Share Issues versus Share Repurchases

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Abstract

Almost all firms repurchase shares through open market repurchase (OMR) programs. In contrast, issue methods are more diverse: both at-the-market offerings, analogous to OMR programs, and SEOs, analogous to rarely-used tender-offer repurchases, are used by significant fractions of firms. Furthermore, average SEOs are larger than at-the-market offerings. We show that this asymmetry in the diversity of transaction methods in issuances and repurchases and the size-method relation in issuances are natural consequences of the single informational friction of a firm having superior information to investors. Moreover, while this friction leads firms to issue inefficiently little, it leads firms to repurchase too little if they maximize long-term shareholder value, but too much if the primary goal is to boost short-term share prices.

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1 Introduction

Public firms often tap into the equity market, both issuing new shares to raise funds, and repurchasing existing shares to return cash to investors. Both issues and repurchases potentially yield transaction surplus; issues raise funds that firms can deploy to investment opportunities, while repurchases disburse funds and avoid various costs associated with holding cash inside a firm. In many ways, issuing and repurchasing shares are mirror images of each other. Both types of transaction are subject to informational frictions arising from firms' superior knowledge. And for both types of transaction, firms choose transaction size and method. Conceptually, share repurchases are simply negative issuances.

In this paper, we analyze the two transactions side-by-side, under the assumption that firms have superior knowledge about their own prospects, and can choose both transaction size and method. Although many papers analyze security transactions under asymmetric information, the comparison of issues and repurchases is new to the literature, and yields fresh insights. Likewise, size and method are two of the main ways in which transactions differ in practice; and correspond to signaling via retention and via money-burning, which have been the focus (largely separately) of prior analysis.[1] Our contribution is to analyze issues, repurchases, transaction size and transaction method in a unified framework, yielding new insights. We emphasize four points.

First, and despite the conceptual symmetry between issue and repurchase transactions, their equilibrium *outcomes* are *not* mirror images of each other. Repurchasing firms cannot signal via the efficiency of transaction method—*money burning*—while issuing firms can. Empirically, almost all firms repurchase via open-market transactions; while issuing firms use both seasoned equity offerings (henceforth, SEOs) and at-the-market offerings (henceforth, ATMs) with significant frequencies, though the latter has received limited academic attention. Our analysis rationalizes both patterns.

Second, and in contrast, reducing repurchase size is a viable signal for repurchasing firms, just as reducing issue size is a viable signal for issuing firms. The predicted patterns of transaction size and market response are consistent with empirical evidence on both issues and repurchases. The contrast between the first and second points highlights that while reducing repurchase size "burns money" by reducing transaction surplus, doing so also has the separate effect of increasing a firm's total value. This additional effect makes size-signalling a possibility in repurchases, while direct "money burning" signalling is impossible.

Third, firms that heavily weight the importance of increasing short-term share prices repurchase inefficiently excessive amounts. Many commentators have claimed that firms engage in excessive buybacks to boost share prices, at the expense of real investment; but the existing academic litera-

[1] See the large literatures following, respectively, Leland and Pyle (1977) on signaling via retention, and Ross (1977) and Bhattacharya (1979) on signaling via money burning.

ture lacks a coherent account of such behavior. In contrast, and regardless of the relative importance firms place on short- vs long-term share prices, issuing firms issue inefficiently too-small amounts.

Fourth, and more conceptually, while the received wisdom is that transaction surplus (NPV) should dictate financial decisions, our analysis isolates a precise formal role for firm value, viz., for any equity transaction under consideration, a manager should ask, "by what percent will this transaction affect firm value?" The point is starkest for the case of repurchases, in which case larger repurchases decrease firm value but often increase transaction surplus. But even for issue decisions, a focus on total firm value sheds light on firms' preferences for signalling-via-issue-size over signalling-via-issue-method, and operationalizes Viswanathan (1995)'s results on the ordering of signals in terms of standard financial quantities.

In addition, our analysis speaks to whether a firm's private information is about its assets-in-place or the profitability of "investment" opportunities, a distinction that dates back at least to Myers and Majluf (1984). Our results suggest that firms have private information about assets-in-place; if instead all private information were about investment opportunities, we obtain the empirically counterfactual prediction that larger issues and smaller repurchases are associated with higher prices (see Section 6).

In more detail, we model issues and repurchases in a unified and symmetric way. A firm privately knows the value of its assets-in-place (Myers and Majluf, 1984), and has a surplus-creating "project" that can only be implemented through trading equity. If the project requires a positive investment, the firm needs to raise capital by issuing shares. In contrast, if the "investment" is negative, then the firm needs to pay out capital by repurchasing equity; here, surplus potentially stems from tax savings and/or the avoidance of wasteful expenditures that would take place if cash were instead retained. In this context, the firm naturally decides how much to invest or pay out and how to carry out equity transactions. The former aspect maps to the transaction's/project's size and the latter corresponds to transaction methods with different levels of efficiency.

The following two points underpin many of our results. First, while higher prices prompt firms to issue more, ceteris paribus, they prompt firms to repurchase *less*.[2] Second, equity transactions mechanically affect total firm value (or equivalently, total assets) even without generating transaction surplus (or equivalently, the NPV of the project). In the textbook case of public information, only transaction surplus (NPV) matters for firm decisions. In contrast, under asymmetric information, total firm value significantly affects firm decisions too. More specifically, an action needs to simultaneously satisfy two conditions to be a viable signal: it decreases surplus, and its effect on the

[2]Empirically, Graham (2022)'s survey shows that 56% of repurchasing firms regard the market price of the stock as an important consideration. Louis and White (2007), Brockman, Khurana, and Martin (2008), Gong, Louis, and Sun (2008), and Chen and Huang (2013) go further, and document that firms release negative information and/or engage in *negative* earnings management prior to repurchases. Chen and Huang (2013) further show that the Sarbanes-Oxley Act reduced the extent of negative earnings management, thereby giving evidence that firms are constrained in their ability to drive down share prices before repurchases.

logarithm of firm value is more favourable to the firms who want to distinguish their types relative to those who want to pool with others (the single crossing property).

On the repurchase side, separation via money-burning cannot occur. The reason is that separation would entail worse firms repurchasing at lower prices, though at the cost of using a more inefficient method. But better firms would be attracted to these inefficient methods, because they would view them as having smaller price impact; and better firms would also care less than worse firms about the cost of inefficiency, because the cost is a smaller fraction of their total value. (More formally: The single-crossing property is violated.)

On the issue side, however, separation via money-burning occurs. In equilibrium, better firms issue at higher prices, though at the cost of using more inefficient methods. Worse firms are deterred from issuing in the same way by the fact that the inefficiency cost represents a greater fraction of their total value.

In contrast to this asymmetry, separation via transaction size occurs for both repurchases and issues. In equilibrium, worse firms repurchase less at lower prices, while better firms issue less at higher prices. In the repurchase decision, a bad firm finds larger repurchases unattractive because they have more price impact and hence are more expensive, which all firms dislike, but worse firms especially dislike because they correspond to repurchasing overvalued shares. Similarly, in the issue decision a good firm finds large issues unattractive because they have more price impact and hence are more dilutive, which all firms dislike, but better firms especially dislike because they correspond to issuing undervalued shares (see Myers and Majluf (1984)).

Finally, we consider the case where firms care not only about long-term shareholder value but also the short-term share price. Another interesting asymmetry emerges: Firms issue too little but repurchase too much. The reason for firms to issue too little is the same as before, as their direct preference for a higher short-term price coincides with their desire to increase long-term shareholder value. In contrast, repurchasing firms must balance their direct preference for higher short-term share prices with the fact that repurchasing at a higher price hurts long-term shareholders' value. When their direct preference for higher short-term prices dominates, the equilibrium outcome is that better firms repurchase excessive amounts. Although worse firms would gain a short-term price increase by mimicking these large repurchases, the cost to long-term shareholder value from overpaying for the shares is too large. Moreover, as in the baseline case of firms focusing exclusively on long-term shareholder value, all firms repurchase using the same method/efficiency; in this case, because separation on (excessive) repurchase size is more efficient than separation on the efficiency of transaction methods.

Our main implications fit well with empirical findings. We start with the asymmetry in transaction methods prediction. In principle, similar transaction methods are available for issuing and repurchasing firms. Specifically, firms can raise equity through an SEO in a one-off transaction, which typically completes in 2-8 weeks (Gao and Ritter, 2010); or more smoothly through ATMs over a couple of years.[3] Likewise, repurchases can be carried out either one-off in tender offers (henceforth, TOR, which are often completed within a month (Masulis, 1980)) or smoothly via open market repurchase (henceforth, OMR) programs that typically last several years (Stephens and Weisbach, 1998). Our asymmetry prediction gives an explanation for the prominent empirical feature that both SEOs and ATMs coexist as frequently observed issue methods, whereas OMR dominates the repurchase market.[4]

Second, the prediction that transaction size reveals firm fundamentals in both issues and repurchases again fits the data well: smaller issues and larger repurchases are both associated with higher prices.[5]

Third, the possibility that firms engage in excessive repurchases in order to push up the share price fits well with many anecdotal/informal accounts (see Section 5). This pattern does not typically emerge from a standard model with complete information, because the direct effect of an inefficient repurchase is a reduction in share prices.

Finally, our model's implication that issuing firms prefer to signal via smaller issues rather than via more inefficient methods implies the following pattern: The worst firms issue the maximum amount using the most efficient method; better firms issue less, still using the most efficient method; and the best firms issue the minimum amount possible to fund the project, but use less efficient issue methods. Consistent with the prediction, empirically the issue method is correlated with issue size. The mapping between efficiency choices in our analysis and empirical choices of transaction method requires placing more structure on the determinants of efficiency. We argue that financial transactions are more efficient when the timing of the cash flows they generate matches that of the firm's "real" operations. In particular, one-off SEOs are more efficient than smoother ATMs, as investment opportunities are typically lumpy, and the former allows the firm to immediately implement the project. In contrast, smooth OMRs are more efficient than one-off TORs, because firms that use repurchases to distribute profits (for example, technology companies such as Apple or Google and banks such as JP Morgan and Bank of America) typically generate these cash flows gradually over time. Combined with the results discussed above, this additional structure on the efficiency of transaction methods yields the empirical predictions that larger issues are carried out via SEOs, smaller issues are carried out via ATMs, and the large majority of repurchases are carried out via OMRs. These predictions are consistent with Billett, Floros, and Garfinkel (2019)'s findings on SEO vs ATM issues,[6] and with the fact that OMR is the overwhelmingly predominant form of

[3]See Billett, Floros, and Garfinkel (2019) for an overview of ATMs, which are growing in popularity.

[4]See the evidence for Prediction 1 in Section 4.

[5]Asquith and Mullins (1986) and Masulis and Korwar (1986) document negative relation between issue size and announcement return. In tender offer repurchases, Vermaelen (1981) find abnormal return is positively related to target tender fraction.

[6]We calculate from Table 2 of Billett, Floros, and Garfinkel (2019) that the average proceeds per SEO are $256 million, whereas average proceeds per ATM program are $92 million. Even though the ratio of proceeds to market equity is roughly the same between the two methods (18% for SEO and 20% for ATM), it is significantly smaller for ATM than for SEO after controlling for other observable factors (see Table 4 of the same paper).

repurchase.

1.1 Related Literature

There is a large literature on firms' capital transaction when they have superior information over investors. When selling securities, costly retention of unsold securities or broadly speaking, transaction size, can be informative signals about firms' hidden quality (see Leland and Pyle (1977), Myers and Majluf (1984), Krasker (1986), and DeMarzo and Duffie (1999)). When repurchasing securities, firms can similarly signal by different repurchase amounts (see Vermaelen (1984), Brennan and Kraus (1987), Ofer and Thakor (1987), Constantinides and Grundy (1989), Chowdhry and Nanda (1994), Lucas and McDonald (1998), and Bond and Zhong (2016)). In general, higher quality firms buy more or sell less (or even not sell at all). In addition to transaction-size signaling, these papers also show that firms can signal through tax-inefficient dividend payouts, or more generally burning cash (for example advertisement signaling in Milgrom and Roberts (1986)), in exchange for a more favorable transaction price. Our analysis contributes to this literature by allowing both size and efficiency signaling simultaneously and compares the two directions of equity transactions (issues and repurchases) side by side. Novel to the literature is the insight that issuing firms use both transaction size and efficiency as signals, whereas repurchasing firms only signal via transaction size. We also establish that issuing firms prefer to signal via issuing less rather than via issuing inefficiently. We show that firms' different objectives to maximize long-term or short-term share prices lead to similar outcomes in issuance, but qualitatively different results in repurchase.

Our analysis covers firms' actions that can be mapped into some combination of retention and money-burning. While this covers a large fraction of firms' decisions in equity transactions, we acknowledge that it doesn't cover everything. A notable case is Oded (2005), in which good firms announce a repurchase program that drives down medium-term prices because investors face adverse selection from trading against a more informed firm, but drives up long-term prices by the amount of the firm's trading profits; the net result is a redistribution away from shareholders hit by liquidity shocks and towards "patient" shareholders. Related, when shareholders are asymmetrically informed or liquidity constrained, the price formation method is also an interesting consideration, e.g., Comment and Jarrell (1991).[7] Our analysis emphasizes different forces and abstracts away from these additional ingredients, and is complementary.

Like us, Babenko, Tserlukevich, and Wan (2020) consider issues and repurchases in a unified model, though from a different perspective. They show that a firm can profitably trade its own equity (market timing), but in doing so harms shareholders who trade against the more informed firm. In contrast, our paper focuses on how these issues and repurchases are carried out, namely the choices

[7] Comment and Jarrell (1991) hypothesize that Dutch-auction TORs and OMRs are weaker signals of undervaluation than fixed-price TORs, and find consistent evidence. We adopt competitive pricing of shares, an standard approach in the literature. It is worth noting that in our model in which participating investors are equally uninformed and unconstrained, many alternative price formation methods are equivalent to competitive pricing.

of transaction size and method (efficiency).

Our paper is also related to the literature on firms' choice of equity transaction methods. Brennan and Thakor (1990) and Oded (2011) study firms' choice between tender offer and open market repurchases. In contrast to our model, which studies firms' choice under private information, these papers consider the interaction between informed and uninformed shareholders in their tendering strategies, and emphasize the role of shareholders' endogenous decision to acquire information. In contrast, when firms raise equity, Burkart and Zhong (2023) compare public offerings and rights offerings. The key driver in their paper is the wealth transfer between constrained and unconstrained shareholders, and the efficiency choice is left out of the model. Chemmanur and Fulghieri (1994) present a model in which investment banks endogenously acquire information as underwriters, and predict that firms choose underwritten issues over direct issues unless they face little information asymmetry or receive too low an evaluation from the investment bank to procure its services. In contrast, abstracting from the role of underwriters, we analyze firms' choices between one-off SEOs and smoother ATMs, emphasizing their differences in efficiency in funding corporate investment.

Our paper also speaks to the literature on multi-dimensional signaling/screening. We defer a fuller discussion of this point until page 16 below.

2 The model

We model share issues and repurchases in a unified framework. Consider a firm with non-negative assets-in-place a and an opportunity to invest i in a new project. The value of assets-in-place, a, is the firm's private information, whereas others only know that a is distributed according to $F(\cdot)$, which admits a density and has support $[a_{\min}, a_{\max}]$. We refer to a as the firm's type.

The firm plays either an issue game or a repurchase game. In both instances, the firm picks "project size" i. In an issue game, i corresponds to funds raised, and is positive: formally, $i \in [I_L, \infty)$ where $I_L \geq 0$ is a minimum project size. In a repurchase game, i correspond to funds paid out, and is negative: formally, $i \in (-\infty, I_L]$ where $I_L \leq 0$ is again a minimum project size. A strictly positive minimum project size $I_L > 0$ in the issue game corresponds to investment opportunities having a minimum viable scale; similarly, a minimum project size in the repurchase game corresponds to a firm being compelled to pay out at least a minimum amount of cash; for example, if retaining cash above some level would lead to extremely wasteful spending.

(We have also fully analyzed the case in which a firm always has the option of choosing $i = 0$, i.e., $|i| \in \{0\} \cup [|I_L|, \infty)$. In particular, this specification is natural to consider for investment projects. The analysis of this case does not yield any additional insights relative to $|i| \in [|I_L|, \infty)$, and so we focus on this latter case both for transparency, and in order to preserve symmetry across the analysis of issues and repurchases.)[8]

Both the issue and repurchase game entail equity transactions, specifically, share issues and share repurchases. For reasons outside the model, the firm prefers to raise funding via equity to other securities, and to pay out cash via repurchases rather than dividends.[9]

In addition to choosing transaction size i, the firm also chooses among equity transaction methods with different levels of efficiency, captured by the variable $\theta \in [0,1]$, with efficiency increasing in θ. At an abstract level, the efficiency choice θ can be mapped to many decisions, including, for example, underwriter choice or lock-up provisions.[10] For some empirical applications we focus on a particular dimension of efficiency, namely whether a transaction method matches the need for investment capital or speed of cash flow generation. See Section 4 for full details.

A transaction i carried out with efficiency θ yields surplus $S(i,\theta)$. A firm's value V is the combination of its assets-in-place a, the funds raised or disbursed by the equity transaction i, and transaction surplus S:

$$V(a,i,\theta) = a + i + S(i,\theta). \tag{3}$$

For share repurchases, surplus stems from cash being more valuable in the hands of shareholders than the firm's, potentially because of taxes, internal agency problems in the firm, or shareholders' liquidity needs.[11] Due to these reasons, a payout of $|i|$ reduces firm value by only $|i| - S(i,\theta)$,

[8]Effectively, for the issue setting, we are assuming, in terms of formal objects defined below, that if $I_L > 0$, then

$$\frac{V(a_{\max}, I_L, 1)}{1 + \frac{I_L}{a_{\min} + S(I_L,1)}} > V(a_{\max}, 0, 1), \tag{1}$$

i.e., the best firm prefers issuing I_L at full efficiency but at the most unfavorable price that can be supported in equilibrium over the alternative of doing nothing; along with the analogous assumption for repurchase: if $I_L < 0$, then

$$\frac{V(a_{\min}, I_L, 1)}{1 + \frac{I_L}{a_{\max} + S(I_L,1)}} > V(a_{\min}, 0, 1). \tag{2}$$

Under these conditions, the results are invariant to adding $i = 0$ to firms' option. See Lemma A2 in the Appendix for details.

[9]For signaling effects of security design, see the large literature following Nachman and Noe (1994). Taxes are commonly invoked as a reason for firms' preference for share repurchases over dividends. For example, in the US dividends are tax-disadvantaged relative to paying out cash through share repurchases, even after the tax changes associated with the 2003 Jobs and Growth Tax Relief Reconciliation Act; see, for example Chetty and Saez (2005) and Blouin, Raedy, and Shackelford (2011). Allowing for dividends would serve to endogenize the lower boundary $a_{\min}$ of the support of firm types in the repurchase game. Firms at this boundary must be indifferent between paying out dividends, avoiding the costs of information asymmetry but incurring tax and other costs; and playing the equilibrium action characterized by our analysis. Similarly, the possibility of debt issues, rights offerings (Burkart and Zhong, 2023), etc. would endogenize the upper boundary $a_{\max}$ in the issue game.

[10]Choosing a more expensive underwriter reduces the issue proceeds and is less efficient. In contrast, the interpretation of a lock-up period is more intricate. The literature has proposed several reasons for lock-up periods (see, e.g., Brau, Lambson, and McQueen (2005) and Karpoff, Lee, and Masulis (2013)): they may directly impose illiquidity costs on the existing owners, thereby decreasing θ; or they may restrict insider trading and alleviate moral hazard problems, thereby increasing θ. Finally, both underwriter choice and lock-up periods may have their own unique signaling features (other than differences in efficiency) that are complementary to our analysis.

[11]See Jensen (1986), Stulz (1990) and Chowdhry and Nanda (1994).

delivering (3). For use throughout, define I^* as the efficiency-maximizing transaction size,

$$I^* = \arg \max_{i:|i| \geq |I_L|} S(i,1).$$

A firm with assets-in-place a can only choose (i,θ) such that $V(a,i,\theta) > 0$, i.e., the firm value must be positive after equity transaction. This rules out paying out more cash than the firm can afford even by liquidating all assets. We assume $V(a,I^*,1) > 0$ for all a. As we will show, this constraint is not binding in equilibrium.

We assume that S is continuously differentiable. Let S_i and S_θ denote the partial derivatives of S with respect to i and θ, and similarly, V_i and V_θ the partial derivatives of V. In addition, S satisfies the following mild assumptions:

Assumption 1. *The surplus function S satisfies*

1. $S(0,\theta) = 0$ *and* $S(i,0) \leq 0$;
2. $S_\theta(i,\theta) > 0$ *for* $i \neq 0$ *and* $S(i,1)$ *is single-peaked in* i;
3. $S_i > -1$, *that is,* $V_i > 0$.

Part 1 and the first half of part 2 are normalizations: zero transaction size and zero efficiency both lead to zero surplus (or less), and surplus is increasing in efficiency. The second half of part 2 is a mild regularity condition. Part 3 says that a larger issue size leads to a higher firm value, and a larger repurchase size leads to a lower firm value. In other words: for issues, even if surplus is decreasing in i for some values, surplus is never so strongly decreasing as to offset the direct effect of adding resources i to the firm. Similarly, the surplus generated by increasing repurchases $|i|$ is never enough to offset the direct effect of paying out resources.

The number of shares outstanding before any issue or repurchase is normalized to 1. Given an equity transaction price p, the firm needs to issue $\frac{i}{p}$ shares to raise capital i, or repurchase $\frac{-i}{p}$ shares for $i < 0$ to disburse $-i$. A firm's long-term investors' value after equity transaction (i,θ) at price p is

$$\Pi(a,i,\theta,p) = \frac{V(a,i,\theta)}{1+\frac{i}{p}}. \tag{4}$$

Our baseline analysis covers the standard case in which firms seek to maximize the payoff of long-term investors, i.e., (4).[12] Under this assumption, firms don't have any direct preference over the short-term share price; instead, the price only matters insofar as it affects the revenue/cost of share

[12] See, for example, Myers and Majluf (1984), Constantinides and Grundy (1989), and Chowdhry and Nanda (1994). When we interpret the surplus S as stemming from repurchases reducing a firm's internal agency problems, we have in mind either repurchase decisions as being made at the board level (in the US, board approval is typically required for repurchase programs) and agency problems occurring below the board and/or repurchase decisions being made by senior executives and agency problems occurring at levels below the firm's senior management.

issues/repurchases.[13] Ceteris paribus, a firm profits more from issuance when its share price is high; but conversely, a firm profits less from a repurchase when its share price is high. Section 5 extends the analysis to consider firms that care about both short- and long-term share prices.

For both intuition and formal analysis, it is convenient to work with the logarithm of the firm's payoff,

$$\ln \Pi\left(a, i, \theta, p\right) = \ln V\left(a, i, \theta\right) - \ln\left(1 + \frac{i}{p}\right). \tag{5}$$

That is, firms trade off percentage changes in firm value V with percentage changes in the number of shares outstanding after the equity transaction. The fact that percentage changes are important stems from our focus on equity transactions.

Equity transactions are carried out at the competitively determined price $P(i, \theta)$. That is: After a firm announces its size and efficiency choices (i, θ), competitive investors update their beliefs about the firm type a, $\mu(i, \theta)$, and offer price $P(i, \theta)$ under which they expect to break even:

$$P\left(i, \theta\right) = E\left[\Pi\left(a, i, \theta, P\left(i, \theta\right)\right)|i, \theta\right], \tag{6}$$

where expectations are taken using beliefs $\mu(i, \theta)$.[14] Equation (6) is equivalent to

$$P\left(i, \theta\right) = E\left[a|i, \theta\right] + S\left(i, \theta\right). \tag{7}$$

The firm chooses (i, θ) to maximize the payoff of its long-term investors, $\Pi(a, i, \theta, P(i, \theta))$ under the equilibrium price function P.

By design, this framework covers both issue and repurchase decisions in a symmetric way. For the remainder of the paper, we refer to the case $i \geq I_L \geq 0$ as the *issue game*, and the case $i \leq I_L \leq 0$ as the *repurchase game.*

2.1 Equilibrium concept

We focus on pure-strategy perfect Bayesian equilibria (PBEs), which consist of each firm-type's choices of size and efficiency, $(i(a), \theta(a))$; investor beliefs $\mu(i, \theta)$ associated with each choice of (i, θ); and competitive investors' price function, $P(i, \theta)$, that satisfy the following conditions:

[13] We assume that managers who make issue/repurchase decisions cannot trade their personal shares before their private information becomes public. For information contents of insider trading around share issues and repurchases, see Leland (1992), Buffa and Nicodano (2008), and Babenko, Tserlukevich, and Vedrashko (2012). We abstract from this aspect and focus on firm's primary equity transactions.

[14] The assumption that transactions are carried at the competitively determined price directly implies that any firm that separates in equilibrium makes zero trading profits from the transaction; though the transaction typically yields some surplus via its effect on firm value.

1. Given $P(i,\theta)$, a firm's equilibrium strategy maximizes its long-term shareholders' payoff:

$$(i(a), \theta(a)) \in \arg\max_{i,\theta} \Pi(a, i, \theta, P(i,\theta)).$$

2. The price function $P(i,\theta)$ satisfies (7) with the expectation taken under beliefs $\mu(i,\theta)$.

3. Investor beliefs $\mu(i,\theta)$ satisfy Bayes' rule for any (i,θ) such that $(i,\theta) = (i(a), \theta(a))$ for some firm type a.

As in many signaling models, there are typically multiple PBEs. In Section 3, we first construct a PBE strategy for each of the repurchase and issue games, and then employ the widely accepted D1 refinement criterion (Cho and Kreps, 1987) to show that the constructed PBE strategies are the only ones supported by "reasonable" off-equilibrium beliefs. Broadly speaking, the D1 refinement says that the off-equilibrium beliefs associated with transaction (i,θ) load on firm types that are most likely to gain by deviating to (i,θ), in the sense that the range of prices for which the deviation is beneficial is largest. Formally, let $\Pi^*(a)$ denote the equilibrium payoff of a type-a firm, and define $D_a(i,\theta)$ as the range of prices p such that firm a prefers equity transaction (i,θ) at price p to the current equilibrium payoff,

$$D_a(i,\theta) = \{p : \Pi(a,i,\theta,p) > \Pi^*(a)\}. \tag{8}$$

A belief about an off-equilibrium choice (i,θ) satisfies D1 if type a is not in the support of $\mu(i,\theta)$ as long as there exists a second type $\tilde{a}$ satisfying $D_a(i,\theta) \subsetneq D_{\tilde{a}}(i,\theta)$. It is worth noting that one of our central results—the impossibility of separation-via-efficiency by repurchasing firms—does not rely on equilibrium refinements.

Remark: Smooth transactions—that is, OMR and ATM programs—also entail optionality, since firms can transact smaller quantities than the initial announcement. As we verify in Online Appendix A, the equilibrium outcomes in Section 3 are robust to firms having this optionality.[15] For brevity, we abstract from this aspect, and assume that firms issue and repurchase the full amount that is announced.

3 Equilibrium characterization

We fully characterize the equilibria of the repurchase and issue games. Specifically, for repurchases we show that separation-via-efficiency is impossible; while separation-via-size naturally arises, with worse firms repurchasing less, at lower prices. For issues, firms separate by issuing different quan-

[15] Specifically, the equilibrium outcomes in Propositions 2 and 4 are also equilibrium outcomes under the following perturbation of the model. Let $\Theta \subset [0,1]$ be the set of choices of θ that entail optionality. As in the main model, a firm publicly announces a transaction plan (i,θ), based on which investors price its shares. Different from the main model, if $\theta \in \Theta$ then the firm can privately choose an actual transaction size smaller than the announced size, viz., choose an actual transaction $\left|i^A\right| \in [|I_L|, |i|]$.

(a) The repurchase game. (b) The issue game.

Figure 1: Equilibrium size (solid line) and efficiency (dashed line) in the repurchase and issue games. Values of $|i(a)|$ and $i(a)$ correspond to the left y-axis, and values of $\theta(a)$ correspond to the right y-axis.

tities, with better firms issuing less, at higher prices; and the best firms further separate by issuing inefficiently, at still higher prices. Figure 1 summarizes these results.

3.1 Full information benchmark

As a benchmark, consider the case in which a firm's assets a are publicly observed. From (3), (4) and (6),

$$P(i,\theta) = \Pi(a,i,\theta,p) = a + S(i,\theta).$$

Hence in this benchmark, and as one would expect, firms choose transaction size $i = I^*$ and efficiency $\theta = 1$ in order to maximize transaction surplus $S(i,\theta)$. Firm value $V(a,i,\theta)$ is irrelevant to the decision.

3.2 Repurchases

We first analyze the behavior of firms wishing to pay out funds by repurchasing shares. We start by showing that repurchasing firms are unable to separate from each other by repurchasing with different efficiency levels. As we will see, this impossibility of separation-via-efficiency contrasts sharply with the possibility of such separation by issuing firms that seek to raise funds.

Proposition 1. *In the repurchase game, in any PBE, all firms that repurchase the same size i choose the same efficiency θ.*

To understand the economics behind Proposition 1, suppose to the contrary that there is an equilibrium in which worse firms repurchase at a lower price $P\left(i,\tilde{\theta}\right) < P(i,\theta)$, though at the cost of using a less efficient method $\tilde{\theta} < \theta$. On the one hand, the resulting sacrifice in firm value V, $S(i,\theta) - S\left(i,\tilde{\theta}\right)$, represents a smaller fraction of V for a better firm. On the other hand, the percentage change in the number of shares is independent of firm type. Consequently, the lower efficiency choice $\tilde{\theta}$ is more attractive for good firms than bad firms (see (5)), and so an equilibrium of this type cannot exist.

In contrast to Proposition 1's result that firms cannot separate via different efficiency levels, it *is* possible for different firms to repurchase different amounts at different prices. We start by constructing a particular equilibrium of this type, and then argue below that it is the one most likely to be played. All firms repurchase in the most efficient way possible, $\theta = 1$. The best firm repurchases the surplus-maximizing amount I^*,

$$\hat{\imath}\left(a_{\max}\right) = I^*. \tag{9}$$

Other firms $a < a_{\max}$ repurchase a lower amount $\left|\hat{\imath}\left(a\right)\right| < \left|I^*\right|$, that is,

$$\hat{\imath}\left(a\right) > I^* \text{ for } a < a_{\max}, \tag{10}$$

determined by the ODE

$$\frac{\partial \hat{\imath}\left(a\right)}{\partial a} = -\frac{\hat{\imath}\left(a\right)}{V\left(a, \hat{\imath}\left(a\right), 1\right) S_i\left(\hat{\imath}\left(a\right), 1\right)} \tag{11}$$

subject to boundary condition (9). There is a unique solution $\hat{\imath}\left(\cdot\right)$ to (9), (10) and (11), which is strictly decreasing.

If $a_{\min}$ is close enough to $a_{\max}$ such that (9), (10) and (11) lead to repurchases above the minimum size ($\left|\hat{\imath}\left(a\right)\right| > \left|I_L\right|$, i.e., $\hat{\imath}\left(a\right) < I_L$) for all firms $a > a_{\min}$, then these repurchases constitute an equilibrium strategy.

The characterization of the size-separation schedule in (9), (10) and (11) is standard (e.g., Mailath (1987)). First, there is no distortion at the "bottom," in this case meaning that the best firm $a_{\max}$ repurchases the surplus-maximizing amount I^*, and hence (9). Second, worse firms separate by repurchasing less, (10), which has the advantage of reducing the repurchase price. Given separation, repurchases are fairly priced, i.e., $P\left(i\left(a\right), \theta\left(a\right)\right) = a + S\left(i\left(a\right), \theta\left(a\right)\right)$. As standard, the equilibrium condition implies that firm a does not gain from mimicking neighboring firms, so that equilibrium strategy $\hat{\imath}\left(a\right)$ solves the differential equation

$$\frac{d}{d\tilde{a}} \Pi\left(a, \hat{\imath}\left(\tilde{a}\right), 1, \tilde{a} + S\left(\hat{\imath}\left(\tilde{a}\right), 1\right)\right)_{\tilde{a}=a} = 0. \tag{12}$$

By straightforward manipulation, (12) simplifies to (11).

Next we consider the case in which $a_{\min}$ is further from $a_{\max}$. In this case, there isn't room for firms to fully separate on repurchase size according to (11). Instead, there must be a lower interval of firms that pool on the smallest repurchase size. Denote the boundary firm by $\hat{a}$; this firm must be indifferent between separating by repurchasing the amount $\hat{\imath}\left(\hat{a}\right)$ determined by (9), (10) and (11), or pooling with firms $[a_{\min}, \hat{a})$ and repurchasing the minimum size I_L.

Formally, define $\hat{a}$ as follows, to encompass both cases above. If $\left|\hat{\imath}\left(a_{\min}\right)\right| \geq \left|I_L\right|$ then simply define $\hat{a} = a_{\min}$. Otherwise, there exists $a_0 > a_{\min}$ with $\hat{\imath}\left(a_0\right) = I_L$, and this firm strictly prefers repurchasing $|I_L|$ at a price pooled with lower types to repurchasing $\left|\hat{i}\left(a_0\right)\right|$ under the fully revealing price:

$$a_0 + S\left(\hat{i}\left(a_0\right), 1\right) < \Pi\left(a_0, I_L, 1, E\left[a|a \leq a_0\right] + S\left(I_L, 1\right)\right). \tag{13}$$

In this case, either there is a unique $\hat{a} \in (a_0, a_{\max})$ such that

$$\hat{a} + S\left(\hat{i}\left(\hat{a}\right), 1\right) = \Pi\left(\hat{a}, I_L, 1, E\left[a|a \leq \hat{a}\right] + S\left(I_L, 1\right)\right), \tag{14}$$

or there is no such $\hat{a}$, in which case define $\hat{a} = a_{\max}$.

It is worth highlighting that when the boundary firm $\hat{a}$ is interior it repurchases discretely more than the minimum repurchase I_L. This is because pooling at I_L generates a discrete improvement in the repurchase price—and so firm $\hat{a}$ is indifferent only if there is also a discrete jump in repurchase amounts. Consequently, firms stop separating on size even before reaching the minimum repurchase size. See Figure 1a for an illustration of this case.

Proposition 2. *The repurchase game has a PBE with the following firm strategy: Firms $a > \hat{a}$ repurchase $\hat{i}\left(a\right)$ and firms $a < \hat{a}$ repurchase I_L, where $\hat{a}$ and $\hat{i}(\cdot)$ are as defined above; all firms repurchase in the most efficient way ($\theta = 1$).*

Why can repurchasing firms separate using size i even though they cannot separate using efficiency θ? The reason is that even though reducing repurchase size from the surplus-maximizing level I^* and reducing efficiency both reduce transaction surplus, the former increases firm value V while the latter decreases V (see Assumption 1). Consider the case of a worse firm repurchasing a smaller amount $|\tilde{i}| < |i|$, i.e., $\tilde{i} > i$, at a lower price $P\left(\tilde{i}, \theta\right) < P\left(i, \theta\right)$. While this smaller repurchase lowers transaction surplus by $S\left(i, \theta\right) - S\left(\tilde{i}, \theta\right)$, it increases firm value by $\tilde{i} + S\left(\tilde{i}, \theta\right) - i - S\left(i, \theta\right)$, because the firm retains more cash. This increase represents a larger fraction of total value for worse firms. So by (5), a smaller repurchase is more attractive to worse firms, making it a viable signal.

Propositions 1 and 2 represent the principle insights of this subsection. First, separation-via-efficiency is impossible for repurchasing firms. Second, and in contrast, separation-via-size is possible. Although scaling down a repurchase reduces the transaction surplus—i.e., "burns money"—just like adopting an inefficient method, doing so *increases* rather than decreases total firm value.

Proposition 2 constructs a particular equilibrium outcome. Importantly, standard refinement arguments suggest that it is the most plausible outcome; specifically, it is the unique outcome to satisfy D1:

Proposition 3. *A unique D1 equilibrium of the repurchase game exists and delivers the outcome characterized in Proposition 2.*

A key implication of the D1 refinement is that all firms that repurchase do so with maximal efficiency $\theta = 1$, which is a strengthening of the no-separation-via-efficiency result in Proposition 1. Similar to the reasoning behind Proposition 1, this follows from the observation that worse firms experience larger (percentage) effects from reducing efficiency. Suppose that, contrary to the claimed result, some firms repurchase using an inefficient method $\tilde{\theta} < 1$. Then these firms would like to deviate and repurchase more efficiently ($\theta = 1$), provided that doing so doesn't significantly increase the repurchase price. The D1 refinement ensures that this condition is met: the gain in firm value, $S(i, 1) - S\left(i, \tilde{\theta}\right)$, is a larger fraction of a worse firm; hence, D1 beliefs about a deviation to $\theta = 1$ are concentrated on worse firms, inducing a price decrease.

3.3 Issues

We now turn to the behavior of firms wishing to raise funds by issuing shares ($i \geq I_L \geq 0$). We establish an asymmetry with respect to repurchases, namely that separation-via-efficiency is feasible for issuing firms, even though it is not for repurchasing firms. In fact, issuing firms separate via both size and efficiency. The economic forces behind issuing firms' separation-via-size and separation-via-efficiency are both standard in the literature. Better firms may separate by retaining a larger fraction of equity, which is more valuable for them (Leland and Pyle, 1977). Better firms may also separate by "burning money," which is less costly for them as a fraction of their firm values.

Parallel to the case of repurchases, we construct a particular equilibrium, illustrated in Figure 1b, and then argue that it is the one most likely to played. First, and as standard, there is no distortion at the bottom: the worst firm $a_{\min}$ issues the surplus-maximizing size I^* with the most efficient method $\theta = 1$:

$$\hat{i}(a_{\min}) = I^*. \tag{15}$$

Second, an interval of firms better than $a_{\min}$ separate by scaling down the project, while retaining maximal issue efficiency $\theta = 1$. The construction is the same as for the equilibrium of the repurchase game, with the exception that it starts from the worst firm $a_{\min}$ rather than the best firm $a_{\max}$. Writing $\hat{i}(a)$ for firm a's issue strategy, the function $\hat{i}(\cdot)$ solves the differential equation (12), which is equivalent to (11), subject to the boundary condition (15) and that

$$\hat{i}(a) < I^* \text{ for } a > a_{\min}. \tag{16}$$

Note that although repurchase and issue sizes share the same differential equation (11), the prediction on transaction size is reversed across the two cases, with better firms repurchasing more (in absolute values) but issuing less.

Third, separation on issue size according to (11) continues as long as there is room. Specifically, if $\hat{i}(a_{\max}) \geq I_L$, all firms separate on issue size; for use below, define $\hat{a} = a_{\max}$. Otherwise, define $\hat{a}$ by $\hat{i}(\hat{a}) = I_L$. Firms better than $\hat{a}$ issue the minimum amount I_L, and separate by adopting less efficient methods. Specifically, a firm $a > \hat{a}$ adopts efficiency level $\hat{\theta}(a)$, determined by the differential equation

$$\frac{d}{d\tilde{a}}\Pi\left(a, I_L, \hat{\theta}(\tilde{a}), \tilde{a} + S\left(I_L, \hat{\theta}(\tilde{a})\right)\right)_{\tilde{a}=a} = 0, \tag{17}$$

subject to the boundary condition

$$\hat{\theta}(\hat{a}) = 1. \tag{18}$$

Equation (17) simplifies to

$$\frac{\partial \hat{\theta}(a)}{\partial a} = -\frac{I_L}{V\left(a, I_L, \hat{\theta}(a)\right) S_\theta\left(I_L, \hat{\theta}(a)\right)}. \tag{19}$$

Recall that we assume that the best firm prefers issuing I_L with efficiency $\theta = 1$ under the worst belief to doing nothing (see footnote 8). Under this assumption, there is enough room in efficiency choices θ for all firms better than $\hat{a}$ to fully separate, i.e., $\hat{\theta}(a)$ remains positive for all $a \in [a_{\min}, a_{\max}]$.

Proposition 4. *The issue game has a PBE with the following firm strategy: Firms $a \leq \hat{a}$ issue $\hat{i}(a)$ in the most efficient way ($\theta = 1$), and firms $a > \hat{a}$ issue $i = I_L$ at efficiency $\hat{\theta}(a)$, where $\hat{a}$, $\hat{i}(\cdot)$, and $\hat{\theta}(\cdot)$ are as defined above.*

In particular, whenever $a_{\min}$ and $a_{\max}$ are sufficiently far apart that $\hat{a} < a_{\max}$, firms $a \in [\hat{a}, a_{\max}]$ adopt different efficiency levels, in contrast to the impossibility of separation-via-efficiency in the repurchase game.

We conclude this subsection by arguing that the equilibrium outcome described in Proposition 4 is the most plausible one, in the sense that it is the only outcome to satisfy the D1 refinement. A first step in this argument is that firms separate via size "before" separating via efficiency:

Lemma 1. *In any D1 equilibrium of the issue game, if a firm issues $i > I_L$ then it uses the most efficient method $\theta = 1$.*

The economic intuition for an issuing firm's preference to separate via size is as follows. Suppose to the contrary that a D1 equilibrium exists in which some firm a issues more than the minimum amount, $i > I_L$, but uses an inefficient method $\theta < 1$. By issuing less but transacting more efficiently, the firm can both increase transaction surplus S and reduce its total value V. That is, there exists a deviation to $\tilde{i} < i$ and $\tilde{\theta} > \theta$ such that

$$S\left(\tilde{i}, \tilde{\theta}\right) \quad > \quad S(i, \theta), \tag{20}$$

$$\tilde{i} + S\left(\tilde{i}, \tilde{\theta}\right) \quad < \quad i + S(i, \theta). \tag{21}$$

The economic principle that makes the combination of (20) and (21) possible is that increasing efficiency θ raises transaction surplus and firm value by the same amount; while issuing less leads to a larger reduction in firm value than in surplus.

The percentage reduction in firm value associated with (21) is smaller for better firms. From (5), it follows from D1 that the beliefs associated with this deviation are at least as good as a. So the deviation $\left(\tilde{i}, \tilde{\theta}\right)$ is at least fairly priced for firm a, and since it strictly raises transaction surplus, it strictly raises firm a's payoff.

Lemma 1 establishes a necessary condition for firms to separate using transaction efficiency, that is, the possibilities from separation on size are exhausted. Proposition 5 shows that this condition is also sufficient: once the ability to separate via size is exhausted, issuing firms indeed switch to separating via transaction efficiency. We therefore have a full characterization of the unique D1 outcome.

Proposition 5. *A unique D1 equilibrium of the issue game exists and delivers the outcome characterized in Proposition 4.*

3.4 Relation to Viswanathan (1995)

Lemma 1's ordering of signaling-via-size versus signaling-via-efficiency can be understood as operationalizing Viswanathan (1995)'s "benefit-cost criterion." When multiple signaling devices are available, Viswanathan establishes that the Pareto-optimal separating equilibrium uses the signal with the highest "benefit-cost ratio." Formally, define $\pi(a,i,\theta,\tilde{a}) = \ln \Pi(a,i,\theta,\tilde{a}+S(i,\theta))$. Viswanathan's benefit-cost ratios for issue size and efficiency are, respectively, $\frac{-\pi_{ai}}{\pi_i}\Big|_{\tilde{a}=a}$ and $\frac{-\pi_{a\theta}}{\pi_\theta}\Big|_{\tilde{a}=a}$.

At first sight, the comparison of these benefit-cost ratios appears opaque. However, this comparison can be expressed entirely in terms of a signal's effect on firm value and transaction surplus. Specifically:

Lemma 2. *The ordering of ratios $\frac{-\pi_{ai}}{\pi_i}\Big|_{\tilde{a}=a}$ and $\frac{-\pi_{a\theta}}{\pi_\theta}\Big|_{\tilde{a}=a}$ coincides with the ordering of ratios $\frac{V_i}{S_i}$ and $\frac{V_\theta}{S_\theta}$.*

In particular, Lemma 2 formalizes the distinct roles of firm value and transaction surplus in determining a signal's attractiveness. Precisely because transaction size i affects firm value not only via transaction surplus S but also directly, in the issue game, on the viable range of size signals ($i \leq I^*$), it is immediate that transaction size has the more attractive benefit-cost ratio,

$$\frac{V_i}{S_i} > \frac{V_\theta}{S_\theta}, \tag{22}$$

consistent with Lemma 1.

Viswanathan (1995) characterizes Pareto-optimal separating equilibria. Abstract papers such as Engers (1987), Cho and Sobel (1990), and Ramey (1996) in turn show that the D1 refinement typically select such equilibria.[16]

[16] For other uses of Pareto-optimality to select among signals in corporate finance settings, see John and Williams (1985), Ambarish, John, and Williams (1987), Besanko and Thakor (1987), Ofer and Thakor (1987), and Williams (1988). Williams (2021) analyzes a seller's choice between signalling via retention and illiquidity in a competitive search model; his results emphasize the role of participation costs of potential investors, which is a dimension that we do not pursue in this paper.

4 Empirical implications

In this section, we explore the empirical implications of our model. Broadly speaking, there are two ways in which firms issue seasoned equity in practice. The first method is a one-off SEO, which is typically completed within several weeks.[17] A lesser known but increasingly popular method is an at-the-market offering (ATM). Billett, Floros, and Garfinkel (2019) provide a nice review of ATMs. In an ATM, the firm first registers new shares with the Securities and Exchange Commission (SEC), and then anonymously sells these shares in the secondary market. Compared to SEOs, ATMs take much longer to complete, on average 6.2 quarters. Similarly, firms can repurchase equity in a one-off fashion through a tender offer repurchase (TOR) within a month.[18] Alternatively, they can carry out an open market repurchase program (OMR) over a horizon of several years.[19]

The starkest prediction to emerge from our analysis (see Propositions 2 and 4) is:

Prediction 1: A greater variety of methods is used in issue transactions than in repurchase transactions.

When firms repurchase shares, Proposition 2 shows that they cannot separate by the efficiency of transaction methods while different repurchase sizes are possible. In contrast, when firms issue shares, Proposition 4 illustrates that firms may adopt both different transaction methods and sizes in equilibrium.

Consistent with this prediction, significant amounts of issuance occur via both SEOs and ATMs. Billett, Floros, and Garfinkel (2019) document that ATMs represented 63% incidences and 26% issue proceeds of those for SEOs in 2016. In addition, economically important quantities of equity are issued via employee stock option grants, restricted stock grants, and in mergers and acquisitions (see Fama and French (2005) and McLean (2011)).

In contrast, an overwhelming fraction of repurchases are OMRs, with only a very small fraction being TORs. For example, in 2004, there were 466 cases of OMR with a total size of $223 billion, while tender offers and Dutch auctions only accounted for 18 and 10 cases, and $1.3 billion and $3.9 billion proceeds respectively (see Banyi, Dyl, and Kahle (2008), and similar patterns documented by Grullon and Ikenberry (2000)). There are also other repurchase methods, including accelerated share repurchase (ASR) or privately negotiated repurchases. King and Teague (2022) show that the dollar amount of ASRs as a fraction of total annual repurchases is consistently less than or around 10%. Furthermore, Bargeron, Kulchania, and Thomas (2011) document that other repurchase methods, such as privately negotiated repurchases, are empirically even rarer than ASRs.

[17] A non-shelf bookbuilt SEO, which accounts for 91% of all SEOs, often takes 2-8 weeks, while an accelerated bookbuilt SEO often takes 2 days from announcement to completion (Gao and Ritter, 2010; Huang and Zhang, 2011).

[18] It takes 25 days on average from announcement of an TOR to the expiration of the offer (Masulis, 1980).

[19] On average, 46.2%, 66.9%, and 73.9% of the target amount is completed by end of the first, second, and third year, respectively (Stephens and Weisbach, 1998).

It is worth highlighting that Prediction 1 is independent of which equity transaction method is more efficient (i.e., the mapping between θ and methods). More specific predictions about transaction methods, and their correlation with transaction size and future outcomes, require us to take a stand on how the efficiency parameter θ in the model maps to different methods. While different assumptions are possible here (for example, see Oded (2011)), we argue that efficiency is enhanced by matching the timing of cash inflows and outflows. Specifically, firms typically generate smooth operating cash flows while investment needs are discrete and lumpy. Hence, a one-off SEO is more efficient because it leads to cash inflows that match the lumpiness of capital expenditure outflows; while a smoother ATM is less efficient because the gradual inflow of issuance proceeds is mismatched to investment expenditure outflows, thereby delaying a firm's investment. In contrast, a smoother OMR is more efficient than a one-off TOR precisely because the smooth OMR generates cash outflows that better match the smooth inflow of earnings; while a one-off TOR results in a timing mismatch, leading to an inefficient build-up of cash inside the firm.[20]

We microfound the efficiency gains from matching inflows and outflows as follows. For the issue setting, consider a firm that encounters an investment opportunity at time 0, but lacks funds to undertake it. The project requires minimum investment I_L and exhibits decreasing returns to scale. The firm chooses both an investment amount i, i.e., project scale; and a time t to start the project. The project can only start after the firm has raised funds i. If implemented at time 0, the project generates payoff $f(i)$, which is increasing and concave. As time passes, the project becomes more obsolete (for example, due to the entry of competitors), and the payoff decreases at the rate of α. Defining $\theta(t) = e^{-\alpha t}$, the project payoff is of the form $f(i)\theta(t)$. As such, an immediate SEO corresponds to the highest efficiency level $\theta = 1$, while smoother ATMs correspond to lower efficiency levels.

For the repurchase setting, consider a firm that generates free cash flows at continuous rate λ over a time interval $[0, T]$. Holding cash inside the firm is costly, either because of internal agency or due to tax inefficiencies, and generates a rate of return strictly lower than investors' opportunity cost of funds. The firm chooses a total amount to repurchase, $|i|$, and how many dates to spread these repurchases over, $n \leq N$; that is, under payout policy $(|i|, n)$ the firm spends $\frac{|i|}{n}$ on repurchases at dates $\frac{T}{n}, 2\frac{T}{n}, \ldots, T$. At a payout date, divestment from the bad project incurs adjustment cost $\kappa\left(\frac{|i|}{n}\right)^{\gamma}$, where $\kappa > 0$ and $\gamma > 1$ are constants. Hence repurchases create surplus by avoiding the wasteful holding of cash in the firm, while larger repurchases $|i|$ carry larger adjustment costs, dissipating the surplus; and more frequent repurchases (higher n) are more efficient because they

[20] Prominent examples of firms using repurchases to distribute smoothly arriving earnings include large technology companies (e.g., Apple, Amazon, Facebook, and Google) and profitable banks (Goldman Sachs, JP Morgan, and Bank of America), among many other companies. Another motive to carry out share repurchases is to distribute one-off windfall cash, such as lump-sum damage awards from legal disputes, spinoff proceeds, sudden capital structure adjustment, and so on. But these are rare events, and we believe our model captures the majority of repurchase activities. Moreover, Blanchard, Lopez-de-Silanes, and Shleifer (1994) document that for firms receiving cash windfalls "repurchases are generally not open-market, but targeted at large outside shareholders of the firm;" this is consistent with our analysis, in that repurchase-efficiency is achieved by matching the timing of cash inflows and outflows.

better match the arrival of cash inflows, thereby allowing the firm to reduce the costs of accumulated cash balances.[21] In terms of our notation, efficiency θ is simply a (scaled) transformation of the number of repurchase dates n. For more details of this microfoundation see Appendix A.

This formalization allows us to replace Prediction 1 with the more specific:

Prediction 1': Firms issue equity using both SEOs and ATMs, while firms repurchase equity via OMRs.

Relative to Prediction 1, the incremental insight of Prediction 1' is that firms repurchase using OMRs. Empirically, this is overwhelmingly the case as suggested by the previous evidence.

Proposition 5 also delivers the cross-sectional prediction that a firm carries out larger issues using efficient methods, which in our interpretation corresponds to an SEO. In contrast, for smaller issues a firm sometimes uses more inefficient methods, corresponding to ATM issues. As graphically illustrated in Figure 1, worse firms separate on size between I^* and I_L, but maintain the most efficient transaction method (SEO). ATMs with varying degrees of efficiency (corresponding to transaction speed) only occur for small-sized issues at I_L. Hence, we have

Prediction 2: SEOs are larger than ATM programs.

Empirically, Billett, Floros, and Garfinkel (2019) document average SEO proceeds of $256 million which is significantly larger than the average ATM program proceeds of $92 million. The result remains robust after controlling for additional observable factors, including size of the issuing firm.[22]

Since worse firms issue more efficiently (SEO), Proposition 5 (Figure 1) also implies

Prediction 3: Firms with better unobservable qualities are more likely to use ATM issues.

Consistent with Prediction 3, Hartzell et al. (2019) show in a dataset of REITs that the announcement returns of ATMs are less negative than of SEOs.[23] Billett, Floros, and Garfinkel (2019) use *future* analyst recommendation updates as their proxy for firm quality unobservable to the market at the time of issuance. Their regression result in Table 4 shows that ATM firms receive better future analyst recommendation updates than SEO firms.

In our model, the degree of the informational friction is captured by the dispersion of firm types, $a_{\max} - a_{\min}$. Proposition 5 predicts that separation via transaction efficiency arises only when the dispersion of firm types is large ($a_{\max} > \hat{a}$ in Figure 1b). Otherwise, all firms use SEO and separate by issue size. Hence, we have

Prediction 4: Firms facing larger informational frictions are more likely to use ATM issues.

[21] In addition, more frequent repurchases also reduce the total impact of convex adjustment costs.

[22] See Tables 2 and 4 in Billett, Floros, and Garfinkel (2019) for details.

[23] That both ATMs and SEOs are followed by negative returns can be generated in our model by relaxing the assumptions of footnote 8, to allow for the possibility that the best firms do not issue any shares.

Consistent with Prediction 4, Billett, Floros, and Garfinkel (2019) show that higher levels of information asymmetry, proxied by unexplained current accruals, are indeed associated with the choice of ATM over SEO.

We close this section with a brief discussion of an implication that superficially appears more testable than we believe it is, viz., the prediction of pooling at the minimum transaction size I_L. The difficulty of testing this prediction is that I_L—in common with all parameters of the model—is common knowledge, and so should be understood as being a function of observable firm characteristics. As such, the prediction of pooling at I_L could only be tested by an econometrician who knows how I_L varies with firm characteristics.

5 Direct preferences for higher short-term share prices

Many commentators suggest that when public firms repurchase shares they are motivated primarily by a desire to boost their short-term share prices, and that inefficiently excessive repurchases are commonplace.[24] The existing academic literature lacks a coherent account of such behavior. In particular, the direct effect of an inefficiently large repurchase is to decrease a firm's share price (in our notation, see (7)),[25] and any explanation for how an excessive repurchase ends up boosting share prices must overcome this direct effect.

In this section, we show that price-motivated inefficiently large repurchases emerge if we perturb our model to one in which firms have a significant direct preference for higher short-term share prices. In this case, better firms signal their quality to investors by repurchasing an excessive amount; the information effect of the signal dominates the direct cost of the inefficient repurchase. This outcome contrasts with the inefficiently small repurchase amounts in our baseline model. At the same time, and in line with our findings on the asymmetry between repurchases and issues, regardless of the weight firms put on short-term share prices, firms issue inefficiently small amounts, just as in the baseline model.

In addition, we show that regardless of the weight that firms place on short- vs long-term share prices, repurchases pool on the choice of the efficiency of transaction method, while issues entail separation in the efficiency of transaction methods.

[24] See, for example, "Are Stock Buybacks Starving the Economy?" (Lowrey, The Atlantic, 2018); "End Stock Buybacks, Save the Economy" (Lazonick and Jacobson, New York Times, 2018); and "Profits Without Prosperity" (Lazonick, Harvard Business Review, 2014), which asserts that "[c]orporate profitability is not translating into widespread economic prosperity. The allocation of corporate profits to stock buybacks deserves much of the blame."

[25] The argument that at least some commentators appears to have in mind for the mechanism via which repurchases boost share prices is that repurchases increase earnings-per-share (EPS). For example, the aforementioned article in the Harvard Business Review states that repurchases "enable [a] company to hit quarterly earnings per share (EPS) targets," while Eisen and Otani (the Wall Street Journal, 2018) state "Share repurchases can play a key role in supporting stock prices because they lower the number of shares outstanding—driving up per-share earnings even without overall profit growth." Related, Almeida, Fos, and Kronlund (2016) present evidence that firms indeed distort decisions in order to meet analysts' EPS forecasts.

Concretely, in this section we analyze an extension of our baseline model in which firms have Cobb-Douglas preferences over short-term and long-term share prices, i.e.,

$$\Pi(a, i, \theta, p) = p^{\epsilon}\left(\frac{V(a, i, \theta)}{1+\frac{i}{p}}\right)^{1-\epsilon}, \tag{23}$$

where the weight $\epsilon \in [0, 1)$ reflects the degree to which firms care about their share prices directly.[26] The case $\epsilon = 0$ corresponds to our baseline analysis that follows the standard modeling assumption that firms aim to maximize the interest of long-term shareholders. In contrast, in cases $\epsilon > 0$ firms have a direct preference for higher short-term share prices.

5.1 Repurchases

We first consider the repurchase game. We establish that, regardless of whether or not firms heavily weight short-term share prices, firms pool on repurchase method and some of them separate by repurchasing different amounts. For separating firms, equilibrium repurchase decisions satisfy the differential equation (12), which rewrites to

$$\frac{\partial \hat{i}(a)}{\partial a} = -\frac{\epsilon V\left(a, \hat{i}(a), 1\right) + (1-\epsilon)\hat{i}(a)}{V\left(a, \hat{i}(a), 1\right) S_i\left(\hat{i}(a), 1\right)}, \tag{25}$$

thereby generalizing (11).

(11) might appear to suggest that the relative weight ϵ that firms place on short- and long-term share prices doesn't qualitatively affect the equilibrium. But this is not the case. Instead, if firms place little weight on short-term prices then, as in our baseline analysis, firms separate by repurchasing inefficiently small amounts. In contrast, if firms heavily weight short-term share prices (ϵ large) then firms separate by repurchasing inefficiently large amounts, $|i| > |I^*|$. Formally, for $\underline{\epsilon} = \min_{i \in [I^*, I_L]} \frac{-i}{a_{\max} + S(i,1)}$ and $\bar{\epsilon} = \frac{-I^*}{a_{\min} + S(I^*,1)}$, which satisfy $0 < \underline{\epsilon} < \bar{\epsilon} < 1$ if $I_L < 0$:[27]

[26] The specific Cobb-Douglas form of preferences in (23) is unimportant, and most of our results extend to linear preferences

$$\Pi(a, i, \theta, p) = \epsilon p + (1-\epsilon)\left(\frac{V(a, i, \theta)}{1+\frac{i}{p}}\right). \tag{24}$$

Specifically, under linear preferences, our central result that repurchasing firms don't separate via the efficiency of transaction methods continues to hold; moreover, under the assumptions that S is concave in i and the extent of asymmetric information $|a_{\max} - a_{\min}|$ isn't too large, the equilibria we characterize in Propositions 6 and 7 remain equilibria. See Online Appendix C for formal results. The main drawback to adopting linear preferences is that characterizing the unique equilibrium via use of the D1 refinement becomes intractable.

[27] Analogous to footnote 8, we make the following simplifying assumptions to ensure all types participate: In the issue game, if $I_L > 0$, then

$$\Pi(a_{\max}, I_L, 1, a_{\min} + S(I_L, 1)) > a_{\max}; \tag{26}$$

In the repurchase game, if $I_L < 0$, then

$$\Pi(a_{\min}, I_L, 1, a_{\max} + S(I_L, 1)) > \Pi(a_{\min}, 0, 1, a_{\max}). \tag{27}$$

Proposition 6. *In the repurchase game:*

1. *For* $\epsilon \in [0,1)$, *in any D1 equilibrium, all repurchasing firms use the fully efficient method* $\theta = 1$.

2. *If* $\epsilon \in [0, \underline{\epsilon})$, *there is a unique D1 equilibrium, in which a firm's strategy takes the same form as in the case of* $\epsilon = 0$ *except that ODE (11) generalizes to (25); In particular,* $|i(a)| < |I^*|$ *for all* $a < a_{\max}$.

3. *If* $\epsilon \in (\bar{\epsilon}, 1)$, *there is a unique D1 equilibrium, in which firms separate on repurchase sizes according to (25), the boundary condition* $\hat{i}(a_{\min}) = I^*$, *and the condition that* $|i(a)| > |I^*|$ *for* $a > a_{\min}$.

To understand the economics behind the contrast between parts 2 and 3 of the proposition, it is easiest to first return to the baseline case of $\epsilon = 0$, and see why separation on transaction size doesn't entail inefficiently large repurchases. Separation of this kind would entail better firms repurchasing more than worse firms, and hence suffering more inefficiency than worse firms. But this cannot be an equilibrium outcome, because better firms would gain from reducing their repurchase amounts and mimicking worse firms; doing so both increases efficiency (higher transaction surplus S) and results in a lower and hence more favorable repurchase price. This logic extends straightforwardly to the case of small positive weights ϵ on the short-term price.

In contrast, consider next the case of firms that heavily weight (large ϵ) the short-term price. In this case, firms' net preference is for a higher short-term price; the direct preference for a high price dominates the cost that high repurchase prices impose on long-term firm value. So the question becomes: if better firms separate by issuing inefficiently large amounts, why don't worse firms mimic them? Analogous to arguments from our baseline analysis, the reason is that an inefficiently large repurchase is proportionally more costly (as a fraction of firm value) for worse firms than for better firms.

As noted, part 3 of Proposition 6 fits well with anecdotal and informal accounts of firms repurchasing inefficiently large amounts in order to boost short-term prices. But these informal accounts skirt over the mechanism via which repurchases actually increase short-term prices, which is far from obvious: after all, a fairly-priced repurchase affects the price only via transaction surplus S, and if this is negative, as alleged, then prices would fall rather than rise.

A more nuanced aspect of the result is that, for large weights ϵ on short-term prices, a good firm could alternatively separate by adopting an inefficient repurchase method (low θ). But the D1 refinement implies that firms separate via inefficiently large repurchases instead of via inefficient methods. The argument is again analogous to those in our baseline analysis. If a firm adopts an inefficient repurchase method, then consider a deviation by that firm to a more efficient method but a larger (and hence more inefficient) repurchase, with the net effect being an increase in transaction surplus but a decrease in firm value. This deviation is proportionally more costly for worse firms, and so by D1 investors interpret such a deviation as coming from good firms, thereby ensuring that the deviation is indeed attractive. Moreover, in this case, firms never "run out of room" to separate on repurchase size, because as repurchases grow larger, the marginal cost of further repurchases grows large, and so even small incremental repurchases carry substantial signaling power.[28]

5.2 Issues

Next, consider the issue game. Here, outcomes under the baseline case of $\epsilon = 0$ remain qualitatively unchanged if instead firms directly care about short-term share prices ($\epsilon > 0$). The reason is that issuing firms prefer to issue at a high price even in the baseline case. Introducing a direct preference for short-term prices only strengthens a firm's desire for higher prices.

Proposition 7. *In the issue game, for any $\epsilon \in [0, 1)$, there is a unique D1 equilibrium, in which a firm's strategy takes the same form as in the case of $\epsilon = 0$ except that ODE (11) generalizes to (25) and ODE (19) generalizes to*

$$\frac{\partial \hat{\theta}(a)}{\partial a} = -\frac{\epsilon V\left(a, I_L, \hat{\theta}(a)\right) + (1-\epsilon) I_L}{V\left(a, I_L, \hat{\theta}(a)\right) S_\theta \left(I_L, \hat{\theta}(a)\right)}. \tag{28}$$

In particular, Proposition 7 establishes that firms' equilibrium issuance decisions are too small, in contrast to the finding for repurchases in the large-ϵ case. Concretely, better firms are unable to separate from worse firms by issuing inefficiently large amounts. This is a consequence of part 3 of Assumption 1, which states that firm value V is increasing in the issuance amount, even though transaction surplus drops for issues $i > I^*$. Consequently, if a good firm a prefers to issue a larger amount over a smaller amount then the same is true for every firm of lower quality $\tilde{a} < a$, for which the increase in firm value is proportionally larger.

6 Private information on project profitability

In our main analysis, a firm's private information is about its assets-in-place a. Here, we summarize the outcomes that arise under the alternative assumption that a firm instead has private information about the profitability of the project to be implemented, which we denote by b. For concreteness, let a firm's post transaction value be

$$V(b, i, \theta) = a_{IL} + bf(C + i, \theta), \tag{29}$$

[28]Economically, it is increasingly costly for worse firms to mimic better firms. Formally, the numerator in the ODE (25) becomes small as $\hat{\imath}$ becomes more negative and V correspondingly decreases. In contrast, in our main analysis issuing firms may run out of room to separate on transaction size because many projects have a natural minimum scale (Propositions 4 and 5).

where a_{IL} and C are the firm's illiquid assets-in-place and cash, f is increasing in i and θ and concave in i, and $b \in [b_{\min}, b_{\max}] \subset (0, \infty)$ controls the profitability of new investments in the issue game, and the gain from paying out cash in repurchase game.[29] Let

$$I^* (b) = \max_i V (b, i, 1) - i$$

denote the efficient transaction size for firm b. We assume that in the issue game, issuing is efficient, i.e., $I^* (b) > 0$ for all b, and in the repurchase game, repurchasing is efficient, i.e., $I^* (b) < 0$ for all b. For the case of repurchases, we assume that excessive repurchases are a possibility for all firms, i.e., $I^* (b) > -C$ for all b; and that the elasticity of $f (x, 1)$ is bounded away from 0 as $x \to 0$, a relatively mild restriction that is satisfied if, for example, $f (\cdot, 1)$ is Cobb-Douglas, or if it is in the CES family with complementarity between capital and other inputs.[30] We emphasize the following implications.

First, and in common with our baseline model in which firms have private information about their assets-in-place, repurchasing firms do not separate via the efficiency choice θ. As such, firms' private information is a robust explanation for the empirical regularity that the vast majority of repurchases are open market repurchases.

Second, many of the other predictions of our analysis "flip." Specifically, in the issue game, firms pool in adopting the most efficient choice θ; better firms issue *more* than worse firms, and they issue inefficiently *excessive* amounts. Similarly, in the repurchase game, worse firms repurchase *more* than better firms, and they repurchase inefficiencly *excessive* amounts. The cross-sectional predictions on issue and repurchase sizes are both natural consequences of better firms putting resources to more productive use than worse firms.

Although the implication that repurchasing firms pool on their efficiency choice matches the data, two of the other implications directly contradict empirical findings. As discussed, empirically, there is heterogeneity in the issue methods that firms adopt, in contrast to the pooling predicted by information asymmetry in b. Perhaps more strikingly, the theoretical prediction from information asymmetry in b that better firms issue larger amounts and worse firms repurchase larger amounts implies that larger issues are associated with higher prices and larger repurchases are associated with lower prices, opposite to empirics.

The combination of the results from our main analysis and those emerging from specification (29) suggests that, empirically, firms' private information *isn't* primarily about the productivity of new projects, but is instead about the value of assets-in-place.

Formally, we establish:

[29]We adopt (29) for transparency. The results discussed here hold more generally; a key condition is that V is log-supermodular in (b, i) and in (b, θ).

[30]Formally, either $f(x, 1) = \kappa_f x^\sigma$ or $f(x, 1) = \left(x^{\frac{\sigma-1}{\sigma}} + \kappa_f\right)^{\frac{\sigma}{\sigma-1}}$ for some constants $\kappa_f > 0$ and $\sigma \in (0, 1)$.

Proposition 8. *Suppose firm value is given by (29) and firm's private information is about b. In both issue and repurchase games, in the unique D1 equilibrium all firms adopt maximal efficiency* $\theta = 1$*. Firms separate in transaction size, with better firms (higher b) issuing more and repurchasing less. Both issues and repurchases are too large, i.e.,* $|i(b)| > |I^*(b)|$ *for almost all firm types b.*

By way of a brief summary: The reason that results flip relative to our main analysis is that in our main analysis a firm's value is log-submodular in its private quality parameter a and equity transaction i, whereas under specification (29) a firm's value is log-supermodular in its private quality parameter b and equity transaction i. The result that repurchasing firms don't separate via efficiency choice stems from a two-step argument. The first step is that repurchasing firms prefer to separate via transaction size rather than efficiency, and stems from the same economic forces as the analogous result in our main analysis that issuing firms prefer to separate via transaction size rather than efficiency. The second step is that repurchasing firms never "run out of room" to separate on transaction size. The reason is that as firms approach the point of paying out all their cash C, the efficiency cost of these large payouts grows very large, and so even small incremental repurchases enable a great deal of separation.

7 Discussion

7.1 Transaction fees

One of our key theoretical insights is that when firms issue or repurchase equity, signaling through cash burning (inefficient transaction methods) is fundamentally different from signaling through reduction in transaction size. Some other actions can be modeled as combinations of these two signals, and hence can be incorporated and understood in our framework. For example, a firm paying a transaction fee c out of the amount of cash to pay out $|i|$ can be interpreted as simultaneously burning repurchase surplus by c and reducing repurchase size to $|i| - c$.[31] If firms can choose the amount of transaction fee c (with i and θ fixed), they may separate on different transaction fees in equilibrium due to the effect of c on repurchase sizes. But when firms can directly choose different repurchase sizes, they separate only on repurchase sizes and pool on these other dimensions such as transaction fees.

[31] Long-term shareholder value in this case will be

$$\begin{aligned}\Pi(a,i,\theta,c,p) &= \frac{V(a,i,\theta)}{1-\frac{|i|-c}{p}} \\ &= \frac{a-|i|+S(i,\theta)}{1-\frac{|i|-c}{p}} \\ &= \frac{a-(|i|-c)+(S(i,\theta)-c)}{1-\frac{|i|-c}{p}}.\end{aligned}$$

7.2 Policy implications

ATM offerings were rarely used until regulatory changes in 2005 and 2008 made them more accessible.[32] Since then, the use of ATMs has risen sharply (Billett, Floros, and Garfinkel, 2019). The regulatory changes reflected the SEC's intention to "allow more companies to benefit from the greater flexibility and efficiency in accessing the public securities market".[33] In line with this intention, Gustafson and Iliev (2017) find that after the 2008 deregulation, the treated firms (listed firms with public floats under \$75 million) raised more public equity and increased their capital expenditure.

Our model implies that even though lifting the barriers to ATMs may allow firms to invest more by issuing more equity, total surplus (welfare) may decrease.[34] To show this, here we consider an "SEO-only" issue game where $\theta = 1$ is the only available issue method (following our interpretation in Section 4), and compare it to our baseline results in Section 3.3, corresponding to the case where the barrier to ATMs is lifted.

As defined in Section 3, let $\hat{a}$ be the cutoff firm type in the baseline model, above which firms separate on issue sizes and use method $\theta = 1$, and below which firms issue I_L and separate on different methods $\theta < 1$. If $\hat{a} = a_{\max}$, then the baseline model and the SEO-only game generate the same outcomes: all firms issue different sizes with SEOs. If $\hat{a} < a_{\max}$, then in the SEO-only game, there is a cutoff firm type $\check{a} < \hat{a}$ such that firms below $\check{a}$ take the same actions as in the baseline model, using SEOs and separating on different issue sizes; whereas types above $\check{a}$ act differently from the baseline model, pooling on issuing the minimum size I_L using SEOs. We show this result in the Online Appendix.

Compared with the SEO-only game, when ATMs are allowed, intermediate firms $a \in (\check{a}, \hat{a})$ issue and invest more, which increases surplus. This is consistent with the above empirical findings of Gustafson and Iliev (2017). On the other hand, the best firms $a > \hat{a}$ use less efficient issue methods. Consequently, the net effect of allowing ATMs on total surplus is ambiguous.[35]

[32] In 2005, the SEC Securities Offering Reform (SOR) liberalized the filing requirements when firms "take securities down" from a "shelf registration" of equity offerings, which allowed takedowns to be done without review or delay by the SEC. This opened the door to ATMs, which involve frequent takedowns off a shelf. In 2008, the SEC expanded the eligibility of shelf offerings including ATMs to firms with public floats under \$75 million.

[33] See the final rule titled "Revisions to the Eligibility Requirements for Primary Security Offerings on Form S-3 and F-3", SEC File No. S7-10-07, December 27, 2007.

[34] Beyond making ATMs practically feasible, the 2005 and 2008 SEC rules also relaxed restrictions on shelf offerings in general. Shelf offerings include shelf SEOs and ATM offerings. Our discussion here is around the effects of allowing ATMs alone.

[35] Our analysis assumes that all firms prefer issuing shares over doing nothing even under the worst market belief (see footnote 8). In general, allowing ATMs could also lead to additional share issuance and investment by firms who would otherwise forgo the investment opportunity.

8 Conclusion

We analyze issue and repurchase transactions side-by-side, under the assumption that firms have superior knowledge about their values, and can choose both transaction size and method. The comparison of issues and repurchases is new to the literature, and yields fresh insights. First: Despite the conceptual symmetry between issue and repurchase transactions, their equilibrium outcomes aren't mirror images of each other. In particular, repurchasing firms do not signal via the efficiency of transaction methods while issuing firms do. Second, and in contrast, reducing repurchase size is a viable signal for repurchasing firms, just as reducing issue size is a viable signal for issuing firms. These implications fit well with empirical evidence. Third, and more conceptually, our analysis isolates a precise formal role for total firm value in equity transactions. Fourth, managers' short v.s. long-term objectives lead to similar outcomes for share issues but qualitatively different outcomes for repurchases; specifically, short-term oriented firms engage in excess repurchases. Finally, a combination of observed empirical regularities with the predictions emerging from extensions of our baseline model suggests that a significant fraction of firms' private information is about their assets-in-place.

While our model is constructed to cover many aspects of firms' issue and repurchase decisions (in particular, we make only mild assumptions on the surplus function S, and the efficiency choice θ can be interpreted in a wide range of ways), it nonetheless omits important topics that would be interesting for future research. One promising such avenue is to consider a firm's choice of cash holdings, which is currently an (implicit) fixed parameter in our analysis. Greater cash holdings reduce inefficiencies for firms with investment opportunities (issuing firms), but increase inefficiencies for firms without investment opportunities (repurchasing firms) because of the costs associated with holding cash. An interesting extension to our current analysis would be to consider how firms trade off these two inefficiencies if they do not know whether or not they will have an investment opportunity in the future.

Another interesting direction to consider is firms' strategic disclosure of information. In this paper, we focus on equity transactions and abstract away from firms' direct information disclosures. For instance, firms may strategically disclose good (bad) information before issuing (repurchasing) shares. How do these activities interact with firms' choices about transaction size and method? We look forward to future work that can speak to these questions.

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Appendix

A Details for the repurchase microfoundation in Section 4

We assume that holding cash inside the firm is costly and generates a negative rate of return $-\beta < 0$ (relative to investors' opportunity cost of funds, which is normalized to 0). Hence if no payouts are made over an arbitrary time interval $[0, t]$, these cash flows accumulate to a date-t value of

$$\int_0^t \lambda e^{-\beta(t-s)} ds = \frac{\lambda}{\beta}\left(1 - e^{-\beta t}\right).$$

Under payout policy $(|i|, n)$, the firm's date-T cash balance is[36]

$$\sum_{m=1}^{n}\left(\frac{\lambda}{\beta}\left(1 - e^{-\beta \frac{T}{n}}\right) - \frac{|i|}{n} - \kappa\left(\frac{|i|}{n}\right)^{\gamma}\right) e^{-\beta\left(T - m\frac{T}{n}\right)}. \tag{A1}$$

Let a_{IL} denote the firm's illiquid assets-in-place, which do not incur the negative return $-\beta$. The firm knows the value of a_{IL} but investors don't. By straightforward evaluation of (A1),[37] the firm's value under the above payout policy is

$$V = a_{IL} + \frac{\lambda}{\beta}\left(1 - e^{-\beta T}\right) - \left(\frac{|i|}{n} + \kappa\left(\frac{|i|}{n}\right)^{\gamma}\right)\frac{1 - e^{-\beta T}}{1 - e^{-\beta \frac{T}{n}}}. \tag{A2}$$

Defining

$$a = a_{IL} + \frac{\lambda}{\beta}\left(1 - e^{-\beta T}\right), \tag{A3}$$

$$\theta(n) = \frac{1 - \frac{1}{n}}{1 - \frac{1}{N}}, \tag{A4}$$

and

$$S = |i| - \left(\frac{|i|}{n} + \kappa\left(\frac{|i|}{n}\right)^{\gamma}\right)\frac{1 - e^{-\beta T}}{1 - e^{-\beta \frac{T}{n}}}, \tag{A5}$$

firm value (3) maps to our general specification of firm value (3). θ increases monotonically from 0 to 1 as n increases from 1 to N. As such, smoother repurchases correspond to higher efficiency levels θ.

We verify that (A2) satisfies Assumption 1. To show $S_\theta > 0$, it suffices to show that $\frac{\partial S}{\partial\left(\frac{1}{n}\right)} < 0$.

[36] The cash flows between dates $(m-1)\frac{T}{n}$ and $m\frac{T}{n}$ accumulate to $\frac{\lambda}{\beta}\left(1 - e^{-\beta\frac{T}{n}}\right)$ at date $m\frac{T}{n}$, and become $\frac{\lambda}{\beta}\left(1 - e^{-\beta\frac{T}{n}}\right) - \frac{|i|}{n} - \kappa\left(\frac{|i|}{n}\right)^{\gamma}$ after divesting and paying out $\frac{|i|}{n}$ at date $m\frac{T}{n}$. This amount of cash kept in the firm becomes $\left(\frac{\lambda}{\beta}\left(1 - e^{-\beta\frac{T}{n}}\right) - \frac{|i|}{n} - \kappa\left(\frac{|i|}{n}\right)^{\gamma}\right) e^{-\beta\left(T - m\frac{T}{n}\right)}$ at date T.

[37] The evaluation uses the fact that $\sum_{m=1}^{n} e^{-\beta\left(T - m\frac{T}{n}\right)} = \frac{1 - e^{-\beta T}}{1 - e^{-\beta\frac{T}{n}}}$.

Differentiating,

$$\frac{\partial S}{\partial\left(\frac{1}{n}\right)}=-\left[|i|+\gamma\kappa\left(\frac{1}{n}\right)^{\gamma-1}|i|^{\gamma}-\frac{\left(\frac{|i|}{n}+\kappa\left(\frac{|i|}{n}\right)^{\gamma}\right)\beta T e^{-\beta\frac{T}{n}}}{1-e^{-\beta\frac{T}{n}}}\right]\frac{1-e^{-\beta T}}{1-e^{-\beta\frac{T}{n}}}.$$

Since $\gamma>1$, it suffices to show that

$$|i|+\kappa\left(\frac{1}{n}\right)^{\gamma-1}|i|^{\gamma}>\frac{\left(\frac{|i|}{n}+\kappa\left(\frac{|i|}{n}\right)^{\gamma}\right)\beta T e^{-\beta\frac{T}{n}}}{1-e^{-\beta\frac{T}{n}}},$$

i.e.,

$$1>\frac{\beta\frac{T}{n}e^{-\beta\frac{T}{n}}}{1-e^{-\beta\frac{T}{n}}},$$

i.e.,

$$e^{\beta\frac{T}{n}}>1+\beta\frac{T}{n},$$

which holds because $\beta\frac{T}{n}>0$.

It is immediate that V is decreasing and concave in $|i|$, and hence increasing and concave in i. Concavity of V in i implies S is single-peaked in i.

To show $I^*<0$, it suffices to show $S_{|i|}(0,1)>0$, where $S_{|i|}$ denote the partial derivative of S with respect to $|i|$. Note

$$S_{|i|}=1-\left(\frac{1}{n}+\gamma\kappa\left(\frac{1}{n}\right)^{\gamma}|i|^{\gamma-1}\right)\frac{1-e^{-\beta T}}{1-e^{-\beta\frac{T}{n}}},$$

and hence

$$S_{|i|}(0,1)=1-\frac{1}{N}\cdot\frac{1-e^{-\beta T}}{1-e^{-\beta\frac{T}{N}}}.$$

It suffices to show

$$1-e^{-\beta\frac{T}{N}}>\frac{1}{N}\left(1-e^{-\beta T}\right).$$

This holds with equality for $\frac{1}{N}=0,1$. The LHS is strictly concave in $\frac{1}{N}$, and the RHS is linear in $\frac{1}{N}$. Hence, the inequality holds given $\frac{1}{N}\in(0,1)$.

B Proof for Section 3

Lemma A1. *In the issue and repurchase games:*

1. If

$$i_1+S(i_1,\theta_1)<i_2+S(i_2,\theta_2), \tag{A6}$$

then

(a) $\Pi(a,i_1,\theta_1,p_1)\geq\Pi(a,i_2,\theta_2,p_2)$ implies $\Pi(a',i_1,\theta_1,p_1)>\Pi(a',i_2,\theta_2,p_2)$ for $a'>a$;

i. $\Pi(a, i_1, \theta_1, p_1) \leq \Pi(a, i_2, \theta_2, p_2)$ implies $\Pi(a', i_1, \theta_1, p_1) < \Pi(a', i_2, \theta_2, p_2)$ for $a' < a$.

(b) If

$$i_1 + S(i_1, \theta_1) = i_2 + S(i_2, \theta_2), \tag{A7}$$

then for any types a and a', $\Pi(a, i_1, \theta_1, p_1) \geq \Pi(a, i_2, \theta_2, p_2)$ if and only if $\Pi(a', i_1, \theta_1, p_1) \geq \Pi(a', i_2, \theta_2, p_2)$.

Proof. (5) implies

$$\ln \Pi(a, i_1, \theta_1, p_1) - \ln \Pi(a, i_2, \theta_2, p_2) = \ln \frac{V(a, i_1, \theta_1)}{V(a, i_2, \theta_2)} - \ln \frac{1 + \frac{i_1}{p_1}}{1 + \frac{i_2}{p_2}}. \tag{A8}$$

(A6) implies $\frac{V(a,i_1,\theta_1)}{V(a,i_2,\theta_2)} \in (0, 1)$ strictly increases in a. Therefore,

$$\ln \Pi(a, i_1, \theta_1, p_1) - \ln \Pi(a, i_2, \theta_2, p_2) \geq 0$$

implies

$$\ln \Pi(a', i_1, \theta_1, p_1) - \ln \Pi(a', i_2, \theta_2, p_2) > 0$$

for $a' > a$. This proves part 1a. Part 1(a)i is then implied by part 1a.

(A7) implies

$$\begin{aligned} &\ln \Pi(a, i_1, \theta_1, p_1) - \ln \Pi(a, i_2, \theta_2, p_2) \\ &= 1 - \ln \frac{1 + \frac{i_1}{p_1}}{1 + \frac{i_2}{p_2}} \end{aligned}$$

is independent of a. This implies part 1b. □

We establish that it is without loss of generality to consider $i \in [I_L, \infty)$ for the issue game and $i \in (\infty, I_L]$ for the repurchase game and ignore firms' option of doing nothing ($i = 0$):

Lemma A2. *In the issue game and the repurchase game, under the assumptions in Footnote 8, if $|I_L| > 0$, all firms transact $|i| \geq |I_L|$ even if they can choose to do nothing ($i = 0$). Conversely, a firm does not benefit from deviating to doing nothing ($i = 0$) as long as it does not benefit from deviating to $(I_L, 1)$.*

Proof. It suffices to prove that when $|I_L| > 0$, all firms prefer the transaction $(I_L, 1)$ under any price to doing nothing.

In the issue game, since $\Pi(a, I_L, 1, p)$ is strictly increasing in p, by issuing I_L with method $\theta = 1$, type $a_{\max}$ has payoff not lower than

$$\Pi(a_{\max}, I_L, 1, a_{\min} + S(I_L, 1)) = \frac{V(a_{\max}, I_L, 1)}{1 + \frac{I_L}{a_{\min} + S(I_L, 1)}}.$$

By (1), this is higher than its payoff from doing nothing. By Assumption 1, part 3,

$$I_L + S(I_L, 1) > S(0, 1).$$

By Lemma A1, all types prefer $(I_L, 1)$ to doing nothing.

In the repurchase game, since $\Pi(a, I_L, 1, p)$ is strictly decreasing in p, by repurchasing I_L with method $\theta = 1$, type $a_{\min}$ has payoff not lower than

$$\Pi(a_{\min}, I_L, 1, a_{\max} + S(I_L, 1)) = \frac{V(a_{\min}, I_L, 1)}{1 + \frac{I_L}{a_{\max}+S(I_L,1)}}.$$

By (2), this is higher than its payoff from doing nothing. By Assumption 1, part 3,

$$I_L + S(I_L, 1) < S(0, 1).$$

By Lemma A1, all types prefer $(I_L, 1)$ to doing nothing. □

Proof of Proposition 1:

Suppose transactions (i, θ) and $\left(i, \tilde{\theta}\right)$ with $\tilde{\theta} < \theta$ are adopted by nonempty sets of firms A and $\tilde{A}$, respectively. Let $\bar{a} = E[a|a \in A]$ and $\tilde{a} = E\left[a|a \in \tilde{A}\right]$. Then $P(i, \theta) = \bar{a} + S(i, \theta)$ and $P\left(i, \tilde{\theta}\right) = \tilde{a} + S\left(i, \tilde{\theta}\right)$.

Since $\tilde{A}$ firms prefer $\left(i, \tilde{\theta}\right)$ over (i, θ), by Lemma A1, firms $a > \inf \tilde{A}$ have the same preference strictly. Hence, $\sup A \leq \inf \tilde{A}$, and consequently, $\bar{a} \leq \tilde{a}$. However, this implies type $\tilde{a}$ strictly prefers (i, θ) to $\left(i, \tilde{\theta}\right)$:

$$\begin{aligned}\Pi(\tilde{a}, i, \theta, P(i, \theta)) &> \Pi(\tilde{a}, i, \theta, \tilde{a} + S(i, \theta)) \\ &= \tilde{a} + S(i, \theta) \\ &> \tilde{a} + S\left(i, \tilde{\theta}\right) \\ &= \Pi\left(\tilde{a}, i, \tilde{\theta}, P\left(i, \tilde{\theta}\right)\right).\end{aligned}$$

The first inequality is due to that $\Pi(a, i, \theta, p)$ is decreasing in p in the repurchase game. By Lemma A1, there is a type in $\tilde{A}$ that strictly prefers (i, θ) to $\left(i, \tilde{\theta}\right)$, leading to a contradiction. This completes the proof.

Lemma A3. *In the repurchase game, there is $a_0 < a_{\max}$ such that conditions (9), (10) and (11) determine a unique $\hat{i}(a) \in [I^*, I_L]$ for each $a \in [a_0, a_{\max}]$, with a_0 satisfying $a_0 = a_{\min}$ or $\hat{i}(a_0) = I_L$.*

Proof. Consider ODE

$$\frac{\partial a(i)}{\partial i} = -\frac{V(a(i), i, 1) S_i(i, 1)}{i} \tag{A9}$$

for $i \in [I^*, I_L]$ with boundary condition

$$a\left(I^*\right) = a_{\max}. \tag{A10}$$

By Assumption 1, part 2, $\frac{\partial a(i)}{\partial i} = 0$ for $i = I^*$ and $\frac{\partial a(i)}{\partial i} < 0$ for $i > I^*$. Hence, (A9) and (A10) determine a unique function $a(i)$ on $[I^*, i_0]$ for some $i_0 \in (I^*, I_L]$, with i_0 satisfying either $a(i_0) = a_{\min}$ or $i_0 = I_L$. By construction, $a(i)$ is strictly decreasing.

Define $a_0 = a(i_0)$. By the Inverse Function Theorem, the inverse function of $a(\cdot)$, denoted by $\hat{\imath}(\cdot)$, is the unique solution to (9), (10) and (11) on $[a_0, a_{\max}]$. By construction, $\hat{\imath}(a_0) = i_0$. If $a_0 > a_{\min}$, then $\hat{\imath}(a_0) = I_L$. □

Lemma A4. *In the issue game and the repurchase game, an interval of firm types A do not benefit from mimicking each other if all the following conditions are satisfied:*

- $i(a)$ is continuous on A;
  - $\frac{\partial i(a)}{\partial a} \leq 0$ for $a \in A$ except for countable points;
  - For a in the interior of A,

$$\frac{dS\left(i\left(a\right), \theta\left(a\right)\right)}{da} = -\frac{i\left(a\right)}{V\left(a, i\left(a\right), \theta\left(a\right)\right)}; \tag{A11}$$

  - If this is a repurchase game, $\theta(a)$ is a constant on A;
  - For $a \in A$, $P(i(a), \theta(a)) = a + S(i(a), \theta(a))$.

Proof. Let $\theta(i, s)$ denotes the θ that satisfies $S(i, \theta) = s$, and consider

$$\begin{aligned}\tilde{\pi}\left(a, i, s, \tilde{a}\right) &= \ln \Pi\left(a, i, \theta\left(i, s\right), \tilde{a} + s\right), \\ &= \ln\left(\tilde{a} + s\right) + \ln\left(a + i + s\right) - \ln\left(\tilde{a} + i + s\right).\end{aligned}$$

Then the partial derivatives of $\tilde{\pi}$ satisfy

$$\tilde{\pi}_i\left(a, i, s, \tilde{a}\right) = \frac{1}{a + i + s} - \frac{1}{\tilde{a} + i + s},$$

$$\tilde{\pi}_s\left(a, i, s, \tilde{a}\right) = \frac{1}{\tilde{a} + s} + \frac{1}{a + i + s} - \frac{1}{\tilde{a} + i + s},$$

and

$$\tilde{\pi}_{\tilde{a}}\left(a, i, s, \tilde{a}\right) = \frac{1}{\tilde{a} + s} - \frac{1}{\tilde{a} + i + s}.$$

Suppose interval A satisfies the conditions in the Lemma. Consider the payoff of a type-a firm from mimicking type $\tilde{a} \in A$: $\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a})$. For $\tilde{a} \in A$ except for countable points,

$$\begin{aligned} &\frac{d}{d\tilde{a}}\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a}) \\ &= \tilde{\pi}_i \cdot \frac{\partial i(\tilde{a})}{\partial \tilde{a}} + \tilde{\pi}_s \cdot \frac{dS(i(\tilde{a}), \theta(\tilde{a}))}{d\tilde{a}} + \tilde{\pi}_{\tilde{a}}, \end{aligned} \tag{A12}$$

and by Condition (A11),

$$\tfrac{d}{d\tilde{a}}\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a})|_{a=\tilde{a}} \quad = 0. \tag{A13}$$

Note that in (A12), $\tilde{\pi}_i$ and $\tilde{\pi}_s$ are strictly decreasing in a, and $\tilde{\pi}_{\tilde{a}}$ is invariant to a. □

- In the issue game, according to (A11), for $\tilde{a}$ in the interior of A, $\frac{dS(i(\tilde{a}),\theta(\tilde{a}))}{d\tilde{a}} < 0$. For $\tilde{a}$ with $\frac{\partial i(\tilde{a})}{\partial \tilde{a}} \leq 0$,

  $$\frac{d}{d\tilde{a}}\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a})$$

  strictly increases with a. Hence, by (A13), for each $a \in A$, for $\tilde{a} \gtreqless a$ in A except for countable points,

  $$\frac{d}{d\tilde{a}}\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a}) \lesseqgtr 0.$$

  This implies for all $a, \tilde{a} \in A$ with $a \neq \tilde{a}$,

  $$\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a}) < \tilde{\pi}(a, i(a), S(i(a), \theta(a)), a).$$

  Hence, firm $a \in A$ does not benefit from mimicking any firm $\tilde{a} \in A$.

  - In the repurchase game, suppose $\theta(a)$ is a constant θ for $a \in A$. For $\tilde{a} \in A$ with $\frac{\partial i(\tilde{a})}{\partial \tilde{a}} \leq 0$,

    $$\frac{dS(i(\tilde{a}), \theta(\tilde{a}))}{d\tilde{a}} = S_i(i(\tilde{a}), \theta)\frac{\partial i(\tilde{a})}{\partial \tilde{a}}.$$

    By (A12),

    $$\begin{aligned} &\frac{d}{d\tilde{a}}\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a}) \\ &= \frac{1 + S_i(i(\tilde{a}), \theta)}{a + i(\tilde{a}) + S(i(\tilde{a}), \theta)}\frac{\partial i(\tilde{a})}{\partial \tilde{a}} + C, \end{aligned} \tag{A14}$$

    where C is independent of a. By Assumption 1, $1 + S_i(i(\tilde{a}), \theta) > 0$. Hence,

    $$\frac{d}{d\tilde{a}}\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a})$$

    is non-decreasing in a. By (A13), for each $a \in A$, for $\tilde{a} > (<)\, a$ in A except for countable points,

    $$\frac{d}{d\tilde{a}}\tilde{\pi}(a, i(\tilde{a}), S(i(\tilde{a}), \theta(\tilde{a})), \tilde{a}) \leq (\geq)\, 0.$$

This implies for all $a, \tilde{a} \in A$,

$$\tilde{\pi}\left(a, i\left(\tilde{a}\right), S\left(i\left(\tilde{a}\right), \theta\left(\tilde{a}\right)\right), \tilde{a}\right) \leq \tilde{\pi}\left(a, i\left(a\right), S\left(i\left(a\right), \theta\left(a\right)\right), a\right).$$

Hence, firm $a \in A$ does not benefit from mimicking any firm $\tilde{a} \in A$.

Lemma A5. *In an equilibrium of the issue or repurchase game, the price of an off-equilibrium choice* (i, θ) *satisfies D1 if and only if both the following conditions are met:*

1. For any firm type a whose equilibrium choice $\left(i\left(a\right), \theta\left(a\right)\right)$ satisfies

$$i + S\left(i, \theta\right) < i\left(a\right) + S\left(i\left(a\right), \theta\left(a\right)\right), \tag{A15}$$

   $P\left(i, \theta\right)$ satisfies

$$P\left(i, \theta\right) \geq a + S\left(i, \theta\right); \tag{A16}$$

   (a) For any firm type a whose equilibrium choice $\left(i\left(a\right), \theta\left(a\right)\right)$ satisfies

$$i + S\left(i, \theta\right) > i\left(a\right) + S\left(i\left(a\right), \theta\left(a\right)\right), \tag{A17}$$

   $P\left(i, \theta\right)$ satisfies

$$P\left(i, \theta\right) \leq a + S\left(i, \theta\right). \tag{A18}$$

Proof. We first show the "only if" part.

Fix an off-equilibrium choice (i, θ) and a type a whose equilibrium choice $\left(i\left(a\right), \theta\left(a\right)\right)$ satisfies (A15). We prove (A16) by showing that any type $\tilde{a} < a$ cannot be associated with (i, θ) under D1. Let p be a price such that

$$\Pi\left(\tilde{a}, i, \theta, p\right) \geq \Pi^*\left(\tilde{a}\right).$$

In equilibrium, type $\tilde{a}$ does not benefit from mimicking type a, implying

$$\Pi^*\left(\tilde{a}\right) \geq \Pi\left(\tilde{a}, i\left(a\right), \theta\left(a\right), P\left(i\left(a\right), \theta\left(a\right)\right)\right).$$

This implies

$$\Pi\left(\tilde{a}, i, \theta, p\right) \geq \Pi\left(\tilde{a}, i\left(a\right), \theta\left(a\right), P\left(i\left(a\right), \theta\left(a\right)\right)\right).$$

By (A15) and Lemma A1, type a strictly prefers to deviate to (i, θ) at price p:

$$\Pi\left(a, i, \theta, p\right) > \Pi\left(a, i\left(a\right), \theta\left(a\right), P\left(i\left(a\right), \theta\left(a\right)\right)\right) = \Pi^*\left(a\right).$$

This implies $D_{\tilde{a}}\left(i, \theta\right) \subsetneq D_a\left(i, \theta\right)$. Under D1, (i, θ) cannot be associated with type $\tilde{a}$.

The other case (A17) is similar: a similar argument to the above yields $D_{\tilde{a}}\left(i, \theta\right) \subsetneq D_a\left(i, \theta\right)$ for $\tilde{a} > a$, implying (i, θ) cannot be associated with $\tilde{a} > a$, and hence (A18).

Next, we prove the "if" part.

Fix an off-equilibrium (i, θ), and fix an equilibrium in which (A15) implies (A16) and (A17) implies (A18). Let a^* be such that

$$P(i, \theta) = a^* + S(i, \theta).$$

Then (A15) implies $a \leq a^*$ and (A17) implies $a \geq a^*$. We show that the price $P(i, \theta)$ satisfies D1. It suffices to show that $D_{\tilde{a}}(i, \theta) \subset D_{a^*}(i, \theta)$ for all $\tilde{a} \neq a^*$, which is equivalent to that for any $\tilde{a} \neq a^*$ and any p,

$$\Pi(\tilde{a}, i, \theta, p) \geq \Pi^*(\tilde{a}) \tag{A19}$$

implies

$$\Pi(a^*, i, \theta, p) \geq \Pi^*(a^*). \tag{A20}$$

□

- Suppose (A19) holds for some $\tilde{a} < a^*$ and p. For any $a \in (\tilde{a}, a^*)$, by hypothesis,

$$i + S(i, \theta) \leq i(a) + S(i(a), \theta(a)). \tag{A21}$$

  Since type $\tilde{a}$ does not mimic type a in equilibrium, by (A19), type $\tilde{a}$ prefers (i, θ) at price p to mimicking type a:

$$\Pi(\tilde{a}, i, \theta, p) \geq \Pi(\tilde{a}, i(a), \theta(a), P(i(a), \theta(a))).$$

  By (A21) and Lemma A1, type a also prefers (i, θ) at price p to its equilibrium choice:

$$\Pi(a, i, \theta, p) \geq \Pi^*(a). \tag{A22}$$

  Suppose (A20) is violated. Then

$$\Pi^*(a^*) > \Pi(a^*, i, \theta, p).$$

  Since $\Pi(a, i(a^*), \theta(a^*), P(i(a^*), \theta(a^*)))$ and $\Pi(a, i, \theta, p)$ are continuous in a, there is $a \in (\tilde{a}, a^*)$ with

$$\Pi(a, i(a^*), \theta(a^*), P(i(a^*), \theta(a^*))) > \Pi(a, i, \theta, p).$$

  By (A22), type a benefits from mimicking type a^*, leading to a contradiction. This shows that (A20) must hold.

  - Suppose (A19) holds for some $\tilde{a} > a^*$ and p. For any $a \in (a^*, \tilde{a})$, by hypothesis,

$$i + S(i, \theta) \geq i(a) + S(i(a), \theta(a)). \tag{A23}$$

    Since type $\tilde{a}$ does not mimic type a, in equilibrium, by (A19), type $\tilde{a}$ prefers (i, θ) at price p to mimicking type a:

$$\Pi\left(\tilde{a}, i, \theta, p\right) \geq \Pi\left(\tilde{a}, i\left(a\right), \theta\left(a\right), P\left(i\left(a\right), \theta\left(a\right)\right)\right).$$

By (A23) and Lemma A1, type a also prefers (i, θ) at price p to its equilibrium choice:

$$\Pi\left(a, i, \theta, p\right) \geq \Pi^{*}\left(a\right). \tag{A24}$$

Suppose (A20) is violated. Then

$$\Pi^{*}\left(a^{*}\right) > \Pi\left(a^{*}, i, \theta, p\right).$$

Since $\Pi\left(a, i\left(a^{*}\right), \theta\left(a^{*}\right), P\left(i\left(a^{*}\right), \theta\left(a^{*}\right)\right)\right)$ and $\Pi\left(a, i, \theta, p\right)$ are continuous in a, there is $a \in \left(a^{*}, \tilde{a}\right)$ with

$$\Pi\left(a, i\left(a^{*}\right), \theta\left(a^{*}\right), P\left(i\left(a^{*}\right), \theta\left(a^{*}\right)\right)\right) > \Pi\left(a, i, \theta, p\right).$$

By (A24), type a benefits from mimicking type a^{*}, leading to a contradiction. This shows that (A20) must hold.

Proof of Proposition 2:

We prove a stronger version of this proposition:

Proposition A1. *In the repurchase game, there exists a D1 equilibrium in which firms follow the strategy characterized in Proposition 2, and off-equilibrium prices are as follows: For off-equilibrium (i, θ) such that there exists $a \in \left(\hat{a}, a_{\max}\right)$ with*

$$i + S\left(i, \theta\right) = \hat{i}\left(a\right) + S\left(\hat{i}\left(a\right), 1\right), \tag{A25}$$

price is based on type a:

$$P\left(i, \theta\right) = a + S\left(i, \theta\right). \tag{A26}$$

For off-equilibrium (i, θ) such that

$$i + S\left(i, \theta\right) \geq \hat{i}\left(\hat{a}\right) + S\left(\hat{i}\left(\hat{a}\right), 1\right), \tag{A27}$$

price is based on type $\hat{a}$:

$$P\left(i, \theta\right) = \hat{a} + S\left(i, \theta\right). \tag{A28}$$

For off-equilibrium (i, θ) with

$$i + S\left(i, \theta\right) \leq I^{*} + S\left(I^{*}, 1\right), \tag{A29}$$

price is based on type $a_{\max}$:

$$P\left(i, \theta\right) = a_{\max} + S\left(i, \theta\right). \tag{A30}$$

Proof. By Lemma A3, the firm strategy is well defined.

Step 1. We show that no firm mimics another type.

It follows Lemma A4 that types $a > \hat{a}$ do not mimic each other.

If $\hat{a} > a_{\min}$, by construction, (14) holds with $\leq$. This implies type $\hat{a}$ weakly prefers $(I_L, 1)$ over $\left(\hat{i}(\hat{a}), 1\right)$, and by Lemma A4, also over $\left(\hat{i}(a), 1\right)$ for any $a \geq \hat{a}$. Lemma A1 hence implies types with $a < \hat{a}$ have the same preference and do not mimic $a \geq \hat{a}$.

If $\hat{a} < a_{\max}$, by construction, (14) holds with $\geq$. In particular, if $\hat{a} = a_{\min}$, then by Assumption 1, part 2,

$$S(I_L, 1) < S\left(\hat{i}(a_{\min}), 1\right),$$

and hence

$$\begin{aligned} a_{\min} + S\left(\hat{i}(a_{\min}), 1\right) &> a_{\min} + S(I_L, 1) \\ &= \Pi(a_{\min}, I_L, 1, a_{\min} + S(I_L, 1)). \end{aligned}$$

That (14) holds with $\geq$ implies type $\hat{a}$ weakly prefers $\left(\hat{i}(\hat{a}), 1\right)$ over $(I_L, 1)$. By Lemma A1, types with $a > \hat{a}$ has the same preference. By Lemma A4, types with $a > \hat{a}$ do not benefit from deviating to $\left(\hat{i}(\hat{a}), 1\right)$, and hence do not deviate to $(I_L, 1)$ mimicking $a < \hat{a}$.

Step 2. We show that no firm deviates to off-equilibrium actions. □

- Consider an off-equilibrium action (i, θ) such that there is $a \in (\hat{a}, a_{\max})$ that satisfies (A25). Given (A26), type a is fairly priced under both (i, θ) and its equilibrium choice $\left(\hat{i}(a), 1\right)$. Suppose $S(i, \theta) > S\left(\hat{i}(a), 1\right)$, then by (A25), $i < \hat{i}(a)$, and by Assumption 1, part 2, $S(i, \theta) < S\left(\hat{i}(a), 1\right)$, leading to a contradiction. Hence, $S(i, \theta) \leq S\left(\hat{i}(a), 1\right)$. Then type a prefers its equilibrium choice $\left(\hat{i}(a), 1\right)$ to (i, θ):

  $$\begin{aligned} \Pi\left(a, \hat{i}(a), 1, P\left(\hat{i}(a), 1\right)\right) &= a + S\left(\hat{i}(a), 1\right) \\ &\geq a + S(i, \theta) \\ &= \Pi(a, i, \theta, P(i, \theta)). \end{aligned}$$

  By Lemma A1, all types have the same preference. Since no type benefits from mimicking type a, no type benefits from deviating to (i, θ).

  - Consider an off-equilibrium action (i, θ) that satisfies (A27). Given (A28), type $\hat{a}$ is fairly priced under both (i, θ) and $\left(\hat{i}(\hat{a}), 1\right)$. By (A27),

    $$i + S(i, 1) \geq \hat{i}(\hat{a}) + S\left(\hat{i}(\hat{a}), 1\right),$$

and by Assumption 1, part 3, $i \geq \hat{i}(\hat{a})$. By Assumption 1, part 2, $S(i,\theta) \leq S\left(\hat{i}(\hat{a}),1\right)$. Hence, type $\hat{a}$ prefers $\left(\hat{i}(\hat{a}),1\right)$ to (i,θ):

$$\begin{aligned}\Pi\left(\hat{a},\hat{i}(\hat{a}),1,P\left(\hat{i}(\hat{a}),1\right)\right) &= \hat{a}+S\left(\hat{i}(\hat{a}),1\right)\\ &\geq \hat{a}+S(i,\theta)\\ &= \Pi(\hat{a},i,\theta,P(i,\theta)).\end{aligned}$$

By Lemma A1, types $a \geq \hat{a}$ have the same preference. Since no type benefits from mimicking type $\hat{a}$ by choosing $\left(\hat{i}(\hat{a}),1\right)$, types $a \geq \hat{a}$ do not benefit from deviating to (i,θ).

If $\hat{a} > a_{\min}$, then by construction, (14) holds with $\leq$, implying type $\hat{a}$ weakly prefers $(I_L,1)$ to $\left(\hat{i}(\hat{a}),1\right)$. Hence, $\hat{a}$ prefers $(I_L,1)$ to (i,θ). By Assumption 1, parts 2 and 3,

$$I_L+S(I_L,1)\geq i+S(i,\theta).$$

By Lemma A1, types $a<\hat{a}$ also weakly prefer $(I_L,1)$ to (i,θ), and hence do not benefit from deviating to (i,θ).

- Consider an off-equilibrium action (i,θ) that satisfies (A29). Given (A30), type $a_{\max}$ is fairly priced under both (i,θ) and $(I^*,1)$. By Assumption 1, $S(i,\theta)\leq S(I^*,1)$. Hence, type $a_{\max}$ prefers $(I^*,1)$ to (i,θ):

$$\begin{aligned}\Pi(a_{\max},I^*,1,P(I^*,1)) &= a_{\max}+S(I^*,1)\\ &\geq a_{\max}+S(i,\theta)\\ &= \Pi(a_{\max},i,\theta,P(i,\theta)).\end{aligned}$$

  * Consider the case where type $a_{\max}$ chooses $(I^*,1)$ in equilibrium. By Lemma A1, all types prefer $(I^*,1)$ to (i,θ). Since no type benefits from mimicking type $a_{\max}$, no type benefits from deviating to (i,θ).
  * Consider the case where type $a_{\max}$ chooses $(I_L,1)$ in equilibrium. Then $\hat{a}=a_{\max}$, and by construction, (14) holds with $\leq$, implying type $a_{\max}$ weakly prefers $(I_L,1)$ to $(I^*,1)$, and hence to (i,θ). By Assumption 1, part 3,

$$I_L+S(I_L,1)>i+S(i,\theta).$$

    By Lemma A1, all types weakly prefer $(I_L,1)$ to (i,θ), and hence do not benefit from deviating to (i,θ).

Proof. Step 3. We show that the off-equilibrium prices satisfy D1.

In the firm strategy, $i(a)+S(i(a),\theta(a))$ is non-increasing in a. By Assumption 1, parts 2 and 3, for any (i, θ),

$$i + S(i, \theta) \leq I_L + S(I_L, 1).$$

It then follows that for each off-equilibrium (i, θ), (A15) implies (A16) and (A17) implies (A18). By Lemma A5, the prices satisfy D1. This completes the proof. □

We establish the following results to prepare for the proof of Proposition 3:

By Lemma A1 and Assumption 1, part 3, we have

Corollary 1. *In any equilibrium of the issue or repurchase game, $i(a) + S(i(a), \theta(a))$ is non-increasing in a. For (i, θ) chosen by some firms in equilibrium, the market belief, represented by $P(i, \theta) - S(i, \theta)$, is non-increasing in $i + S(i, \theta)$.*

Lemma A6. *In a D1 equilibrium of the repurchase game, all repurchasing firms choose the maximal efficiency $\theta = 1$.*

Proof. Suppose transaction (i, θ) with $\theta < 1$ is chosen by a non-empty set of firms A in equilibrium. Consider firm $a \in A$ with $a \leq E[a|a \in A]$. Since

$$\begin{aligned} P(i, \theta) &= E[a|a \in A] + S(i, \theta), \\ &\geq a + S(i, \theta), \end{aligned}$$

and since $\Pi(a, i, \theta, p)$ decreases in p in the repurchase game,

$$\begin{aligned} \Pi^*(a) &\leq \Pi(a, i, \theta, a + S(i, \theta)) \\ &= a + S(i, \theta). \end{aligned}$$

By Assumption 1, part 2, $S(i, \theta) < S(i, 1)$, and hence $i + S(i, \theta) < i + S(i, 1)$. By Lemma A5, under D1, the price of $(i, 1)$ satisfies

$$P(i, 1) \leq a + S(i, 1).$$

But under this price, type a benefits from deviating to $(i, 1)$:

$$\begin{aligned} \Pi(a, i, 1, P(i, 1)) &\geq \Pi(a, i, 1, a + S(i, 1)) \\ &= a + S(i, 1) \\ &> a + S(i, \theta) \\ &\geq \Pi^*(a), \end{aligned}$$

leading to a contradiction. □

Lemma A7. *In a D1 equilibrium of the repurchase game, there is a firm type $\hat{a} \in [a_{\min}, a_{\max}]$ such that firms $a > \hat{a}$ repurchase I_L and firms $a < \hat{a}$ separate on different sizes $i(a) < I_L$ with $i(a)$ continuous and strictly decreasing.*

Proof. By Lemma A6, all repurchasing firms use method $\theta = 1$ in a D1 equilibrium. By Corollary 1, there is $\hat{a} \in [a_{\min}, a_{\max}]$ such that firms $a < \hat{a}$ repurchase I_L and firms $a > \hat{a}$ repurchase $i(a) < I_L$ with $i(a)$ non-increasing.

Step 1. We show that for $a > \hat{a}$, $i(a)$ is strictly decreasing.

Suppose $i < I_L$ is chosen by an interval A of firm types and A is not a singleton. Then $P(i, 1) > \inf A + S(i, 1)$. Since $\Pi(\bar{a}, i, 1, p)$ strictly decreases in p, for $a \in A$,

$$\begin{aligned} \Pi^*(a) &= \Pi(a, i, 1, P(i, 1)) \\ &< \Pi(a, i, 1, \inf A + S(i, 1)). \end{aligned} \tag{A31}$$

For $i' > i$, by Assumption 1, part 3,

$$i' + S(i', 1) > i + S(i, 1).$$

By Lemma A12,

$$P(i', 1) \leq \inf A + S(i', 1).$$

Since $\Pi(a, i', 1, p)$ strictly decreases in p,

$$\Pi(a, i', 1, P(i', 1)) \geq \Pi(a, i', 1, \inf A + S(i', 1)).$$

Since

$$\lim_{i' \downarrow i} \Pi(a, i', 1, \inf A + S(i', 1)) = \Pi(a, i, 1, \inf A + S(i, 1)),$$

by (A31), there is $i' > i$ with

$$\Pi(a, i', 1, P(i', 1)) > \Pi^*(a).$$

This implies type a benefits from deviating to $(i', 1)$, leading to a contradiction.

Step 2. We show that for $a > \hat{a}$, $i(a)$ is continuous.

Since $i(a)$ is non-increasing, right and left limits exist, and it is sufficient to rule out jumps. Suppose there is $\tilde{a} > \hat{a}$ such that

$$\bar{i} \equiv \lim_{a \uparrow \tilde{a}} i(a) > \underline{i} \equiv \lim_{a \downarrow \tilde{a}} i(a).$$

Since $i(a)$ is strictly decreasing for $a > \hat{a}$, prices for these types are fully revealing, and hence $\Pi^*(a) = a + S(i(a), 1)$. This implies

$$\lim_{a \downarrow \tilde{a}} \Pi^*(a) = \tilde{a} + S(\underline{i}, 1), \tag{A32}$$

$$\lim_{a \uparrow \tilde{a}} \Pi^* (a) = \tilde{a} + S\left(\bar{i}, 1\right). \tag{A33}$$

By Lemma A12, for $i \in \left[\underline{i}, \bar{i}\right]$,

$$P(i, 1) = \tilde{a} + S(i, 1),$$

and hence

$$\begin{aligned} \lim_{a \to \tilde{a}} \Pi(a, i, 1, P(i, 1)) &= \Pi(\tilde{a}, i, 1, P(i, 1)) \\ &= \tilde{a} + S(i, 1). \end{aligned} \tag{A34}$$

Let $i^* = \arg\max_{i \in [\underline{i}, \bar{i}]} S(i, 1)$. Since $S(i, 1)$ is single-peaked, at least one of $S(\underline{i}, 1)$ and $S\left(\bar{i}, 1\right)$ is strictly smaller than $S(i^*, 1)$. If $S(\underline{i}, 1) < S(i^*, 1)$, by (A32) and (A34), there is $a > \tilde{a}$ with

$$\Pi^* (a) < \Pi(a, i^*, 1, P(i^*, 1)),$$

implying type a benefits from deviating to $(i^*, 1)$, leading to a contradiction. If $S\left(\bar{i}, 1\right) < S(i^*, 1)$, by (A32) and (A34), there is $a \in (\hat{a}, \tilde{a})$ with

$$\Pi^* (a) < \Pi(a, i^*, 1, P(i^*, 1)),$$

implying type a benefits from deviating to $(i^*, 1)$, leading to a contradiction. □

Lemma A8. *In a D1 equilibrium of the repurchase game, if firm $a_{\max}$ repurchases $i(a_{\max}) < I_L$, then $i(a_{\max}) = I^*$.*

Proof. Suppose firm $a_{\max}$ repurchases $i < I_L$ with $i \neq I^*$ using method θ. By Assumption 1, part 2, $S(i, \theta) < S(I^*, 1)$. By Lemma A7, type $a_{\max}$ is fairly priced and has equilibrium payoff

$$\Pi^* (a_{\max}) = a_{\max} + S(i, \theta).$$

Since $\Pi(a_{\max}, I^*, 1, p)$ strictly decreases in p in the repurchase game, if type $a_{\max}$ deviates to $(I^*, 1)$, it has payoff

$$\begin{aligned} \Pi(a_{\max}, I^*, 1, P(I^*, 1)) &\geq \Pi(a_{\max}, I^*, 1, a_{\max} + S(I^*, 1)) \\ &= a_{\max} + S(I^*, 1) \\ &> a_{\max} + S(i, \theta). \end{aligned}$$

Hence, type $a_{\max}$ benefits from deviating to $(I^*, 1)$, leading to a contradiction. □

Lemma A9. *In an equilibrium of the issue or repurchase game, if there is an interval of firm types A on which the size and method choices $i(a)$ and $\theta(a)$ are continuous, and the choices fully reveal the firm types, then (A11) holds on A.*

Proof. Suppose on an interval of firm types A, $i(a)$ and $\theta(a)$ are continuous, and prices are fully revealing. Then for $a \in A$,

$$\Pi^*(a) = P(i(a), \theta(a)) = a + S(i(a), \theta(a)).$$

Consider two firm types $a_1, a_2 \in A$ with $a_2 > a_1$. Let their equilibrium choices be denoted by (i_1, θ_1) and (i_2, θ_2). For $k = 1, 2$, let p_k denote $P(i_k, \theta_k)$, s_k denote $S(i_k, \theta_k)$, and v_k denote $V(a_k, i_k, \theta_k)$. Equilibrium choices imply

$$\Pi(a_1, i_2, \theta_2, p_2) \leq \Pi^*(a_1), \tag{A35}$$

$$\Pi(a_2, i_1, \theta_1, p_1) \leq \Pi^*(a_2). \tag{A36}$$

With (3) and (4), condition (A35) can be written as

$$\begin{aligned} & p_2(v_2 - a_2 + a_1) \leq p_1 v_2, \\ \iff \quad & (p_2 - p_1) v_2 \leq (a_2 - a_1) p_2, \\ \iff \quad & (s_2 - s_1) v_2 \leq (a_2 - a_1)(-i_2), \\ \iff \quad & \tfrac{s_2 - s_1}{a_2 - a_1} \leq \tfrac{-i_2}{v_2}, \\ \iff \quad & \tfrac{S(i_2,\theta_2) - S(i_1,\theta_1)}{a_2 - a_1} \leq \tfrac{-i_2}{V(a_2,i_2,\theta_2)}. \end{aligned} \tag{A37}$$

Similarly, condition (A36) yields

$$\frac{S(i_1, \theta_1) - S(i_2, \theta_2)}{a_1 - a_2} \geq \frac{-i_1}{V(a_1, i_1, \theta_1)}, \tag{A38}$$

and hence

$$\frac{-i_1}{V(a_1, i_1, \theta_1)} \leq \frac{S(i_2, \theta_2) - S(i_1, \theta_1)}{a_2 - a_1} \leq \frac{-i_2}{V(a_2, i_2, \theta_2)}. \tag{A39}$$

Since $i(a)$ and $\theta(a)$ are continuous on A, $V(a, i(a), \theta(a))$ is continuous on A. The squeeze theorem implies (A11). □

Proof of Proposition 3:

Proof. The existence of the D1 equilibrium is shown in Proposition 2. We establish the uniqueness here.

By Corollary 1, Lemma A6, A7, A8, and A9, in a D1 equilibrium, firms must follow the firm strategy characterized in Proposition 2 for some $\hat{a} \in [a_{\min}, a_{\max}]$.

By Assumption 1, parts 2 and 3, $i(a) + S(i(a), \theta(a))$ is strictly decreasing in a for $a < \hat{a}$, and for any $(i, \theta) \neq (I_L, 1)$,

$$i + S(i, \theta) < I_L + S(I_L, 1).$$

By Lemma A5, the prices characterized in Proposition A1 are the unique D1 prices.

We next show that $\hat{a}$ is unique. Let $\hat{\imath}(a)$ be determined by (9), (10), and (11), and define

$$\begin{aligned} g(\tilde{a}) = &\Pi(\tilde{a}, I_L, 1, E[a|a \leq \tilde{a}] + S(I_L, 1)) \\ &- \left[\tilde{a} + S\left(\hat{\imath}(\tilde{a}), 1\right)\right]. \end{aligned}$$

Step 1. We establish that $g(a_1) \leq 0$ implies $g(a_2) < 0$ for $a_2 > a_1$.

Suppose $g(a_1) \leq 0$, implying type a_1 weakly prefers $\left(\hat{\imath}(a_1), 1\right)$ at price $a_1 + S\left(\hat{\imath}(a_1), 1\right)$ to $(I_L, 1)$ at price $E[a|a \leq a_1] + S(I_L, 1)$. By Assumption 1, part 3,

$$\hat{\imath}(a_1) + S\left(\hat{\imath}(a_1), 1\right) \leq I_L + S(I_L, 1).$$

By Lemma A1, type $a_2 > a_1$ strictly prefers $\left(\hat{\imath}(a_1), 1\right)$ at price $a_1 + S\left(\hat{\imath}(a_1), 1\right)$ to $(I_L, 1)$ at price $E[a|a \leq a_1] + S(I_L, 1)$. By Lemma A4, type a_2 weakly prefers $\left(\hat{\imath}(a_2), 1\right)$ at price $a_2 + S\left(\hat{\imath}(a_2), 1\right)$ to $\left(\hat{\imath}(a_1), 1\right)$ at price $a_1 + S\left(\hat{\imath}(a_1), 1\right)$. Since $\Pi_p < 0$, for the repurchase $(I_L, 1)$, type a_2 strictly prefers price $E[a|a \leq a_1] + S(I_L, 1)$ to price $E[a|a \leq a_2] + S(I_L, 1)$. These imply type a_2 strictly prefers $\left(\hat{\imath}(a_2), 1\right)$ at price $a_2 + S\left(\hat{\imath}(a_2), 1\right)$ to $(I_L, 1)$ at price $E[a|a \leq a_2] + S(I_L, 1)$, and hence $g(a_2) < 0$.

Step 2. We show that if $\hat{\imath}(a_{\min}) \leq I_L$, then $\hat{a} = a_{\min}$.

If $\hat{\imath}(a_{\min}) \leq I_L$,

$$\begin{aligned} \Pi(a_{\min}, I_L, 1, a_{\min} + S(I_L, 1)) &= a_{\min} + S(I_L, 1) \\ &\leq a_{\min} + S\left(\hat{\imath}(a_{\min}), 1\right), \end{aligned}$$

and hence $g(a_{\min}) \leq 0$. Suppose $\hat{a} > a_{\min}$. Then $\hat{\imath}(\hat{a}) < I_L$, and by step 1, $g(\hat{a}) < 0$. By the unique D1 prices,

$$P(I^*, 1) = E[a|a \leq \hat{a}] + S(I_L, 1)$$

and

$$P\left(\hat{\imath}(\hat{a}), 1\right) = \hat{a} + S\left(\hat{\imath}(\hat{a}), 1\right).$$

That $g(\hat{a}) < 0$ implies

$$\Pi(\hat{a}, I_L, 1, P(I^*, 1)) < \Pi\left(\hat{a}, \hat{\imath}(\hat{a}), 1, P\left(\hat{\imath}(\hat{a}), 1\right)\right).$$

Since Π is continuous in a, there is $a < \hat{a}$ with

$$\Pi^*(a) = \Pi(a, I_L, 1, P(I^*, 1)) < \Pi\left(a, \hat{\imath}(\hat{a}), 1, P\left(\hat{\imath}(\hat{a}), 1\right)\right),$$

implying type a benefits from deviating to $\left(\hat{\imath}(\hat{a}), 1\right)$, leading to a contradiction.

Step 3. We show that if there is $a_0 > a_{\min}$ with $\hat{\imath}(a_0) = I_L$, then $g(a_0) > 0$. Either $g(a) > 0$ for all $a > a_0$, in which case $\hat{a} = a_{\max}$, or $\hat{a}$ is uniquely determined by $\hat{a} > a_0$ and $g(\hat{a}) = 0$.

Suppose there is $a_0 > a_{\min}$ with $\hat{\imath}(a_0) = I_L$. Then $\hat{\imath}(a)$ is only defined for $a \geq a_0$, and it follows that $\hat{a} \geq a_0$.

Since $\Pi_p < 0$ and $E[a|a < a_0] < a_0$, $g(a_0) > 0$. □

- Consider the case in which $g(a) > 0$ for all $a > a_0$. Suppose $\hat{a} < a_{\max}$. Then

  $$P(I_L, 1) = E[a|a \leq \hat{a}] + S(I_L, 1).$$

  Since $g(\hat{a}) > 0$,
  $$\Pi(\hat{a}, I_L, 1, P(I_L, 1)) > \hat{a} + S\left(\hat{\imath}(\hat{a}), 1\right).$$

  Since $\Pi(a, I_L, 1, P(I_L, 1))$ and $a + S\left(\hat{\imath}(a), 1\right)$ are continuous in a, there is $a > \hat{a}$ with

  $$\Pi(a, I_L, 1, P(I_L, 1)) > a + S\left(\hat{\imath}(a), 1\right) = \Pi^*(a),$$

  implying type a benefits from deviating to $(I_L, 1)$, leading to a contradiction. Hence, $\hat{a}$ must be $a_{\max}$.

  – Consider the case in which there is $a > a_0$ with $g(a) \leq 0$. Then by step 1, there is a unique type $\tilde{a} > a_0$ with $g(\tilde{a}) = 0$. Suppose $\hat{a} \in (a_0, \tilde{a})$. Then $g(\hat{a}) > 0$. As shown above, this implies there is $a > \hat{a}$ who benefits from deviating to $(I_L, 1)$, leading to a contradiction. Suppose $\hat{a} > \tilde{a}$. Then $g(\hat{a}) < 0$. As shown in step 2, this implies there is $a < \hat{a}$ who benefits from deviating to $\left(\hat{\imath}(\hat{a}), 1\right)$, leading to a contradiction. Hence, $\hat{a}$ must be $\tilde{a}$.

Proof. The above implies $\hat{a}$ is unique in each case, and completes the proof. □

Lemma A10. *In the issue game, there is $\hat{a} > a_{\min}$ such that conditions (11) and (15), and (16) determine a unique $\hat{\imath}(a) \in [I_L, I^*]$ for each $a \in [a_{\min}, \hat{a}]$, with $\hat{a}$ satisfying $\hat{a} = a_{\max}$ or $\hat{\imath}(\hat{a}) = I_L > 0$. If $\hat{a} < a_{\max}$, then there is a unique function $\hat{\theta}(a) \in (0, 1]$ for $a \in [\hat{a}, a_{\max}]$ that satisfy (18) and (19).*

Proof. Consider ODE (A9) for $i \in [I_L, I^*]$ with boundary condition

$$a(I^*) = a_{\min}. \tag{A40}$$

By Assumption 1, part 2, $\frac{\partial a(i)}{\partial i} = 0$ for $i = I^*$ and $\frac{\partial a(i)}{\partial i} < 0$ for $i < I^*$. Hence, (A9) and (A40) determine a unique function $a(i)$ on $[i_0, I^*]$ for some $i_0 \in [I_L, I^*)$, with i_0 satisfying either $i_0 > I_L$ and $a(i_0) = a_{\max}$, or $i_0 = I_L$. By construction, $a(i)$ is strictly decreasing.

Let $\hat{a} = a(i_0)$. By the Inverse Function Theorem, the inverse function of $a(\cdot)$, denoted by $\hat{i}(\cdot)$, is the unique solution to (11), (15), and (16) on $[a_{\min}, \hat{a}]$. By construction, $\hat{i}(\hat{a}) = i_0$. If $\hat{a} < a_{\max}$, then $\hat{i}(\hat{a}) = I_L$.

If $I_L = 0$, (11) is zero for $\hat{i}(a) = I_L = 0$, and hence the ODE never reaches I_L. In this case, $\hat{i}(a) > I_L$ for all a, and $\hat{a} = a_{\max}$.

By the above, if $\hat{a} < a_{\max}$, then $I_L > 0$, and (19) is strictly negative. Hence, (18) and (19) determines a unique function $\hat{\theta}(a)$ for $a \geq \hat{a}$. We argue that there is enough space on $\theta \in (0,1]$ for these types to separate by contradiction. If not, then there is type $a_0 > \hat{a}$ such that $\hat{\theta}(a_0) = 0$. By Assumption (1) in Footnote (8), type $a_{\max}$ strictly prefers issuing $(I_L, 1)$ at price $a_{\min} + S(I_L, 1)$ to doing nothing. By Lemma A1, type a_0 has the same preference:

$$\Pi(a_0, I_L, 1, a_{\min} + S(I_L, 1)) > \Pi(a_0, 0, 1, a_{\max}) \geq a_0.$$

By Lemma A4, type a_0 weakly prefers issuing I_L with efficiency $\hat{\theta}(a_0) = 0$ at price $a_0 + S(I_L, 0)$ to issuing I_L with efficiency $\hat{\theta}(\hat{a}) = 1$ at price $\hat{a} + S(I_L, 1)$:

$$\begin{aligned}\Pi(a_0, I_L, 0, a_0 + S(I_L, 0)) &= a_0 + S(I_L, 0) \\ &\geq \Pi(a_0, I_L, 1, \hat{a} + S(I_L, 1)) \\ &> \Pi(a_0, I_L, 1, a_{\min} + S(I_L, 1)).\end{aligned}$$

The last inequality is because $\Pi(a_0, I_L, 1, p)$ is strictly increasing in p. The above implies

$$a_0 + S(I_L, 0) > a_0,$$

contradicting Assumption 1, part 1. Hence, there is no such a_0, and $\hat{\theta}(a) > 0$ for $a \in [\hat{a}, a_{\max}]$. □

Proof of Proposition 4:

We prove a stronger version of this proposition:

Proposition A2. *In the issue game, there exists a D1 equilibrium in which firms follow the strategy characterized in Proposition 4, and off-equilibrium prices are as follows: For off-equilibrium (i, θ) such that there exists a with*

$$i + S(i, \theta) = i(a) + S(i(a), \theta(a)), \tag{A41}$$

the price is based on type a:

$$P(i, \theta) = a + S(i, \theta). \tag{A42}$$

For off-equilibrium (i, θ) such that

$$i + S(i, \theta) \geq I^* + S(I^*, 1), \tag{A43}$$

the price is based on type $a_{\min}$*:*

$$P(i,\theta) = a_{\min} + S(i,\theta). \tag{A44}$$

For off-equilibrium (i,θ) *with*

$$i + S(i,\theta) \leq i(a_{\max}) + S(i(a_{\max}), \theta(a_{\max})), \tag{A45}$$

the price is based on type $a_{\max}$*:*

$$P(i,\theta) = a_{\max} + S(i,\theta). \tag{A46}$$

Proof. By Lemma A10, the firm strategy is well defined. By Lemma A4, firms do not mimic each other. Since $i(a) + S(i(a), \theta(a))$ is strictly decreasing in a, the off-equilibrium prices satisfy D1 by Lemma A5.

We show that no firm deviates to off-equilibrium actions. □

- Consider an off-equilibrium action (i,θ) such that there exists a that satisfies (A41). Given (A42), type a is fairly priced under both (i,θ) and its equilibrium choice $(i(a), \theta(a))$. Suppose $S(i,\theta) > S(i(a), \theta(a))$, then by (A41), $i < i(a)$, and hence $i(a) > I_L$ and $\theta(a) = 1$, and by Assumption 1, part 2, $S(i,\theta) < S(i(a), \theta(a))$, leading to a contradiction. Hence, $S(i,\theta) \leq S(i(a), \theta(a))$. Then type a prefers its equilibrium choice $(i(a), \theta(a))$ to (i,θ):

$$\begin{aligned}\Pi(a, i(a), \theta(a), P(i(a), \theta(a))) &= a + S(i(a), \theta(a)) \\ &\geq a + S(i,\theta) \\ &= \Pi(a, i, \theta, P(i,\theta)).\end{aligned}$$

  By Lemma A1, all types have the same preference. Since no type benefits from mimicking type a, no type benefits from deviating to (i,θ).

  - Consider an off-equilibrium action (i,θ) that satisfies (A43). Given (A44), type $a_{\min}$ is fairly priced under both (i,θ) and $(I^*, 1)$. By Assumption 1, part 2, $S(i,\theta) \leq S(I^*, 1)$. Hence, type $a_{\min}$ prefers $(I^*, 1)$ to (i,θ):

$$\begin{aligned}\Pi(a_{\min}, I^*, 1, P(I^*, 1)) &= a_{\min} + S(I^*, 1) \\ &\geq a_{\min} + S(i,\theta) \\ &= \Pi(a_{\min}, i, \theta, P(i,\theta)).\end{aligned}$$

    By Lemma A1, all firms have the same preference. Since no type benefits from mimicking type $a_{\min}$ by choosing $(I^*, 1)$, no type benefits from deviating to (i,θ).

  - Consider an off-equilibrium action (i,θ) that satisfies (A45). Given (A46), type $a_{\max}$ is fairly priced both in equilibrium and under (i,θ). If $S(i,\theta) > S(i(a_{\max}), \theta(a_{\max}))$, then by (A45), $i < i(a_{\max})$, and hence $i(a_{\max}) > I_L$ and $\theta(a_{\max}) = 1$, and by Assumption

1, part 2, $S(i,\theta) < S(i(a_{\max}),\theta(a_{\max}))$, leading to a contradiction. Hence, $S(i,\theta) \leq S(i(a_{\max}),\theta(a_{\max}))$. Type $a_{\max}$ prefers $(i(a_{\max}),\theta(a_{\max}))$ to (i,θ):

$$\begin{aligned}\Pi\left(a_{\max}, i\left(a_{\max}\right), \theta\left(a_{\max}\right), P\left(i\left(a_{\max}\right), \theta\left(a_{\max}\right)\right)\right) &= a_{\max} + S\left(i\left(a_{\max}\right), \theta\left(a_{\max}\right)\right) \\ &\geq a_{\max} + S\left(i, \theta\right) \\ &= \Pi\left(a_{\max}, i, \theta, P\left(i, \theta\right)\right).\end{aligned}$$

By Lemma A1, all types have the same preference. Since no type benefits from mimicking type $a_{\max}$, no type benefits from deviating to (i,θ).

Proof of Lemma 1:

Suppose in a D1 equilibrium, a non-empty set of firm types A choose (i,θ) with $i > I_L$ and $\theta < 1$. One can find $\tilde{i} < i$ and $\tilde{\theta}$ that satisfy (20) and (21) in the following way:

- If there is $\tilde{i} \in [I_L, i)$ with $S\left(\tilde{i},\theta\right) \geq S(i,\theta)$, then by $V_i > 0$,

  $$\tilde{i} + S\left(\tilde{i},\theta\right) < i + S(i,\theta),$$

  and since $S_\theta > 0$, there is $\tilde{\theta} > \theta$ such that (20) and (21) are satisfied.

- Otherwise, $S(I_L,\theta) < S(i,\theta)$. Since $V_i > 0$,

  $$I_L + S(I_L,\theta) < i + S(i,\theta).$$

  - If there is $\theta' \in (\theta, 1)$ such that $S(I_L,\theta') = S(i,\theta)$, then

    $$I_L + S(I_L,\theta') < i + S(i,\theta).$$

    Since $S_\theta > 0$, there is $\tilde{\theta} \in (\theta', 1)$ that satisfies (20) and (21) with $\tilde{i} = I_L$.

  - Otherwise, by continuity, $S(I_L, 1) \leq S(i,\theta)$. Since $S_\theta > 0$,

    $$S(i,1) > S(i,\theta) \geq S(I_L,1).$$

    By continuity, there is $i' \in [I_L, i)$ with $S(i',1) = S(i,\theta)$. This implies

    $$i' + S(i',1) < i + S(i,\theta).$$

    By Assumption 1, $i' \neq I^*$, and hence $|S_i(i',1)| > 0$. There is $\tilde{i}$ near i' that satisfies (20) and (21) with $\tilde{\theta} = 1$.

Consider firm type $a \in A$ with

$$P(i,\theta) \leq a + S(i,\theta).$$

Lemma A5 implies

$$P\left(\tilde{i},\tilde{\theta}\right)\geq a+S\left(\tilde{i},\tilde{\theta}\right)>a+S\left(i,\theta\right).$$

Then firm $\tilde{a}$ benefits from deviating to $\left(\tilde{i},\tilde{\theta}\right)$, leading to a contradiction:

$$\begin{aligned}\Pi\left(a,\tilde{i},\tilde{\theta},P\left(\tilde{i},\tilde{\theta}\right)\right) &> \Pi\left(a,\tilde{i},\tilde{\theta},a+S\left(\tilde{i},\tilde{\theta}\right)\right)\\ &= a+S\left(\tilde{i},\tilde{\theta}\right)\\ &> a+S\left(i,\theta\right)\\ &= \Pi\left(a,i,\theta,a+S\left(i,\theta\right)\right)\\ &\geq \Pi\left(a,i,\theta,P\left(i,\theta\right)\right),\end{aligned}$$

where the first and third inequality are because $\Pi\left(a,i,\theta,p\right)$ is strictly increasing in p.

Proof of Proposition 5:

By Lemma 1, a firm chooses either $(i,1)$ for some i or (I_L,θ) for some θ. By Corollary 1, $i\left(a\right)+S\left(i\left(a\right),\theta\left(a\right)\right)$ is non-increasing in a. Hence, there is type $\hat{a}$ such that types $a<\hat{a}$ choose $\left(i\left(a\right),1\right)$ with non-increasing $i\left(a\right)$ and types $a>\hat{a}$ choose $\left(I_L,\theta\left(a\right)\right)$ with non-increasing $\theta\left(a\right)$.

Step 1. We show that no type chooses $(I_L,0)$ in equilibrium.

By Assumption (1) of Footnote 8, type $a_{\max}$ strictly prefers $(I_L,1)$ to $(0,1)$ (doing nothing) under any prices. Type $a_{\max}$ prefers $(0,1)$ under price $P\left(0,1\right)=a_{\max}$ over $(I_L,0)$ under any price:

$$\begin{aligned}\Pi\left(a_{\max},0,1,a_{\max}\right) &= a_{\max}\\ &\geq a_{\max}+S\left(I_L,0\right)\\ &= \Pi\left(a_{\max},I_L,0,a_{\max}+S\left(I_L,0\right)\right)\\ &\geq \Pi\left(a_{\max},I_L,0,P\left(I_L,0\right)\right)\end{aligned}.$$

The first inequality is due to Assumption 1, part 1, and the second inequality is because $\Pi\left(a_{\max},I_L,0,p\right)$ is non-decreasing in p. Hence, type $a_{\max}$ strictly prefers $(I_L,1)$ to $(I_L,0)$ under any prices. By Lemma A1, all types have the same preference. Hence, no type chooses $(I_L,0)$ in equilibrium.

Step 2. All types separate on different pairs of (i,θ), and hence $i\left(a\right)+S\left(i\left(a\right),\theta\left(a\right)\right)$ is strictly decreasing in a.

Suppose types in a non-singleton interval A pool on (i,θ) in equilibrium. Then

$$P\left(i,\theta\right)<\sup A+S\left(i,\theta\right).$$

By step 1, $(i,\theta)\neq(I_L,0)$. There is $\left(\tilde{i},\tilde{\theta}\right)$ that satisfies

$$\tilde{i}+S\left(\tilde{i},\tilde{\theta}\right)<i+S\left(i,\theta\right).$$

By Lemma A5 and Corollary 1,

$$P\left(\tilde{i},\tilde{\theta}\right) \geq \sup A + S\left(\tilde{i},\tilde{\theta}\right).$$

Types in A benefit from deviating to such $\left(\tilde{i},\tilde{\theta}\right)$ close to (i,θ), which leads to marginal changes in i and $S(i,\theta)$ but a positive jump in price. This leads to a contradiction.

Step 3. Type $a_{\min}$ chooses $(I^*,1)$.

Suppose $a_{\min}$ chooses $(i,\theta) \neq (I^*,1)$. By Assumption 1, part 2, $S(i,\theta) < S(I^*,1)$. By step 2, type $a_{\min}$ has equilibrium payoff

$$\Pi^*(a_{\min}) = a_{\min} + S(i,\theta).$$

It benefits from deviating to $(I^*,1)$:

$$\begin{aligned}\Pi\left(a_{\min}, I^*, 1, P\left(I^*,1\right)\right) &\geq \Pi\left(a_{\min}, I^*, 1, a_{\min} + S\left(I^*,1\right)\right) \\ &= a_{\min} + S\left(I^*,1\right) \\ &> a_{\min} + S(i,\theta),\end{aligned}$$

leading to a contradiction.

Step 4. $i(a)$ and $\theta(a)$ are continuous.

Given $i(a)$ and $\theta(a)$ are non-increasing, it suffices to exclude jumps. Suppose there is $\tilde{a} \in (a_{\min}, a_{\max})$ such that

$$\lim_{a\uparrow\tilde{a}} i(a) > \lim_{a\downarrow\tilde{a}} i(a)$$

or

$$\lim_{a\uparrow\tilde{a}} \theta(a) > \lim_{a\downarrow\tilde{a}} \theta(a).$$

Let $\bar{i} \equiv \lim_{a\uparrow\tilde{a}} i(a)$, $\underline{i} \equiv \lim_{a\downarrow\tilde{a}} i(a)$, $\bar{\theta} \equiv \lim_{a\uparrow\tilde{a}} \theta(a)$, $\underline{\theta} \equiv \lim_{a\downarrow\tilde{a}} \theta(a)$. Then $\bar{i} \geq \underline{i}$ and $\bar{\theta} \geq \underline{\theta}$ with one and only one inequality holding strictly. By steps 2 and 3, $\bar{i}, \underline{i} < I^*$.

By step 2, as a approaches $\tilde{a}$ from above, their equilibrium payoff, $a + S(i(a), \theta(a))$, approaches $\tilde{a} + S(\underline{i}, \underline{\theta})$:

$$\lim_{a\downarrow\tilde{a}} \Pi^*(a) = \lim_{a\downarrow\tilde{a}} \left[a + S(i(a), \theta(a))\right] = \tilde{a} + S(\underline{i}, \underline{\theta}).$$

By Lemma A5,

$$P\left(\bar{i},\bar{\theta}\right) = \tilde{a} + S\left(\bar{i},\bar{\theta}\right).$$

As a approaches $\tilde{a}$ from above, their payoff from deviating to $\left(\bar{i},\bar{\theta}\right)$ approaches $\tilde{a} + S\left(\bar{i},\bar{\theta}\right)$:

$$\lim_{a\downarrow\tilde{a}} \Pi\left(a, \bar{i}, \bar{\theta}, P\left(\bar{i},\bar{\theta}\right)\right) = \Pi\left(\tilde{a}, \bar{i}, \bar{\theta}, P\left(\bar{i},\bar{\theta}\right)\right) = \tilde{a} + S\left(\bar{i},\bar{\theta}\right).$$

By Assumption 1, part 2,

$$\tilde{a}+S\left(\underline{i},\underline{\theta}\right)<\tilde{a}+S\left(\bar{i},\bar{\theta}\right).$$

Hence, there is $a>\tilde{a}$ with

$$\Pi^{*}\left(a\right)<\Pi\left(a,\bar{i},\bar{\theta},P\left(\bar{i},\bar{\theta}\right)\right),$$

implying type a benefits from deviating to $\left(\bar{i},\bar{\theta}\right)$, leading to a contradiction.

Step 5. It then follows Lemma A9 that $i\left(a\right)$ satisfies ODE (11) for types $a\leq\hat{a}$; the cutoff type $\hat{a}$ is uniquely determined by the condition "either $\hat{i}\left(\hat{a}\right)=I_L$ or $\hat{a}=a_{\max}$", and $\theta\left(a\right)$ satisfies ODE (19) for $a>\hat{a}$.

Proof of Lemma 2:

Since

$$\pi\left(a,i,\theta,\tilde{a}\right)=\ln V\left(a,i,\theta\right)-\ln\left(1+\frac{i}{\tilde{a}+S\left(i,\theta\right)}\right),$$

$$\pi_a\left(a,i,\theta,\tilde{a}\right)=\frac{V_a\left(a,i,\theta\right)}{V\left(a,i,\theta\right)}=\frac{1}{V\left(a,i,\theta\right)},$$

and

$$\pi_{ai}=\frac{-V_i\left(a,i,\theta\right)}{V\left(a,i,\theta\right)^2}.$$

Moreover,

$$\begin{aligned}\pi_i\left(a,i,\theta,a\right)&=\frac{\partial\left[\ln V\left(a,i,\theta\right)-\ln\left(1+\frac{i}{a+S(i,\theta)}\right)\right]}{\partial i}\\&=\frac{\partial\ln\left(a+S\left(i,\theta\right)\right)}{\partial i}\\&=\frac{S_i\left(i,\theta\right)}{a+S\left(i,\theta\right)}.\end{aligned}$$

Hence,

$$\left.\frac{-\pi_{ai}\left(a,i,\theta,\tilde{a}\right)}{\pi_i\left(a,i,\theta,\tilde{a}\right)}\right|_{\tilde{a}=a}=\frac{V_i\left(a,i,\theta\right)}{S_i\left(a,i,\theta\right)}\cdot\frac{a+S\left(i,\theta\right)}{V\left(a,i,\theta\right)^2}.$$

Similarly,

$$\left.\frac{-\pi_{a\theta}\left(a,i,\theta,\tilde{a}\right)}{\pi_\theta\left(a,i,\theta,\tilde{a}\right)}\right|_{\tilde{a}=a}=\frac{V_\theta\left(a,i,\theta\right)}{S_\theta\left(a,i,\theta\right)}\cdot\frac{a+S\left(i,\theta\right)}{V\left(a,i,\theta\right)^2}.$$

Since $\frac{a+S(i,\theta)}{V(a,i,\theta)^2}>0$,

$$\left.\frac{-\pi_{ai}\left(a,i,\theta,\tilde{a}\right)}{\pi_i\left(a,i,\theta,\tilde{a}\right)}\right|_{\tilde{a}=a}-\left.\frac{-\pi_{a\theta}\left(a,i,\theta,\tilde{a}\right)}{\pi_\theta\left(a,i,\theta,\tilde{a}\right)}\right|_{\tilde{a}=a}$$

has the same sign as

$$\frac{V_i\left(a,i,\theta\right)}{S_i\left(a,i,\theta\right)}-\frac{V_\theta\left(a,i,\theta\right)}{S_\theta\left(a,i,\theta\right)}.$$