Market Run-Ups, Market Freezes, Inventories, and Leverage*

Philip Bond
University of Washington

Yaron Leitner†
Federal Reserve Bank of Philadelphia

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†Corresponding author: Yaron Leitner, Federal Reserve Bank of Philadelphia, Research Department, Ten Independence Mall, Philadelphia, PA 19106, phone: 215-574-3963.
email: YaronLeitner@gmail.com.

Abstract

We study trade between an informed seller and an uninformed buyer who have existing inventories of assets similar to those being traded. We show that these inventories may induce the buyer to increase the price (a "run-up") but may also make trade impossible (a "freeze") and hamper information dissemination. Competition may amplify the run-up by inducing buyers to purchase assets at a loss to prevent competitors from purchasing at lower prices and releasing bad news about inventory values. We analyze a dynamic extension and discuss robustness to other trading environments. We also discuss empirical and policy implications.

1. Introduction

Consider the sale of mortgages by a loan originator to a buyer. As widely noted, the originator has a natural information advantage and knows more about the quality of the underlying assets than do other market participants. One consequence, which has been much discussed, is that the originator will attempt to sell only the worst mortgages.[1] However, a second important feature of this transaction has received much less attention. Both the buyer and the seller may hold significant inventories of mortgages similar to those being sold, and they may care about the market valuation of these inventories, which affects how much leverage they can take. Consequently, they may care about the dissemination of any information that affects market valuations of their inventories. In this paper, we analyze how inventories affect trade—in particular, prices and information dissemination. Our setting applies to the sale of mortgage-related products, but more broadly, to situations in which the seller has more information about the value of the asset being traded.

Our main result is that the effect of inventories on trade depends on the buyer's and seller's initial leverage, or more precisely, on how tight their capital constraints are. When capital constraints are moderately tight, concerns about the value of existing inventories lead to higher prices (a market "run-up"). However, when capital constraints are very tight (i.e., initial leverage is very high), trade becomes impossible (a market "freeze") and information dissemination ceases.

Our results cast light on several features of the market for structured financial

[1] See, for example, Ashcraft and Schuermann (2008), and Downing, Jaffee, and Wallace (2009).

products that have attracted much attention. First, it is widely believed that these products were overpriced in the period leading up to the financial crisis. This is illustrated by Fig. 1, which shows a sharp divergence between mortgage delinquency rates and mortgage bond prices. It is also consistent with many anecdotal accounts. For example, Lewis (2010, page 164) suggests that "from mid-2005 until early 2007, there had been this growing disconnect between the price of subprime mortgage bonds and the value of the loans underpinning them." Second, this market collapsed in the financial crisis, as illustrated by Fig. 2. Third, the collapse of this market attracted concern not just because of the associated fall in potentially socially beneficial trade, but also because it severely hampered the dissemination of information (see, e.g., Scott and Taylor, 2009).

In our basic model, there is one buyer and one seller, and the (uninformed) buyer makes a take-it-or-leave-it offer to the informed seller. The motive for trade is that the buyer values the asset by $\Delta > 0$ more than the seller. The buyer has existing inventories of the traded assets, and he incurs a large cost if the market value of his inventories falls below some threshold. For example, creditors may not roll over the buyer's debt and the buyer may go bankrupt. We refer to this as the buyer's capital constraint and assume that, before trade begins, the capital constraint is satisfied.

The intuition for our main results is as follows. Whenever the seller agrees to sell at a price p, the market infers that the value of the asset is less than p, and so the value of the buyer's existing inventories drops. This may lead to a violation of the buyer's capital constraint. When the buyer's capital constraint has sufficient slack, the buyer can prevent a violation of his constraint by increasing the price while still maintaining positive profits; hence, we obtain a price run-up. However, when the capital constraint is tight, the buyer can no longer increase the price without losing money. At this point, the buyer prefers not to make any offer; hence, trade completely breaks down, and the market learns nothing about the value of the asset.[2] We obtain similar results when the seller is subject to a capital constraint and must retain a fraction of his assets on his balance sheet.

In a dynamic extension of our basic model (Section 4), we show that when the buyer has high leverage and holds inventories of two assets with independent values, trade in the first asset leads to a tightening of the buyer's capital constraint, which in turn leads the buyer to either increase the price he offers for the second asset, or else stop trading. One implication of our dynamic setting is that high leverage may lead to a run-up in prices that is followed by a market freeze.

We also study an extension in which two buyers compete by making offers to a single seller. In this case a buyer may be forced to acquire assets at a loss-making price, just to make sure that a competing buyer does not acquire them at a lower price (Proposition 3). The key insight here is that a purchase by one buyer leads to the release of information that may cause a violation of the capital constraint of a competing buyer, and this forces the competing buyer to increase the price.

[2] Of course, asymmetric information by itself can lead to a reduction in trade, even absent inventories and capital constraints. However, when there are strictly positive gains from trade for even the lowest-valuation seller, as there are in our setting, some trade still survives absent inventories and capital constraints. In particular, sellers with sufficiently low valuations still trade. In contrast, inventories and capital constraints can lead to a complete market breakdown.

In our model, prices are important because market participants infer information from prices about asset values. These inferences affect a market participant's ability to borrow against the value of his existing assets. In practice, transaction prices may also be important because of accounting rules that rely on the actual price rather than what can be inferred from it. Milbradt (2012) focuses on such an environment. In his setting, a regulated financial institution is subject to a capital constraint that is based on mark-to-market accounting. Milbradt shows that such an accounting rule may induce the financial institution to suspend trade so that losses are not reflected on its balance sheet.[3] One might be tempted to conclude that the problem is flawed regulation and/or flawed accounting rules. Our model, which is based on rational expectations, shows that the problem is more fundamental. Our model also addresses situations in which market participants can strategically choose ("manipulate") prices. In our model the price is endogenous and may not reflect the true value. In contrast, in Milbradt the price is exogenous and always reflects the true value, by assumption.

Endogenizing the price leads to new predictions that are absent in Milbradt. Under many conditions, these new predictions continue to hold even if the capital constraint is based on mark-to-market accounting rather than rational inferences. The first new prediction is the price run-up and the related implications. The second new prediction is the possibility of a complete market breakdown and lack of

[3]Other papers that discuss problems that arise from mark-to-market accounting rules include Allen and Carletti (2008), Heaton, Lucas, and McDonald (2010), and Plantin, Sapra, and Shin (2008).

information dissemination. We discuss the robustness of these new predictions in Section 5.

As noted above, one application of our analysis is to the market for structured financial products. Our analysis provides an explanation for prices that change over time without any change in the distribution of asset values; for a price run up; for a market breakdown; and for the lack of information revelation in the market breakdown. Related, our analysis predicts that tight capital constraints are associated both with a market breakdown and high asset prices immediately before the breakdown.

A separate application is to the effects of broker-dealer inventories on prices. By interpreting buyers in our model as market-makers, our model predicts that higher inventories may lead to higher prices, consistent with the empirical findings in Manaster and Mann (1996). In contrast, previous models of the effect of market-maker inventories on prices, such as Amihud and Mendelson (1980) and Ho and Stoll (1981, 1983), assume symmetric information and predict that prices fall as inventories increase. Prices fall in these models either because the dealer is risk averse and concerned about future price movements, or because he is not allowed to carry too much inventory.

We also use our model to discuss implications for regulatory intervention in illiquid markets (Section 7). On the buyer's side, our analysis highlights the potential role of a large investor unencumbered by existing inventories (the government, for example). One implication is that by purchasing assets, the government may impose a cost on potential buyers who choose not to trade. On the seller's side, our analysis suggests potential limitations to the standard prescription that sellers should retain a stake in the assets they sell. Under some circumstances, this prescription may lead to a market breakdown.

1.1. Related literature

Our paper relates to the literature on trade under asymmetric information, in which the seller is better informed (e.g., Akerlof, 1970; Samuelson, 1984). We show that adding inventories and capital constraints to this standard problem can lead to a complete market breakdown even if the gains from trade for the lowest-valuation seller are strictly positive.[4] Moreover, we show that high leverage may lead to higher prices. In our setting, trade is always efficient, so increasing the price increases welfare (as the probability that the seller will accept the buyer's offer increases). In this sense, our paper differs from papers in which price manipulation creates distortions that are suboptimal from a social point of view.[5,6]

[4] Akerlof (1970) provides an example in which the market breaks down completely, but in his example there are no gains from trade between the buyer and the seller with the lowest possible valuation. Glode, Green, and Lowery (2012) endogenize the extent of adverse selection and show that firms may overinvest in financial expertise. The outcome of this is that if uncertainty increases, the probability of efficient trade is reduced; but in contrast to our setting, the market does not completely break down.

[5] Examples include Allen and Gale (1992), Brunnermeier and Pedersen (2005), and Goldstein and Guembel (2008).

[6] Also related, Clayton and Ravid (2002) study the effect of leverage on bidding behaviors in private-value auctions. They show that higher leverage reduces the amount that a firm is willing to bid. However, in their setting, trade does not affect the value of existing inventories. Instead,

Our competing-buyers result illustrates a situation in which trade between the seller and one buyer has externalities for other buyers, and hence relates to existing auction-theoretic papers dealing with externalities (e.g., Jehiel, Moldovanu, and Stacchetti, 1996). In contrast to this literature, the externality in our paper depends on the price paid rather than on simply whether another buyer obtains the asset.

Two recent papers obtain periods of no trade in a dynamic lemons problem. To do so, these papers add the assumption that some noisy information about the asset quality is revealed (exogenously). In Kremer and Skrzypacz (2007), information is revealed at some future date T, and there exists $t < T$ such that trade ceases over the time interval $[t, T]$. In Daley and Green (2012), information is revealed gradually. Instead, we obtain a no-trade result by adding inventories and capital constraints to a standard lemons problem.[7]

Our paper also relates to the literature that explores the link between leverage and trade. For example, Shleifer and Vishny (1992) show that high leverage may force firms to sell assets at fire-sale prices, while Diamond and Rajan (2011) show that the prospect of fire sales may lead to a market freeze. Other papers explore feedback effects between asset prices and leverage: Low prices reduce borrowing capacity, and hence asset holdings and prices also (see, e.g., Kiyotaki and Moore, 1997). In contrast, we model a situation in which firms can meet their financial needs by staying with the status quo. Therefore, there is no need for fire sales or "cash in

they focus on a debt-overhang problem.

[7]An alternative explanation for a complete market breakdown in situations in which there are gains from trade involves Knightian uncertainty; see, e.g., Easley and O'Hara (2010).

the market" pricing, as in Allen and Gale (1994). In addition, in our setting, prices may increase even after valuations of existing inventories fall. As noted earlier, our paper also relates to Milbradt (2012), to the literature on mark-to-market accounting, and to the market microstructure literature that links market-maker inventories to prices.

Finally, our paper relates to the literature on equity issuance, in which the issuing firm cares about the market valuation of its remaining equity.[8] However, in our main analysis, we do not focus on signaling. Instead, we focus on the bidding strategies of uninformed buyers. Section 5 discusses the case in which informed sellers make offers.

2. The basic model

The model is based on a simple variant of Akerlof's (1970) "lemons problem." The new feature is that the buyer has an inventory of the traded asset and is subject to a capital constraint.

In the basic model, an uninformed buyer makes a take-it-or-leave-it offer to buy one unit of an asset from an informed seller. The buyer and seller are risk neutral. The value of the asset is v to the seller and $v + \Delta$ to the buyer, where $\Delta > 0$ denotes the gains from trade. Since $\Delta > 0$, trade is always efficient. Both Δ and the distribution of v are common knowledge, but the exact value of v is the seller's private information. Consequently, trade affects posterior beliefs about v and hence

[8]See, for example, Allen and Faulhaber (1989), Grinblatt and Hwang (1989), and Welch (1989).

the market value of each unit of asset. For simplicity, we assume that v is drawn from a uniform distribution on $[0, 1]$. The buyer's offer and the seller's response are publicly observable (see also footnote 14).

In one interpretation, the seller is a loan originator, and the gains from trade reflect the fact that the buyer has a lower cost than the seller of retaining risky assets on the balance sheet. For example, the buyer may face lower borrowing costs or less stringent regulation. Alternatively, the buyer might be a broker-dealer who helps with the matching process between the seller and other investors who have higher valuations for the asset.

The buyer has an inventory of M units of the asset, which he acquired earlier.[9] The buyer also has cash and a short-term debt liability. The liability net of cash holdings is L, so the buyer can roll over his liabilities only if the value of his noncash assets exceeds L. Assume, for simplicity, that the buyer holds only the traded asset and that the purchase of additional units is financed out of existing cash holdings and/or new short-term borrowing.

Specifically, suppose the buyer purchases $q \in \{0, 1\}$ additional units at a price per unit p, and let h denote the "market value" of the asset, defined as the expected value of $v + \Delta$, conditional on the trading outcome, using Bayes' rule. Then the buyer can roll over his debt if

$$h(M + q) \geq L + pq, \tag{1}$$

[9] We obtain qualitatively similar results if, instead, the buyer's inventory consists of assets whose values are correlated with the value of the asset being traded.

where $M + q$ is the buyer's total inventory of assets net of trade, and $L + pq$ is the buyer's total liabilities net of trade. We refer to equation (1) as the buyer's *capital constraint*.

(Implicit here is that the threat of losing the asset induces the buyer to pay his obligations, so the capital constraint is based on the value of the asset to the buyer. The nature of the results remains under a more general formulation in which h is the expected value of $\alpha(v + \gamma\Delta)$, where $\alpha \leq 1$ reflects constraints on the buyer's ability to pledge future cash flows, and $\gamma \in [0, 1]$ allows the capital constraint to be based on the value of the asset to creditors, who may have lower valuations than the buyer. An earlier draft, available upon request, contains full details.)

If the buyer violates his capital constraint, he defaults and incurs a cost κ, which represents lost growth opportunities due to bankruptcy or closure by a regulator. Alternatively, one can think of a situation in which the buyer must raise L dollars to invest in a profitable opportunity with cash flows that cannot be promised to others (e.g., because of nonobservability). In this case, the cost of violating the capital constraint arises from the fact that the buyer cannot take advantage of the investment opportunity.

We make the following parameter assumptions:

Assumption 1 $(\frac{1}{2} + \Delta)M \geq L$.

Assumption 2 *The cost κ of violating the capital constraint is very high.*

Assumption 3 $\Delta < \frac{1}{2}$.

Finally, with the exception of a brief analysis of the benchmark case $M = 0$ in Section 3.1, we assume:

Assumption 4 *$M > 1$.*

Assumption 1 says that the capital constraint is satisfied before trading begins (i.e., when $q = 0$ and $h = \frac{1}{2} + \Delta$, so assets are evaluated at the prior). This assumption allows us to focus on the question of how the buyer changes his behavior to avoid violating the capital constraint, rather than on the much studied fire sales that follow when the constraint is violated.

Assumption 2 implies that the buyer's first priority is to satisfy his constraint.[10] Consequently, the buyer maximizes expected trading profits subject to the constraint that his offer satisfies the capital constraint.

Assumption 3 says that the gains from trade are not too high. Our results on price run-ups do not depend on this assumption, but the market freeze result does. In particular, when the gains from trade are high, $\Delta \geq \frac{1}{2}$, the outcome in which the seller sells at price 1 satisfies the seller's participation constraint for each seller type, and it also gives the buyer positive profits, since the buyer acquires an asset with an expected value of $\frac{1}{2} + \Delta$ for a price of 1.

Finally, Assumption 4 says that the quantity of the asset available for trade is smaller than the buyer's existing asset holdings. This assumption ensures that inventory valuation effects are important and, in particular, implies that the buyer's capital constraint puts a lower bound on the price (see below).

[10] For example, for the results in Section 3, it is enough to assume that $\kappa \geq 1 + \Delta$.

The assumption that the buyer can purchase only one unit is made for simplicity. The nature of the results remains if the asset is divisible and the buyer can choose a quantity in addition to a price.[11]

We discuss extensions of our basic model and the robustness of our main results to other trading environments in Sections 3.4, 4, and 5.

3. Analysis

We start with the benchmark case $M = 0$, in which the buyer has no inventories. Then we analyze the main case with inventories, $M > 0$.

3.1. Benchmark: Buyer does not have inventories

Absent inventories, the buyer offers to purchase the asset at a price p, which maximizes his expected profits. The seller accepts the offer if and only if $v \leq p$, which happens with probability p, since v is uniform on $[0, 1]$.[12] Conditional on the seller accepting the offer, the expected value of the asset to the buyer is $\frac{1}{2}p + \Delta$. Since the buyer pays p, his expected profit per unit bought is $\Delta - \frac{1}{2}p$. Taking into account the probability of trade, the buyer's expected profit is $\pi(p) \equiv p(\Delta - \frac{1}{2}p)$. The buyer's profit-maximizing bid is $p = \Delta$.

[11] An earlier draft, available upon request, contains a full analysis of this case.

[12] If $p > 1$, the acceptance probability is simply 1. However, by Assumption 3, offers of $p > 1$ generate negative profits, since the buyer pays $p > 1$ for an asset with an expected value of $\Delta + \frac{1}{2} < 1$.

Lemma 1 *In the benchmark case of no inventories, the buyer offers to buy the asset at a price Δ. The seller accepts this offer if and only if $v \leq \Delta$.*

3.2. Buyer cares about the value of his inventory

When a seller accepts an offer, the market infers that v is below p. Hence, the market value of existing inventories falls from the prior of $(\frac{1}{2}+\Delta)M$ to $(\frac{1}{2}p+\Delta)M$. To ensure that the capital constraint continues to hold after the offer is accepted, the offer p must satisfy

$$(\frac{1}{2}p + \Delta)(M + 1) \geq L + p, \tag{2}$$

which follows from equation (1) with $h = \frac{1}{2}p + \Delta$ and $q = 1$.

Define $\delta \equiv \frac{L}{(\frac{1}{2}+\Delta)M}$, a measure of both the buyer's initial leverage and the tightness of his capital constraint before trade begins. Assumption 1 is equivalent to $\delta \leq 1$. By Assumption 4, the capital constraint (2) puts a lower bound on the price. Specifically, by defining

$$P(\delta) \equiv \frac{\delta M(1 + 2\Delta) - 2\Delta(M + 1)}{M - 1}, \tag{3}$$

the capital constraint (2) simplifies to

$$p \geq P(\delta). \tag{4}$$

If instead the seller rejects the offer, the market infers that v is above p and the market value of existing inventories rises to $(\frac{1}{2}p + \frac{1}{2} + \Delta)M$, relaxing the capital constraint.

Hence, the buyer faces a constrained optimization problem: namely, choose a price p to maximize expected profits $\pi(p)$ subject to the constraint that either he makes no offer, $p = 0$, or else the offer satisfies equation (4).[13,14] Observe that the lower bound on the price $P(\delta)$ is increasing in the buyer's initial leverage δ.

Fig. 3 illustrates the optimal solution to the buyer's constrained optimization problem. The parabola represents the profits function $\pi(p)$, and the vertical lines represent the lower bound $P(\delta)$ for three different values of δ. If leverage is low, i.e., $P(\delta) \leq \Delta$, the capital constraint is not binding, and the buyer makes his benchmark offer Δ. If leverage is intermediate, $P(\delta) \in (\Delta, 2\Delta)$, the capital constraint binds, and it is optimal to offer $p = P(\delta)$, since a higher price would reduce profits. Finally, if leverage is high, $P(\delta) > 2\Delta$, and the buyer would lose money if he offers $p \geq P(\delta)$. In this case, the buyer prefers not to trade. In other words, the market "freezes."

The condition $P(\delta) \leq 2\Delta$ reduces to $\delta \leq \frac{4\Delta}{1+2\Delta}$. Hence, we have established:

Proposition 1 *In the basic model of a monopolist buyer who cares about the value*

[13]From the law of iterated expectations, the value of inventoried assets equals its prior. Hence, maximizing expected trading profits $\pi(p)$ is the same as maximizing the expected value of the buyer's assets.

[14]We obtain similar results if the market observes only the terms of accepted offers but not the terms of rejected offers. In particular, since there is a positive probability that the buyer's offer is rejected along the equilibrium path, inferences regarding the asset value are the same as in the case in which the market observes the terms of rejected offers. Moreover, if the capital constraint is satisfied after accepted offers, it is also satisfied after rejected offers. Consequently, the analysis below remains unchanged.

of his inventories, trade can happen if and only if the buyer's initial leverage is not too high, i.e., $\delta \leq \frac{4\Delta}{1+2\Delta}$*. In this case, the buyer offers to purchase the asset at a price* $\max\{\Delta, P(\delta)\}$*, which increases in his initial leverage.*

Proposition 1 says that when the capital constraint is moderately tight (i.e., leverage is intermediate), the price, and hence the probability of trade, is increasing in leverage and is above the benchmark bid Δ. However, when the capital constraint is tight (i.e., leverage is high), trade completely breaks down.

An immediate corollary to Proposition 1 concerns the effect of high leverage and the corresponding market breakdown on the revelation of the seller's information about asset values:

Corollary 1 *If initial leverage is high,* $\delta > \frac{4\Delta}{1+2\Delta}$*, market participants learn nothing about the value* v *of the asset.*

3.3. Implications

Although simple, the basic model delivers many of the main implications. First, the market price is not determined solely by the distribution of asset values, but is also affected by the tightness of the buyer's capital constraint. In particular, the price increases in the tightness of this constraint. Second, and related, the buyer's expected return from holding the asset is decreasing in the tightness of his capital constraint. Third, very tight capital constraints lead to market breakdown and prevent information dissemination.

3.4. Remarks

Remark 1: The nature of the results above continues to hold if the seller is capital constrained and can sell only a fraction of his assets. In particular, since accepting a low offer reduces the market value of the units that the seller retains, a seller who is highly leveraged will accept an offer $p \geq v$ only if the price is sufficiently high.[15]

Remark 2: The nature of the results above continues to hold if the market value h in the capital constraint is based on actual transaction prices, as in mark-to-market accounting, rather than inferences. To see this, substitute $h = p$ in the capital constraint (1) and observe that it becomes $p \geq \frac{L}{M}$, so the capital constraint continues to put a lower bound on the price, and the lower bound increases as the buyer becomes more leveraged.

4. Dynamic run-ups and breakdowns

The static model is suggestive of a dynamic process in which the buyer increases leverage and prices until the market breaks down eventually. To model this explicitly, we extend our single-period model to a two-period model in which the monopolist buyer trades sequentially with two potential sellers. Each seller sells a different asset,

[15]Specifically, if the seller has M_s units of the asset but can sell only $x < M_s$, then p must satisfy $h(M_s - x) \geq L_s - px$, where L_s denotes the seller's liabilities. This constraint puts a lower bound on p, and this lower bound increases in the seller's initial leverage. In particular, if h is the expected value of $v + \gamma\Delta$ for some $\gamma \in [0, 1]$, then the seller's initial leverage is $\delta_s \equiv \frac{L_s}{(\frac{1}{2}+\gamma\Delta)M_s}$ and the capital constraint reduces to $p \geq P_s(\delta_s) \equiv \frac{\delta_s(\frac{1}{2}+\gamma\Delta)M_s - \gamma\Delta(M_s - x)}{\frac{1}{2}(M_s + x)}$.

and the values of the two assets are assumed to be independent.[16] Hence, one cannot infer anything about the value of one asset by observing trade in the other asset. This allows us to focus on the effect of leverage, which changes endogenously.

Specifically, seller i ($i = 1, 2$) can sell one unit of asset i and can trade only in period i. Before trading begins, the buyer has inventories of M units of each asset. The value (per unit) of asset i is v_i to the seller and $v_i + \Delta$ to the buyer, where v_1, v_2 are independent random variables drawn from a uniform distribution on $[0, 1]$. In period $i = 1, 2$, the buyer makes a take-it-or-leave-it offer p_i to seller i. The discount rate is assumed to be 1.

The two-period capital constraint is analogous to the one-period constraint (1):

$$h_1(M + q_1) + h_2\,(M + q_2) \geq L + p_1 q_1 + p_2 q_2, \tag{5}$$

where $q_i \in \{0, 1\}$ is the amount of asset i that the buyer purchases, and h_i is the market value of asset i conditional on the trading outcome. Specifically, if the buyer offers $p_i > 0$ and the seller accepts the offer, then $h_i = \frac{1}{2}p_i + \Delta$; if the seller rejects the offer, then $h_i = \frac{1}{2}(1 + p_i) + \Delta$; and if $p_i = 0$, then $h_i = \frac{1}{2}$.

Parallel to Assumption 1, we assume that the capital constraint (5) is satisfied absent new trade (i.e., if $q_1 = q_2 = 0$ and $h_1 = h_2 = \frac{1}{2} + \Delta$). The buyer's initial leverage is $\bar{\delta} \equiv \frac{L}{(\frac{1}{2}+\Delta)(2M)}$. We assume that the cost κ (Assumption 2) is incurred

[16] One can think of the two assets as idiosyncratic components of the same class of assets, e.g., mortgage-backed securities. Moreover, a positive correlation between the two values would only strengthen our results below, because trade in the first period would reduce not only the market value of the first asset but also the market value of the second asset. Hence, the buyer's capital constraint would be tightened by a larger amount after his first offer is accepted.

if the capital constraint is violated at the end of the second period. We obtain the same outcome if the buyer also incurs a cost κ for violating the constraint after the first period. (The proof of Proposition 2 contains more details.)

The buyer's second-period offer p_2 can depend on the outcome of trade with the first seller. We denote by p_a (respectively, p_r) the second-period offer after the first seller accepts (rejects) the offer. Since sellers' acceptance decisions are simple cutoff rules (seller i accepts offer p_i if and only if $v_i \leq p_i$), the buyer's expected profits are

$$\pi(p_1) + p_1\pi(p_a) + (1 - p_1)\pi(p_r). \tag{6}$$

The buyer's problem is to choose prices (p_1, p_a, p_r) to maximize his expected profits subject to satisfying the capital constraint with probability 1.[17]

We show in the Appendix that if the first offer is rejected, the capital constraint becomes sufficiently slack so the buyer can make his unconstrained optimal bid of Δ in the second period (i.e., $p_r = \Delta$). It remains to characterize p_1 and p_a.

Define

$$H \equiv \frac{(\frac{1}{2} + \Delta)2M\bar{\delta} - 2\Delta(M + 1)}{\frac{1}{2}(M - 1)}, \tag{7}$$

which is an increasing function of the buyer's initial leverage $\bar{\delta}$, and let $\sigma \equiv \frac{\frac{1}{2}M-\Delta}{\frac{1}{2}(M-1)}$. Our main result in this section is:

Proposition 2 *In the two-period model, there exist $\hat{H} \in (2\Delta, 4\Delta)$ and $\check{H} > 2\Delta + \sigma > \hat{H}$ (that depend on M and Δ) such that:*

[17] The solution to this constrained maximization problem does not depend on whether the buyer can commit to his date 2 offers p_a and p_r. If (p_1, p_a, p_r) maximizes profits when the buyer can commit, then clearly the buyer has no incentive to deviate from p_a and p_r at date 2.

(1) If $H \leq 2\Delta$, the buyer bids $(p_1, p_a) = (\Delta, \Delta)$.

(2) If $H \in \left(2\Delta, \hat{H}\right)$, the buyer bids (p_1, p_a) with $p_a > p_1 > \Delta$ (i.e., the buyer increases his bid if his first offer is accepted).

(3) If $H \in \left(\hat{H}, \check{H}\right)$, the buyer bids $p_1 = \max\left\{H - \sigma, \Delta - \frac{1}{2}\Delta^2\right\}$ and then withdraws from the market if this offer is accepted, i.e., $p_a = 0$.

(4) If $H > \check{H}$, the buyer does not bid at all.

(At the boundaries $H \in \left\{\hat{H}, \check{H}\right\}$ the buyer is indifferent between the two alternate strategies.)

Proposition 2 captures a few aspects of a dynamic behavior. If the buyer's initial leverage is not too high (Parts 1 and 2), the buyer has enough slack in his capital constraint to make two rounds of offers. In Part 1 (low leverage), the capital constraint is not binding and the buyer makes the benchmark bid Δ in both periods. In Part 2 (moderate leverage), the capital constraint is binding. This induces the buyer to bid more than the benchmark price in the first period ($p_1 > \Delta$), and if the first bid is accepted, the capital constraint is tightened, forcing the buyer to bid even more in the second period ($p_a > p_1$). Hence, prices increase even though, by assumption, there is no change in the distribution of asset values.

If instead initial leverage is higher (Part 3), there are two consequences. First, the buyer prefers to have at most one offer accepted because if two offers are accepted, the bid prices must be very high to satisfy the capital constraint, and the buyer is left with very low or even negative profits. Hence, if the buyer's first offer is accepted, there is no trade in the second period. Second, a tight capital constraint pushes the initial bid high, for the same reasons as in the benchmark case. Consequently, high leverage generates a market freeze that is preceded by high prices (a "run-up"). Specifically:

Corollary 2 *If* $H \in (\max\{\hat{H}, \Delta + \sigma\}, \check{H}\}$*, then the buyer bids more than the benchmark in the first period* ($p_1 = H - \sigma > \Delta$)*, and if the first offer is accepted, the buyer does not bid in the second period. That is, a market freeze is preceded by high prices.*

Note that while the buyer never bids above his valuation in the second period, it might be optimal for him to do so in the first period, even though he loses money if his offer is accepted. In particular, if $H \in (2\Delta + \sigma, \check{H})$, the buyer bids $p_1 = H - \sigma > 2\Delta$ and $p_a = 0$. The advantage of such a loss-making offer in the first period is that, if the offer is rejected, the market valuation of the buyer's inventory rises, thereby relaxing the buyer's capital constraint. This allows the buyer to make a profitable offer in the second period.[18]

[18]Note that if $\hat{H} < \Delta + \sigma$, then Part 3 of Proposition 2 includes an interval of leverage levels in which the price before the freeze is below the benchmark bid Δ. The intuition is that since the buyer wants to have only one offer accepted, the sellers in periods 1 and 2 are effectively in competition, which puts downward pressure on the price. However, this interval does not always exist. In particular, we show in the Appendix that if Δ is sufficiently high (though still satisfies Assumption 3) then $\hat{H} > \Delta + \sigma$, and this case does not arise. Moreover, this interval would not exist in perturbations of the environment that weaken the competition effect. Two examples are introducing more buyers to restore "competitive balance" between the two sides of the market; and changing the dynamic model to one in which there are two different buyers (who have inventories of both assets), with each buyer active only in one period.

Finally, if initial leverage is too high (Part 4), the market completely breaks down.

Remark 3: The nature of the results in Proposition 2 continues to hold if the capital constraint is based on actual transaction prices (as in mark-to-market accounting) rather than inferences (Bayes' rule), but only if the initial book value of the first asset is sufficiently high (e.g., more than 2Δ). In this case, the capital constraint is tightened after the first offer is accepted, which forces the buyer to either increase his next bid or else stop bidding.[19] If instead the initial book value is low, an accepted offer may relax rather than tighten the capital constraint, and we may obtain different predictions. In particular, the buyer will neither stop bidding after the first offer is accepted nor increase his bid.

5. Discussion

In our basic model, a single buyer makes a take-it-or-leave-it offer to a single informed seller. In this section, we discuss the extent to which our main results, namely price run-ups and market freezes, extend beyond our basic model. We also obtain new results for the case of multiple competing buyers.

[19] Since rejected offers do not affect the capital constraint, the buyer will not make loss-making offers. However, in an earlier draft, we showed that the buyer can benefit from making loss-making offers if he can offer to purchase just a small quantity of the asset. This is because acceptance of such offers can relax the capital constraint.

5.1. *Price run-ups*

The run-up result hinges on two important properties of our basic model. First, the positive surplus the buyer gets from trade varies with leverage. Second, and related, the asset does not trade at its true value v. Our dynamic run-up result also makes use of the property that trade tightens the capital constraint. We discuss each of these three properties in turn.

5.1.1. *What if the buyer does not get positive surplus?*

A natural case in which the buyer does not get positive surplus is when the seller (rather than the buyer) makes a take-it-or-leave-it offer. In this case, our run-up result may not hold. For example, if the condition for trade in our basic setting holds, then there is an equilibrium in which the seller sells the asset for a price 2Δ if $v \leq 2\Delta$ and does not sell if $v > 2\Delta$.[20] In particular, the price is independent of the buyer's leverage, and the buyer's expected profits are zero.

Another case in which the buyer does not get positive surplus is when he is in competition with other buyers. However, in this case, our run-up result may actually

[20] Because the seller is the informed party, the result is a signaling game with many equilibria. Any equilibrium with trade is characterized by a price p such that, if $v \leq p$, the seller sells at price p and, if $v > p$, the seller does not sell. In the equilibrium that is most preferred by the seller, $p = 2\Delta$. Equilibria in which sellers with different valuations sell for different prices do not exist because a seller with a low valuation would deviate by acting as if he has a high valuation. Similarly, there are no equilibria in which the buyer trades with a seller with a high valuation but does not trade with a seller with a low valuation.

be amplified. In particular, a highly leveraged buyer may engage in a negative-surplus trade to prevent the other buyer from purchasing the asset at a low price. To see this, consider an extension of our basic model in which two buyers compete by making simultaneous offers to the informed seller. The two buyers have different liabilities (L_1 and L_2, respectively) but the same inventory (M) and the same valuation for the asset ($v + \Delta$); hence, buyer i's leverage is $\delta_i = \frac{L_i}{(\frac{1}{2}+\Delta)M}$. Then:

Proposition 3 *Suppose there are two competing buyers.*

(1) If $\max\{P(\delta_1), P(\delta_2)\} \leq 2\Delta$ *(i.e., both buyers have low leverage), the unique equilibrium price is* 2Δ.

(2) If $P(\delta_1) \leq 2\Delta < P(\delta_2) \leq 1$ *(i.e., buyer 1 has low leverage and buyer 2 has high leverage), the unique equilibrium price is* $P(\delta_2)$*, which increases in buyer 2's leverage. In this case, whenever the seller sells the asset, he sells it to buyer* 2*, who makes negative profits.*

The proof is in the Appendix. Part (1) reflects the simple intuition that competition may drive the price to the zero-profit price. Part (2) illustrates that the combination of competing buyers and capital constraints may push the price even higher than the zero-profit price so that the price again increases in leverage. In particular, if buyer 2 has high leverage but buyer 1 has low leverage, buyer 2 may be forced to make a high offer, $P(\delta_2)$, since otherwise the seller would trade with buyer 1 at a low price, which would lead to a violation of buyer 2's capital constraint. Buyer 2 makes this loss-making offer, even though he would not bid at all if he were the only buyer.[21]

More generally, if the two buyers have different valuations for the asset, it is possible that the seller sells the asset to the less leveraged buyer at a price that is determined by the leverage of the highly-leveraged buyer. We show this in an earlier draft.[22]

5.1.2. What if the asset trades at the true value?

If trade always occurs at a price that equals the asset's true value v, the run-up result does not hold.[23] Under our informational assumption that the seller knows more about the asset value than the buyer, the most natural force that would push the price to equal v is when multiple informed sellers (with assets of equal value v) compete to sell to the buyer. We make the following two observations about this setting.

First, under the relatively mild assumption that the buyer has some way to

[21] For the result in Part (2) in Proposition 3, it is important that the buyer can purchase the entire amount for sale without violating his capital constraint. If the buyer is too leveraged ($P(\delta_2) > 1$) or if the aggregate amount for sale is too high (e.g., there are many informed sellers), the buyer cannot prevent a violation of his capital constraint by making a high bid. If he purchases the entire amount, his losses are too large and lead to a violation of the constraint. But if he purchases less than the full amount, the less leveraged buyer (or buyers) will purchase the remaining units at a low price.

[22] The earlier draft contains a full analysis of the more general case in which Δ and M vary across the two buyers, in addition to L.

[23] For example, this is a feature of Milbradt (2012).

publicly declare that he is "out of the market," an equilibrium in which the asset *always* trades at a price equal to v can exist only if the buyer's capital constraint is very slack. If instead the capital constraint is tight, trading at the true value would violate the buyer's capital constraint when the true value is low. Foreseeing this, the buyer would rather stay out of the market.

Second, under the mild assumption above, the capital constraint continues to put a lower bound on the price. In particular, the only way that a leveraged buyer can trade with a low-valuation seller is if this seller pools with a seller with a high valuation. Higher leverage necessitates pooling with sellers with higher valuations and, hence, a higher pooling price. In this sense, a form of price run-up still exists.[24]

5.1.3. What if trade relaxes the capital constraint?

If trade relaxes rather than tightens the capital constraint, our dynamic run-up result may not hold. One example is when the capital constraint is based on mark-to-market accounting and the initial price is low, as discussed in Remark 3. Another example is when there are multiple competing sellers, as in Section 5.1.2. As suggested above, in this case, there is an equilibrium in which high-valuation sellers sell their assets for the true value. When this occurs for sellers with $v > \frac{1}{2}$, the effect is to relax capital constraints, thus eliminating the force that pushes up prices over time.

[24] There are also potentially equilibria in which sellers with low valuations do not trade at all. We discuss the robustness of equilibria of this type in Section 5.2 below.

5.2. Market freezes

Two key properties generate market freezes in our basic model. First, abstaining from trade leads the capital constraint to be satisfied. Second, trade leads to a tightening of the capital constraint and, in particular, may lead to violation of the capital constraint.

Given Assumption 1, the first property is likely to hold in any setting with a single buyer. However, with multiple buyers, the actions of one buyer may lead to a violation of a second buyer's capital constraint, even if the second buyer abstains from trade. One implication is that buyers may engage in loss-making offers, as illustrated in Proposition 3. A second implication is that even if *all* competing buyers are very leveraged and would each prefer to abstain from trade, there are still equilibria in which the market does not freeze. In these equilibria, one buyer makes a low (latent) offer, while another buyer makes a high and loss-making offer to prevent trade from occurring at the low price.[25] However, we show in an earlier draft that, in general, these equilibria are not robust in the sense that they do not survive iterated elimination of weakly dominated strategies. Formally, in the notation of Proposition 3, when $\min\{P(\delta_1), P(\delta_1)\} > 2\Delta$ and $P(\delta_1) \neq P(\delta_1)$, the unique equilibrium that survives iterated elimination of weakly dominated strategies is a market freeze.

The second property may not hold if there are competing sellers as in Section

[25]In our model, latent offers induce entry. This is different from existing literature on nonexclusive contracting, in which latent offers deter entry (see, e.g., Bisin and Guaitoli, 2004; Attar, Mariotti, and Salanie, 2011; Ales and Maziero, 2011).

5.1.2. In this case, under some conditions there are equilibria in which the market does not completely freeze even when leverage is high. In particular, if leverage is sufficiently high, there is an equilibrium in which trade occurs at the true value when the value is high but does not occur when the value is low.[26] This equilibrium has some features that resemble the equilibrium outcome in Milbradt (2012), who assumes that sellers can choose whether to trade or not but that trade occurs at a price that equals the true value. However, in our setting with endogenous prices, this equilibrium has the following unattractive feature. Consider any offer p that the buyer accepts in equilibrium. To prevent a deviation in which a low-valuation seller offers to trade at price p, the buyer's strategy must entail rejecting the offer p if it comes from just one seller but accepting it if it comes from two sellers. So taking the

[26]Such an equilibrium exists if the capital constraint is based on the actual transaction price, as in Remark 2, or if the capital constraint is based on inferences but rejected offers are not observed by the market. For the first case, define v' as the solution to $(v' + \Delta)M \geq L$, and for the second case define v' as the solution to $(\frac{1}{2}v' + \Delta)M \geq L$. Then there is an equilibrium such that, if $v \geq v'$, every seller offers to sell his asset for price v (and trade occurs), and if $v < v'$, no seller makes an offer.

However, if the capital constraint is based on inferences and the market observes rejected offers—as is the case in our benchmark model—there is no equilibrium of the type above because the buyer accepts any off-equilibrium offer below Δ regardless of what beliefs are ascribed to this offer. In particular, since the buyer is uninformed and the offer is observed regardless of the buyer's response, market beliefs are independent of the buyer's response. Hence, acceptance of an offer below Δ not only leads to positive profits but it also increases the left-hand side of the capital constraint (1). Consequently, any equilibrium with trade must include trade by sellers with a valuation below Δ.

buyer's strategy as given, low-valuation sellers play a complete information game with multiple equilibria. If both make an offer that the buyer accepts in equilibrium, both end up with strictly positive (expected) profits. But if both play some other action, trade does not occur and each obtains a zero payoff. The unattractive feature is that low-valuation sellers play the payoff-dominated equilibrium instead of coordinating on the price p that leads to strictly positive profits. This is despite the fact that low-valuation sellers do not face a prisoners' dilemma: Unilateral deviations pose no risk for a seller, since low-valuation sellers already receive their minimal payoff of 0.[27]

6. Empirical implications

In this section we collect our model's main predictions and, where possible, provide supporting evidence.

Prices increase even though valuations do not increase: A basic implication of our analysis is that the price of an asset may respond to variables other than beliefs about its true value. In particular, as the buyer's capital constraint tightens, he offers to pay a higher price even though his beliefs regarding the asset value are unchanged.

[27] Sellers with high valuations may also attempt to coordinate on a higher price, so this argument would also rule out an equilibrium in which sellers with valuations above some cutoff trade at exactly the true value. However, an equilibrium in which high-valuation sellers sell for ε more than the true value would not suffer from this problem, since here sellers would face the prisoners' dilemma problem of unilateral deviations leading to lower payoffs.

In Section 4 we show how this force can lead to prices that increase over time even without a change in the "fundamentals" (i.e., the value distribution). Although hard to definitively test, there is at least some evidence consistent with the view that prices in the market for structured financial products were partially divorced from fundamentals in the run-up to the financial crisis. We discussed some evidence in the introduction (see also Coval et al., 2009).[28]

Tight capital constraints lead to market breakdown: Adrian and Shin (2010) document the sharp increase in dealers' leverage prior to the recent financial crisis, while Fig. 2 illustrates the collapse of the market for mortgage-backed securities during the crisis. More anecdotally, many market observers expressed the view that concerns about the value of inventories induced firms not to sell their assets. For example, an analyst was quoted in *American Banker*[29] as saying that "[other companies] may be wary of selling assets for fear of establishing a market-clearing price that could force them to mark down the carrying value of their nonperforming portfolio."Also related is the view expressed in Lewis (2010, pages 184-185) that dealers who sold credit default swaps on subprime mortgage bonds did not make a market in these

[28]Less direct evidence is provided by Faltin-Traeger et al. (2010), who show that the prices of asset-backed securities failed to reflect relevant and observable information about sponsor credit quality, and by Ashcraft et al. (2010) and Rajan et al. (2012), who show that subordination levels failed to reflect the riskiness of underlying cash flows. Finally, the claim that structured financial products were overpriced is closely related to the widely held view that these same products received excessively favorable credit ratings. See, for example, Griffin and Tang (2012) for evidence.

[29]"Nonperformance Space: Risky Assets Find Market" (*American Banker*, August 19, 2009).

securities to prevent information from being revealed and a subsequent "mark down" of their positions.

Market breakdown is associated with a loss of information: In our analysis, trade is a source of information, so when the market freezes, information dissemination ceases. Loss of information was a primary concern expressed by observers during the financial crisis. For just one example, see Scott and Taylor (2009).

The tightness of capital constraints affects expected holding returns: This prediction relates to the prediction that the tightness of capital constraints affects prices. Corollary 2 implies that when capital constraints are sufficiently tight, the market freezes and prices before the freeze are high; so one should see low average returns on assets purchased shortly before a market freeze.

In broker-dealer markets, prices are increasing in dealer inventories: This prediction is essentially a special case of the first implication above. As discussed in the introduction, it is a prediction consistent with the empirical findings of Manaster and Mann (1996), which are not easily explained by previous models of market-maker inventories. More formally, this implication is a corollary of our Proposition 1. To see this, observe that the derivative of $P(\delta)$ with respect to inventories M has the same sign as $4\Delta - \delta(1 + 2\Delta)$, which by Proposition 1 is positive whenever trade occurs.[30]

[30] Note that here we are characterizing only the direct effect of inventories, in the sense that we are holding leverage δ constant. If changes in inventory levels also affect leverage, there is an additional indirect effect on prices. This indirect effect reinforces the direct effect if higher inventories are associated with greater leverage (as seems likely).

Competition may induce highly leveraged buyers to purchase assets at a loss. This prediction follows from Proposition 3. More generally, the leverage of one buyer may affect the price offered by another buyer.

7. Policy implications

Our analysis has implications for government attempts to defrost markets and for regulatory proposals aimed at improving market functioning.

7.1. Defrosting frozen markets

Consider the case in which trade has completely broken down because the buyer's capital constraint is too tight.[31] One option open to a government is to offer to buy the seller's assets. A central question is whether such government purchases can succeed without taxpayer subsidies (in expectation). Our model has two implications in this respect. First, if the government faces the same lemons problem that potential buyers do, a subsidy-free purchase scheme is possible only if the asset is worth more to the government than to the seller. Second, even if this condition holds, a government purchase may impose a cost on the original buyer. In particular, unless the government has a strictly higher valuation than the original buyer, then any subsidy-free purchase scheme either leads to a violation of the buyer's capital constraint, or else it forces him to purchase the asset at a loss. This is similar to the negative externality that one buyer imposes on another buyer in Proposition 3.

[31] The discussion can easily be extended to the case of more than one buyer.

Another option is to remove assets from the buyer's balance sheet: that is, to replace assets with cash. If the buyer can borrow against the full value of his assets, as assumed in our analysis so far, then again, purchasing assets from the buyer can relax his capital constraint only if the purchase involves a taxpayer subsidy. If instead the buyer has limited borrowing capacity, purchasing assets from the buyer might relax his capital constraint even if the purchase does not involve a taxpayer subsidy.

7.2. Should regulation mandate some retention of the asset by the seller?

A commonly voiced regulatory proposal is that sellers of assets subject to asymmetric information problems, such as issuers of asset-backed securities, should be required to retain some stake in the assets they sell.[32] Our analysis identifies a potential cost to this proposal: namely, that under some circumstances it leads to a market breakdown. Specifically, if the seller is highly leveraged and must retain a large fraction of his assets on his balance sheet, the seller may prefer not to trade because trade may reduce the market value of the assets that he retains and this may lead to a violation of his capital constraint.[33]

The goal that regulators appear to have in mind with this regulation is to reduce moral hazard on the part of asset sellers: for example, to discourage loan originators

[32] See, for example, section 15G of the Investor Protection and Securities Reform Act of 2010.

[33] Formally, it follows from footnote 15 that trade is possible only if $P_s(\delta_s) \leq 2\Delta$. If $\gamma < 1$, this condition reduces to $x \geq \frac{\delta_s(\frac{1}{2}+\gamma\Delta)M_s-(1+\gamma)\Delta M_s}{(1-\gamma)\Delta.}$.

from making bad loans and/or shirking on monitoring later on. Our analysis does not speak to this issue, and it seems likely that the regulation will have its intended effect in this regard. Our point here is instead to draw attention to a potentially significant cost of this regulation: namely, that it can lead to the breakdown of socially efficient trade.

8. Summary

We analyze how existing stocks of assets—inventories—affect prices and information dissemination. When market participants are highly leveraged (i.e., capital constraints are tight), concerns about the revelation of bad news prevent socially beneficial trade and information dissemination. However, when market participants are only moderately leveraged, inventories lead to higher equilibrium prices and stimulate socially beneficial trade. Competition may amplify this effect by inducing highly leveraged buyers to purchase assets at a loss. We also analyze a dynamic extension and show that a market freeze may be preceded by a run-up in prices.

The model's predictions are consistent with several features of the market for structured financial products during the recent financial crisis, such as prices that remain unchanged even as fundamentals worsen, a market breakdown that is preceded by high prices, and the lack of information dissemination during a breakdown. Our model also predicts that asset prices are increasing in broker-dealer inventories, consistent with empirical evidence but different from the predictions of existing market microstructure models. Our model also provides policy implications for government interventions in illiquid markets.

Appendix

Proof of Proposition 2. For use throughout, note that $\pi'(p) = \Delta - p$, $\pi''(p) = -1$, $\pi(\Delta) = \frac{1}{2}\Delta^2$, and that $\pi(p)$ can be written as

$$\pi(p) = \pi(\Delta) + \frac{1}{2}\pi'(p)(p - \Delta). \tag{8}$$

We first show that $p_r = \Delta$, and that p_1 and p_a satisfy the capital constraint (5) if and only if

$$p_1 \geq \begin{cases} H - p_a & \text{if } p_a > 0 \\ H - \sigma & \text{if } p_a = 0. \end{cases} \tag{9}$$

Consider a buyer's strategy (p_1, p_a, p_r) that satisfies (5) with probability 1. Then (5) must be satisfied when offers are accepted. In particular, if $p_a = 0$ (and $p_1 > 0$), we must have

$$(\frac{1}{2}p_1 + \Delta)(M + 1) + (\frac{1}{2} + \Delta)M \geq L + p_1, \tag{10}$$

and if $p_a > 0$, we must have

$$(\frac{1}{2}p_1 + \Delta)(M + 1) + (\frac{1}{2}p_a + \Delta)(M + 1) \geq L + p_1 + p_a. \tag{11}$$

By Assumption 4, and using the definitions for H and σ, (10) reduces to $p_1 \geq H - \sigma$, and (11) reduces to $p_1 \geq H - p_a$.

We can assume, without loss of generality, that $p_1 \leq 1$ and $p_a \leq 1$. By Assumption 3, $\sigma \geq 1$. Hence, (11) implies (10). (As an aside, note that (10) means that the capital constraint is satisfied at the end of the first period, as claimed in the main text.)

By Assumption 4, (10) implies that

$$(\frac{1}{2}p_1 + \Delta)(M+1) + (\frac{1}{2} + \Delta)M + \frac{1}{2}\Delta(M-1) + \frac{1}{2}p_1 \geq L + p_1, \tag{12}$$

or equivalently

$$(\frac{1}{2} + \frac{1}{2}p_1 + \Delta)M + (\frac{1}{2}\Delta + \Delta)(M+1) \geq L + \Delta. \tag{13}$$

Hence, if (p_1, p_a, p_r) satisfies (5) then (p_1, p_a, Δ) does also. Since $p_r = \Delta$ maximizes single-period profits, it follows that offering $p_r = \Delta$ is optimal. Finally, since rejected offers increase the value of existing assets, if (p_1, p_a) satisfies (9) and $p_r = \Delta$, then the capital constraint (5) is satisfied with probability 1.

Given $p_r = \Delta$, the buyer's expected two-period profits are

$$V(p_1, p_a) \equiv \pi(p_1) + p_1 \pi(p_a) + (1 - p_1)\pi(\Delta). \tag{14}$$

Hence, the buyer's problem reduces to choosing $(p_1, p_a) \in [0,1] \times [0,1]$ to maximize $V(p_1, p_a)$ such that either $p_1 = 0$ or else (9) is satisfied.

Note immediately that the solution to this problem must have $p_a \leq 2\Delta$, as follows. If, to the contrary, the solution is (p_1, p_a) with $p_a \in (2\Delta, 1]$, then $V(p_1, p_a) < V(p_1, 0)$ and $p_1 \geq H - p_a > H - \sigma$. But this contradicts the optimality of (p_1, p_a).

Part 1. If $H \leq 2\Delta$, then $p_1 = p_a = \Delta$ is uniquely optimal, since this satisfies the capital constraint (9) and attains the unconstrained maximum profits of $2\pi(\Delta)$.

Part 2. Suppose $H \in (2\Delta, 4\Delta]$. Since $\sigma > 1 > 2\Delta$ (Assumption 3), it follows that $H - \sigma < 2\Delta$, so the pair $(p_1 = H - \sigma, p_a = 0)$ is feasible and leads to positive profits. Hence, $p_1 > 0$.

The heart of the proof is in comparing strategies with $p_a = 0$ and $p_a > 0$. Accordingly, define maximized profits from each of these two strategies respectively by

$$\begin{aligned} v_1(H) &= \max_{p_1 \in [\max\{H-\sigma,0\},1]} V(p_1, 0) && (15) \\ v_2(H) &= \max_{p_1, p_a \in [0,1] \text{ s.t. } p_1 + p_a \geq H} V(p_1, p_a). && (16) \end{aligned}$$

Note that both v_1 and v_2 are continuous.

The profit function v_1 is straightforward to evaluate. The derivative of $V(p_1, 0)$ with respect to p_1 is $\pi'(p_1) - \pi(\Delta) = \Delta - p_1 - \pi(\Delta)$. Hence $V(p_1, 0)$ is increasing in p_1 up to $\Delta - \pi(\Delta)$ and decreasing thereafter. So $v_1(H) = V(\max\{H - \sigma, \Delta - \pi(\Delta)\}, 0)$, and

$$v_1'(H) = \begin{cases} 0 & \text{if } H \leq \Delta - \pi(\Delta) + \sigma \\ \pi'(H - \sigma) - \pi(\Delta) & \text{if } H \geq \Delta - \pi(\Delta) + \sigma \end{cases}. \tag{17}$$

Note that this characterization of v_1 makes no use of $H \in (2\Delta, 4\Delta]$.

Next, we evaluate the profit function v_2 when $H \in (2\Delta, 4\Delta]$. First observe that

$$v_2(H) = \max_{p_a \in [\max\{H-1,0\}, \min\{H,1\}]} V(H - p_a, p_a), \tag{18}$$

as follows. Since $H > 2\Delta$, if $p_a > 0$ then $p_1 > \Delta$ and/or $p_a > \Delta$. Note that $\frac{\partial}{\partial p_a} V(p_1, p_a) = p_1 \pi'(p_a)$ and $\frac{\partial}{\partial p_1} V(p_1, p_a) \leq \pi'(p_1)$. Hence at least one of $\frac{\partial}{\partial p_a} V(p_1, p_a)$ and $\frac{\partial}{\partial p_1} V(p_1, p_a)$ is strictly negative, implying that the solution to the maximization problem that defines $v_2(H)$ must have a binding capital constraint, i.e., $p_1 + p_a = H$.

Next we show that $V(H-p_a,p_a)$ has a unique local maximum, which we denote by $p_a(H)$. We also show that the local maximum lies in the interval $\left(\frac{H}{2},H-\Delta\right]$, approaches Δ as $H\to 2\Delta$, and is strictly increasing in H. The proof is as follows. $V(H-p_a,p_a)$ is a cubic in p_a with a positive coefficient on the cubic term. Hence, if a local maximum exists, it is the smaller root of the convex quadratic in p_a defined by

$$F(p_a,H)\equiv\frac{d}{dp_a}V(H-p_a,p_a)=-\pi'(H-p_a)-(\pi(p_a)-\pi(\Delta))+(H-p_a)\pi'(p_a). \tag{19}$$

By (8), the function F simplifies to

$$F(p_a,H)=-\pi'(H-p_a)+\left(H-p_a-\frac{1}{2}(p_a-\Delta)\right)\pi'(p_a). \tag{20}$$

Using this relation, observe that, for $H\in(2\Delta,4\Delta]$,

$$F\left(\frac{H}{2},H\right)=\left(\frac{H}{4}+\frac{1}{2}\Delta-1\right)\pi'\left(\frac{H}{2}\right)>0 \tag{21}$$

$$F(H-\Delta,H)=\left(\Delta-\frac{1}{2}(H-2\Delta)\right)\pi'(H-\Delta)\leq 0, \tag{22}$$

where (21) uses Assumption 3.

Hence $p_a(H)$ is well defined, with

$$p_a(H)\in\left(\frac{H}{2},H-\Delta\right] \tag{23}$$

and

$$p_a(H)\to\Delta \text{ as } H\to 2\Delta. \tag{24}$$

To establish that $p_a(H)$ is strictly increasing in H, note that $\frac{\partial F}{\partial p_a} < 0$, and so

$$sign\left(p_a'(H)\right) = sign\left(\frac{\partial F}{\partial H}\right) = sign\left(-\pi'' + \pi'\left(p_a(H)\right)\right) = sign\left(1 + \Delta - p_a(H)\right) > 0. \tag{25}$$

Next, we show that if $v_2(H) \geq v_1(H)$, then $v_2(H) = V(H - p_a(H), p_a(H))$. To see that, note first that from (23) and Assumption 3, $p_a(H) \geq \frac{H}{2} \geq \max\{H-1, 0\}$. Consequently, the solution to the maximization problem (18) is either $p_a(H)$ or $\min\{H, 1\}$. Since $\min\{H, 1\} > 2\Delta$ (by Assumption 3), and since we know that the solution to the buyer's problem satisfies $p_a \leq 2\Delta$, it follows that the solution is $p_a(H)$.

Hence, it follows from (23) that whenever $H \in (2\Delta, 4\Delta)$ and $v_2(H) > v_1(H)$, the optimal solution to the buyer's maximization problem satisfies $p_a > p_1 > \Delta$.

To complete the proof of Part 2, we need to show that there exists $\hat{H} \in (2\Delta, 4\Delta)$ such that $v_2(H) > v_1(H)$ if $H \in \left(2\Delta, \hat{H}\right)$ and $v_1(H) > v_2(H)$ if $H \in (\hat{H}, 4\Delta]$. This result follows from the continuity of v_1 and v_2 and the following observations, which are proved below: (i) $v_2(H) > v_1(H)$ for all H sufficiently close to 2Δ; (ii) $v_1(H) > v_2(H)$ at $H = 4\Delta$; and (iii) if $v_2(H) \geq v_1(H)$ over some interval, then $v_2(H) - v_1(H)$ is either monotone strictly decreasing; or strictly decreasing then strictly increasing; or monotone strictly increasing. The first observation follows since as $H \to 2\Delta$, $v_2(H)$ approaches the unconstrained optimum $V(\Delta, \Delta)$ while $v_1(H)$ does not. The second observation follows since $p_a(4\Delta) > 2\Delta$, by (23).

Finally, the third claim is proved as follows: First, if $H \leq \Delta - \pi(\Delta) + \sigma$, then $v_1(H)$ is constant, from (17), while $v_2(H)$ is decreasing, since the constraint that

$p_1 + p_a \geq H$ is binding. Second, consider the case $H > \Delta - \pi(\Delta) + \sigma$. Since $v_2(H) \geq v_1(H)$, we know from above that $v_2(H) = V(H - p_a(H), p_a(H))$. Given this, standard envelope arguments imply

$$v_2'(H) = \pi'(H - p_a(H)) + \pi(p_a(H)) - \pi(\Delta), \tag{26}$$

so that from (17),

$$\begin{aligned} v_2'(H) - v_1'(H) &= \pi'(H - p_a(H)) + \pi(p_a(H)) - \pi'(H - \sigma) \\ &= p_a(H) - \sigma + \pi(p_a(H)), \end{aligned} \tag{27}$$

and hence

$$v_2''(H) - v_1''(H) = (1 + \pi'(p_a(H)))\, p_a'(H) > 0, \tag{28}$$

where the inequality follows since $p_a(H) \leq 1$ and $p_a'(H) > 0$ as shown above. Hence $v_2(H) - v_1(H)$ is either monotone strictly decreasing; or strictly decreasing then strictly increasing; or strictly monotone increasing. This completes the proof of Part 2.

Parts 3 and 4. From the proof of Part 2, we know that whenever $H \in (\hat{H}, 4\Delta]$, the buyer chooses $p_1 > 0$ and $p_a = 0$, and that whenever $p_1 > 0$, the buyer bids $p_1 = \max\{H - \sigma, \Delta - \frac{1}{2}\Delta^2\}$. The rest of this part focuses on $H > 4\Delta$. First, observe that $v_1(H) > v_2(H)$, as follows. Consider any offer (p_1, p_a) such that $p_1 + p_a \geq H > 4\Delta$. Since $p_a \leq 2\Delta$, we know $p_1 > 2\Delta \geq p_a$, but then the offer (p_1, p_a) is strictly dominated by $(p_a, 0)$, since

$$\begin{aligned} & \pi(p_1) + p_1 \pi(p_a) + (1 - p_1)\pi(\Delta) - \pi(p_a) - (1 - p_a)\pi(\Delta) \\ = \; & \pi(p_1) - (1 - p_1)\pi(p_a) - (p_1 - p_a)\pi(\Delta) \end{aligned} \tag{29}$$

is strictly negative; and moreover, $(p_a, 0)$ satisfies the capital constraint.

Hence, when $H > \hat{H}$, the buyer chooses $p_a = 0$. If $v_1(H) > 0$, it is optimal to choose $p_1 > 0$, and if $v_1(H) < 0$ and/or $H - \sigma > 1$, it is optimal to choose $p_1 = 0$. The offer $(p_1, p_a) = (2\Delta, 0)$ yields strictly positive profits. Consequently, for all $H \leq 2\Delta + \sigma$ the buyer can make strictly positive profits while satisfying the capital constraint. To show that there exists some $\check{H} \in (2\Delta + \sigma, 1 + \sigma)$, such that if $H > \check{H}$ the buyer bids $p_1 = 0$ and if $H \in (\hat{H}, \check{H})$ the buyer bids $p_1 > 0$, observe that $v_1(H)$ is strictly decreasing when $H > 2\Delta + \sigma$, from (17), and that $v_1(1 + \sigma) < 0$. Finally, observe that $v_1(H)$ and $v_2(H)$ do not depend on L, so $\check{H}$ and $\hat{H}$ depend only on M and Δ.

Remarks for footnote 18. We claimed in this footnote that if Δ is sufficiently high (but satisfies Assumption 3) then $\hat{H} > \Delta + \sigma$. Consider first $\Delta = \frac{1}{2}$. Note that $\sigma = 2\Delta = 1$. Hence at $H = \Delta + \sigma$, we know $v_2(H) > V(\Delta, 2\Delta) = V(\Delta, 0) > v_1(H)$, since the offer $(p_1, p_1) = (\Delta, 2\Delta)$ satisfies (9) and is strictly dominated by the offer $(\Delta + \varepsilon, 2\Delta - \varepsilon)$ when ε is sufficiently small. So, by continuity, $v_2(\Delta + \sigma) > v_1(\Delta + \sigma)$ for all Δ sufficiently close to $\frac{1}{2}$. Hence for all Δ sufficiently close to $\frac{1}{2}$, $\hat{H} > \Delta + \sigma$.

Proof of Proposition 3. As a preliminary, note that if buyer $-i$ offers price p_{-i} and the seller accepts the offer, the market learns that $v \leq p_{-i}$. Hence, the capital constraint of buyer i, whose offer is not accepted, becomes $(\frac{1}{2}p_{-i} + \Delta)M \geq L_i$ (i.e., constraint (1) with $q = 0$ and $v = \frac{1}{2}p_{-i} + \Delta$). This is equivalent to $p_{-i} \geq \delta_i(1 + 2\Delta) - 2\Delta$. Hence, if $p_{-i} \geq 2\Delta$, the capital constraint of buyer i is satisfied if $\delta_i \leq \frac{4\Delta}{1+2\Delta}$, which is equivalent to $P(\delta_i) \leq 2\Delta$. Conversely, if $P(\delta_i) > 2\Delta$ then the capital constraint of buyer i is satisfied only if $p_{-i} > 2\Delta$.

Part (1). Clearly, there is an equilibrium in which both buyers offer $p = 2\Delta$ because, in equilibrium, the capital constraint of both buyers is satisfied and, given that one buyer offers 2Δ, the other buyer cannot increase his expected profits by offering $p \neq 2\Delta$.

There is no equilibrium in which the equilibrium price is less than 2Δ because if the buyer that offers the highest price offers $p < 2\Delta$, the other buyer (with leverage δ_i) can strictly increase his expected profits without violating his capital constraint by offering $\max\{p + \varepsilon, P(\delta_i)\}$, where ε is sufficiently small.

Finally, there is also no equilibrium in which the equilibrium price is more than 2Δ. To see why, suppose in contradiction that there is an equilibrium in which buyer i offers $p_i > 2\Delta$ and buyer $-i$ offers $p_{-i} \leq p_i$. In this equilibrium, buyer i makes negative profits. If instead buyer i offers 2Δ, he makes zero profits and guarantees that his capital constraint will be satisfied, as follows. If $p_{-i} \geq 2\Delta$ and the seller accepts the offer of buyer $-i$, the capital constraint of buyer i is satisfied by the preliminary argument above. If $p_{-i} \leq 2\Delta$, and the seller accepts the offer of buyer i, the capital constraint of buyer i is satisfied since $P(\delta_i) \leq 2\Delta$.

Part (2). There is an equilibrium in which buyer 1 offers $P(\delta_1)$ and buyer 2 offers $P(\delta_2)$. In this equilibrium, the capital constraints of both buyers are satisfied, buyer 1 makes zero profits (since his offer is never accepted), and buyer 2 makes negative expected profits. Buyer 1 cannot gain by offering $p \geq P(\delta_2)$ because he will make negative profits. Buyer 1 cannot gain by offering $p < P(\delta_2)$, since his offer will not be accepted. Buyer 2 cannot gain by reducing the price because his capital constraint will be violated. In particular, if the seller accepts an offer $p' < P(\delta_2)$, the capital constraint of buyer 2 is violated from the definition of $P(\cdot)$, and if the seller accepts the offer from buyer 1, the capital constraint of buyer 2 is violated from the preliminary observations above. Finally, buyer 2 cannot gain by offering more than $P(\delta_2)$ because doing so increases his expected losses.

There is no equilibrium in which the equilibrium price is more than $P(\delta_2)$ because the buyer who offers the highest price can reduce his expected losses by reducing the price slightly.

There is no equilibrium in which the equilibrium price is less than $P(\delta_2)$, as follows. If the highest price is from buyer 2, buyer 2's capital constraint is violated, and buyer 2 can strictly gain by increasing the price to $P(\delta_2)$. If instead the highest price is from buyer 1 and the price is more than 2Δ, buyer 1 can reduce his losses without violating his constraint by reducing the price to 2Δ. If the highest price is from buyer 1 and it is less than 2Δ, the capital constraint of buyer 2 is violated, and buyer 2 can gain by increasing the price to $P(\delta_2)$, so that his capital constraint is never violated.

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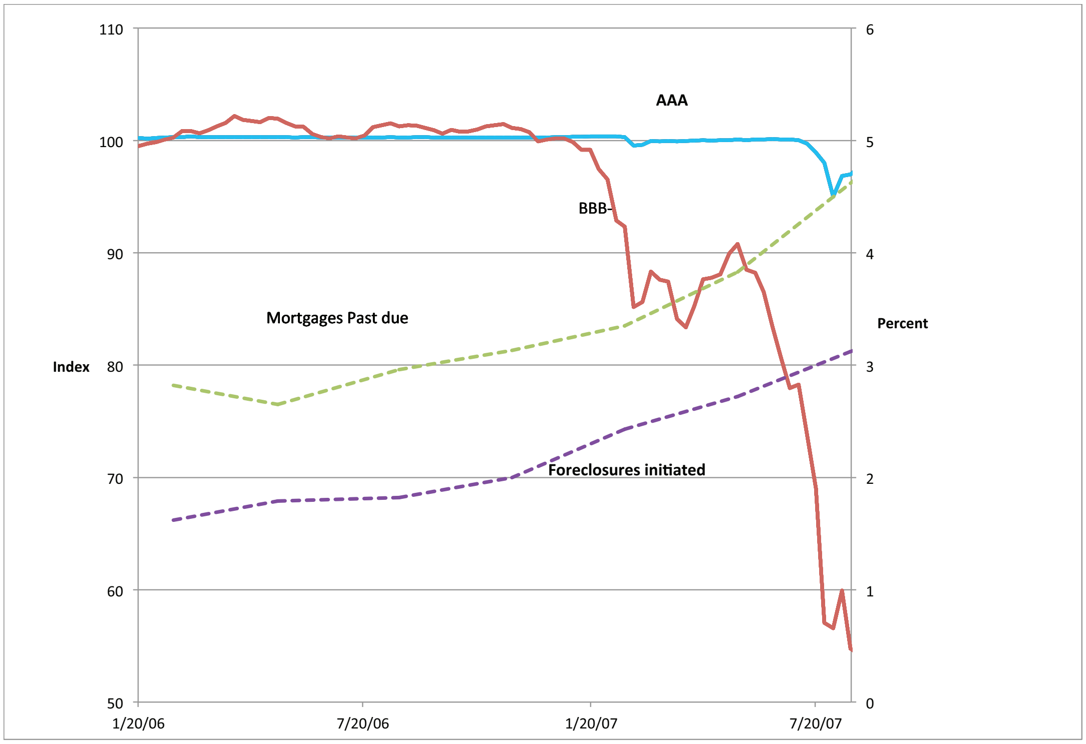

Figure 1: The solid lines plot the Markit ABX.HE indexes for residential mortgage-backed securities rated AAA and BBB-. The dashed lines plot quarterly data from the Mortgage Bankers Association on subprime mortgage payments: specifically, the percentage rate of mortgages that are past due 90 days or more, and the percentage rate of loans for which a foreclosure has been initiated during the quarter (both are seasonably adjusted).

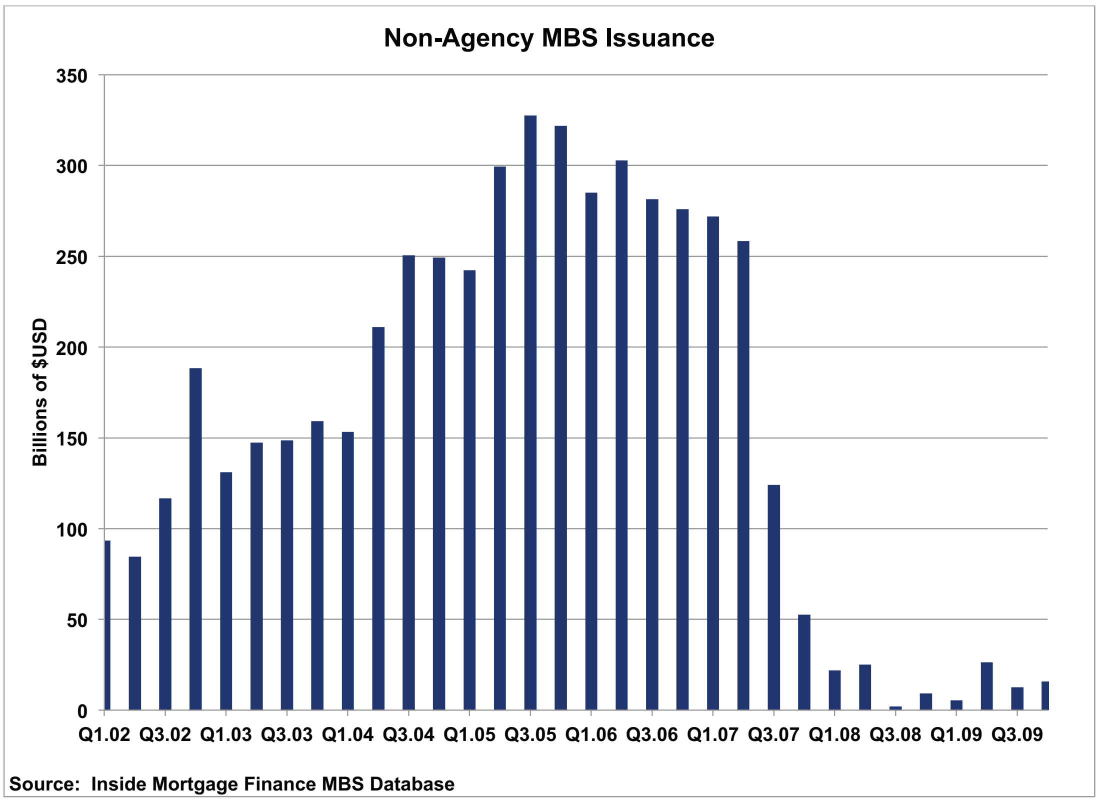

Figure 2: Total issuance of non-agency mortgage-backed securities (MBS), 2002-2009.

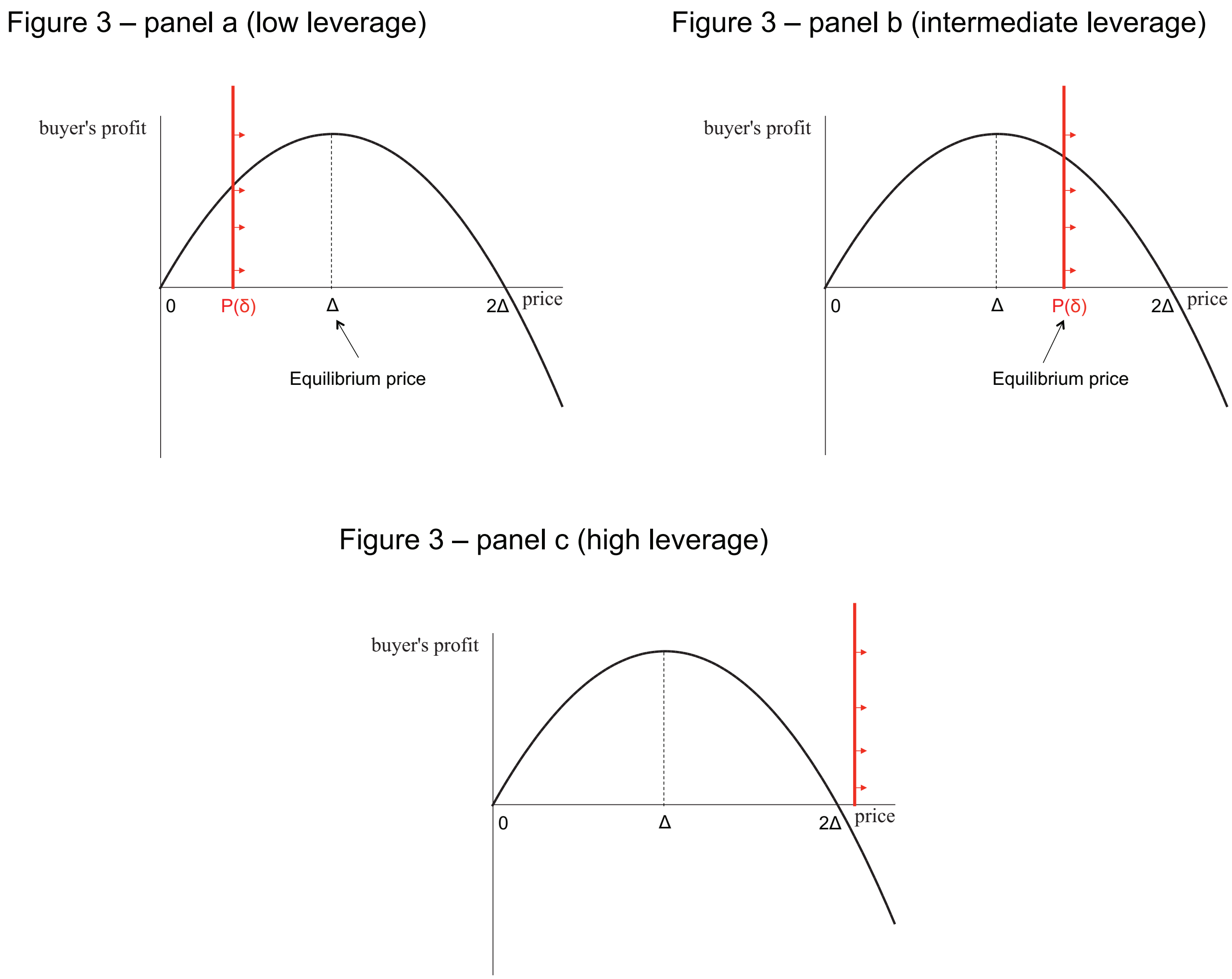

Figure 3: Each panel shows the buyer's profit function as a function of the buyer's bid price. The vertical line represents the capital constraint. In panel a, the capital constraint is not binding, in panel b the capital constraint binds, and in panel c, the capital constraint is so tight that the buyer prefers not to bid and the market freezes.