Silence is safest: non-disclosure when the audience's preferences are uncertain*

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Abstract

We examine voluntary disclosure in a setting where the would-be discloser ("sender") is risk-averse and faces uncertainty about the audience's ("receiver's") preference ordering over different sender-types. We show that some senders abstain from disclosing in equilibrium, in contrast to classic "unravelling" results. In such equilibria, senders with extreme types do not disclose, while senders with intermediate types disclose. Non-disclosure reduces the sensitivity of a sender's payoff to the receiver's preference ordering, which is attractive to risk-averse senders. Increased sender risk-aversion reduces equilibrium disclosure by sender-types who bear a higher risk under disclosure than non-disclosure. In contrast, non-disclosure exposes receivers to risk by reducing their ability to differentiate between sender types, and consequently, increased receiver risk-aversion increases equilibrium disclosure. The model accommodates many applications in finance and economics.

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1 Introduction

An important and long-standing question in the economics of information is whether voluntary disclosure leads to full disclosure. A compelling and intuitive argument, often described as the "unravelling" argument, suggests yes.[1] In brief, the argument is that the firm, or more generally the "sender," with the most favorable information will voluntarily disclose. So the audience for the disclosure—the "receiver"—will interpret non-disclosure as indicating that the firm does not have the most favorable information. But given this, the firm with the second most favorable piece of information will disclose, and so on. All the firms thus disclose in the end.

Despite the force of the unravelling argument, the prediction of full disclosure appears too strong. There are many cases in which valuable information that is potentially disclosable is not disclosed. Firms do not voluntarily reveal all value-relevant information. In legal proceedings, defendants frequently opt for silence. Politicians do not always voluntarily reveal past tax returns. In such cases, potential disclosers believe that non-disclosure is in their best interests, even though audiences often interpret non-disclosure with skepticism.

In this paper, we give a simple explanation for non-disclosure, which captures the idea that non-disclosure is the safest option available. Our explanation has two key ingredients. The first ingredient is that the sender is unsure of the receiver's preferences. For example, a firm selling financial securities may be unsure whether the buyers of its securities desire higher or lower cash flow variance. Even if the firm's private information is about the first rather than second moment of cash flows, the firm's disclosure preferences will differ depending on whether the "buyer" of the firm's securities is an investor or a tax authority (or regulator, or union). A politician who is considering disclosing past tax returns may be unsure whether voters wish to see high income (thereby indicating that he is rich and successful) or low income (thereby excusing the low taxes he is known to have paid).

By itself, however, sender uncertainty about receiver preferences is not enough to generate non-disclosure. The reason is that the expected payoff from disclosure can still be ordered, so that one can still identify senders with the highest incentive to disclose, and the unravelling argument still applies. Thus, the second key ingredient

[1] See Viscusi (1978), Grossman and Hart (1980), Milgrom (1981), Grossman (1981), and Milgrom and Roberts (1986). Dranove and Jin (2010) provide a recent survey of the literature.

for our explanation is sender risk-aversion, so that senders care about the risk of payoffs induced by disclosure relative to non-disclosure, in addition to simply the expected payoff. We show that non-disclosure arises precisely when it is safer than disclosure, in the sense of leading to lower risk in payoffs.

In a little more detail, consider, for example, a firm with private information about its cash flow variance represented by $x \in [0, 1]$, which it can voluntarily disclose. The firm is selling securities. It knows that some investors would prefer high-variance cash flows (high x), while other investors would prefer low-variance cash flows (low x), but does not know which type of investor it is dealing with. Thus, a disclosing firm faces a lottery over different prices that it may receive for its securities, where the prices depend on the corresponding investor's preferences over cash-flow variance. Firms with extreme values of cash flow variance—i.e., either very high or very low variance—face a particularly high-risk lottery over prices, because in these cases the valuation of an investor who likes high-variance cash flows diverges widely from the valuation of an investor who dislikes cash flow variance.

In a non-disclosure equilibrium, firms with extreme information stay silent and do not disclose, while firms with intermediate information disclose. Investors correctly interpret non-disclosure as indicating extreme information—in this example, either very low or very high cash flow variance. So the price they are willing to pay is based on an average of these extremes. In particular, this means that the price paid if investors want high variance is close to the price paid if investors want low variance. So non-disclosure generates a lower-risk lottery over prices for firms with extreme information, relative to the alternative of disclosing.

Given the economic forces underlying equilibrium non-disclosure and the failure of unravelling, it is natural to conjecture that non-disclosure becomes more likely as sender risk-aversion increases. Our analysis makes this intuition precise. Similarly, non-disclosure exposes receivers to risk by reducing their ability to differentiate between different sender types. Consequently, non-disclosure becomes less likely as receiver risk-aversion increases.

Previous research has identified other possible reasons for why full unravelling may not occur, and some senders choose to remain silent instead of disclosing. Perhaps the most widely applicable existing explanations are that full unravelling does not occur if disclosure is costly (Grossman and Hart, 1980, Jovanovic, 1982); and that full unravelling does not occur if there is some probability that the sender is unable to disclose (Dye, 1985).

While the assumptions of costly disclosure and unobservably impossible disclosure are certainly satisfied in some settings, there are also many settings in which disclosure is costless, and there is no uncertainty as to whether the sender is able to disclose, but voluntary disclosure does not generate full disclosure. For example, disclosure of tax returns by a politician is both costless, and known to be feasible with complete certainty. Hence, our paper can explain non-disclosure in settings where previous explanations cannot. Moreover, it captures precisely the idea that staying silent and not disclosing is the "safest" course of action.

Unravelling results have been generalized to wider classes of economies by papers such as Okuno-Fujiwara, Postlewaite and Suzumura (1990) and Seidmann and Winter (1997).[2] Okuno-Fujiwara, et. al. (1990) stress the importance of sender payoff monotonicity, and exhibit examples in which a failure of monotonicity blocks unravelling and leads to complete non-disclosure. However, we show that payoff non-monotonicity alone is not sufficient to block unravelling. Our paper can be viewed as identifying a set of economically relevant conditions under which partial non-disclosure emerges as an equilibrium outcome in a natural setting.

The literature on disclosure is large, and has suggested a number of other alternative explanations of non-disclosure as surveyed in Dranove and Jin (2010). Among them, some share our focus on receiver heterogeneity, though rely on very different economic forces. For example, Fishman and Hagerty (2003) show that non-disclosure arises if some receivers are unable to process the information content of disclosure. Harbaugh and To (2017) consider a setting in which the sender's type is drawn from the interval $[0, 1]$, but disclosures are restricted to specifying which element of a finite partition of $[0, 1]$ the type belongs to. Moreover, the receiver is endowed with a private signal about the sender's type. Consequently, the best senders in a partition element may prefer to remain silent in order to avoid mixing with mediocre senders in the same partition element, and thus the unraveling argument breaks down.

In the accounting literature, Dutta and Trueman (2002) and Suijs (2007) analyze relatively special situations in which the sender is unsure how the receiver will respond

[2]Giovannoni and Seidmann (2007) study a setting similar to Seidmann and Winter (1997), and characterize conditions under which no disclosure occurs. Differently from our paper, the sender knows the receiver's preferences. Instead, non-disclosure in Giovannoni and Seidmann arises because the sender's "ideal action exceeds the informed [r]eceiver's ideal action if and only if the [s]ender's type is low."

to a disclosure. However, Dutta and Trueman (2002) assume that there is a strictly positive probability that the sender has nothing to disclose, and state that this is critical for their results. In Suijs (2007)'s environment (unlike ours), there is a direct gain to non-disclosure.[3]

2 Model

Consider a firm—henceforth, the sender—that sells an item with characteristic x to a buyer—henceforth, the receiver. The sender has type $x \in X$, where X is an interval of the real line. The prior distribution of x is continuous over X, with full support and a strictly positive density function, which we denote by f. We normalize $\inf X = 0$ and $\sup X = 1$. We make no assumption as to whether X is open, closed, or half-closed.

The sender's type is private information to the sender. The sender can, at zero cost, credibly disclose his type x to the receiver, or not disclose any information. The sender's payoff is determined by the receiver's type, and the receiver's beliefs about the sender's type, as described below.

The receiver can be of $n \geq 2$ types, denoted $i \in N \equiv \{1, 2, \ldots, n\}$. The probability of type i is q_i. The preferences of a receiver of type i are given by $u_i\left(g_i(x) - p_i\right)$, where p_i is the price receiver type i pays to the sender, and u_i is continuous, strictly increasing and weakly concave, capturing the receiver's risk-aversion, and g_i is also continuous, differentiable, and weakly concave. Note that we impose no assumption on the relationship between different g_i's or as to whether g_i is monotone or not.

If the receiver is type i, the price p_i is determined by the competitive condition

$$E_x\left[u_i\left(g_i(x) - p_i\right) | \mathcal{I}\right] = u_i\left(0\right), \tag{1}$$

where $\mathcal{I}$ is the receiver's information (i.e., either the particular x the sender discloses, or nothing). Note that, for clarity, we typically write E_x to make clear the expectation is being taken over sender types x, and correspondingly write E_i when the expectation is taken over receiver types $i \in N$.

[3] To be specific, in Suijs's model, disclosure gives a payoff of either $U\left(0\right)$ or $U\left(1\right)$, with probabilities $1 - p\left(\phi\right)$ and $p\left(\phi\right)$ respectively, where ϕ is the sender's type. Non-disclosure gives payoffs of $U\left(\frac{1}{2}\right)$ and something at least $U\left(0\right)$, with corresponding probabilities, and *regardless* of receiver inferences about what non-disclosure means. So if the type space is such that $1 - p\left(\phi\right)$ is sufficiently high for all types, non-disclosure is an equilibrium.

Consequently, any disclosure decision by the sender leads to a lottery over prices $(p_i)_{i \in N}$, where p_i is received with probability q_i. The sender's expected utility from this lottery is

$$E_i [p_i] = \sum_{i \in N} q_i v (p_i) ,$$

where v is strictly increasing. As we will establish, whether the sender's utility function v is concave or convex is an important driver of equilibrium disclosure outcomes.

For use throughout, we denote the expected utility of sender disclosing x by $V^D (x)$. This quantity is straightforward to calculate, since in this case the price the sender gets from a sender of type i is simply $p_i = g_i (x)$, and thus

$$V^D (x) \equiv \sum_{i \in N} q_i v (g_i (x)) = E_i [v (g_i (x))] .$$

We say an equilibrium features *full disclosure* if the probability that the sender discloses is 1. We say an equilibrium features *non-disclosure* if the probability that the sender discloses is strictly less than 1.

Throughout, we write $\left(p_i^{ND}\right)_{i \in N}$ for the prices received from the different receiver types following non-disclosure. Note that these prices are endogenous, and are determined in equilibrium.

We make the following mild regularity assumptions. First, no receiver has flat preferences over the sender's type:

Assumption 1 *For any $i \in N$, $\frac{\partial}{\partial x} g_i(x) = 0$ holds for at most one value of x.*

Second, we rule out the non-generic case in which the expected payoff (as opposed to utility) from disclosure is constant across sender types. When the sender is risk neutral, this non-generic case can generate economically uninteresting outcomes in which all senders are indifferent between disclosure and non-disclosure. (Note that we nonetheless allow this non-generic case when the sender is strictly risk-averse, since it allows for a simple illustrative example that we use below.)

Assumption 2 *Either the sender is strictly risk-averse (v is strictly concave), or else $\frac{\partial}{\partial x} E_i [g_i (x)] = 0$ holds for at most one value of x.*

2.1 A preliminary result

Before proceeding, we note the following straightforward result, which is directly implied by the receiver's (weak) risk-aversion, and which we use repeatedly:

Lemma 1 *For $i \in N$,*

$$p_i \quad \leq \quad E_x\left[g_i(x)|\mathcal{I}\right] \leq g_i\left(E_x\left[x|\mathcal{I}\right]\right), \tag{2}$$

where the first inequality is strict if u_i is strictly concave and the posterior of x given information $\mathcal{I}$ is non-degenerate.

3 Model applications

Our model is general enough to accommodate many economically relevant applications. We have described the baseline model setting in terms of the sender being a firm that sells an item with characteristic x to buyers (the receivers). The seller discloses some feature of the item. Different buyers have different preferences over the characteristic x. To give a few examples, before a potential disclosure, a start-up firm selling equity may be unsure about the risk-return preferences of potential investors; a financial advisor may be unsure about clients' preferences; and a target firm may be unsure as to whether the bidding firms' technology is a complement or a substitute to its own technology.

Below, we expand on four applications of our model that are perhaps less obvious:

3.1 Conflict between debt and equity

A leading case of distinct investor preferences in financial economics is that between equity- and bond-holders, where different preferences stem from the different structure of these securities.

A firm anticipates that it will need to raise funding in the future. With probability q_1 it will prefer to issue equity, but with probability q_2 it will prefer to issue debt, where for simplicity we take the firm's preference between debt and equity as exogenous.

The firm's cash flow y is a random variable. The firm does not know its future cash flow realization, but it does know its type, x, which determines the distribution of y. For example, x may represent the firm's choice of projects, which affect both the mean and variance of cash flows. Investors do not directly observe y. The firm can disclose x.

The firm has some outstanding debt. The value of its existing equity, debt, and total value is given by $E(x)$, $D(x)$, $V(x)$, respectively, where as usual, $V(x) = E(x) + D(x)$. For simplicity, we assume that the firm's future issue of debt or equity is sufficiently small that the new issue does not affect prices. Hence if κ_1 and κ_2 denote the small amount of equity and debt that the firm will issue, then $g_1(x) = \kappa_1 E(x)$ and $g_2(x) = \kappa_2 D(x)$. Provided that $E(x)$ is strictly monotone in x, and $V\left(E^{-1}\left(\tilde{E}\right)\right)$ is concave in $\tilde{E}$, this falls in our framework. In particular, if firm value V is constant, and x captures cash flow variance, then E is increasing in x and D is decreasing in x, and both E and D are weakly concave in x.

3.2 Investors and regulators (or tax authority, or bargaining employees)

A distinct application in corporate finance is that of a firm disclosing, but being unsure whether its disclosure will be received by investors, or by a regulator. This application has the feature that if the receiver is a regulator then the sender's payoff does not stem from an explicit price paid by the regulator.

To fix ideas, consider a firm disclosing its expected cash flow. Let type-1 receivers be investors, and the type-2 receiver be a regulator. For simplicity, consider the case of a firm deciding whether or not to disclose its cash flow x. Also for simplicity, we focus on an all equity firm, and assume that the firm benefits from a higher share price (either because it intends to issue more equity, or because of managerial compensation contracts).

In this case, $g_1(x) = x$, so that when the receiver is an investor (who we continue to assume is risk-neutral), the share price is $E[x|\mathcal{I}]$. When the receiver is a regulator, in contrast, the regulator may take some action that affects the firm, based on its belief about the firm's cash flow. So by setting u_2 to be linear, and assuming the firm's share price is $E[x]$ if investors do not pay attention to the disclosure/non-disclosure, the firm's expected payoff is

$$q_1 v\left(E[x|\mathcal{I}]\right) + q_2 v\left(E[x] + E\left[g_2(x)|\mathcal{I}\right]\right).$$

In particular, g_2 may be a decreasing function of x, representing the idea that a regulator will treat a firm more harshly if it believes the firm cash flow is high.

By relabeling, this structure also covers cases in which the type-2 receiver is instead a tax authority, or a group of employees bargaining with the firm.

Moreover, one can also consider cases in which both investors and a regulator receive the disclosure. For example, suppose investors receive the disclosure with probability $\tilde{q}_1 > 0$, and a regulator receives the disclosure with probability $\tilde{q}_2 > 0$, and these events are independent. Then the firm's payoff is

$$\begin{aligned} & \tilde{q}_1 \tilde{q}_2 v\left(E\left[x|\mathcal{I}\right] + E\left[g_2\left(x\right)|\mathcal{I}\right]\right) + \tilde{q}_1\left(1-\tilde{q}_2\right) v\left(E\left[x|\mathcal{I}\right]\right) \\ + \quad & \left(1-\tilde{q}_1\right) \tilde{q}_2 v\left(E\left[x\right] + E\left[g_2\left(x\right)|\mathcal{I}\right]\right) + \left(1-\tilde{q}_1\right)\left(1-\tilde{q}_2\right) v\left(E\left[x\right]\right). \end{aligned}$$

In particular, if either $\tilde{q}_1 < 1$ or $\tilde{q}_2 < 1$ then this falls within our framework. Note that if both $\tilde{q}_1 < 1$ and $\tilde{q}_2 < 1$, then there are effectively three receiver types, which also fits in our framework.[4]

3.3 Political elections

We next consider a second important case in which the sender's payoffs do not stem from prices paid by buyers, that is, political elections. This case also illustrates that the concavity of sender's preference function v need not stem from fundamental risk preferences. We present a very stripped down model of elections, though (as with elsewhere) it could be straightforwardly enriched.

Consider a political candidate facing a pool of voters. The candidate has an attribute (either innate, or a policy position) $x \in (0,1)$. For example, x may represent the strength of a candidate's links to some industry; or his stance on trade agreements; or his personal income. The candidate does not know how voters respond to this attribute. In particular, with probability q_1, voters are of type 1 in the sense that they may like this attribute, and respond positively to higher values of x. In contrast, with probability q_2, voters are of type 2 in the sense that they may dislike this attribute, and respond negatively.

In addition, and regardless of whether the pool of voters is type 1 or 2, voters also weight other factors when deciding whether to vote the candidate. These other factors are represented by δ, which is uniformly distributed over $[0,1]$. Specifically, if

[4] More precisely, the three receiver types are given by $g_1(x) = x$, $g_2(x)$ as previously defined (may differ by a constant $E[x]$), and $g_3(x) = x + g_2(x)$.

the pool of voters is type i, the candidate wins the election if

$$\log \left(E_x \left[g_i(x)|\mathcal{I}\right] + \kappa_a\right) + \log \delta \geq \log \kappa_b,$$

where κ_a and κ_b are parameters capturing details of the political process, and the characteristics of the candidate's opponent(s). Consequently, the candidate wins the election if $\delta \geq \frac{\kappa_b}{E[g_i(x)|\mathcal{I}]+\kappa_a}$, and so has a winning probability of

$$1 - \frac{\kappa_b}{E_x \left[g_i(x)|\mathcal{I}\right] + \kappa_a}.$$

Normalizing the candidate's winning payoff to 1, and defining $v\left(p\right) = 1 - \frac{\kappa_b}{p+\kappa_a}$, the candidate's expected payoff is hence

$$\sum_{i=1,2} q_i v \left(E_x \left[g_i(x)|\mathcal{I}\right]\right),$$

which falls within our framework. Note that v is strictly increasing, and concave.

3.4 Differences in assessments of signal accuracy

Finally, we consider a case in which the sender discloses a signal x of some underlying attribute y. All receivers agree that higher values of y are desirable. In this case, differences in the receivers stem from differences in their assessment of the accuracy of the signal x.

A variety of applications fit this specific framework. To give just one concrete example, consider the case of restaurants considering whether to disclose food safety ratings, which are subject to a concern of rating inflation.

To illustrate the mechanism, we give a tightly parameterized example of how this case can arise. But the general idea is that the posterior expectation $E\left[y|x\right]$ may be non-monotone with respect to the signal x. This general point is well understood; in particular, Dawid (1973) gives conditions under which $E\left[y|x\right] \to E\left[y\right]$ as $x \to \infty$.

In more detail: the sender has an underlying attribute, denoted y, and receives a rating, denoted x ("type"). Neither the sender nor receivers can observe y. The receiver observes the rating x, and must decide whether or not to disclose it.

The rating x is either completely accurate (i.e., $x = y$), or is pure noise (distributed independently from y). A receiver of type $i \in N$ attaches probability p_i to the rating being accurate, where p_i differs across receiver types. The attribute y is distributed over $[0,1]$ according to the density function $h_y(y) = 2(1-y)$, so that $E[y] = \frac{1}{3}$. When the rating is noise, it is distributed over $[0,1]$ according to density $h_x(x) = 2x$. That is, the true type y is drawn from a distribution weighted towards low values, while the rating is drawn from a distribution weighted towards high values.

A receiver's expectation of attribute y conditional on the rating being x is

$$\begin{aligned} E[y|x] &= \Pr(\text{accurate}|x) E[y|x, \text{accurate}] + \Pr(\text{inaccurate}|x) E[y|x, \text{inaccurate}] \\ &= \Pr(\text{accurate}|x) x + \Pr(\text{inaccurate}|x) E[y] \\ &= \Pr(\text{accurate}|x)(x - E[y]) + E[y]. \end{aligned}$$

Moreover, by Bayes' rule,

$$\begin{aligned} \Pr(\text{accurate}|x) &= \frac{\Pr(\text{accurate}) \Pr(x|\text{accurate})}{\Pr(\text{accurate}) \Pr(x|\text{accurate}) + \Pr(\text{inaccurate}) \Pr(x|\text{inaccurate})} \\ &= \frac{p_i h_y(x)}{p_i h_y(x) + (1-p_i) h_x(x)} \\ &= \frac{p_i(1-x)}{p_i(1-x) + (1-p_i)x}. \end{aligned}$$

In particular, a receiver's belief that the rating is accurate is a decreasing function of the rating.

It follows that

$$E[y|x] = \frac{p_i(1-x)\left(x - \frac{1}{3}\right)}{p_i(1-x) + (1-p_i)x} + \frac{1}{3}. \tag{3}$$

Straightforward though tedious calculation (see appendix) establishes that $E[y|x]$ is strictly concave as a function of x, and obtains it maximum in the interior of $(0,1)$, at $\frac{\sqrt{1-p_i^2}-p_i}{\sqrt{3}(1-2p_i)}$ if $p_i \neq \frac{1}{2}$, and at $x = \frac{2}{3}$ if $p_i = \frac{1}{2}$.

Consequently, by setting u_i linear, and $g_i(x)$ equal to (3), this fits within our framework.

4 Equilibria with non-disclosure

We start by showing how equilibria with non-disclosure can emerge in our setting. It is useful to start by considering the following simple example, which illustrates the main features that deliver non-disclosure in equilibrium:

Example: There are two receiver types ($n = 2$), both of whom are risk-neutral (u_i is linear for $i = 1, 2$) and their preferences over sender types are linear and symmetric ($g_1(x) = x$ and $g_2(x) = 1 - x$); the sender is strictly risk-averse (v is strictly concave); there is an equal probability of each receiver type ($q_1 = q_2 = \frac{1}{2}$); and the unconditional mean $E[x]$ of the sender's type is $\frac{1}{2}$.

Under these conditions, there is an equilibrium with no disclosure at all, as follows. In such an equilibrium, non-disclosure results in prices

$$\begin{aligned} p_1^{ND} &= E[x] = \frac{1}{2} \\ p_2^{ND} &= E[1-x] = \frac{1}{2}, \end{aligned}$$

and so the sender's expected utility from non-disclosure is simply $v\left(\frac{1}{2}\right)$.

On the other hand, if a sender of type x discloses, he faces a lottery over prices x and $1 - x$, with a probability $\frac{1}{2}$ of each outcome. This lottery has an expected payoff of $\frac{1}{2}$. So, since the sender is risk-averse, he strictly prefers non-disclosure to disclosure.[5]

Note that the Example also illustrates that our setting regularly has multiple equilibria. Full-disclosure can always be supported as an equilibrium, simply by assigning off-equilibrium beliefs on non-disclosure that load on the type with the lowest utility from disclosure. Accordingly, our main results are concerned with characterizing non-disclosure equilibria when they exist, and with comparative statics on non-disclosure equilibria.

As the Example makes clear, the three key properties driving equilibrium non-disclosure are (I) different preferences of different receivers, which gives rise to a lottery over prices; (II) the preferences deliver different orderings over sender types, at least over some range; and (III) sender risk-aversion. We next establish the necessity of these three properties.

[5]A sender with type $x = \frac{1}{2}$ is indifferent.

4.1 Necessary conditions for non-disclosure

First, non-disclosure can only arise if the sender's expected utility from disclosure, V^D, is non-monotone:

Proposition 1 *If the disclosure payoff V^D is either strictly increasing or strictly decreasing over X, then disclosure occurs with probability* 1*.*

Second, non-disclosure can only arise if types differ in the sense that there must exist at least two receiver types g_i and g_j, $i \neq j$, who have different preference orderings:

Proposition 2 *If there is no uncertainty over receiver preference orderings, i.e., g_i is ordinally equivalent to g_j in the sense that $g_i(x) < (\leq) g_i(\tilde{x})$ if and only if $g_j(x) < (\leq) g_j(\tilde{x})$ for any $x, \tilde{x} \in X$ and for all $i, j \in N$, then disclosure occurs with probability* 1*.*

We highlight that Proposition 2 is true even if g_i is non-monotone, suggesting that non-monotone receiver preferences (and hence non-monotone sender payoffs) alone are not sufficient to generate non-disclosure in equilibrium.

Third, non-disclosure can only arise if the sender is strictly risk-averse (v strictly concave).

Proposition 3 *If the sender's payoff function v is linear or strictly convex then disclosure occurs with probability* 1*.*

The basic intuition for each of Propositions 1, 2, and 3 is the same. In each case, one can think of changing variables in the description of a firm's type, and applying the standard unravelling argument to the new "type." The "types" in the three cases are, respectively, $g_1(x)$, $V^D(x)$, and $E_i[g_i(x)]$.

4.2 Properties of equilibria with non-disclosure

In light of Proposition 3, for the remainder of the paper we impose:

Assumption 3 *The sender's payoff function v is strictly concave.*

The Example above has no disclosure at all. However, this is an unusual case, in the sense that it can arise only if

$$\max_{\tilde{x}} E_i \left[v \left(g_i \left(\tilde{x} \right) \right) \right] \leq E_i \left[v \left(p_i^{ND} \right) \right],$$

which by Lemma 1 implies

$$\max_{\tilde{x}} E_i \left[v \left(g_i \left(\tilde{x} \right) \right) \right] \leq E_i \left[v \left(p_i^{ND} \right) \right] \leq E_i \left[v \left(g_i \left(E_x \left[x \right] \right) \right) \right],$$

which requires the knife-edge condition $E_x [x] = \arg\max_{\tilde{x}} E_i [v (g_i (\tilde{x}))]$.

More generally, non-disclosure equilibria entail some sender types disclosing and other types not disclosing. We define a marginal discloser as a sender type that is indifferent between disclosing or not in such an equilibrium:

Definition 1 *In any equilibrium with non-disclosure, a sender type x is a marginal discloser if and only if $V^D(x) = E_i \left[v \left(p_i^{ND} \right) \right]$.*

An important result is the following:

Proposition 4 *Any equilibrium with non-disclosure is of one of the following two types:*

(A) Complete non-disclosure: no sender discloses.

(B) Partial non-disclosure: both high and low extreme sender types do not disclose, i.e., the set of non-disclosing senders is of the form

$$X \backslash \left[\underline{x}, \bar{x} \right]$$

for some $0 < \underline{x} < \bar{x} < 1$, where $\underline{x}$ and $\bar{x}$ are the two marginal disclosers, and consequently,

$$V^D \left(\underline{x} \right) = V^D \left(\bar{x} \right) = E_i \left[v \left(p_i^{ND} \right) \right]. \tag{4}$$

Figure 1 illustrates a generic partial non-disclosure equilibrium.

The proof of Proposition 4 is intuitive. It relies on the fact that if types $\underline{x}$ and $\bar{x}$ disclose and $\underline{x} < \bar{x}$, then any type $x \in (\underline{x}, \bar{x})$ in the middle must also disclose, as follows. The fact that $\underline{x}$ and $\bar{x}$ disclose implies

$$\min \left\{ V^D \left(\underline{x} \right), V^D \left(\bar{x} \right) \right\} \geq E_i \left[v \left(p_i^{ND} \right) \right],$$

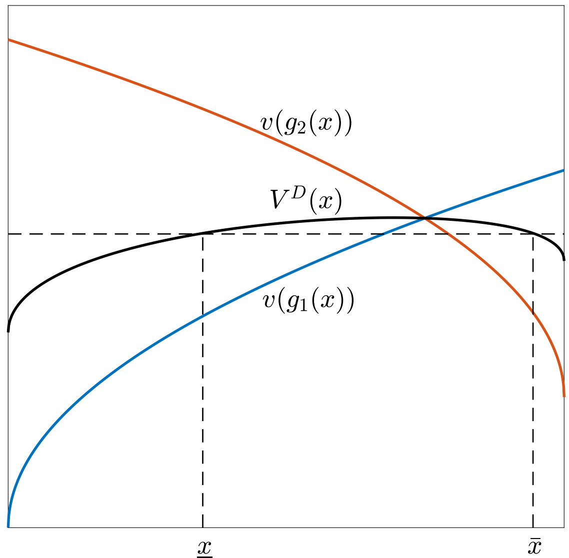

Figure 1: Illustration of a generic partial non-disclosure equilibrium

and so Assumption 2 in turn implies that, for any $x \in (\underline{x}, \bar{x})$,

$$V^D(x) > E_i\left[v\left(p_i^{ND}\right)\right].$$

The formal proof in the appendix further rules out the cases of $\underline{x} = 0$ or $\bar{x} = 1$, that is, it must be both high and low extreme sender types that choose to keep silent. The intuition is as follows. For specificity, we present the intuition for why the non-disclosure set cannot be a lower interval of X, of the form $\{x \in X | x \leq \bar{x}\}$. There are two cases. If V^D is decreasing at $\bar{x}$, then senders with type just below $\bar{x}$ would gain from deviating and disclosing. If instead V^D is increasing at $\bar{x}$, then since V^D is increasing for all $x \leq \bar{x}$, and the unravelling argument operates within $\{x \in X | x \leq \bar{x}\}$ (formally, see Proposition 1).

Consistent with this result, Luca and Smith (2015) find that top business schools are least likely to disclose their rankings, whereas mid-ranked schools are most likely to disclose. Similarly, Bederson, et. al. (2017) find that the highest-rated restaurants do not disclose their hygiene ratings, while relatively higher-rated restaurants disclose to stand out from other lower-rated ones.

Economically, non-disclosure is attractive for extreme sender types because, given the equilibrium form of both extreme types not disclosing, receivers interpret non-disclosure as meaning that the sender must either have a very low or very high type, and so on average is of an intermediate type. Consequently, non-disclosure allows an extreme type agent to replace a very risky lottery over prices $(g_i(x))_{i \in N}$ with a safer lottery over more similar prices $\left(p_i^{ND}\right)_{i \in N}$.

Our next result formalizes the idea that the lottery over $\left(p_i^{ND}\right)_{i \in N}$ is safer for at least one of the two marginal disclosers. For use both here and below, we state the following mild condition, which guarantees strictness of some key inequalities:

Condition 1 *There is at least one receiver type i for which g_i is strictly concave.*

Proposition 5 *Consider a partial non-disclosure equilibrium in which the non-disclosure set is $[0, \underline{x}) \cup (\bar{x}, 1]$ with $0 < \underline{x} < \bar{x} < 1$. Then*

$$E_x \left[x | [0, \underline{x}) \cup (\bar{x}, 1] \right] \quad \in \quad [\underline{x}, \bar{x}] \tag{5}$$

and moreover, for some marginal discloser $x \in \{\underline{x}, \bar{x}\}$,

$$E_i \left[p_i^{ND} \right] \leq E_i \left[g_i \left(x \right) \right], \tag{6}$$

where the inequality is strict if Condition 1 holds.

Equation (5) in Proposition 5 formalizes the idea that non-disclosure is attractive because receivers' equilibrium expectation of the sender's type given non-disclosure lies between the marginal discloser types $\underline{x}$ and $\bar{x}$. Inequality (6) says that the non-disclosure lottery is safer than the disclosure lottery of at least one of the marginal disclosers, in the following sense: since the lotteries provide the same expected utility to the sender (by the equilibrium condition (4)), a lower expected payoff implies that the lottery must be safer. In words, "silence is safest."

4.3 Existence of non-disclosure equilibria

Propositions 4 and 5 characterize non-disclosure equilibria, conditional on such equilibria existing. In general, an equilibrium with non-disclosure indeed exists provided that (I) receivers have different preference orderings over extreme sender types; (II) the probability of different receiver types is such that extreme sender types dislike disclosure sufficiently equally; and (III) receivers are not too risk-averse. Proposition 6 establishes existence under these conditions.

The result requires some mild regularity conditions on the prior density of sender types at the extremes, and on receiver preferences over extreme sender types. For clarity, we state these regularity assumptions separately.

Assumption 4 *For any constant $\kappa > 0$, $\lim_{x\to 0} \frac{f(x)}{f(1-\kappa x)}$ exists and is strictly positive.*

Assumption 5 *For all $i \in N$, the set $v(g_i(X))$ is bounded, and moreover, there exist $\underline{v}_i > 0$ and $\bar{v}_i < \infty$ such that $\frac{\partial}{\partial x} v(g_i(x)) \in [\underline{v}_i, \bar{v}_i]$ for all $x \in (0,1)$.*

Note that Assumption 5 is required only in cases where the sender's type space X is non-compact.

In addition, recall that at this point in the paper we have imposed Assumption 3, which states that the sender is strictly risk averse.

Proposition 6 *Suppose that there are receiver types $i, j \in N$ such that $g_i(0) < g_i(1)$ and $g_j(0) > g_j(1)$. Then an equilibrium with non-disclosure exists if the distribution of receiver types $(q_i)_{i\in N}$ is such that $\left|\lim_{x\to 0} V^D(x) - \lim_{x\to 1} V^D(x)\right|$ is sufficiently small, and all receiver types are sufficiently close to risk-neutral.*

The proof of Proposition 6 is based on standard fixed-point arguments, and we sketch a special case here to illustrate how it works. Let everything be the same as in the above Example, with the exception that now $E_x[x] \neq \frac{1}{2}$. An important property of the Example, which considerably simplifies the argument below, is that $p_2^{ND} = 1 - p_1^{ND}$, so that the non-disclosure payoff is simply $q_1 v\left(p_1^{ND}\right) + q_2 v\left(1 - p_1^{ND}\right) = V^D\left(p_1^{ND}\right)$.[6]

To show that an equilibrium exists, we look for a candidate equilibrium in which types $X \backslash [\underline{x}, \bar{x}]$ stay silent and do not to disclose, while types $[\underline{x}, \bar{x}]$ disclose. From Proposition 4, we know that any non-disclosure equilibrium is of this type. To this end, we vary the candidate value of $\underline{x}$ continuously from $\arg\max_{\tilde{x}} V^D(\tilde{x}) = \frac{1}{2}$ down to 0. The corresponding candidate value of $\bar{x} > \frac{1}{2}$ is determined by the equilibrium condition $V^D(\underline{x}) = V^D(\bar{x})$. Given candidate values of $\underline{x}$, $\bar{x}$, the corresponding payoffs associated with non-disclosure are $p_1^{ND} = E_x[x | X \backslash [\underline{x}, \bar{x}]]$ and $p_2^{ND} = E_x[1 - x | X \backslash [\underline{x}, \bar{x}]]$.

[6]The proof in the appendix does not rely on this property.

On the one hand, at $\underline{x} = \bar{x} = \frac{1}{2}$, we know $p_1^{ND} = E_x[x] \neq \frac{1}{2}$, so that $V^D(\underline{x}) > V^D\left(p_1^{ND}\right)$. That is, the sender $\underline{x}$ strictly prefers disclosure to non-disclosure, implying that full non-disclosure is not an equilibrium.

On the other hand, as $\underline{x}$ approaches 0, $\bar{x}$ approaches 1. So provided the density of x behaves similarly at the extremes of $[0,1]$, it follows that p_1^{ND} is bounded away from both 0 and 1. Consequently, for all $\underline{x}$ sufficiently close to 0, we know $V^D(\underline{x}) < V^D\left(p_1^{ND}\right)$, since V^D obtains its minimum value at the extremes $x = 0, 1$. In words, the sender $\underline{x}$ strictly prefers non-disclosure to disclosure as $\underline{x}$ approaches 0.

By continuity, it follows that there is at least one candidate equilibrium $\underline{x} \in \left(0, \frac{1}{2}\right)$ that satisfies the equilibrium condition $V^D(\underline{x}) = V^D(\bar{x}) = V^D\left(p_1^{ND}\right)$.

Among other things, the above argument highlights the role of the condition in Proposition 6 that $\left|V^D(0) - V^D(1)\right|$ needs to be sufficiently small. This condition ensures that for any candidate specification of a marginal discloser with low type (i.e., a small $\underline{x}$), it remains possible to find a corresponding marginal discloser with high type (i.e., a large $\bar{x}$).

At the same time, it is worth emphasizing that Proposition 6 states just one set of sufficient conditions for non-disclosure. Non-disclosure equilibria can certainly exist even when $V^D(0)$ and $V^D(1)$ are very different.

5 Comparative statics

Given that a key economic force driving equilibrium non-disclosure is that non-disclosure reduces the risk faced by senders, especially those with extreme types, it is natural to conjecture that non-disclosure is increasing in sender risk-aversion. Propositions 7 and 8 make this intuition precise. It is also natural to expect that non-disclosing is decreasing in receiver risk-aversion because a more risk-averse receiver is less willing to pay a high price to a non-disclosing sender. This is formalized in Proposition 9.

5.1 Increasing sender risk-aversion

Proposition 5 says that in a partial non-disclosure equilibrium, non-disclosure reduces risk for at least one of the marginal disclosing types $\underline{x}$ and $\bar{x}$. Given this, a natural conjecture is that as seller risk-aversion increases, senders are less likely to disclose, and more likely to remain silent.

As is widely appreciated, it is relatively hard to produce general comparative statics of choices between risky lotteries with respect to risk aversion (see, e.g., Ross (1981) for a discussion of this point), without imposing significant structure on either the utility function or on the distribution of payoffs. The following result uses a notion of increased risk aversion related to (though more restrictive than) that used in Ross (1981). Proposition 8 below instead uses the more general notion of Pratt (1964), but applies only to the case of two receiver types ($n = 2$).

Proposition 7 *Suppose that Condition 1 holds, and that an equilibrium with both non-disclosure and disclosure exists when the sender's preferences are given by v. If the sender's preferences change to $\tilde{v}$, where $\tilde{v}$ is more risk averse than v in the sense that $v(x) = \alpha \tilde{v}(x) + x$ at all $x \in X$ for some constant $\alpha > 0$, then there is a marginal discloser x_m for whom non-disclosure is safer than disclosure in the original equilibrium, i.e., $E_i \left[p_i^{ND} \right] < E_i \left[g_i(x_m) \right]$, and a new non-disclosure equilibrium under preferences $\tilde{v}$, such that non-disclosure increases in the neighborhood of x_m.*

In words, the comparison of risk aversion used in Proposition 7 amounts to saying: preferences represented by $\tilde{v}$ are more risk averse than preferences represented by v if v corresponds to a mixture of $\tilde{v}$ and risk neutral preferences. In particular, for mean variance preferences, this comparison corresponds to a greater dislike of variance.

In the case of just two receiver types ($n = 2$), we can substantially generalize Proposition 7. The key reason is that for $n = 2$, both disclosure and non-disclosure induce binary lotteries; and moreover (as we show in the proof of Proposition 8 below), for at least one of the marginal disclosers $x_m \in \{\underline{x}, \bar{x}\}$, it is the case that $p_1^{ND}, p_2^{ND} \in [\min\{g_1(x_m), g_2(x_m)\}, \max\{g_1(x_m), g_2(x_m)\}]$. This additional structure on the lotteries associated with disclosure and non-disclosure allows us to apply Pratt's (1964) more general ordering of risk preferences:

Proposition 8 *Suppose that $n = 2$, Condition 1 holds, and that an equilibrium with both non-disclosure and disclosure exists when the sender's preferences are given by v. If the sender's preferences change to $\tilde{v}$, where $\tilde{v}$ is more risk averse than v in the sense that there exists an increasing concave function ϕ such that $\tilde{v}(x) = \phi(v(x))$ at all $x \in X$, then there is a marginal discloser x_m for whom non-disclosure is safer* *than disclosure in the original equilibrium, i.e.,* $E_i\left[p_i^{ND}\right] < E_i\left[g_i\left(x_m\right)\right]$*, and a new non-disclosure equilibrium under preferences* $\tilde{v}$*, such that non-disclosure increases in the neighborhood of* x_m*.*

5.2 Increasing receiver risk-aversion

Similarly, we consider an increase in risk-aversion by the receiver by applying Prat's (1964) general ordering of risk preferences.

Intuitively, while non-disclosure helps risk-averse senders by delivering a safer lottery, it hurts risk-averse receivers, because it means that they buy an item of uncertain quality. Consequently, an increase in receiver risk-aversion reduces the prices paid to a non-disclosing sender. Hence higher risk-aversion of receivers make non-disclosure less attractive for senders. Hence an increase in receiver risk-aversion reduces non-disclosure:

Proposition 9 *Suppose that Condition 1 holds, and that an equilibrium with both non-disclosure and disclosure exists when the receivers' preferences are given by* u_i*. If receiver* i*'s preferences changes to* $\tilde{u}_i$*, where* $\tilde{u}_i$ *is more risk averse than* u_i *in the sense that there exists an increasing concave function* ϕ *such that* $\tilde{u}_i(x) = \phi(u_i(x))$ *at all* $x \in X$*, then the new equilibrium features more disclosure.*

Note that in our setting, disclosure by a sender eliminates all risk for a receiver. However, the economic force in Proposition 9 continues to hold even in situations where disclosure reduced the risk faced by receivers, instead of completely eliminating it.

6 Generalized disclosure

Thus far, we have considered the case in which the sender either discloses that his type is in the singleton set $\{x\}$, or else discloses nothing. In this section, we consider instead the case in which the sender can disclose any member A of some family of sets $\mathcal{X}$, provided that $x \in A$. We assume that, at a minimum, $\mathcal{X}$ contains all singletons, all closed subintervals of the interval X, and all binary unions of closed subintervals of X. To avoid economically uninteresting mathematical complications, we assume that all members of $\mathcal{X}$ are closed; and moreover (and for this section only) that X itself is closed, i.e., $X = [0, 1]$. Note that no-disclosure simply corresponds to disclosing X.

This enlarged set of disclosure possibilities is most likely to be relevant if disclosure takes the form of a trustworthy auditor reporting a sender's type x to receivers. If instead disclosure takes the form of simply displaying some attribute to receivers (e.g., a food safety rating, a tax return, etc.), then our analysis so far covers the relevant case.

Note that this expansion of the sender's disclosure possibilities does not affect standard unravelling results. Indeed, it is straightforward to adapt the proofs of Propositions 2, 1, and 3 to show that, under the conditions stated in these three results, in any equilibrium a sender discloses $\{x\}$ with probability one.

Our main result in this section is that, given the expanded set of disclosure possibilities, an equilibrium with less than full disclosure exists under a very wide range of circumstances if the key conditions we identify in this paper are satisfied, namely sender risk aversion, differences in receiver preferences, and receivers who are not too risk averse. In particular, we are able to establish existence of an equilibrium with less than full disclosure without imposing the sufficient condition that $V^D(0)$ is sufficiently close to $V^D(1)$, which we used to establish Proposition 6.[7]

Proposition 10 *If (A) there exist $\underline{\xi}, \bar{\xi} \in (0, 1)$ and a pair of receiver types i, j such that $\underline{\xi} \neq \bar{\xi}$, $V^D\left(\underline{\xi}\right) = V^D\left(\bar{\xi}\right)$, and $g_i(x) \neq g_j(x)$ for $x = \underline{\xi}, \bar{\xi}$, and (B) all receiver types are sufficiently close to risk neutral, then there is an equilibrium with less than full disclosure, i.e., there is a positive probability of a sender disclosing a signal other than $\{x\}$.*

It is worth stressing that the condition (A) is satisfied whenever receivers have different preferences (g_i differs from g_j for at least some i, j), and these different preferences generate non-monotonicity of the expected utility from disclosing $\{x\}$, as given by the function V^D.

The proof of Proposition 10 is very close to previous analysis, and we give it here. We establish the existence of an equilibrium characterized by $\underline{x}, \bar{x} \in \left(\underline{\xi}, \bar{\xi}\right)$, in which senders with $x \in (\underline{x}, \bar{x})$ and $x \in X \setminus \left[\underline{\xi}, \bar{\xi}\right]$ disclose their exact type $\{x\}$; while the remaining senders with $x \in \left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$ disclose simply $\left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$.

[7] We are also able to drop the regularity conditions contained in Assumptions 4 and 5, for reasons that should be clear below.

The proof of Proposition 10 builds on the proof of 6. First, if one restricts senders to disclose either $\{x\}$ or $\left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$, the proof is the same as that of Proposition 10.[8]

It then remains to ensure that senders do not deviate to other disclosures. The equilibrium is supported by the following off-equilibrium beliefs: If the sender discloses $A \in \mathcal{X}$, and $A \neq \left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$, off-equilibrium beliefs place full mass on the sender's type being in $\arg\min_{\tilde{x} \in A} V^D(\tilde{x})$. These off-equilibrium beliefs immediately imply that senders with $x \in X \backslash \left(\left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]\right)$ do not have a profitable deviation. For senders with $x \in \left[\underline{\xi}, \underline{x}\right] \cup \left[\bar{x}, \bar{\xi}\right]$, note that these off-equilibrium beliefs ensure that any deviation is at least weakly worse than the deviation of disclosing $\{x\}$—which has already been established to be an unprofitable deviation, by the first step of the proof.

7 Welfare consequences of mandated disclosure

In many circumstances, regulations and laws mandate disclosure. In cases where the standard unravelling argument applies, such regulations should have little effect on equilibrium outcomes and utilities. In contrast, in the cases we have characterized where the equilibrium outcome is less than full disclosure, such regulations clearly increase disclosure. This affects welfare differently for senders and receivers.

For senders, mandated disclosure can only lower welfare, since an unregulated sender always has the option of not-disclosing.

Under the competitive condition (1), receiver utility is always simply $u_i(0)$, so that receiver utility is unaffected by mandated disclosure. But more generally, one could imagine replacing (1) with alternative assumptions that leave receivers some surplus. (Such a change would not affect the key economic forces in our analysis.) In this case, mandated disclosure has the potential to increase receiver welfare, by reducing the risk to which they are exposed.

[8] Indeed, the fact that $\underline{\xi}, \bar{\xi} \in (0,1)$ means that the proof avoids the complications of what happens to utility and density functions as $x \to 0, 1$, which is what allows use to dispense with the regularity conditions contained in Assumptions 4 and 5, as noted in footnote 7.

8 Conclusion

There are many settings in which voluntary disclosure is possible, but in which disclosure occurs with probabilities below 1, despite classic unravelling arguments. In this paper we explore a possible explanation, which is new to the literature, namely that potential disclosers do not know the preference ordering of the people they are disclosing to, and because of risk-aversion they dislike the risk that this imposes. We show how these two features together naturally deliver equilibrium non-disclosure.

In contrast to existing leading explanations of non-disclosure, our explanation does not require disclosure to be either costly, or impossible for some (unobservable) subset of would-be disclosers. As such, our paper can explain non-disclosure even in settings where disclosure is costless, and there is no uncertainty about whether disclosure is possible.

Non-disclosure is attractive because it reduces the risk borne by potential disclosers with extreme information. Consequently, disclosure decreases when potential disclosers grow more risk-averse, in a sense we make precise. On the other hand, non-disclosure reduces the information available to the audience for disclosures, thereby increasing the risk borne by the audience. Because of this, potential disclosers benefit more from disclosing when audiences grow more risk averse, leading to increased equilibrium disclosure.

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Appendix

Throughout the appendix, denote by ND the set of sender types which do not disclose.

Results omitted from main text

Lemma A-1 *Let Assumptions 1 and 3 hold. If an equilibrium non-disclosure set is of the form* $X \backslash [\underline{x}, \bar{x}]$ *where* $\underline{x} < \bar{x}$ *then* $E_x[x|ND] \in [\underline{x}, \bar{x}]$.

Proof of Lemma A-1: Under Assumptions 1 and 3, V^D is strictly concave. Since $V^D(\underline{x}) = V^D(\bar{x})$, it follows that V^D is strictly increasing for $x \leq \underline{x}$ and strictly decreasing for $x \geq \bar{x}$. So if $E_x[x|ND] < \underline{x}$ then

$$V^D(E_x[x|ND]) < V^D(\underline{x}) = E_i\left[v\left(p_i^{ND}\right)\right],$$

while if instead $E_x[x|ND] > \bar{x}$ then

$$V^D(E_x[x|ND]) < V^D(\bar{x}) = E_i\left[v\left(p_i^{ND}\right)\right].$$

However, Lemma 1 implies that

$$V^D(E_x[x|ND]) \geq E_i\left[v\left(p_i^{ND}\right)\right].$$

The contradiction completes the proof.

Proofs of results stated in main text

Proof of Lemma 1: Concavity of u_i and Jensen's inequality imply

$$u_i(E_x[g_i(x) - p_i|\mathcal{I}]) \quad \geq \quad E_x[u_i(g_i(x) - p_i)|\mathcal{I}] = u_i(0)$$

which in turn implies $p_i \leq E_x[g_i(x)|\mathcal{I}]$. The second inequality in (2) is immediately from Jensen's inequality, completing the proof.

Details for Subsection 3.4:

Collecting terms in condition (3),

$$E[y|x] = -p_i \frac{\frac{1}{3} - \frac{4}{3}x + x^2}{p_i + (1-2p_i)x} + \frac{1}{3}.$$

Hence differentiation yields

$$\begin{aligned}\frac{\partial}{\partial x}E\left[y|x\right] &= -p_i\frac{\left(2x-\frac{4}{3}\right)\left(p_i+\left(1-2p_i\right)x\right)-\left(1-2p_i\right)\left(\frac{1}{3}-\frac{4}{3}x+x^2\right)}{\left(p_i+\left(1-2p_i\right)x\right)^2}\\ &= -p_i\frac{\left(1-2p_i\right)x^2+2p_ix-\frac{1}{3}-\frac{2}{3}p_i}{\left(p_i+\left(1-2p_i\right)x\right)^2}\end{aligned}$$

and

$$\begin{aligned}&\frac{\partial^2}{\partial x^2}E\left[y|x\right]\\ = &-p_i\frac{\left(2\left(1-2p_i\right)x+2p_i\right)\left(p_i+\left(1-2p_i\right)x\right)^2-2\left(1-2p_i\right)\left(p_i+\left(1-2p_i\right)x\right)\left(\left(1-2p_i\right)x^2+2p_ix-\frac{1}{3}-\frac{2}{3}p_i\right)}{\left(p+\left(1-2p\right)x\right)^4}\\ = &-2p_i\frac{\left(\left(1-2p_i\right)x+p_i\right)\left(p+\left(1-2p_i\right)x\right)-\left(1-2p_i\right)\left(\left(1-2p_i\right)x^2+2p_ix-\frac{1}{3}-\frac{2}{3}p_i\right)}{\left(p_i+\left(1-2p_i\right)x\right)^3}\\ = &-2p_i\frac{p_i^2+\frac{1}{3}\left(1-2p_i\right)\left(1+2p_i\right)}{\left(p_i+\left(1-2p_i\right)x\right)^3}=-2p\frac{\frac{1}{3}-\frac{1}{3}p_i^2}{\left(p_i+\left(1-2p_i\right)x\right)^3}.\end{aligned}$$

Note that $\frac{\partial^2}{\partial x^2}E\left[y|x\right]<0$, establishing strict concavity. Moreover, $\frac{\partial}{\partial x}E\left[y|x\right]>0$ at $x=0$ and $\frac{\partial}{\partial x}E\left[y|x\right]=-\frac{2}{3}\frac{p}{1-p}<0$ at $x=1$, implying that $E\left[y|x\right]$ obtains its maximum in the interior of $(0,1)$. The expression for the location of the maximum then follows from the condition $\frac{\partial}{\partial x}E\left[y|x\right]=0$.

Proof of Proposition 1: We establish the result for the case of V strictly increasing. (The proof of V strictly decreasing is analogous.). Suppose that, contrary to the claimed result, the probability of non-disclosure is strictly positive. So there exists some subset $ND\subset[0,1]$ of sender-types who disclose with probability below 1. Moreover, there must exist some $x'\in ND$ such that $x'>E_x[x|ND]$. Since V^D is strictly increasing,

$$\begin{aligned}V^D(x') &> V^D\left(E_x\left[x|ND\right]\right)\\ &= E_i\left[v\left(g_i\left(E_x\left[x|ND\right]\right)\right)\right]\\ &\geq E_i\left[v\left(p_i^{ND}\right)\right],\end{aligned}$$

where the second inequality follows from Lemma 1. So x' is strictly better off deviating and disclosing, which contradicts the equilibrium condition and completes the proof.

Proof of Proposition 2: The cases of $(g_i)_{i \in N}$ all strictly increasing, and all strictly decreasing, are covered by Proposition 1. So it remains to consider the case in which $(g_i)_{i \in N}$ are all non-monotone. By concavity and Assumption 1, it must be that $(g_i)_{i \in N}$ are all first strictly increasing and then strictly decreasing.

Suppose to the contrary that the probability of non-disclosure is strictly positive. So there exists some non-zero-measure subset $ND \subset [0,1]$ of sender types who do not disclose. Let $\overline{ND}$ be the closure of ND in X. By continuity, the equilibrium condition implies $E_i\left[v\left(p_i^{ND}\right)\right] \geq E_i\left[v\left(g_i\left(x\right)\right)\right]$ for all $x \in \overline{ND}$. From Lemma 1, $p_i^{ND} \leq E_x\left[g_i(x)|ND\right] = E_x\left[g_i(x)|\overline{ND}\right]$. Let $x = \arg\max_{\tilde{x} \in \overline{ND}} g_1(\tilde{x})$, where we have used the fact that g_1 is strictly increasing then strictly decreasing to ensure that x is well-defined.[9] By ordinal equivalence, $x = \arg\max_{\tilde{x} \in \overline{ND}} g_i(\tilde{x})$ for all receiver types i. By Assumption 1, it follows that $p_i^{ND} < g_i(x)$ for all receiver types i. But then $E_i\left[v\left(p_i^{ND}\right)\right] < E_i\left[v\left(g_i\left(x\right)\right)\right]$, and the contradiction completes the proof.

Proof of Proposition 3: Suppose to the contrary that the probability of non-disclosure is strictly positive. So there exists some non-zero-measure subset $ND \subset [0,1]$ of sender-types who disclose with probability below 1. Since any sender type $x' \in ND$ prefers non-disclosure to disclosure, Lemma 1 implies

$$E_i\left[v\left(g_i\left(x'\right)\right)\right] \leq E_i\left[v\left(p_i^{ND}\right)\right] \leq E_i\left[v\left(E_x\left[g_i(x)|ND\right]\right)\right].$$

Since v is weakly convex,

$$E_i\left[v\left(E_x\left[g_i(x)|ND\right]\right)\right] \leq E_i\left[E_x\left[v\left(g_i(x)\right)|ND\right]\right] = E_x\left[E_i\left[v\left(g_i\left(x\right)\right)\right]|ND\right].$$

Combining these two inequalities implies that, for any $x' \in ND$,

$$E_i\left[v\left(g_i\left(x'\right)\right)\right] \leq E_x\left[E_i\left[v\left(g_i\left(x\right)\right)\right]|ND\right].$$

If v is linear, this is a contradiction by Assumption 2. If instead v is strictly convex, the above inequality is strict, which again gives a contradiction, completing the proof.

Proof of Proposition 4: Given the argument in the main text, it remains only to rule out the cases $\underline{x} = 0$ and $\bar{x} = 1$. If $\underline{x} = 0$ and $\bar{x} < 1$ then $E_x\left[x|ND\right] > \bar{x}$,

[9] Specifically, we need to handle cases such as: X is open at its upper end and $\sup ND = 1$. In this case, $\sup \overline{ND} = 1$, but $1 \notin \overline{ND}$. But $\arg\max_{\tilde{x} \in \overline{ND}} g_1(\tilde{x})$ is still guaranteed to exist since g_1 is strictly decreasing as x approaches 1.

contradicting to Lemma A-1. Similarly, if $\underline{x} > 0$ and $\bar{x} = 1$ then $E_x [x|ND] < \underline{x}$, again contradicting to Lemma A-1, and completing the proof.

Proof of Proposition 5: Inequality (5) is established by Lemma A-1. To establish (6), suppose to the contrary that

$$E_i \left[p_i^{ND}\right] > \max \left\{E_i \left[g_i \left(\underline{x}\right)\right], E_i \left[g_i \left(\bar{x}\right)\right]\right\}. \tag{A-1}$$

By Lemma 1, it follows that

$$E_i \left[g_i \left(E \left[x|ND\right]\right)\right] > \max \left\{E_i \left[g_i \left(\underline{x}\right)\right], E_i \left[g_i \left(\bar{x}\right)\right]\right\}.$$

Given concavity of g_i and (5), it follows that $E_i [g_i (x)]$ obtains its maximum in the interval $[\underline{x}, \bar{x}]$, and hence is weakly increasing over $[0, \underline{x}]$ and weakly decreasing over $[\bar{x}, 1]$. Hence (A-1) implies that

$$E_i \left[p_i^{ND}\right] > E_i \left[g_i \left(\tilde{x}\right)\right] \text{ for all } \tilde{x} \in [0, \underline{x}] \cup [\bar{x}, 1].$$

Another application of Lemma 1 then implies that

$$E_i \left[E_x \left[g_i (x) \,|\, [0, \underline{x}] \cup [\bar{x}, 1]\right]\right] > E_i \left[g_i \left(\tilde{x}\right)\right] \text{ for all } \tilde{x} \in [0, \underline{x}] \cup [\bar{x}, 1].$$

The contradiction establishes (6). Finally, an easy adaptation of the above argument establishes that (6) is strict when at least one g_i is strictly concave, completing the proof.

Proof of Proposition 6: If $0 \notin X$ we write $V^D (0) = \lim_{x \to 0} V^D (x)$, with $V^D (1)$ treated similarly. Under the stated conditions, there exists some distribution of receiver types $(q_i)_{i \in N}$ such that $V^D (0) = V^D (1)$. We establish the existence of a non-disclosure equilibrium for this distribution, and for the case in which all receiver types are risk neutral (u_i linear for all $i \in N$). The general result then follows by continuity.

Because receivers are risk neutral, non-disclosure prices are simply given by $p_i^{ND} = E_x [g_i (x) \,|ND]$.

Note that Assumptions 1 and 3 imply that V^D is strictly concave. Define $x_{\max} = \arg\max_{\tilde{x}} V^D (\tilde{x})$.

If $V^D(x_{\max}) \leq E_i[v(E_x[g_i(x)])]$ then there is an equilibrium in which no sender discloses, and the proof is complete. So for the remainder of the proof, we consider the case in which

$$V^D(x_{\max}) > E_i[v(E_x[g_i(x)])]. \tag{A-2}$$

For any $\underline{x} \in (0, x_{\max})$, define $\eta(\underline{x}) \in (x_{\max}, 1)$ by $V^D(\eta(\underline{x})) = V^D(\underline{x})$. Note that $\eta(\underline{x})$ exists and is unique, since $V^D(0) = V^D(1)$ and V^D is strictly concave. Moreover, η is continuous, with $\eta(\underline{x}) \to 1$ as $\underline{x} \to 0$, and

$$\frac{\partial}{\partial \underline{x}} \eta(\underline{x}) = \frac{\left.\frac{\partial}{\partial x} V^D(x)\right|_{x=\underline{x}}}{\left.\frac{\partial}{\partial x} V^D(x)\right|_{x=\eta(\underline{x})}}.$$

By Assumption 5, $\frac{\partial}{\partial \underline{x}} \eta(\underline{x})$ remains bounded away from both 0 and $-\infty$ as $\underline{x} \to 0$. Assumption 4 and l'Hôpital's rule then imply that the following limit exists, and is bounded away from 0:

$$\lim_{\underline{x} \to 0} \frac{\int_0^{\underline{x}} f(x)\,dx}{\int_{\eta(\underline{x})}^1 f(x)\,dx} = -\lim_{\underline{x} \to 0} \frac{f(\underline{x})}{f(\eta(\underline{x}))\frac{\partial}{\partial \underline{x}} \eta(\underline{x})}.$$

Strict concavity of v (Assumption 3) and the condition that there are receiver types $i, j \in N$ such that $g_i(0) < g_i(1)$ and $g_j(0) > g_j(1)$ then implies that

$$\lim_{\underline{x} \to 0} E_i[v(E_x[g_i(x)\,|X \backslash [\underline{x}, \eta(\underline{x})]])] - E_i[E_x[v(g_i(x))\,|X \backslash [\underline{x}, \eta(\underline{x})]]] > 0. \tag{A-3}$$

Also note that

$$\begin{aligned}
& E_i[E_x[v(g_i(x))\,|X \backslash [\underline{x}, \eta(\underline{x})]]] \\
= \; & E_x[E_i[v(g_i(x))]\,|X \backslash [\underline{x}, \eta(\underline{x})]] \\
= \; & E_x\left[V^D(x)\,|X \backslash [\underline{x}, \eta(\underline{x})]\right].
\end{aligned}$$

Hence, and using $V^D(0) = V^D(1)$,

$$\lim_{\underline{x} \to 0} \left(E_i[E_x[v(g_i(x))\,|X \backslash [\underline{x}, \eta(\underline{x})]]] - V^D(\underline{x})\right) = 0. \tag{A-4}$$

It follows by (A-3) that

$$V^D(\underline{x}) - E_i\left[v\left(E_x\left[g_i(x)\,|X\backslash\left[\underline{x},\eta(\underline{x})\right]\right]\right)\right] < 0$$

for all $\underline{x}$ sufficiently close to 0.

Combined with (A-2), continuity then implies that there exists some $\underline{x} \in (0, x_{\max})$ such that

$$V^D(\underline{x}) = V^D(\eta(\underline{x})) = E_i\left[v\left(E_x\left[g_i(x)\,|X\backslash\left[\underline{x},\eta(\underline{x})\right]\right]\right)\right].$$

Hence there is an equilibrium in which senders $[\underline{x}, \eta(\underline{x})]$ disclose, while senders $X\backslash[\underline{x},\eta(\underline{x})]$ remain silent and do not disclose, completing the proof.

Proof of Proposition 7: Given Proposition 4, when the sender's preferences are given by v, consider an equilibrium in which senders in $[0,\underline{x}) \cup (\bar{x}, 1]$ do not disclose. By Proposition 5, for some $x_m \in \{\underline{x}, \bar{x}\}$,

$$E_i\left[p_i^{ND}\right] < E_i\left[g_i(x_m)\right]. \tag{A-5}$$

It follows that

$$E_i\left[\tilde{v}\left(p_i^{ND}\right)\right] > E_i\left[\tilde{v}\left(g_i(x_m)\right)\right], \tag{A-6}$$

since otherwise (A-5) and the definition that $v(x) = \alpha\tilde{v}(x) + x$ at all $x \in X$ implies that

$$E_i\left[v\left(p_i^{ND}\right)\right] < E_i\left[v\left(g_i(x_m)\right)\right],$$

contradicting the equilibrium condition when the sender's preferences are given by v.

Given inequality (A-6), a straightforward modification of the argument in the proof of equilibrium existence in Proposition 6 implies that, for preferences $\tilde{v}$, there exists an equilibrium in which senders $[0, \underset{\sim}{x}) \cup (\tilde{x}, 1]$ do not disclose, where if $x_m = \underline{x}$ then $\underset{\sim}{x} > \underline{x}$, and if $x_m = \bar{x}$ then $\tilde{x} < \bar{x}$. This completes the proof.

Proof of Proposition 8: Consider any non-disclosure equilibrium, with a non-disclosure set $[0,\underline{x}) \cup (\bar{x}, 1]$.

Claim A: For any receiver type i, $p_i^{ND} \leq \max\{g_i(\underline{x}), g_i(\bar{x})\}$.

Proof of claim: If g_i is monotone over $[\underline{x}, \bar{x}]$, then

$$p_i^{ND} \leq E_x[g_i(x)|ND] \leq g_i(E_x[x|ND]) \leq \max\{g_i(\underline{x}), g_i(\bar{x})\},$$

where the first inequality follows Lemma 1, the second inequality follows Jensen's inequality and the concavity of g_i, and the last inequality follows Lemma A-1 and the monotonicity of g_i over $[\underline{x}, \bar{x}]$.

If instead g_i is non-monotone over $[\underline{x}, \bar{x}]$, then by concavity, it is strictly increasing over $[0, \underline{x})$ and strictly decreasing over $(\bar{x}, 1]$. Hence $g_i(x) < \max\{g_i(\underline{x}), g_i(\bar{x})\}$ for all $x \in [0, \underline{x}) \cup (\bar{x}, 1]$. So by Lemma 1,

$$p_i^{ND} \leq E_x[g_i(x)|ND] < \max\{g_i(\underline{x}), g_i(\bar{x})\}.$$

Claim B: For some $x \in \{\underline{x}, \bar{x}\}$, $p_1^{ND}, p_2^{ND} \in [\min\{g_1(x), g_2(x)\}, \max\{g_1(x), g_2(x)\}]$.

Proof of Claim: Now consider any non-disclosure equilibrium in which the non-disclosure set is $[0, \underline{x}) \cup (\bar{x}, 1]$. The equilibrium condition implies that $g_1(\bar{x}) - g_1(\underline{x})$ and $g_2(\bar{x}) - g_2(\underline{x})$ have opposite signs. Without loss, assume $g_1(\underline{x}) \leq g_1(\bar{x})$ and $g_2(\bar{x}) \leq g_2(\underline{x})$. So Claim A implies $p_1^{ND} \leq g_1(\bar{x})$ and $p_2^{ND} \leq g_2(\underline{x})$. The equilibrium condition then implies $p_2^{ND} \geq g_2(\bar{x})$ and $p_1^{ND} \geq g_1(\underline{x})$, and so $p_1^{ND} \in [g_1(\underline{x}), g_1(\bar{x})]$ and $p_2^{ND} \in [g_2(\bar{x}), g_2(\underline{x})]$.

If the sets $[g_1(\underline{x}), g_1(\bar{x})]$ and $[g_2(\bar{x}), g_2(\underline{x})]$ are ranked by the strong set order (Veinott, 1989) then the result is straightforward: If $[g_1(\underline{x}), g_1(\bar{x})] \preceq [g_2(\bar{x}), g_2(\underline{x})]$ under this order, then $p_1^{ND}, p_2^{ND} \in [g_1(\underline{x}), g_2(\underline{x})]$; while if instead $[g_2(\bar{x}), g_2(\underline{x})] \preceq [g_1(\underline{x}), g_1(\bar{x})]$, then $p_1^{ND}, p_2^{ND} \in [g_2(\bar{x}), g_1(\bar{x})]$.

Next, consider the cases where the two sets $[g_1(\underline{x}), g_1(\bar{x})]$ and $[g_2(\bar{x}), g_2(\underline{x})]$ are not ranked by the strong set order. There are two sub-cases. In the first sub-case, $[g_1(\underline{x}), g_1(\bar{x})] \subset [g_2(\bar{x}), g_2(\underline{x})]$, and so either $p_2^{ND} \in [g_2(\bar{x}), g_1(\bar{x})]$ or $p_2^{ND} \in [g_1(\underline{x}), g_2(\underline{x})]$ (or both), while both $p_1^{ND} \in [g_2(\bar{x}), g_1(\bar{x})]$ and $p_1^{ND} \in [g_1(\underline{x}), g_2(\underline{x})]$. In the second sub-case, $[g_2(\bar{x}), g_2(\underline{x})] \subset [g_1(\underline{x}), g_1(\bar{x})]$, and so either $p_1^{ND} \in [g_1(\underline{x}), g_2(\underline{x})]$ or $p_1^{ND} \in [g_2(\bar{x}), g_1(\bar{x})]$ (or both), while both $p_2^{ND} \in [g_1(\underline{x}), g_2(\underline{x})]$ and $p_2^{ND} \in [g_2(\bar{x}), g_1(\bar{x})]$.

Claim C: If $x_m \in \{\underline{x}, \bar{x}\}$ and $p_1^{ND}, p_2^{ND} \in [\min\{g_1(x_m), g_2(x_m)\}, \max\{g_1(x_m), g_2(x_m)\}]$ then $E_i\left[p_i^{ND}\right] \leq E_i[g_i(x_m)]$.

Proof of Claim: If instead $E_i\left[p_i^{ND}\right] > E_i[g_i(x_m)]$ then Theorem 3 of Hammond (1974) implies that $E_i\left[v\left(p_i^{ND}\right)\right] > E_i[v(g_i(x_m))]$, contradicting the equilibrium condition.

Completing the proof: From above, for at least one $x_m \in \{\underline{x}, \bar{x}\}$, we know $p_1^{ND}, p_2^{ND} \in [\min\{g_1(x_m), g_2(x_m)\}, \max\{g_1(x_m), g_2(x_m)\}]$ and $E_i\left[p_i^{ND}\right] \leq E_i\left[g_i^{ND}(x_m)\right]$, along with the equilibrium condition $E_i\left[v\left(p_i^{ND}\right)\right] = E_i\left[v\left(g_i\left(x\right)\right)\right]$. So for any strictly concave function ϕ, Theorem 3 of Hammond (1974) implies that $E_i\left[\phi\left(v\left(p_i^{ND}\right)\right)\right] \geq E_i\left[\phi\left(v\left(g_i\left(x\right)\right)\right)\right]$. An easy adaption of the argument above establishes a strict inequality $E_i\left[\phi\left(v\left(p_i^{ND}\right)\right)\right] > E_i\left[\phi\left(v\left(g_i\left(x\right)\right)\right)\right]$ when at least one g_i is strictly concave.

Proof of Proposition 9: Consider the initial equilibrium with the highest probability of non-disclosure, which satisfies the equilibrium condition (4). If the risk-aversion of any receiver-type j increases, by Lemma 1, the corresponding p_j^{ND} strictly decreases. Hence, for both $x_m \in \{\underline{x}, \bar{x}\}$ we have $E_i\left[v\left(p_i^{ND}\right)\right] < E_i\left[v\left(g_i\left(x_m\right)\right)\right]$, implying that senders in both the neighborhoods of $\underline{x}$ and $\bar{x}$ disclose in the new non-disclosure equilibrium. As a result, the new equilibrium features more disclosure, completing the proof.