclosers, in the following sense: since the lotteries provide the same expected utility to the sender (this is the definition of a marginal discloser), a lower expected payment implies that the lottery must be safer. In words, "silence is safest."

5.3 Existence of non-disclosure equilibria

Propositions 3 and 4 characterize non-disclosure equilibria, conditional on such equilibria existing. In general, an equilibrium with non-disclosure indeed exists provided that (I) receivers have different preference orderings over extreme sender-types; (II) the probability of different receiver-types is such that extreme sender-types dislike disclosure sufficiently equally; and (III) receivers are not too risk-averse. Proposition 5 establishes existence of non-disclosure equilibria under these conditions.

The result requires some mild regularity conditions on receiver preferences over extreme sender-types, and on the prior density f of extreme sender-types. For clarity, we state these regularity assumptions separately.

Assumption 5 *For all receiver-types i, the derivative $\frac{\partial v(g_i(x))}{\partial x}$ remains bounded as $x \to 0, 1$.*

Assumption 6 *For any constant $\kappa > 0$, $\lim_{x \to 0} \frac{f(x)}{f(1-\kappa x)}$ exists and is strictly positive.*

In addition, recall that at this point in the paper we have imposed Assumption 3, which states that the sender is strictly risk-averse.

Proposition 5 *Suppose that there are receiver-types $i, j \in N$ such that $g_i(0) < g_i(1)$ and $g_j(0) > g_j(1)$. Then an equilibrium with non-disclosure exists if the distribution of receiver-types $(q_i)_{i \in N}$ is such that $\left|V^D(0) - V^D(1)\right|$ is sufficiently small, and all receiver-types are sufficiently close to risk-neutral.*

The proof of Proposition 5 is based on standard fixed-point arguments, and we sketch a special case here to illustrate how it works. Let everything be the same as in the above Example, with the exception that now $E_x[x] \neq \frac{1}{2}$. An important property of the Example, which considerably simplifies the argument below, is that $p_2^{ND} = 1 - p_1^{ND}$, so that the non-disclosure expected utility is simply $q_1 v\left(p_1^{ND}\right) + q_2 v\left(1 - p_1^{ND}\right) = V^D\left(p_1^{ND}\right)$.[12] Note that the condition that $\left|V^D(1) - V^D(0)\right|$ is sufficiently small is certainly satisfied, since in the Example $V^D(0) = V^D(1)$.

To show that an equilibrium exists, we look for a candidate equilibrium in which types $X \setminus [\underline{x}, \bar{x}]$ stay silent and do not disclose, while types $[\underline{x}, \bar{x}]$ disclose. From Proposition 3, we know that any non-disclosure equilibrium has this structure. To this end,

[12]The proof in the appendix is general and does not rely on this property.

we vary the candidate value of $\underline{x}$ continuously from $\arg\max_{\tilde{x}} V^D(\tilde{x}) = \frac{1}{2}$ down to 0. The corresponding candidate value of $\bar{x} > \frac{1}{2}$ is determined by the equilibrium condition $V^D(\underline{x}) = V^D(\bar{x})$. Given candidate values of $\underline{x}$, $\bar{x}$, the corresponding payoffs associated with non-disclosure are $p_1^{ND} = E_x[x|X \backslash [\underline{x}, \bar{x}]]$ and $p_2^{ND} = E_x[1-x|X \backslash [\underline{x}, \bar{x}]]$.

On the one hand, at $\underline{x} = \bar{x} = \frac{1}{2}$, we know $p_1^{ND} = E_x[x] \neq \frac{1}{2}$, so that $V^D(\underline{x}) > V^D\left(p_1^{ND}\right)$. That is, the sender $\underline{x}$ strictly prefers disclosure to non-disclosure, implying that full non-disclosure is not an equilibrium.

On the other hand, as $\underline{x}$ approaches 0, $\bar{x}$ approaches 1. Under the regularity conditions on the tails of the density function of Assumption 6, it follows that p_1^{ND} is bounded away from both 0 and 1. Consequently, for all $\underline{x}$ sufficiently close to 0, we know $V^D(\underline{x}) < V^D\left(p_1^{ND}\right)$, since V^D obtains its minimum value at the extremes $x = 0, 1$. In words, the sender $\underline{x}$ strictly prefers non-disclosure to disclosure as $\underline{x}$ approaches 0.

By continuity, it follows that there is at least one candidate equilibrium $\underline{x} \in \left(0, \frac{1}{2}\right)$ that satisfies the equilibrium condition $V^D(\underline{x}) = V^D(\bar{x}) = V^D\left(p_1^{ND}\right)$.

Among other things, the above argument highlights the role of the condition in Proposition 5 that $\left|V^D(0) - V^D(1)\right|$ needs to be sufficiently small. This condition ensures that for any candidate specification of a marginal discloser with low type (i.e., a small $\underline{x}$), it remains possible to find a corresponding marginal discloser with high type (i.e., a large $\bar{x}$).

At the same time, it is worth emphasizing that Proposition 5 states just one set of sufficient conditions for non-disclosure. Non-disclosure equilibria can certainly exist even when $V^D(0)$ and $V^D(1)$ are very different.

6 Comparative statics

Given that a key economic force driving equilibrium non-disclosure is that non-disclosure reduces the risk faced by senders, especially those with extreme types, it is natural to conjecture that non-disclosure is increasing in sender risk-aversion. Propositions 6 and 7 make this intuition precise. It is also natural to expect that disclosure is increasing in receiver risk-aversion because non-disclosure exposes receivers to risk by reducing their ability to differentiate between different sender-types, and thus a more risk-averse receiver is less willing to pay a high price to a non-disclosing sender. This is formalized in Proposition 8.

6.1 Increasing sender risk-aversion

Proposition 4 says that in a partial non-disclosure equilibrium, non-disclosure reduces risk for at least one of the marginal disclosers $\underline{x}$ and $\bar{x}$. Given this, a natural conjecture is that as seller risk-aversion increases, senders close to this marginal discloser are less likely to disclose, and more likely to remain silent.

For the case of two receiver-types ($n = 2$), we can establish this result using Pratt's (1964) general ordering of risk preferences.

Proposition 6 *Suppose that $n = 2$, Condition 1 holds, and that an equilibrium with partial non-disclosure exists when the sender's preferences are given by v. Suppose that the sender's preferences change to $\tilde{v} = \phi \circ v$ for some increasing and strictly concave ϕ, corresponding to greater risk aversion. Then there is a marginal discloser x_m for whom non-disclosure is safer than disclosure in the original equilibrium, i.e., $E_i\left[p_i^{ND}\right] < E_i\left[g_i\left(x_m\right)\right]$, and a new non-disclosure equilibrium under preferences $\tilde{v}$, such that non-disclosure strictly increases in the neighborhood of x_m.*

The restriction to two receiver-types in Proposition 6 is needed because, as is widely appreciated, it is hard to produce general comparative statics on choices between risky lotteries with respect to risk preferences (see, e.g., Ross (1981) for a discussion of this point), without imposing significant structure on either the utility function or on the distribution of payoffs. Specifically, with just two receiver-types we are able to show that, for at least one of the marginal disclosers $x_m \in \{\underline{x}, \bar{x}\}$, the prices associated with non-disclosure, i.e., p_1^{ND}, p_2^{ND}, lie within the range of possible prices associated with disclosure, i.e., lie in the interval $\left[\min\left\{g_1\left(x_m\right), g_2\left(x_m\right)\right\}, \max\left\{g_1\left(x_m\right), g_2\left(x_m\right)\right\}\right]$. This property allows us to apply results based on Pratt's ordering of risk preferences (specifically, Hammond (1974)).

For more than two receiver-types, we are unable to guarantee this property. Since we then lack structure on the distribution of payoffs, we must instead impose more structure on the set of utility functions to produce similar comparative statics with respect to sender risk-aversion. We have the following result:

Proposition 7 *Suppose that Condition 1 holds, and that an equilibrium with partial non-disclosure exists when the sender's preferences are given by v. Suppose that* *the sender's preferences change to $\tilde{v}$, where $\alpha\tilde{v}(x) + x = v(x)$ for some constant $\alpha > 0$, corresponding to greater risk aversion. Then there is a marginal discloser x_m for whom non-disclosure is safer than disclosure in the original equilibrium, i.e., $E_i\left[p_i^{ND}\right] < E_i\left[g_i(x_m)\right]$, and a new non-disclosure equilibrium under preferences $\tilde{v}$, such that non-disclosure strictly increases in the neighborhood of x_m.*

In words, the comparison of risk preferences used in Proposition 7 amounts to saying: preferences represented by $\tilde{v}$ are more risk-averse than preferences represented by v if v corresponds to a mixture of $\tilde{v}$ and risk neutral preferences. This ordering is closely related to Ross's (1981) notion of preferences becoming "strongly more risk averse." Note that in the specific case of mean variance preferences, our comparison corresponds to a greater dislike of variance.

6.2 Increasing receiver risk-aversion

Similarly, we consider an increase in receiver risk-aversion, again in the sense of Pratt. Intuitively, while non-disclosure helps risk-averse senders by delivering a safer lottery, it hurts risk-averse receivers, because it means that they buy an item of uncertain quality. Consequently, an increase in receiver risk-aversion reduces the prices paid to a non-disclosing sender. Hence higher risk-aversion of receivers makes non-disclosure less attractive for senders, and consequently, an increase in receiver risk-aversion reduces non-disclosure:

Proposition 8 *Suppose that Condition 1 holds and an equilibrium with non-disclosure exists when receivers' preferences are given by $\{u_i\}$. Suppose that receiver j's preferences change to $\tilde{u}_j = \phi \circ u_j$ for some increasing and strictly concave ϕ, corresponding to greater risk aversion. Then all equilibria feature strictly more disclosure than the equilibrium with the least amount of disclosure under $\{u_i\}$.*

Note that, in our setting, disclosure by a sender eliminates all risk for a receiver. However, the economic force in Proposition 8 continues to hold even in situations where disclosure reduces the risk faced by receivers, instead of completely eliminating it.

7 Extensions and discussion

7.1 Generalized disclosure

Thus far, we have considered the case in which the sender either discloses that his type is in the singleton set $\{x\}$, or else discloses nothing. In this section, we consider instead the case in which the sender can disclose any member A of some family of sets $\mathcal{X}$, provided that $x \in A$. We assume that, at a minimum, $\mathcal{X}$ contains all singletons, all closed subintervals of the interval X, and all binary unions of closed subintervals of X. To avoid economically uninteresting mathematical complications, we assume that all members of $\mathcal{X}$ are closed. Note that no-disclosure simply corresponds to disclosing X.

This enlarged set of disclosure possibilities is most likely to be relevant if disclosure takes the form of a trustworthy auditor reporting a sender's type x to receivers; or alternatively, if severe ex-post penalties can be inflicted on senders who are found to have lied (see discussion in Glode et al (2018)). If instead disclosure takes the form of simply displaying some attribute to receivers (e.g., a food safety rating, a tax return, etc.), then our benchmark analysis so far covers the relevant case.[13]

Note that this expansion of the sender's disclosure possibilities does not affect standard unravelling results. Indeed, it is straightforward to adapt the proofs of Propositions 1 and 2 to show that, under the conditions stated in these results, in any equilibrium a sender discloses $\{x\}$ with probability one.

Our main result in this section is that, given the expanded set of disclosure possibilities, an equilibrium with less than full disclosure exists under a very wide range of circumstances if the key conditions we identify in this paper are satisfied, namely sender risk-aversion, differences in receiver preferences, and receivers who are not too risk-averse. In particular, we are able to establish existence of an equilibrium with less than full disclosure without imposing the sufficient condition that $V^D(0)$ is sufficiently close to $V^D(1)$, which we used to establish Proposition 5.

[13]Specifically, Glode et al (2018) analyze a setting in which the sender can disclose any subset of the type space that includes his own type. Their analysis also differs from ours in two other important respects. First, the receiver has all the bargaining power, which implies that any sender obtains zero surplus if he fully discloses his type. Second, their paper is primarily concerned with the case in which the sender can commit to a disclosure rule before seeing his type. As an extension, they also consider the non-commitment case, and show that partial disclosure survives as an equilibrium, since given the bargaining power assumption the sender prefers to preserve some uncertainty about his type in order to obtain at least some informational rent.

Proposition 9 *If (A) there exist* $\underline{\xi}, \bar{\xi} \in (0,1)$ *and a pair of receiver-types* i, j *such that* $\underline{\xi} \neq \bar{\xi}$, $V^D\left(\underline{\xi}\right) = V^D\left(\bar{\xi}\right)$, *and* $g_i(x) \neq g_j(x)$ *for* $x = \underline{\xi}, \bar{\xi}$, *and (B) all receiver-types are sufficiently close to risk neutral, then there is an equilibrium with less than full disclosure, i.e., there is a positive probability of a sender disclosing a signal other than* $\{x\}$.

It is worth stressing that the condition (A) is satisfied whenever receivers have different preferences (g_i differs from g_j for at least some i, j), and these different preferences generate non-monotonicity of the expected utility from disclosing $\{x\}$, as given by the function V^D.

The proof of Proposition 9 is very close to previous analysis, and we give it here. We establish the existence of an equilibrium characterized by $\underline{x}, \bar{x} \in \left(\underline{\xi}, \bar{\xi}\right)$, in which senders with $x \in (\underline{x}, \bar{x})$ and $x \in X \backslash \left[\underline{\xi}, \bar{\xi}\right]$ disclose their exact type $\{x\}$; while the remaining senders with $x \in \left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$ disclose simply $\left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$.

The proof of Proposition 9 builds on the proof of Proposition 5. First, if one restricts senders to disclose either $\{x\}$ or $\left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$, the proof is the same as that of Proposition 5.[14]

It then remains to ensure that senders do not deviate to other disclosures. The equilibrium is supported by the following off-equilibrium beliefs: If the sender discloses $A \in \mathcal{X}$, and $A \neq \left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$, off-equilibrium beliefs place full mass on the sender's type being in $\arg\min_{\tilde{x} \in A} V^D(\tilde{x})$. These off-equilibrium beliefs immediately imply that senders with $x \in X \backslash \left(\left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]\right)$ do not have a profitable deviation. For senders with $x \in \left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$, note that these off-equilibrium beliefs ensure that any deviation is at least weakly worse than the deviation of disclosing $\{x\}$—which has already been established to be an unprofitable deviation, by the first step of the proof.

7.2 Welfare consequences of mandated disclosure

In many circumstances, regulations and laws mandate disclosure. In cases where the standard unravelling argument applies, such regulations should have little effect on equilibrium outcomes and utilities. In contrast, in the cases we have characterized

[14]Indeed, the fact that $\underline{\xi}, \bar{\xi} \in (0,1)$ means that the proof avoids the complications of what happens to utility and density functions as $x \to 0, 1$, which is what allows use to dispense with the regularity conditions contained in Assumptions 5 and 6.

where the equilibrium outcome is less than full disclosure, such regulations clearly increase disclosure. This affects welfare differently for senders and receivers.

For senders, mandated disclosure can only lower welfare, since an unregulated sender always has the option of staying silent.

Under the competitive condition (1), receiver utility is always simply $u_i(0)$, so that receiver utility is unaffected by mandated disclosure. But more generally, one could imagine replacing (1) with alternative assumptions that leave receivers some surplus. (Such a change would not affect the key economic forces in our analysis.) In this case, mandated disclosure has the potential to increase receiver welfare, by reducing the risk to which they are exposed.

7.3 Direct benefits to non-disclosure

The analysis of Sections 5 and 6 is all conducted under Assumption 4, which states that the payoff functions g_i are weakly concave. In this subsection, we briefly relax this assumption and explore the opposite case in which the payoff functions are strictly convex. As we noted when introducing Assumption 4, convexity of g_i introduces a direct gain to non-disclosure. Although this is not uninteresting, this force is separate from the effects due to sender uncertainty about the receiver's type, and sender risk-aversion, both of which are necessary for non-disclosure, and so are central effects we wish to study.

We focus on the specific case in which, for all receiver-types i, there is a constant α_i such that $g_i(x) = v^{-1}(\alpha_i x)$. Since v is strictly concave, this implies that g_i is strictly convex. In this analytically very tractable case we show how the convexity of g_i generates a direct gain to non-disclosure, and in turn leads to an equilibrium with full non-disclosure. (In contrast, recall that, under Assumption 4, full non-disclosure is non-generic in the space of probability distributions over receiver-types.)

In this case, the sender's expected utility from disclosure, $V^D(x)$, is clearly linear. Assuming that α_i does not have the same sign for all receiver-types (see Proposition 1), we can choose probabilities $\{q_i\}$ such that V^D has a slope arbitrarily close to 0. And whenever the slope is sufficiently close 0, there is an equilibrium in which no one discloses, as we next show.

If no sender-type discloses, the non-disclosure expected utility is

$$E_i\left[v\left(E_x\left[g_i(x)\right]\right)\right].$$

The expected utility gain from non-disclosure (if no one discloses) relative to disclosure for a given sender-type $\hat{x}$ is

$$E_i\left[v\left(E_x\left[g_i\left(x\right)\right]\right)\right]-V^D\left(\hat{x}\right)=E_i\left[v\left(E_x\left[g_i\left(x\right)\right]\right)\right]-E_i\left[v\left(g_i\left(E_x\left[x\right]\right)\right)\right]+V^D\left(E_x\left[x\right]\right)-V^D\left(\hat{x}\right). \tag{13}$$

The sense in which convexity of g_i generates a direct benefit to non-disclosure is then that, since g_i is strictly convex (since v is strictly concave), for any receiver-type,

$$E_x\left[g_i\left(x\right)\right]-g_i\left(E_x\left[x\right]\right)>0.$$

Thus, the first difference in (13) is the direct benefit to non-disclosure induced by the convexity of g_i, which is bounded away from 0 for all possible probabilities $\{q_i\}$. The second term in (13) approaches 0 as the slope of V^D approaches 0. So provided probabilities $\{q_i\}$ are chosen so that V^D has a slope sufficiently close to 0, there is indeed an equilibrium in which no one discloses. As discussed, this equilibrium outcome is driven by the fact that non-disclosure generates a direct benefit.

8 Conclusion

There are many settings in which voluntary disclosure is possible, but in which disclosure occurs with probabilities below 1, despite classic unravelling arguments. In this paper we explore a possible explanation, which is new to the literature, namely that potential disclosers do not know the preference ordering of the people they are disclosing to, and because of risk-aversion they dislike the risk that this imposes. We show how these two features together naturally deliver equilibrium non-disclosure.

In contrast to existing leading explanations of non-disclosure, our explanation does not require disclosure to be either costly, or impossible for some (unobservable) subset of would-be disclosers. As such, our paper can explain non-disclosure even in settings where disclosure is costless, and there is no uncertainty about whether disclosure is possible.

Our explanation captures the intuitive notion that a sender may prefer to stay silent because anything that he says will make some people very unhappy, while staying silent avoids this extreme outcome. That is, silence is safest. Specifically, silence reduces the risk borne by potential disclosers with extreme information. Consequently, disclosure decreases when potential disclosers grow more risk-averse, in a sense we make precise. On the other hand, non-disclosure reduces the information available to the audience for disclosures, thereby increasing the risk borne by the audience. Because of this, potential disclosers benefit more from disclosing when audiences grow more risk-averse, leading to increased equilibrium disclosure.

References

Bederson, Benjamin B., Ginger Zhe Jin, Phillip Leslie, Alexander J. Quinn, and Ben Zou (2018), "Incomplete Disclosure: Evidence of Signaling and Counter-signaling," *American Economic Journal: Microeconomics*, Vol. 10, pp. 41-66.

Celik, Levent. (2014), "Information Unraveling Revisited: Disclosure of Horizontal Attributes." *Journal of Industrial Economics*, Vol. 62, pp. 113-136.

Dawid, A. P. (1973), "Posterior Expectations for Large Observations," *Biometrika*, Vol. 60, No. 3, pp. 664-667.

Dranove, David and Ginger Z. Jin (2010), "Quality Disclosure and Certification: Theory and Practice," *Journal of Economic Literature*, Vol. 48, pp. 935-963.

Dutta, Sunil, and Brett Trueman (2002), "The Interpretation of Information and Corporate Disclosure Strategies," *Review of Accounting Studies*, Vol. 7, pp. 75-96.

Fishman, Michael J. and Kathleen M. Hagerty (2003), "Mandatory versus Voluntary Disclosure in Markets with Informed and Uninformed Customers," *Journal of Law, Economics, & Organization*, Vol. 19, pp. 45-63.

Giovannoni, Francesco and Daniel J. Seidmann (2007), "Secrecy, Two-sided Bias and the Value of Evidence," *Games and Economic Behavior*, Vol. 59, pp. 296-315.

Glode, Vincent, Christian Opp and Xingtan Zhang (2018), "Voluntary Disclosure in Bilateral Transactions," *Journal of Economic Theory*, Vol. 175, pp. 652-688.

Grossman, Sanford J. (1981), "The Informational Role of Warranties and Private Disclosure about Product Quality," *Journal of Law & Economics*, Vol. 24, pp. 461-483.

Grossman, Sanford. J. and Oliver. D. Hart (1980), "Disclosure Laws and Takeover Bids," *Journal of Finance*, Vol. 35, pp. 323-334.

Hammond, John S. (1974), "Simplifying the Choice between Uncertain Prospects Where Preference Is Nonlinear," *Management Science*, Vol. 20, pp. 1047-1072.

Harbaugh, Rick, and Theodore To (2017), "False Modesty: When Disclosing Good News Looks Bad," Working Paper.

Jovanovic, Boyan (1982), "Truthful Disclosure of Information," *Bell Journal of Economics*, Vol. 13, pp. 36-44.

Luca, Michael and Jonathan Smith (2015), "Strategic Disclosure: the Case of Business School Rankings," *Journal of Economic Behavior & Organization*, Vol. 112, pp. 17-25.

Milgrom, Paul R. (1981), "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, Vol. 12, pp. 380-391.

Milgrom, Paul R. and John Roberts (1986), "Relying on the Information of Interested Parties," *RAND Journal of Economics*, Vol. 17, pp. 18-32.

Okuno-Fujiwara, Masahiro, Andrew Postlewaite and Kotaro Suzumura (1990), "Strategic Information Revelation," *Review of Economic Studies*, Vol. 57, pp. 25-47.

Pratt, John W. (1964), "Risk Aversion in the Small and in the Large," *Econometrica*, Vol. 32, pp. 122-136.

Quigley, Daniel and Ansgar Walther (2018), "Inside and Outside Information: Fragility and Stress Test Design," Working Paper.

Ross, Stephen A. (1981), "Some Stronger Measures of Risk Aversion in the Small and the Large with Applications," *Econometrica*, Vol. 49, pp. 621-638.

Seidmann, Daniel J. and Eyal Winter (1997), "Strategic Information Transmission with Verifiable Messages," *Econometrica*, Vol. 65, pp. 163-169.

Suijs, Jeroen (2007), "Voluntary Disclosure of Information when Firms are Uncertain of Investor Response," *Journal of Accounting and Economics*, Vol. 43, pp. 391-410.

Viscusi, Kip W. (1978), "A Note on 'Lemons' Markets with Quality Certification," *Bell Journal of Economics*, Vol. 9, pp. 277-279.

Veinott, Arthur F. (1989), "Lattice Programming," Unpublished notes from lectures delivered at Johns Hopkins University.

Warren, J. Donald Jr., Kevin C. Moffitt, and Paul Byrnes (2015), "How Big Data Will Change Accounting," *Accounting Horizons*, Vol. 29, pp. 397-407.

Appendix

Throughout the appendix, denote by ND the set of sender-types which do not disclose.

Results omitted from main text

Lemma A-1 *Let Assumptions 1, 3 and 4 hold. Let* $\underline{x}, \bar{x}$ *be such that* $0 \leq \underline{x} < \bar{x} \leq 1$*; all senders in* $(\underline{x}, \bar{x})$ *disclose; and all senders* $x < \underline{x}$ *and* $x > \bar{x}$ *do not disclose. Then* $E_x[x|ND] \in [\underline{x}, \bar{x}]$*. Under Condition 1, moreover,* $E_x[x|ND] \in (\underline{x}, \bar{x})$*.*

Proof of Lemma A-1: Given the assumptions, V^D is strictly concave.

Case 1, $0 < \underline{x} < \bar{x} < 1$*:* In this case, $V^D(\underline{x}) = V^D(\bar{x}) = E_i\left[v\left(p_i^{ND}\right)\right]$, and V^D is strictly increasing for $x \leq \underline{x}$ and strictly decreasing for $x \geq \bar{x}$. So if $E_x[x|ND] < \underline{x}$ then

$$V^D(E_x[x|ND]) < V^D(\underline{x}) = E_i\left[v\left(p_i^{ND}\right)\right], \tag{A-1}$$

while if instead $E_x[x|ND] > \bar{x}$ then

$$V^D(E_x[x|ND]) < V^D(\bar{x}) = E_i\left[v\left(p_i^{ND}\right)\right]. \tag{A-2}$$

However, (7) implies that

$$V^D(E_x[x|ND]) = E_i[v(g_i(E_x[x|ND]))] \geq E_i\left[v\left(p_i^{ND}\right)\right], \tag{A-3}$$

delivering a contradiction.

Case 2, $0 = \underline{x} < \bar{x} < 1$*:* In this case, $V^D(\bar{x}) = E_i\left[v\left(p_i^{ND}\right)\right]$ and V^D is strictly decreasing for $x \geq \bar{x}$. So if $E_x[x|ND] > \bar{x}$ then (A-2) and (A-3) deliver a contradiction.

Case 3, $0 < \underline{x} < \bar{x} = 1$*:* In this case, $V^D(\underline{x}) = E_i\left[v\left(p_i^{ND}\right)\right]$ and V^D is strictly increasing for $x \leq \underline{x}$. So if $E_x[x|ND] < \underline{x}$ then (A-1) and (A-3) deliver a contradiction.

Finally, to show that $E_x[x|ND] \in (\underline{x}, \bar{x})$ under Condition 1, note that Condition 1 implies that inequality (A-3) holds strictly.

Details for Section 3

Subsection 3.1: Provided that $E(x)$ is strictly monotone in x, and $V\left(E^{-1}\left(\tilde{E}\right)\right)$ is concave in $\tilde{E}$, this falls in our framework (including satisfying Assumption 4) after a change in variables in which the type is the equity value $\tilde{E} = E(x)$, since the debt value is $V\left(E^{-1}\left(\tilde{E}\right)\right) - \tilde{E}$. In particular, if V is constant in x, as discussed in the main text, then $V\left(E^{-1}\left(\tilde{E}\right)\right)$ is constant and hence concave in $\tilde{E}$.

Subsection 3.2: In the main text we formalized a setting where the firm is uncertain about how harshly a regulator will treat it. Here, we present an alternative formalization in which the firm is instead uncertain about who will receive its disclosure.

Consider a firm choosing whether to disclose its expected cash flow, which we denote by x. Let type-1 receivers be investors, who for simplicity we assume are risk-neutral, and the type-2 receiver be a regulator. Also for simplicity, we focus on an all-equity firm, and assume that the firm benefits from a higher share price (either because it intends to issue more equity, or because of managerial compensation contracts).

In this case, $g_1(x) = x$, so that when the receiver is an investor, the share price is $E[x|\mathcal{I}]$. When the receiver is a regulator, in contrast, the regulator may take some action that affects the firm, based on its belief about the firm's cash flow. So by setting u_2 to be linear, and assuming the firm's share price is $E[x]$ if investors do not pay attention to the disclosure/non-disclosure, the firm's expected utility is

$$q_1 v\left(E[x|\mathcal{I}]\right) + q_2 v\left(E[x] + E[g_2(x)|\mathcal{I}]\right).$$

In particular, g_2 may be a decreasing function of x, representing the idea that a regulator will treat a firm more harshly if it believes the firm's cash flow is higher.

More generally, one can also consider cases in which both investors and a regulator receive the disclosure. For example, suppose investors receive the disclosure with probability $\tilde{q}_1 > 0$, and a regulator receives the disclosure with probability $\tilde{q}_2 > 0$, and these events are independent. Then the firm's expected utility is

$$\begin{aligned} & \tilde{q}_1\tilde{q}_2 v\left(E[x|\mathcal{I}] + E[g_2(x)|\mathcal{I}]\right) + \tilde{q}_1(1-\tilde{q}_2) v\left(E[x|\mathcal{I}]\right) \\ + \quad & (1-\tilde{q}_1)\tilde{q}_2 v\left(E[x] + E[g_2(x)|\mathcal{I}]\right) + (1-\tilde{q}_1)(1-\tilde{q}_2) v\left(E[x]\right). \end{aligned}$$

In particular, if either $\tilde{q}_1 < 1$ or $\tilde{q}_2 < 1$ then this falls within our framework.[15]

Subsection 3.4: Evaluating, receiver i's unconditional expectation of the attribute y is

$$E^i[y] = (1-\alpha_i)\frac{1}{3} + \alpha_i\frac{2}{3} = \frac{1+\alpha_i}{3}, \tag{A-4}$$

and the substituting (5) and (6) into (3) implies that, upon observing signal x, receiver i assesses the probability that it is perfectly correlated with the underlying attribute as

$$\frac{\lambda_i(2(1-x)(1-\alpha_i)+2x\alpha_i)}{\lambda_i(2(1-x)(1-\alpha_i)+2x\alpha_i)+(1-p_i)2x} = \frac{\lambda_i(1-\alpha_i+x(2\alpha_i-1))}{\lambda_i(1-\alpha_i+x(2\alpha_i-1))+(1-\lambda_i)x}. \tag{A-5}$$

As one would expect, this probability is increasing in λ_i, the receiver's prior that the signal x is perfectly correlated with the attribute y; and is also increasing in α_i for high signals $x > \frac{1}{2}$. By straightforward differentiation, it is decreasing in x (and strictly so if $\alpha_i < 1$).

Substituting (A-4) and (A-5) into (4) yields

$$\begin{aligned} E^i[y|x] &= \frac{\lambda_i(1-\alpha_i+x(2\alpha_i-1))}{\lambda_i(1-\alpha_i+x(2\alpha_i-1))+(1-\lambda_i)x}\left(x-\frac{1+\alpha_i}{3}\right)+\frac{1+\alpha_i}{3} \\ &= \lambda_i\frac{(2\alpha_i-1)x^2-\frac{2}{3}(\alpha_i^2+2\alpha_i-2)x-\frac{1}{3}(1-\alpha_i^2)}{x(1-2\lambda_i(1-\alpha_i))+\lambda_i(1-\alpha_i)}+\frac{1+\alpha_i}{3}. \end{aligned}$$

Differentiation yields[16]

$$\begin{aligned} \frac{\partial}{\partial x}E^i[y|x] &= \lambda_i\frac{(2\alpha_i-1)(1-2\lambda_i(1-\alpha_i))x^2+2(2\alpha_i-1)\lambda_i(1-\alpha_i)x}{(\lambda_i(1-\alpha_i)+x(1-2\lambda_i(1-\alpha_i)))^2} \\ &+ \lambda_i\frac{\frac{1}{3}(1-\alpha_i^2)(1-2\lambda_i(1-\alpha_i))-\frac{2}{3}(\alpha_i^2+2\alpha_i-2)\lambda_i(1-\alpha_i)}{(\lambda_i(1-\alpha_i)+x(1-2\lambda_i(1-\alpha_i)))^2} \end{aligned}$$

[15] Note that if both $\tilde{q}_1 < 1$ and $\tilde{q}_2 < 1$, then there are effectively three receiver-types, with receiver payoff functions given by x, $E[x]+g_2(x)$, and $x+g_2(x)$.

[16] To obtain the following expressions, note that for arbitrary constants a, b, c, d, e,

$$\begin{aligned} \frac{\partial}{\partial x}\frac{ax^2+bx+c}{dx+e} &= \frac{adx^2+2aex+be-cd}{(dx+e)^2}, \\ \frac{\partial^2}{\partial x^2}\frac{ax^2+bx+c}{dx+e} &= 2\frac{ae^2-d(be-cd)}{(dx+e)^3}. \end{aligned}$$

$$\begin{aligned}
&= \lambda_i \frac{(2\alpha_i - 1)(1 - 2\lambda_i(1-\alpha_i))x^2 + 2(2\alpha_i - 1)\lambda_i(1-\alpha_i)x}{(\lambda_i(1-\alpha_i) + x(1 - 2\lambda_i(1-\alpha_i)))^2} \\
&+ \lambda_i \frac{\frac{1}{3}(1-\alpha_i)(1 + 2\lambda_i + \alpha_i(1 - 4\lambda_i))}{(\lambda_i(1-\alpha_i) + x(1 - 2\lambda_i(1-\alpha_i)))^2}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2}{\partial x^2} E^i[y|x] &= 2\lambda_i \frac{(2\alpha_i - 1)\lambda_i^2(1-\alpha_i)^2 - (1 - 2\lambda_i(1-\alpha_i))\frac{1}{3}(1-\alpha_i)(1 + 2\lambda_i + \alpha_i(1 - 4\lambda_i))}{(\lambda_i(1-\alpha_i) + x(1 - 2\lambda_i(1-\alpha_i)))^3} \\
&= 2\lambda_i(1-\alpha_i)\frac{(2\alpha_i - 1)\lambda_i^2(1-\alpha_i) - \frac{1}{3}(1 - 2\lambda_i(1-\alpha_i))(1 + 2\lambda_i + \alpha_i(1 - 4\lambda_i))}{(\lambda_i(1-\alpha_i) + x(1 - 2\lambda_i(1-\alpha_i)))^3}.
\end{aligned}$$

First, we show that $\frac{\partial^2}{\partial x^2} E^i[y|x] < 0$. The denominator term $\lambda_i(1-\alpha_i) + x(1 - 2\lambda_i(1-\alpha_i))$ is positive, since it is just a rewriting of $\lambda_i\psi_i(x) + (1-\lambda_i)\phi(x)$. The numerator is negative, as follows. Note first that the numerator term is a quadratic in λ_i, which at $\lambda_i = 0$ evaluates as $-\frac{1}{3}(1+\alpha_i) < 0$ and at $\lambda_i = 1$ evaluates as $(1-\alpha_i)((2\alpha_i - 1) - (1 - 2(1-\alpha_i))) = 0$. So it is sufficient to show that the numerator is increasing in λ_i at $\lambda_i = 1$. The derivative of the numerator term with respect to λ_i is

$$2\lambda_i(2\alpha_i - 1)(1-\alpha_i) + \frac{2}{3}(1-\alpha_i)(1 + 2\lambda_i + \alpha_i(1 - 4\lambda_i)) - \frac{1}{3}(1 - 2\lambda_i(1-\alpha_i))(2 - 4\alpha_i),$$

which at $\lambda_i = 1$ evaluates as

$$2(2\alpha_i - 1)(1-\alpha_i) + 2(1-\alpha_i)^2 - \frac{2}{3}(1 - 2(1-\alpha_i))(1 - 2\alpha_i) = \frac{2}{3}\left(a_i^2 + a_i + 1\right) > \frac{2}{3}\left(a_i + \frac{1}{2}\right)^2 \geq 0,$$

completing the proof that $\frac{\partial^2}{\partial x^2} E^i[y|x] < 0$.

Next, we show that there exists some $\hat{\alpha} < \frac{1}{2}$ such that, if $\alpha_i < \hat{\alpha}$, $E^i[y|x]$ obtains its maximum at a signal value strictly below 1. At $x = 0$,

$$\frac{\partial}{\partial x} E^i[y|x] = \lambda_i \frac{\frac{1}{3}(1-\alpha_i)(1 + 2\lambda_i + \alpha_i(1 - 4\lambda_i))}{(\lambda_i(1-\alpha_i) + x(1 - 2\lambda_i(1-\alpha_i)))^2} > 0,$$

where the inequality follows from the fact that $1 + 2\lambda_i + \alpha_i(1 - 4\lambda_i)$ is positive at both $\lambda_i = 0$ and $\lambda_i = 1$. At $x = 1$,

$$\frac{\partial}{\partial x} E^i[y|x] = \lambda_i \frac{(2\alpha_i - 1) + \frac{1}{3}(1-\alpha_i)(1 + 2\lambda_i + \alpha_i(1-4\lambda_i))}{(\lambda_i(1-\alpha_i) + x(1 - 2\lambda_i(1-\alpha_i)))^2}.$$

Note that if $\alpha_i = 0$ then this expression is strictly negative, while if $\alpha_i = \frac{1}{2}$ it is strictly positive. Hence there is some $\hat{\alpha} < \frac{1}{2}$ such that, if $\alpha_i < \hat{\alpha}$, $E^i[y|x]$ obtains its maximum at a signal value strictly below 1.

Finally, we show that for $\alpha_i < \hat{\alpha}$, $\arg\max E^i[y|x]$ is increasing in λ_i. To do so, it suffices to show that the denominator term of $E^i[y|x]$,

$$\begin{aligned} & (2\alpha_i - 1)(1 - 2\lambda_i(1-\alpha_i))x^2 + 2(2\alpha_i - 1)\lambda_i(1-\alpha_i)x \\ + & \frac{1}{3}(1-\alpha_i)(1 + 2\lambda_i + \alpha_i(1-4\lambda_i)), \end{aligned}$$

is increasing in λ_i, i.e., that

$$-2(2\alpha_i - 1)(1-\alpha_i)x^2 + 2(2\alpha_i - 1)(1-\alpha_i)x + \frac{1}{3}(1-\alpha_i)(2 - 4\alpha_i) > 0,$$

i.e. (and recalling that $1 - 2\alpha_i > 0$),

$$x^2 - x + \frac{1}{3} > 0.$$

This is indeed true since

$$x^2 - x + \frac{1}{3} > x^2 - x + \frac{1}{4} = \left(x - \frac{1}{2}\right)^2 \geq 0.$$

Proofs of results stated in main text

Proof of Lemma 1: Concavity of u_i and Jensen's inequality imply

$$u_i(E_x[g_i(x) - p_i|\mathcal{I}]) \geq E_x[u_i(g_i(x) - p_i)|\mathcal{I}] = u_i(0),$$

which in turn implies $p_i \leq E_x[g_i(x)|\mathcal{I}]$.

Proof of Proposition 1: Suppose to the contrary that the probability of non-disclosure is strictly positive. So there exists some non-zero-measure subset $ND \subset [0,1]$ of sender-types who do not disclose.

We recursively define $x_1, \ldots, x_n \in ND$ as follows. First, by Assumption 1, define $x_1 \in ND$ such that $g_1(x_1) > E_x[g_1(x)|ND]$. Next, suppose that $x_1, \ldots, x_{k-1}$ are defined, with the properties that $x_{k-1} \in ND$, and $g_i(x_{k-1}) > E_x[g_i(x)|ND]$ for all receiver-types $i = 1, \ldots, k-1$. Then, define $x_k \in ND$ such that $g_k(x_k) \geq g_k(x_{k-1})$ and $g_k(x_k) > E_x[g_k(x)|ND]$. To see that such a choice is possible, note that if $g_k(x_{k-1}) > E_x[g_k(x)|ND]$ then one can simply set $x_k = x_{k-1}$; while if instead $E_x[g_k(x)|ND] \geq g_k(x_{k-1})$, by Assumption 1 there must exist $x_k \in ND$ with $g_k(x_k) > E_x[g_k(x)|ND] \geq g_k(x_{k-1})$. Since $g_k(x_k) \geq g_k(x_{k-1})$, by ordinal equivalence $g_i(x_k) \geq g_i(x_{k-1})$ for any receiver-type i, and hence $g_i(x_k) > E_x[g_i(x)|ND]$ for all receiver-types $i = 1, \ldots, k$, establishing the recursive step.

So in particular, $v(g_i(x_n)) > v(E_x[g_i(x)|ND])$ for all receiver-types $i \in N$. By Lemma 1, $E_x[g_i(x)|ND] \geq p_i^{ND}$. Hence $v(g_i(x_n)) > v\left(p_i^{ND}\right)$ for all receiver-types $i \in N$. But this violates the equilibrium condition $E_i\left[v\left(p_i^{ND}\right)\right] = E_i[v(g_i(x_n))]$ for the sender $x_n \in ND$, completing the proof.

Proof of Proposition 2: Suppose to the contrary that the probability of non-disclosure is strictly positive. So there exists some non-zero-measure subset $ND \subset [0,1]$ of sender-types who disclose with probability below 1. Since any sender-type $x' \in ND$ prefers non-disclosure to disclosure, Lemma 1 implies

$$E_i[v(g_i(x'))] \leq E_i\left[v\left(p_i^{ND}\right)\right] \leq E_i[v(E_x[g_i(x)|ND])].$$

Since v is weakly convex,

$$E_i[v(E_x[g_i(x)|ND])] \leq E_i[E_x[v(g_i(x))|ND]] = E_x[E_i[v(g_i(x))]|ND].$$

Combining these two inequalities implies that, for any $x' \in ND$,

$$E_i[v(g_i(x'))] \leq E_x[E_i[v(g_i(x))]|ND].$$

If v is strictly convex, the above inequality is strict, giving a contradiction. If instead v is linear, this inequality contradicts Assumption 2, completing the proof.

Proof of Proposition 4: First, consider the case in which the equilibrium features full non-disclosure, i.e., $\underline{x} = \bar{x}$. Then (9) implies

$$E_i \left[v \left(g_i \left(E_x \left[x \right] \right) \right) \right] = \max_{\tilde{x}} E_i \left[v \left(g_i \left(\tilde{x} \right) \right) \right] = E_i \left[v \left(p_i^{ND} \right) \right]. \tag{A-6}$$

Since V^D is strictly concave, $E_x [x]$ is the unique maximizer of V^D, and hence $\bar{x} = E_x [x]$. Moreover, (A-6) combines with (7) to imply $p_i^{ND} = g_i (E_x [x])$ for all receiver-types i, completing the proof of this case.

Next, consider the case of an equilibrium with partial non-disclosure. Inequality (11) is established by Lemma A-1, and is strict under Condition 1. To establish (12), suppose to the contrary that

$$E_i \left[p_i^{ND} \right] > \max \left\{ E_i \left[g_i \left(\underline{x} \right) \right], E_i \left[g_i \left(\bar{x} \right) \right] \right\}. \tag{A-7}$$

By (7), it follows that

$$E_i \left[g_i \left(E \left[x | ND \right] \right) \right] > \max \left\{ E_i \left[g_i \left(\underline{x} \right) \right], E_i \left[g_i \left(\bar{x} \right) \right] \right\}.$$

Given concavity of g_i and (11), it follows that $E_i [g_i (x)]$ obtains its maximum in the interval $[\underline{x}, \bar{x}]$, and hence is weakly increasing over $[0, \underline{x}]$ and weakly decreasing over $[\bar{x}, 1]$. Hence (A-7) implies that

$$E_i \left[p_i^{ND} \right] > E_i \left[g_i \left(\tilde{x} \right) \right] \text{ for all } \tilde{x} \in [0, \underline{x}] \cup [\bar{x}, 1].$$

Another application of Lemma 1 then implies that

$$E_i \left[E_x \left[g_i (x) \,|\, [0, \underline{x}] \cup [\bar{x}, 1] \right] \right] > E_i \left[g_i \left(\tilde{x} \right) \right] \text{ for all } \tilde{x} \in [0, \underline{x}] \cup [\bar{x}, 1].$$

The contradiction establishes (12). Finally, an easy adaptation of the above argument establishes that (12) is strict under Condition 1, completing the proof.

Proof of Proposition 5: Under the stated conditions, there exists some distribution of receiver-types $(q_i)_{i \in N}$ such that $V^D (0) = V^D (1)$. We establish the existence of a non-disclosure equilibrium for this distribution, and for the case in which all receiver-types are risk neutral (u_i linear for all $i \in N$). The general result then follows by continuity.

Because receivers are risk neutral, non-disclosure prices are simply given by $p_i^{ND} =$

$E_x\left[g_i(x)\,|ND\right]$.

Note that Assumptions 1 and 3 imply that V^D is strictly concave. Define $x_{\max} = \arg\max_{\tilde{x}} V^D(\tilde{x})$.

If $V^D(x_{\max}) \leq E_i\left[v\left(E_x\left[g_i(x)\right]\right)\right]$ then there is an equilibrium in which no sender discloses, and the proof is complete. So for the remainder of the proof, we consider the case in which

$$V^D(x_{\max}) > E_i\left[v\left(E_x\left[g_i(x)\right]\right)\right]. \tag{A-8}$$

For any $\underline{x} \in (0, x_{\max})$, define $\eta(\underline{x}) \in (x_{\max}, 1)$ by $V^D(\eta(\underline{x})) = V^D(\underline{x})$. Note that $\eta(\underline{x})$ exists and is unique, since $V^D(0) = V^D(1)$ and V^D is strictly concave. Moreover, η is continuous, with $\eta(\underline{x}) \to 1$ as $\underline{x} \to 0$, and

$$\frac{\partial}{\partial \underline{x}}\eta(\underline{x}) = \frac{\left.\frac{\partial}{\partial x}V^D(x)\right|_{x=\underline{x}}}{\left.\frac{\partial}{\partial x}V^D(x)\right|_{x=\eta(\underline{x})}}.$$

Since $V^D(0) = V^D(1)$, and V^D is strictly concave, $\frac{\partial}{\partial x}V^D(x)$ remains bounded away from 0 as $x \to 0, 1$. Assumption 5 then implies that $\frac{\partial}{\partial \underline{x}}\eta(\underline{x})$ remains bounded away from both 0 and $-\infty$ as $\underline{x} \to 0$. Assumption 6 and l'Hôpital's rule then imply that the following limit exists, and is bounded away from 0:

$$\lim_{\underline{x}\to 0}\frac{\int_0^{\underline{x}} f(x)\,dx}{\int_{\eta(\underline{x})}^1 f(x)\,dx} = -\lim_{\underline{x}\to 0}\frac{f(\underline{x})}{f(\eta(\underline{x}))\,\frac{\partial}{\partial \underline{x}}\eta(\underline{x})}.$$

Strict concavity of v (Assumption 3) and the condition that there are receiver-types $i, j \in N$ such that $g_i(0) < g_i(1)$ and $g_j(0) > g_j(1)$ then implies that

$$\lim_{\underline{x}\to 0} E_i\left[v\left(E_x\left[g_i(x)\,|X\backslash\left[\underline{x}, \eta(\underline{x})\right]\right]\right)\right] - E_i\left[E_x\left[v\left(g_i(x)\right)|X\backslash\left[\underline{x}, \eta(\underline{x})\right]\right]\right] > 0. \tag{A-9}$$

Also note that

$$E_i\left[E_x\left[v\left(g_i(x)\right)|X\backslash\left[\underline{x}, \eta(\underline{x})\right]\right]\right] = E_x\left[E_i\left[v\left(g_i(x)\right)\right]|X\backslash\left[\underline{x}, \eta(\underline{x})\right]\right] = E_x\left[V^D(x)\,|X\backslash\left[\underline{x}, \eta(\underline{x})\right]\right].$$

Hence, and using $V^D(0) = V^D(1)$,

$$\lim_{\underline{x}\to 0}\left(E_i\left[E_x\left[v\left(g_i(x)\right)|X\backslash\left[\underline{x}, \eta(\underline{x})\right]\right]\right] - V^D(\underline{x})\right) = 0. \tag{A-10}$$

It follows by (A-9) that

$$V^D(\underline{x}) - E_i\left[v\left(E_x\left[g_i(x)\,|X\backslash\left[\underline{x},\eta(\underline{x})\right]\right]\right)\right] < 0$$

for all $\underline{x}$ sufficiently close to 0.

Combined with (A-8), continuity then implies that there exists some $\underline{x} \in (0, x_{\max})$ such that

$$V^D(\underline{x}) = V^D(\eta(\underline{x})) = E_i\left[v\left(E_x\left[g_i(x)\,|X\backslash\left[\underline{x},\eta(\underline{x})\right]\right]\right)\right].$$

Hence there is an equilibrium in which senders $[\underline{x}, \eta(\underline{x})]$ disclose, while senders $X\backslash[\underline{x}, \eta(\underline{x})]$ remain silent and do not disclose, completing the proof.

Proof of Proposition 6: Consider any non-disclosure equilibrium, with a non-disclosure set $[0, \underline{x}) \cup (\bar{x}, 1]$.

Claim A: For any receiver-type i, $p_i^{ND} \leq \max\{g_i(\underline{x}), g_i(\bar{x})\}$.

Proof of claim: If g_i is monotone over $[\underline{x}, \bar{x}]$, then

$$p_i^{ND} \leq E_x[g_i(x)|ND] \leq g_i(E_x[x|ND]) \leq \max\{g_i(\underline{x}), g_i(\bar{x})\},$$

where the first inequality follows from Lemma 1, the second inequality follows from Jensen's inequality and the concavity of g_i, and the last inequality follows from Proposition 4 and the monotonicity of g_i over $[\underline{x}, \bar{x}]$.

If instead g_i is non-monotone over $[\underline{x}, \bar{x}]$, then by concavity, it is strictly increasing over $[0, \underline{x})$ and strictly decreasing over $(\bar{x}, 1]$. Hence $g_i(x) < \max\{g_i(\underline{x}), g_i(\bar{x})\}$ for all $x \in [0, \underline{x}) \cup (\bar{x}, 1]$. So by Lemma 1,

$$p_i^{ND} \leq E_x[g_i(x)|ND] < \max\{g_i(\underline{x}), g_i(\bar{x})\}.$$

Claim B: For some $x \in \{\underline{x}, \bar{x}\}$, $p_1^{ND}, p_2^{ND} \in [\min\{g_1(x), g_2(x)\}, \max\{g_1(x), g_2(x)\}]$.

Proof of Claim: Now consider any non-disclosure equilibrium in which the non-disclosure set is $[0, \underline{x}) \cup (\bar{x}, 1]$. The equilibrium condition implies that $g_1(\bar{x}) - g_1(\underline{x})$ and $g_2(\bar{x}) - g_2(\underline{x})$ have opposite signs. Without loss, assume $g_1(\underline{x}) \leq g_1(\bar{x})$ and $g_2(\bar{x}) \leq g_2(\underline{x})$. So Claim A implies $p_1^{ND} \leq g_1(\bar{x})$ and $p_2^{ND} \leq g_2(\underline{x})$. The equilibrium condition then implies $p_2^{ND} \geq g_2(\bar{x})$ and $p_1^{ND} \geq g_1(\underline{x})$, and so $p_1^{ND} \in [g_1(\underline{x}), g_1(\bar{x})]$ and $p_2^{ND} \in [g_2(\bar{x}), g_2(\underline{x})]$.

If the sets $[g_1(\underline{x}), g_1(\bar{x})]$ and $[g_2(\bar{x}), g_2(\underline{x})]$ are ranked by the strong set order

(Veinott, 1989) then the result is straightforward: If $[g_1(\underline{x}), g_1(\bar{x})] \preceq [g_2(\bar{x}), g_2(\underline{x})]$ under this order, then $p_1^{ND}, p_2^{ND} \in [g_1(\underline{x}), g_2(\underline{x})]$; while if instead $[g_2(\bar{x}), g_2(\underline{x})] \preceq [g_1(\underline{x}), g_1(\bar{x})]$, then $p_1^{ND}, p_2^{ND} \in [g_2(\bar{x}), g_1(\bar{x})]$.

Next, consider the cases where the two sets $[g_1(\underline{x}), g_1(\bar{x})]$ and $[g_2(\bar{x}), g_2(\underline{x})]$ are not ranked by the strong set order. There are two sub-cases. In the first sub-case, $[g_1(\underline{x}), g_1(\bar{x})] \subset [g_2(\bar{x}), g_2(\underline{x})]$, and so either $p_2^{ND} \in [g_2(\bar{x}), g_1(\bar{x})]$ or $p_2^{ND} \in [g_1(\underline{x}), g_2(\underline{x})]$ (or both), while both $p_1^{ND} \in [g_2(\bar{x}), g_1(\bar{x})]$ and $p_1^{ND} \in [g_1(\underline{x}), g_2(\underline{x})]$. In the second sub-case, $[g_2(\bar{x}), g_2(\underline{x})] \subset [g_1(\underline{x}), g_1(\bar{x})]$, and so either $p_1^{ND} \in [g_1(\underline{x}), g_2(\underline{x})]$ or $p_1^{ND} \in [g_2(\bar{x}), g_1(\bar{x})]$ (or both), while both $p_2^{ND} \in [g_1(\underline{x}), g_2(\underline{x})]$ and $p_2^{ND} \in [g_2(\bar{x}), g_1(\bar{x})]$.

Claim C: If $x_m \in \{\underline{x}, \bar{x}\}$ and $p_1^{ND}, p_2^{ND} \in [\min\{g_1(x_m), g_2(x_m)\}, \max\{g_1(x_m), g_2(x_m)\}]$ then $E_i\left[p_i^{ND}\right] \leq E_i[g_i(x_m)]$.

Proof of Claim: If instead $E_i\left[p_i^{ND}\right] > E_i[g_i(x_m)]$ then Theorem 3 of Hammond (1974) implies that $E_i\left[v\left(p_i^{ND}\right)\right] > E_i[v(g_i(x_m))]$, contradicting the equilibrium condition.

Completing the proof: From above, for at least one $x_m \in \{\underline{x}, \bar{x}\}$, we know $p_1^{ND}, p_2^{ND} \in [\min\{g_1(x_m), g_2(x_m)\}, \max\{g_1(x_m), g_2(x_m)\}]$ and $E_i\left[p_i^{ND}\right] \leq E_i\left[g_i^{ND}(x_m)\right]$, along with the equilibrium condition $E_i\left[v\left(p_i^{ND}\right)\right] = E_i[v(g_i(x_m))]$. So for any increasing and strictly concave function ϕ, Theorem 3 of Hammond (1974) implies that

$$E_i\left[\phi\left(v\left(p_i^{ND}\right)\right)\right] \geq E_i[\phi(v(g_i(x_m)))]. \tag{A-11}$$

Moreover, under Condition 1, Claim A holds strictly (by Proposition 4), and hence Claims B and C hold strictly also, and so (A-11) likewise holds strictly.

Given inequality (A-11), a straightforward modification of the argument in the proof of equilibrium existence in Proposition 5 implies that, for preferences $\tilde{v}$, there exists an equilibrium in which senders $[0, \underset{\sim}{x}) \cup (\tilde{x}, 1]$ do not disclose, where if $x_m = \underline{x}$ then $\underset{\sim}{x} > \underline{x}$, and if $x_m = \bar{x}$ then $\tilde{x} < \bar{x}$. This completes the proof.

Proof of Proposition 7: Given Proposition 3, when the sender's preferences are given by v, consider an equilibrium in which senders in $[0, \underline{x}) \cup (\bar{x}, 1]$ do not disclose. By Proposition 4, for some $x_m \in \{\underline{x}, \bar{x}\}$,

$$E_i\left[p_i^{ND}\right] < E_i[g_i(x_m)]. \tag{A-12}$$

It follows that

$$E_i\left[\tilde{v}\left(p_i^{ND}\right)\right] > E_i\left[\tilde{v}\left(g_i\left(x_m\right)\right)\right], \tag{A-13}$$

since otherwise (A-12) and the definition that $v(x) = \alpha\tilde{v}(x) + x$ at all $x \in X$ implies that

$$E_i\left[v\left(p_i^{ND}\right)\right] < E_i\left[v\left(g_i\left(x_m\right)\right)\right],$$

contradicting the equilibrium condition when the sender's preferences are given by v. Given (A-13), the result follows as in the last step of the proof of Proposition 6.

Proof of Proposition 8: Consider the equilibrium with the least amount of disclosure. For any marginal discloser x_m the equilibrium condition $E_i\left[v\left(p_i^{ND}\right)\right] = E_i\left[v\left(g_i\left(x_m\right)\right)\right]$ holds. Following the increase in receiver j's risk-aversion, if the non-disclosure set stays unchanged then p_j^{ND} strictly decreases. Hence, for both marginal disclosers $x_m \in \{\underline{x}, \bar{x}\}$ we have $E_i\left[v\left(p_i^{ND}\right)\right] < E_i\left[v\left(g_i\left(x_m\right)\right)\right]$. The result follows as in the last step of the proof of Proposition 6.