Silence is safest: information disclosure when the audience's preferences are uncertain*

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Abstract

We examine voluntary disclosure decisions when firms are uncertain about audience preferences, and risk averse. In contrast to classic "unraveling" results, some firms remain silent in equilibrium. Silence is safer than disclosure; silence reduces the sensitivity of a firm's payoff to audience preferences. Increases in firm (audience) risk-aversion reduce (increase) disclosure. Our model explains why some firms do not disclose earnings breakdowns, executive compensation, and ratings when they face diverse audiences; and why they disclose less under regulatory rules mandating that disclosure be entirely public.

Keywords: information disclosure, risk-aversion, uncertainty, preferences.

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1 Introduction

Firms possess large amounts of information that is relevant to investors, customers, and other stakeholders, and that firms could voluntarily disclose if they wished (e.g., Graham et al, 2005). However, there are many cases in which valuable information that is potentially disclosable is not disclosed, and firms instead stay silent. As examples: firms frequently report only aggregate earnings, without geographic or business segment decompositions; provide little guidance about future earnings; minimize the information they disclose about executive compensation; and refrain from reporting Environmental and Social Governance (ESG) ratings.[1] This silence on the part of firms with respect to value-relevant information is puzzling in light of the well-known "unraveling" argument that predicts that, in equilibrium, firms disclose all information that they are able to.[2] In brief, the unraveling argument is that the firm with the most favorable information certainly discloses; the audience for the disclosure then interprets silence as indicating that the firm does not have the most favorable information; but then the firm with the second most favorable piece of information also discloses, and so on.

In this paper, we argue that in many settings firms stay silent because doing so is safer than disclosure; specifically, firms are uncertain about what it would be most beneficial for their audiences to believe, and silence reduces this risk. For example, a firm making large profits in a specific market would like to convey this information to its investors; but would often like an array of other economic agents, including competitors, tax authorities, regulators, and employee unions, to believe that profits in this market are low. If the firm is uncertain about the relative importance of these different parts of its audience, it is accordingly uncertain about whether it is better to try to convince its combined audience that profits in this market are high, or low. We show that, in many cases, firms respond to this uncertainty by staying silent, because doing so reduces the variance of firm payoffs. Relative to existing leading explanations, our analysis is able to account for silence even when disclosure has no direct cost, and even when there is no uncertainty that the firm possesses information

[1] See, respectively, and for example: Hope et al (2013), Harris (1998), Botosan and Stanford (2005), Bova et al (2015), Murphy (2012), Amel-Zadeh and Serafeim (2018). We detail our analysis's application to these specific examples in Section 7.

[2] See Viscusi (1978), Grossman and Hart (1980), Milgrom (1981), Grossman (1981), and Milgrom and Roberts (1986). Dranove and Jin (2010) survey the literature.

to disclose (see Grossman and Hart, 1980; Jovanovic, 1982; Dye, 1985).

Our analysis further implies that silence is driven by a firm's uncertainty about what its audience wants to see. A number of extant empirical studies are consistent with this prediction (Section 7). For example, firm silence is empirically associated with employees with more bargaining power; exposure to public disapproval of tax avoidance via "income shifting"; and fears of competition. Our analysis also provides a simple explanation for the increasing willingness of firms to disclose ESG performance, namely, increasing homogeneity of audience preferences; and is consistent with the view that mandatory disclosure of executive compensation is costly to firms because it exposes them to disapproval from outside the firm.

Closely related, our analysis predicts that when targeted disclosure to specific subsets of economic agents is possible, firms will regularly avail themselves of this opportunity, because doing so reduces their uncertainty about what an audience wants to see. For this reason, regulations that make targeted disclosure more difficult, such as Regulation Fair Disclosure in the U.S., may end up reducing disclosure. Similarly, and perhaps paradoxically, technological change that reduces frictions in sharing information[3] may result in less disclosure, because it makes targeted disclosure harder. Indeed, anecdotal accounts suggest that firms and CEOs have become more reluctant to make public remarks and instead are increasingly "acting like a politician" due to the increasing use of digital communication and recordings, which allows gaffes to go viral and trigger backlash from unfavorable audiences.[4]

Related Literature:

Our paper contributes to the large literature on information disclosure. In our reading, the explanations of silence with widest applicability are that disclosure may be costly (Grossman and Hart, 1980; Jovanovic, 1982) and that some firms may be exogenously unable to disclose, leading to endogenous silence by some firms that are able to disclose (Dye, 1985).[5]As we noted, we believe that an attractive feature of our

[3]See, for example, Warren et al (2015) for a discussion of "How Big Data Will Change Accounting," including the prediction that "Big Data will improve the quality and relevance of accounting information, thereby enhancing transparency and stakeholder decision making."

[4]See "Hold Your Peace," *The Economist*, Vol. 429, Issue 9115.

[5]Unraveling results have been generalized to wider classes of economies by papers such as Okuno-Fujiwara et al (1990) and Seidmann and Winter (1997). Okuno-Fujiwara et al (1990) stress the importance of the monotonicity of the sender's expected utility in the receiver's beliefs, and include examples in which a failure of this property leads to full silence. Our paper can be viewed as identifying a set of natural economic conditions that generate non-monotonicity of the sender's

analysis is its ability to explain silence in settings in which disclosure is both cheap and known to be feasible.

The literature has also suggested a number of further alternative explanations of silence, as surveyed in Dranove and Jin (2010). Among them, some share our focus on audience heterogeneity, though rely on very different economic forces. For example, Fishman and Hagerty (2003) show that silence arises if some audience members are unable to process the information content of disclosure. Harbaugh and To (2020) consider a setting in which the sender's type is drawn from the interval $[0, 1]$, but disclosures are restricted to specifying which element of a finite partition of $[0, 1]$ the type belongs to. Moreover, the audience is endowed with a private signal about the sender's type. Consequently, the best senders in a partition element may prefer to remain silent in order to avoid mixing with mediocre senders in the same partition element, and thus the unraveling argument breaks down. Similarly, Quigley and Walther (2020) show that when disclosing is costly while the audience observes a separate noisy signal about the sender, the best sender may remain silent, rely on the audience's signal, and thus save the disclosure cost. This then generates "reverse unraveling" in which other sender-types also remain silent in order to pool with higher sender-types.

Dutta and Trueman (2002), Suijs (2007), and Celik (2014) all analyze relatively special situations in which the firm as the sender is unsure how the audience will respond to a disclosure. In Dutta and Trueman (2002), the firm has two pieces of information, one representing a "fact" about the firm and another governing how the audience would interpret the fact; the firm can only disclose the former. However, there is a strictly positive probability that the firm has no "fact" to disclose, so that the economic forces that generate silence in Dye (1985) operate in their paper also.[6] In Suijs (2007)'s environment (unlike ours), there is a direct benefit to silence.[7] In Celik (2014), the firm as a seller chooses whether to disclose a location on a Hotelling

expected utility in receiver beliefs. In doing so, we characterize the extent of silence—typically, partial rather than full—along with comparative statics with respect to sender and receiver risk aversion.

[6]Specifically, Dutta and Trueman (2002) state that if the probability of the firm knowing the "fact" is 1, unraveling always happens in equilibrium.

[7]Specifically, in Suijs (2007)'s model, disclosure gives a payoff of either $U(0)$ or $U(1)$, with probabilities $1 - p(\phi)$ and $p(\phi)$ respectively, where ϕ is the sender's type. Silence gives payoffs of $U\left(\frac{1}{2}\right)$ and something at least $U(0)$, with corresponding probabilities, and *regardless* of audience inferences about what silence means. So if the type space is such that $1 - p(\phi)$ is sufficiently high for all types, silence is an equilibrium.

line, and *also* makes a take-it-or-leave-it price offer to a buyer whose location on the Hotelling line is assumed to follow a uniform distribution.[8] The details of price formation are important: if instead there were several buyers in competition, the only equilibrium would be full disclosure.

2 Example

We start with an illustrative example. We emphasize that the example's functional form choices and distributional assumptions are not essential, as our subsequent formal results demonstrate.

A firm can disclose to an audience value-relevant information x—for example, profits in a particular market. The value of x lies in $[0,1]$, and the audience's priors about x are given by the density function

$$f(x) = 1 - a(1 - 2x), \tag{1}$$

where $a \in [-1,1]$ is a parameter. The case $a = 0$ is the uniform distribution, while $a = -1, 1$ respectively are lower and upper triangular distributions.

The audience for the firm's disclosure consists of investors, and another party who we label an *antagonist*, and depending on the application may variously represent a regulator, tax authority, employee group, or competitor. Let μ denote the audience's beliefs about x, which depend on whether the firm discloses or stays silent. The firm is uncertain whether the antagonist is *passive* or *aggressive* and attaches probability $\frac{1}{2}$ to each possibility. The firm's risk preferences are represented by a strictly concave function v. If the antagonist is passive, the firm's value is $v(E[x-1|\mu])$, while if the antagonist is aggressive, the firm's value is $v(E[x-2x|\mu])$. Concretely, one can interpret these payoffs as being composed of $E[x|\mu]$ from investors, and either -1 or $E[-2x|\mu]$ from passive and aggressive antagonists, respectively.

We highlight three features of the example that are important. First, the firm benefits from investors believing that x is high, but benefits from the antagonist believing that x is low. Second, the firm is uncertain about exactly how much it will benefit from the antagonist believing that x is low. Third, the firm is effectively risk-

[8]These assumptions imply that disclosing sellers at the ends of the line face a severe trade off between proposing a higher price and achieving a reasonable sale probability.

averse (either because of intrinsic preferences or contracting frictions) over outcomes.

Figure 1 plots $v(x-1)$ and $v(x-2x)$. "Extreme" firms that have high or low values of x face the most uncertainty related to the audience's identity (specifically, whether the antagonist is passive or aggressive). Firms with intermediate values face little uncertainty; and the firm $x=\frac{1}{2}$ faces no uncertainty at all.

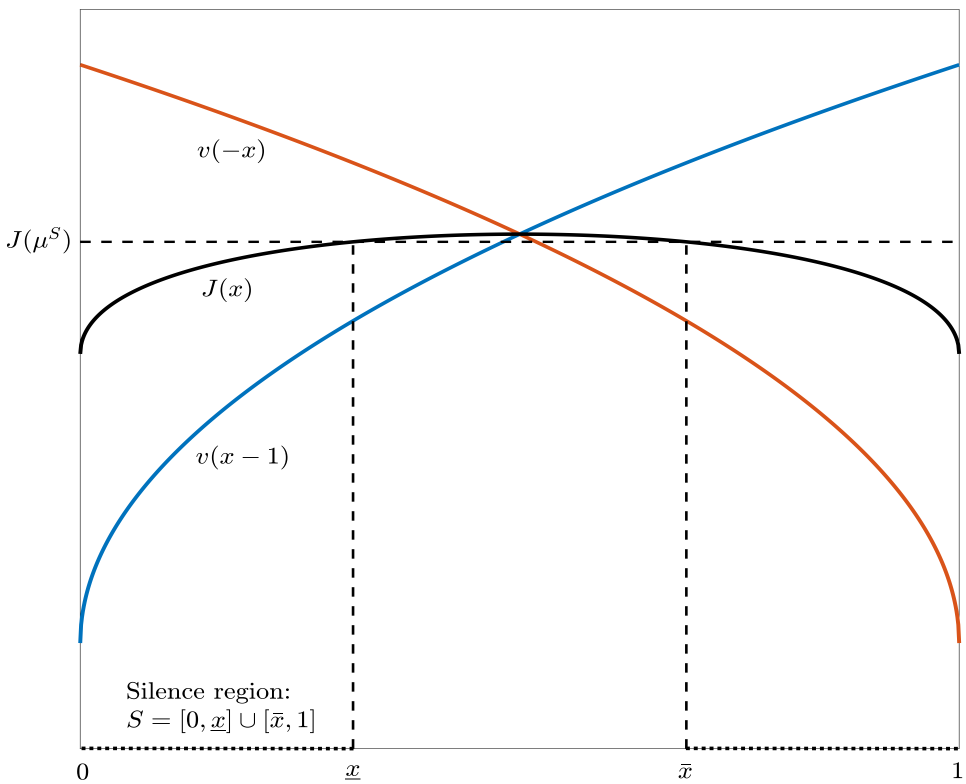

Figure 1: Graphical illustration of example of Section 2

We write $J(\mu)$ for the firm's expected value under audience beliefs μ:

$$J(\mu) \equiv \frac{1}{2} v(E[x-1|\mu]) + \frac{1}{2} v(E[-x|\mu]).$$

If the firm discloses x, the audience's beliefs are concentrated on x, and with slight abuse of notation the firm's expected value is

$$J(x) = \frac{1}{2} v(x-1) + \frac{1}{2} v(-x).$$

Figure 1 also plots $J(x)$, the firm's value from disclosure. It is a strictly concave function of x. Additionally, and special to this example, it is symmetric about $x = \frac{1}{2}$.

An equilibrium is characterized by the set of firms $S \subset [0, 1]$ that stay silent. All silent firms face the same audience beliefs, which we denote by μ^S; and hence all silent firms have the same payoff $J\left(\mu^S\right)$.

2.1 Silence of "extreme" firms

An immediate implication is that if an equilibrium entails silence, the silence set consists of "extreme" firms with high or low values of x. That is, there are $\underline{x}$ and $\bar{x}$ such that the silence set is

$$S = [0, \underline{x}) \cup (\bar{x}, 1].$$

Moreover, firms $\underline{x}$ and $\bar{x}$ are indifferent between silence and disclosure:

$$J(\underline{x}) = J(\bar{x}) = J\left(\mu^S\right). \tag{2}$$

2.2 Silence is safest

We next rewrite (2) more explicitly, focusing on the case of $a \geq 0$, so that the audience's prior has an upwards sloping density. (The case of $a \leq 0$ is directly analogous.) Two features specific to the example are very helpful in rewriting (2). First, the symmetry of $J(x)$ immediately implies that

$$\bar{x} = 1 - \underline{x}. \tag{3}$$

Second, a firm's value after silence equals the value from disclosing an x equal to the expected value of silent firms, $E\left[x|\mu^S\right]$:

$$J\left(\mu^S\right) = J\left(E\left[x|\mu^S\right]\right). \tag{4}$$

The symmetry property (3) and the focus on upwards sloping densities ($a \geq 0$) together imply that the average type of a silent firm is above $\frac{1}{2}$. From (4), it follows that the equilibrium condition (2) can be written simply as

$$1 - \underline{x} = E\left[x|x \leq \underline{x} \text{ or } x \geq 1 - \underline{x}\right]. \tag{5}$$

That is, silence induces audience beliefs such that the expected value of x of a silent firm coincides with a firm that is happy to disclose. Disclosing firms are intermediate firms, which face less uncertainty from the audience's identity. By staying silent, extreme firm achieve safer outcomes, which they prefer because of risk aversion—that is, silence is safest.

2.3 Equilibrium silence

If $a = 0$, the audience's prior is uniform, and $\underline{x} = \frac{1}{2}$ solves (5). In this case, there is an equilibrium in which all firms other than $x = \frac{1}{2}$ remain silent; and even firm $x = \frac{1}{2}$ is indifferent between silence and disclosure.

For $a \in (0, 1)$, the lefthand side (LHS) of (5) is less than the RHS at $\underline{x} = \frac{1}{2}$. On the other hand, as $\underline{x} \to 0$, the LHS approaches 1, while the RHS approaches $\frac{f(1)}{f(0)+f(1)}$, which is strictly less than 1, because silence pools firms with low and high values of x together. So by continuity, there exists $\underline{x} \in (0, 1)$ that solves (5), corresponding to an equilibrium in which some firms stay silent and some disclose.

Moreover, substitution of the density function into (5) delivers the explicit solution:

$$\underline{x} = \frac{-\left(1 - |a|\right) + \sqrt{\left(1 - |a|\right)\left(1 + \frac{1}{3}|a|\right)}}{\frac{4}{3}|a|}.$$

It is straightforward to show that $\underline{x} \in \left[0, \frac{1}{2}\right]$, with $\underline{x} \to 0, \frac{1}{2}$ as $|a| \to 1, 0$.

In particular, the benefit of silence lies in extreme firms pooling together so that the audience believes they are average. This benefit is largest when the audience's prior beliefs attach similar probabilities to both "low" and "high" types, leading to greater equilibrium silence.

3 Model

We now state our formal model, which generalizes the example. A firm has a type x drawn from a compact set $X \subset \Re$, which we normalize to $X = [0, 1]$. The firm is privately informed about its type x, which the audience does not know. The audience's prior of x is given by a probability measure μ_0, which has full support over X, and admits a density function f.

The firm can costlessly disclose x to an audience, or alternatively, stay silent. Subsequent to a firm's disclosure or silence, audience beliefs are given by a probability measure μ. Specifically, if a firm discloses x, audience beliefs are concentrated on x. If instead a firm stays silent, audience beliefs are given by μ^S, which is obtained from the initial beliefs μ_0 after conditioning on x belonging the set of firms that stay silent in equilibrium, which we denote by S.

The firm is uncertain about its audience. The firm's payoff depends on the realized identity of its audience, and on the audience's beliefs about its type. The set of possible audiences is N and a specific audience is denoted by i, and has probability $\Pr(i)$. Let $p_i(\mu)$ be the firm's *payoff* from an audience i with beliefs μ about the firm's type. We assume that $p_i(\mu)$ is continuous as a function of μ, i.e., if μ_n converges weakly to μ then $p_i(\mu_n) \to p_i(\mu)$. We write $p_i(x)$ for the case in which the firm discloses and so μ is concentrated on x. Note that $p_i(x)$ is continuous in x. The payoff function p_i summarizes how audience i's actions given beliefs μ affect the firm.

We assume that audiences are (weakly) risk-averse in the sense that they dislike uncertainty about the firm's type, and this in turn negatively impacts the firm:

$$p_i(\mu) \leq E[p_i(x)|\mu]. \tag{6}$$

Audience risk-neutrality corresponds to (6) holding with equality. We emphasize that $p_i(x)$ may be increasing, decreasing, or even non-monotonic in x. Note that the assumption of audience risk-aversion makes silence costly for the firm, in turn making it harder for silence to arise in equilibrium.

Because the firm is uncertain about its audience, the firm's expected value depends on its risk preferences, which are captured by a strictly increasing function v, henceforth the firm's *value*. For now, we allow for v to be either concave or convex. The firm's *expected value* if the audience has beliefs μ is hence

$$J(\mu) \equiv E[v(p_i(\mu))] = \sum_{i \in N} \Pr(i)\, v(p_i(\mu)).$$

We write $J(x)$ for the firm's expected value after disclosing x, henceforth the firm's *disclosure value.*

As much as possible, we express results in terms of the expected value function J. Note that $J(x)$ inherits continuity from $p_i(x)$. As noted, at this point we have made no assumptions on the shape of $p_i(x)$.

To rule out economically uninteresting cases in which $J(x)$ is flat, or oscillates infinitely often, we impose the following very mild regularity assumption.

Assumption 1 *$J(x)$ has only a finite number of extrema.*

An equilibrium is characterized by a "silence" set S of firm types that do not disclose, and stay silent. The remaining firms $X \backslash S$ disclose. The equilibrium condition is that each firm's decision between disclosure and silence is optimal, i.e.,

$$\begin{aligned} J(x) &\leq J\left(\mu^S\right) \text{ for all } x \in S \\ J(x) &\geq J\left(\mu^S\right) \text{ for all } s \notin S. \end{aligned}$$

Note that if all firms disclose, $S = \emptyset$, and $J\left(\mu^S\right)$ is not defined. Indeed, full disclosure can always be supported as an equilibrium by assigning off-equilibrium-path beliefs in which the audience interprets silence as meaning that the firm's type is $\arg\min_{x \in X} J(x)$. Our analysis characterizes when equilibria with silence exist, and the form they take. We refer to any equilibrium with $\mu_0(S) > 0$ as a *silence* equilibrium; and further distinguish between equilibria with *full silence*, i.e., $\mu_0(S) = 1$, and with *partial silence*, i.e., $0 < \mu_0(S) < 1$. Similarly, an equilibrium with $\mu_0(S) = 0$ has *full disclosure.*

4 Silence is safest

We first characterize an important feature of silence equilibria, namely, a sense in which "silence is safest." We do so by highlighting a simple property of the firm's expected value function J. Section 5 gives necessary and sufficient conditions for silence equilibria to exist.

Specifically, we explore the implications of the firm's expected value function satisfying the following property, which we label as "Average is Better" (AB):

$$J(\mu) \leq J(E[x|\mu]). \tag{AB}$$

Property (AB) says that if the audience's beliefs about the firm are given by μ, the firm would (weakly) benefit from the audience instead treating the firm as the average of these types, $E[x|\mu]$. This property can be viewed as a strengthening of audience risk aversion (6). That is: if the payoff functions $p_i(x)$ are weakly concave (see discussion further below) then audience risk-aversion (6) implies

$$p_i(\mu) \leq p_i(E[x|\mu]) \tag{7}$$

for each audience i. Inequality (7) immediately implies (AB).

Property (AB) directly implies a key property of any silence equilibrium:

Proposition 1 *Let (AB) hold. In any silence equilibrium, a firm with type equal to the average type of silent firms, $E\left[x|\mu^S\right]$, weakly prefers disclosure to silence; and the set of firm types that strictly prefer silence to disclosure is not an interval.*

The first statement in Proposition 1 is $J\left(E\left[x|\mu^S\right]\right) \geq J\left(\mu^S\right)$, which is simply a special case of (AB). For the second statement, suppose to the contrary that the stated set is an interval. By Assumption 1, only a finite number of firm types can be indifferent between silence and disclosure, implying that $E\left[x|\mu^S\right]$ belongs to the interval, contradicting the first statement.

Proposition 1 says that in any silence equilibrium there are firms sandwiched between silent firms that are happy to disclose. The advantage of silence is that the audience interprets it as meaning that the average type of silent firms, $E\left[x|\mu^S\right]$, corresponds to a happy-to-disclose type. This averaging effect is the "safety" that a firm gains from staying silent; in other words, "silence is safest."[9] Section 6 develops this point further.

As noted, a sufficient condition for (AB) is that the payoff functions $p_i(x)$ are weakly concave. To give economic meaning to the concavity of $p_i(x)$, consider the case of audiences consisting of a mixture of investors and antagonists (see Section 2). Suppose that there is an audience in which the antagonist is passive, so that if the firm knew it faced this audience, it would focus on pleasing investors. This investor-dominated audience can be taken as a "numeraire" audience: without loss, denote

[9] We also note that if (AB) holds strictly for any μ with non-null support, then Proposition 1 can be straightforwardly strengthened to state: "In any silence equilibrium, a firm with type $E\left[x|\mu^S\right]$ strictly prefers disclosure to silence; and the set of firm types that weakly prefer silence to disclosure is not an interval." Along the same lines, if one assumes that a firm always breaks indifference in favor of disclosure (a heuristic argument outside our model is that the firm knows its disclosure payoff from a given audience, while its silence payoff depends on audience beliefs) then Assumption 1 can be dropped, and Proposition 1 can be written simply as "In any silence equilibrium, a firm with type $E\left[x|\mu^S\right]$ discloses; and the silence set is not an interval."

this audience as audience 1, and identify a firm's type with the reaction of investors, i.e., $p_1(x) = x$. Then the concavity of $p_i(x)$ corresponds to antagonists growing increasingly unhappy at marginal improvements in investor payoffs, i.e., increasing marginal disutility.

For much of our analysis we focus on the case of concave payoff functions $p_i(x)$, both because we believe it to be economically relevant, and also because if $p_i(x)$ are instead strictly convex, silence can arise for more mechanical reasons, a point we explore in Appendix A. But we also highlight that we establish some of our central results about equilibrium existence below (Propositions 2-4) independently of the concavity of $p_i(x)$ and of (AB).

5 When do equilibria with silence emerge?

5.1 Necessary conditions for silence

Proposition 1 characterizes silence equilibria, conditional on such equilibria existing. We next derive necessary conditions for such equilibria to exist. To state our results, it is useful to first express the unraveling condition (i.e., when full disclosure must happen) in terms of the firm's expected value function J:

Condition 1 *For any non-null set S, there exists $x \in S$ such that $J\left(\mu^S\right) < J(x)$.*

Condition 1 says that for any mix of firm types μ^S staying silent, there is always a firm type $x \in S$ that would benefit from separating itself from the other firms and disclosing. If Condition 1 holds, it is immediate that the only equilibrium has full disclosure.

Equilibrium silence can only exist if Condition 1 is violated, as in the example of Section 2. The key ingredients in the example are that (I) the firm is unsure whether it would benefit from convincing the audience that its type is high, low, or perhaps intermediate, and (II) firm risk-aversion. Moreover, it is important that (III) audiences are not so risk-averse that they respond extremely negatively to the uncertainty that silence leaves them with. All the three conditions are necessary, as we next establish.

5.1.1 Uncertain about audience preferences

First, silence only arises if at least some audiences differ in their preference orderings:

Proposition 2 *If there is no uncertainty over audience preference orderings, i.e., p_i is ordinally equivalent to p_j in the sense that $p_i(x) < (\leq) p_j(\tilde{x})$ if and only if $p_i(x) < (\leq) p_j(\tilde{x})$ for any $i, j \in N$, then Condition 1 holds and the only equilibrium is full disclosure.*

By Proposition 2, uncertainty over only the *strength* of audience preferences for a higher value of x is insufficient to generate silence, since in this case all the audiences have ordinally equivalent preferences, and a version of the standard unraveling proof applies. In contrast, silence requires the firm to be unsure about whether an audience values higher or lower values of x, at least over some range. For instance, if the example of Section 2 is perturbed so that the firm's payoff is either $E[x-1|\mu]$ or $E\left[x-\frac{1}{2}x|\mu\right]$, depending on the audience, then the only equilibrium is full disclosure.

We also highlight that Proposition 2 is true even if $p_i(x)$ is non-monotone in x, illustrating that non-monotone audience preferences alone are insufficient to generate silence in equilibrium. Roughly speaking, if $p_i(x)$ is non-monotone, but all audiences have ordinally equivalent preferences, the unraveling argument still applies after a change in variables from x to $p_i(x)$.

5.1.2 Firm risk aversion

We now turn to our second necessary condition, firm risk-aversion. Recall that firm risk-aversion naturally arises from any of: concentrated ownership; managerial risk-aversion coupled with internal agency fictions; external financing frictions. If the firm is either risk-neutral or risk-loving, then unraveling occurs, and all firms disclose:

Proposition 3 *If the firm is either risk neutral or risk loving (i.e., v weakly convex) then Condition 1 holds and the only equilibrium is full disclosure.*

In particular, if the firm is risk neutral (v linear) and the payoff functions p_i are linear, then one can simply switch variables from x to $E[p_i(x)]$, and apply the standard unraveling argument with respect to $E[p_i(x)]$. The proof of Proposition 3 extends this argument to cover convex v functions and arbitrary p_i functions.

5.1.3 Audience risk aversion

A third necessary condition is that audiences cannot be too risk-averse. Recall that the risk-aversion of audience i is embodied in the relation between $E\left[p_i\left(x\right)|\mu\right]$ and $p_i\left(\mu\right)$; greater risk aversion corresponds to a larger value of $E\left[p_i\left(x\right)|\mu\right] - p_i\left(\mu\right)$,[10] with risk neutrality corresponding to this expression equalling zero. So as to avoid imposing functional forms on p_i, we focus on the extreme case of infinite audience risk aversion, and show that in this case the only equilibrium is full disclosure. Formally, infinite risk-aversion corresponds to[11]

$$p_i\left(\mu\right) = \inf_{\tilde{x}\in supp(\mu)} p_i\left(\tilde{x}\right). \tag{8}$$

Proposition 4 *If audiences are infinitely risk averse in the sense of (8) then Condition 1 holds and the only equilibrium is full disclosure.*

Intuitively, silence is unlikely to be attractive if audiences are very risk-averse, because in such cases it imposes so much risk on audiences that it harms firms by more than they gain by pooling and reducing their own risk stemming from uncertainty about audience preferences.

5.1.4 Non-monotonicity of disclosure value $J\left(x\right)$

Our final necessary condition, which the example in Section 2 illustrates, is that the firm's disclosure value $J(x)$ must be non-monotone. Note that this is a necessary condition only for the case that we focus on in the paper, namely that in which (AB) holds, discussed in Section 4.[12] Recall that under (AB), silence equilibria entail disclosure by intermediate types, and silence by more extreme types. The only way this can occur is if the disclosure value $J\left(x\right)$ is non-monotone:

Proposition 5 *If (AB) holds and the disclosure value $J\left(x\right)$ is monotone then Condition 1 holds and the only equilibrium is full disclosure.*

[10] See also subsection 6.3, and the associated Appendix B.

[11] Note that infinite risk-aversion violates the continuity axiom, and so does not admit an expected utility representation.

[12] Appendix A presents an example in which $J(x)$ is monotone, (AB) is violated, and full silence arises.

Propositions 2 and 3 already establish that both uncertainty about audience preferences and firm risk-aversion are necessary for silence. Proposition 4 shows that these conditions are not sufficient. In particular, these conditions generate silence only if they generate a non-monotone disclosure value $J(x)$.

Whether or not uncertainty about audience preferences and firm risk-aversion indeed generate a non-monotone disclosure value $J(x)$ depends on the probability distribution of different audiences. Lemma 1 immediately below shows that there exist probability distributions under which $J(x)$ is non-monotone, at least for the case of concave payoff functions that is our main focus.[13]

Lemma 1 *If the payoff functions $p_i(x)$ are concave, the firm is risk-averse, and there is uncertainty about audience preferences (i.e., there exist firm types x, $\tilde{x}$ and audiences i, j such that $p_i(x) < p_j(\tilde{x})$ and $p_i(\tilde{x}) < p_j(x)$), then there is a neighborhood of probability distributions over audiences such that $J(x)$ is non-monotone.*

5.2 Sufficient conditions for silence

We next turn to sufficient conditions for the existence of silence equilibria. To establish that such silence equilibria exist in general (i.e., beyond the example of Section 2), Proposition 6 below establishes that the following conditions are sufficient for silence equilibria (in addition to some mild regularity conditions stated further below): (I) At least some pair of audiences has differing preference orderings over extreme firm-types; (II) Firm risk-aversion; (III) Audiences are not too risk-averse; and (IV) The probability of different audiences is such that extreme firm-types dislike disclosure close-to-equally. These four sufficient conditions are the counterpoints of the necessary conditions stated in, respectively, Propositions 2, 3, 4, and 5. As such, our previous results about necessity show that if these conditions are sufficiently far from holding, then no silence equilibrium exists.

As noted, we also need some mild regularity conditions on audience preferences over extreme firm-types, and on the prior density f of extreme firm-types. For clarity, we state these regularity assumptions separately. Both are satisfied by the example of Section 2, for density parameters $|a| \neq 1$.

[13]We also note that non-monotonicity of $J(x)$ does not nest uncertainty about audience preferences. In particular, non-monotonicity of $J(x)$ can easily arise even if the sender knows the audience's preferences; if, for example, there is only one audience with non-monotone preferences.

Assumption 2 *For all audiences $i \in N$, the derivative $\frac{\partial v(p_i(x))}{\partial x}$ remains bounded as $x \to 0, 1$.*

Assumption 3 *For any constant $\kappa > 0$, $\lim_{x \to 0} \frac{f(x)}{f(1-\kappa x)}$ exists and is strictly positive.*

Finally, we also suppose that the payoff functions $p_i(x)$ are weakly concave. For the reasons discussed in Section 4, this is the case that we generally focus on. But we also emphasize that this property plays a much more minor role in Proposition 6 than the features (I)-(IV) that we emphasize, and could be straightforwardly replaced with considerably weaker conditions.[14]

Proposition 6 *Suppose the payoff functions $p_i(x)$ are weakly concave, along with: (I) There are audiences i, j such that $p_i(0) < p_i(1)$ and $p_j(0) > p_j(1)$; (II) The firm's value function v is strictly concave; (III) All audiences are sufficiently close to risk-neutral; and (IV) The distribution of audiences $\{Pr(i)\}$ is such that $|J(0) - J(1)|$ is sufficiently small. Then a silence equilibrium exists.*

The proof of Proposition 6 is a generalization of the fixed point argument described at the start of subsection 2.3 in the context of the example.

In general, further results on sufficient conditions require considerably more parametric structure on the economy. That said, a very simple sufficient condition arises if (AB) holds with equality.

Proposition 7 *Suppose the firm's value function v is strictly concave and (AB) holds with equality. If $J(1) > (<) J(0)$ and $\arg\max_x J(x)$ is interior and weakly less than (greater than) $E[x|\mu_0]$ then there exists a silence equilibrium.*

Concretely, (AB) holds with equality if audiences are risk neutral and the payoff functions $p_i(x)$ are linear (as in the example of Section 2).

By Proposition 5, we know that silence only arises if uncertainty about audience preferences[15] leads to a non-monotone disclosure value $J(x)$. Proposition 7 gives a simple lower bound on an "amount" of non-monotonicity that is enough to deliver silence. That is, if $J(1) > J(0)$, so that overall the disclosure value $J(x)$ slopes up

[14] Specifically, it is straightforward to replace the weak concavity of $p_i(x)$ in Proposition 6 with the much milder assumption that $J(x)$ has a minimum at either $x = 0$ or 1.

[15] Linearity of the payoff functions $p_i(x)$ and the condition that $\arg\max_x J(x)$ is interior imply that the firm is uncertain about audience preferences.

from left to right, then the departure from monotonicity must be large enough that the peak of the disclosure value function $J(x)$ lies to the left of the average type $E[x|\mu_0]$.

Remark (Aside): Although our focus in this paper is on the case in which firms cannot commit to disclosure policies, one can also ask what disclosure policy a firm would commit to if commitment were feasible prior to learning its type x. When (AB) holds with equality, and a firm is risk averse, the answer is that a firm would commit to full silence, since for any possible silence set S that the firm commits to,[16]

$$J(\mu_0) > E[J(x)|\mu^{X\backslash S}]\Pr(X\backslash S) + E[J(\mu^S)]\Pr(S).$$

6 Characterization of silence equilibria

We further characterize silence equilibria, focusing on the relationship between risk-aversion and "silence is safest." Given the analysis in Sections 4 and 5, for the remainder of the paper we impose the following pair of assumptions. First, we focus on strictly concave firm value functions v, since otherwise silence does not arise (Proposition 3). Second, and as discussed in Section 4, we focus on weakly concave payoff functions $p_i(x)$, in order to rule out more mechanical benefits of silence.

Assumption 4 *The firm's value function v is strictly concave.*

Assumption 5 *The payoff functions $p_i(x)$ are weakly concave.*

Assumptions 4 and 5 imply that $J(x)$ is strictly concave, and in particular single-peaked. As noted earlier, Assumption 5 implies both (7) and (AB).

[16]The following inequality is a consequence of

$$\begin{aligned} J(\mu_0) &= J(E[x|\mu_0]) \\ &> E[J(E[x|\mu^{X\backslash S}])]\Pr(X\backslash S) + E[J(E[x|\mu^S])]\Pr(S) \\ &> E[J(x)|\mu^{X\backslash S}]\Pr(X\backslash S) + E[J(E[x|\mu^S])]\Pr(S) \\ &= E[J(x)|\mu^{X\backslash S}]\Pr(X\backslash S) + E[J(\mu^S)]\Pr(S), \end{aligned}$$

where the two inequalities follow from Jensen's inequality, and the two equalities are (AB).

6.1 Silence is safest revisited

This last pair of assumptions allows us to more tightly characterize silence equilibria. As a preliminary: at various points below we make use of the following mild condition, which implies that for any μ with a non-null support, (7) holds strictly for some audience i, and hence that (AB) also holds strictly; and in turn guarantees strictness of some key inequalities:

Condition 2 *There exists at least one audience i for which $p_i(x)$ is strictly concave.*

First, note that, since $J(x)$ is single-peaked, the structure of a silence equilibrium can be immediately be strengthened to:

Corollary 1 *In a silence equilibrium S, there are $\underline{x}, \bar{x}$ such that $S = [0, \underline{x}) \cup (\bar{x}, 1]$;*

$$\underline{x} \leq E\left[x|\mu^S\right] \leq \bar{x}; \tag{9}$$

and

$$J\left(\underline{x}\right) = J\left(\bar{x}\right) = J\left(\mu^S\right). \tag{10}$$

If silence is partial silence ($\underline{x} < \bar{x}$) and Condition 2 holds then both inequalities in (9) are strict.

In Corollary 1, firms $\underline{x}$ and $\bar{x}$ are *marginal disclosers*, in the sense of being indifferent between disclosure and silence, as in (10).

Corollary 1 further implies:

Corollary 2 *In any silence equilibrium S there is at least one marginal discloser x_m for which*

$$E\left[p_i\left(\mu^S\right)\right] \leq E\left[p_i\left(x_m\right)\right]. \tag{11}$$

Moreover, the inequality is strict if silence is partial and Condition 2 holds.

Corollary 2 says that the silence lottery is safer than the disclosure lottery of at least one of the marginal disclosers, in the following sense: since the lotteries provide the same expected value to a marginal discloser, a lower expected payment implies that the lottery must be safer. In this sense, Corollary 2 is a more explicit demonstration that silence is safest.

6.2 Comparative statics with respect to firm risk-aversion

To further reinforce the point that a key economic force behind silence is the reduction in risk it engenders, we next consider comparative statics in firm risk-aversion.

Specifically, Corollary 2 says that silence reduces risk for at least one of the marginal disclosers $\underline{x}$ and $\bar{x}$. We show that as firm risk-aversion increases, firms close to this marginal discloser switch from disclosure to silence. Concretely, variations in firm risk-aversion correspond to variation in ownership concentration, managerial risk-version, internal agency frictions, or external financing frictions.

For the case of two audiences ($|N| = 2$), we establish this result using Pratt's (1964) general ordering of risk preferences.

Proposition 8 *Suppose that $|N| = 2$, Condition 2 holds, and that a partial silence equilibrium exists when the firm's value function is v. Suppose that the firm's value function changes to $\tilde{v} = \phi \circ v$ for some increasing and strictly concave ϕ, corresponding to greater risk-aversion. Then there is a marginal discloser x_m for which silence is safer than disclosure in the original equilibrium, i.e., $E\left[p_i\left(\mu^S\right)\right] < E\left[p_i\left(x_m\right)\right]$, and a new silence equilibrium under $\tilde{v}$, such that silence strictly increases in the neighborhood of x_m.*

The restriction to two audiences in Proposition 8 is needed because, as is widely appreciated, it is hard to produce general comparative statics on choices between risky lotteries with respect to risk preferences (see, e.g., Ross (1981) for a discussion of this point), without imposing significant structure on either preference or on the distribution of payoffs. Specifically, with just two audiences, we are able to show that, for at least one of the marginal disclosers $x_m \in \{\underline{x}, \bar{x}\}$, the payoffs associated with silence, i.e., $p_1\left(\mu^S\right), p_2\left(\mu^S\right)$, lie within the range of possible payoffs associated with disclosure, i.e., lie in the interval $[\min\{p_1(x_m), p_2(x_m)\}, \max\{p_1(x_m), p_2(x_m)\}]$. This property allows us to apply results based on Pratt's ordering of risk preferences (specifically, Hammond (1974)).

For more than two audiences, we are unable to guarantee this property. Since we then lack structure on the distribution of payoffs, we must instead impose more structure on the risk-aversion ordering:

Proposition 9 *Suppose that Condition 2 holds, and that a partial silence equilibrium exists when the firm's value function is v. Suppose that the firm's value function* *changes to $\tilde{v}$, where $\alpha\tilde{v}(z) + z = v(z)$ for some constant $\alpha > 0$, corresponding to greater risk-aversion. Then there is a marginal discloser x_m for which silence is safer than disclosure in the original equilibrium, i.e., $E\left[p_i\left(\mu^S\right)\right] < E\left[p_i\left(x_m\right)\right]$, and a new silence equilibrium under $\tilde{v}$, such that silence strictly increases in the neighborhood of x_m.*

In words, the comparison of risk preferences used in Proposition 9 amounts to saying: preferences represented by $\tilde{v}$ are more risk-averse than preferences represented by v if v corresponds to a mixture of $\tilde{v}$ and risk neutral preferences. This ordering is closely related to Ross's (1981) notion of preferences becoming "strongly more risk averse." Note that in the specific case of mean variance preferences, this comparison corresponds to a greater dislike of variance.

6.3 Comparative statics with respect to audience risk-aversion

While silence has the potential benefit of reducing risk for firms, it has the cost of increasing risk for audiences. If audiences are risk-averse, this in turn reduces firms' benefit from silence.

As noted above, greater audience risk aversion corresponds to larger values of $E\left[p_i\left(x\right)|\mu\right] - p_i\left(\mu\right)$. Equivalently, holding $p_i\left(x\right)$ constant, strictly greater audience risk aversion corresponds to strictly lower values of $p_i\left(\mu^S\right)$ for any non-null S. In Appendix B we show that this definition is equivalent to Pratt's risk-aversion ordering in a standard willingness-to-pay model.

Proposition 10 *Suppose that Condition 2 holds and a silence equilibrium exists. Suppose that audience j's risk aversion increases. Then all equilibria feature more disclosure than the equilibrium with the least amount of disclosure under audience j's original risk preferences; and the relation is strict if the original equilibrium has partial silence.*

Note that, in our setting, disclosure by a firm eliminates all risk for the audiences. However, the economic force in Proposition 10 continues to hold even in situations where disclosure reduces the risk faced by the audiences, instead of completely eliminating it.

7 Empirical evidence and applications

7.1 Silence when disclosure is costless and known to be feasible

An immediate implication of our analysis is that silence can arise even when disclosure is costless, and even when disclosure is known to be feasible. As discussed in the introduction, silence in these circumstances is often viewed as puzzling. As an example, a firm can certainly disclose the full details of its CEO's compensation package, and in many cases the direct costs of doing so are extremely low; but yet firms very frequently remain silent about many compensation details.

7.2 Disclosure and uncertainty about audience preferences

Beyond the existence of silence, the primary empirical prediction of our analysis is that silence is related to firm uncertainty about what it would be most beneficial to communicate to its audience (see, in particular, Proposition 2). This prediction is supported by a number of empirical studies, as we review below, which mostly fall under the rubric of a firm disclosing to a mix of investors and "antagonists."

Bova et al (2015) present evidence that firms facing employees with greater bargaining power (union representation, or tight local labor markets) are less likely to disclose "management guidance" to investors. In terms of our model: firms face an audience composed of a mixture of investors and employees. If firms are sufficiently uncertain about the relative desirability of moving investor and employee beliefs about future cash flows, our analysis implies that they choose silence over disclosure. In contrast, firms for which wage rates are determined primarily by employees' outside options do not face this uncertainty, and standard unraveling arguments predict that such firms disclose. Additionally, the authors find that greater employee stock ownership increases disclosure. In terms of our model, greater stock ownership removes the firm's uncertainty about what it would like to communicate to its audience; specifically, it would like to convince all audience members that future cash flows are high.

Hope et al (2013) present evidence that multinational firms that are likely using geographic "income shifting" to reduce taxes are less likely to disclose the geographic breakdown of earnings. In terms of our model: such firms face an audience composed of a mixture of investors, who would like to know to the geographic breakdown of earnings, and a mixture of "policy makers," "citizen groups" and "foreign tax authorities." As the authors put it, disclosure of "abnormally high geographic earnings in low-tax jurisdictions" would "potentially garner negative publicity from policy makers and citizen groups, attract the attention of foreign tax authorities, and possibly damage the manager's and the firm's reputation." If firms are sufficiently uncertain about the relative pros and cons of pleasing different parts of their audiences, our analysis implies that that will choose silence over disclosure. In contrast, firms that are not income-shifting do not face this uncertainty, and standard unraveling arguments predict that such firms disclose.[17]

Studying a period in which US firms had substantial discretion over whether or not to decompose operating performance across business segments, Harris (1998) and Botosan and Stanford (2005) present evidence that firms were less likely to report such a decomposition when some segments were operating in relatively uncompetitive industries. In terms of our model: a firm that has a business segment in an industry with little competitive pressure would like to convince investors that profits in this industry are high, but would like to convince potential entrants that profits in this industry are low. If such a firm is uncertain about the strength of latent competition from new entrants, our analysis predicts it is more likely to stay silent about its operating performance in this industry. Related also, many respondents in Graham et al's (2005) survey of executives cite a "concern that some disclosures might jeopardize the firm's competitive position in the product market" as a reason for non-disclosure.[18]

Firms are frequently silent about the details of executive compensation. In response, the U.S. has introduced a sequence of disclosure mandates, starting in the 1930s, as reviewed by Murphy (2012). Our analysis is consistent with Murphy's observation that, once disclosed, "executive contracts in publicly held corporations are not a private matter between employers and employees but are rather influenced by the media, labor unions, and by political forces operating inside and outside companies." If firms are unsure what the aggregate audience reaction will be to compensation disclosure, our analysis predicts that some firms stay silent—unless regulation forces disclosure.

[17]It is worth noting that Hope et al (2013) are very clear in *not* attributing their findings to a direct need of firms to hide income-shifting because it is in fact illegal.

[18]Returning to the discussion following Proposition 2: By itself, competitive pressures are not enough to generate silence, because if firms were simply interested in deterring competitors then they would try to convince outsiders that earnings are low, and the usual unraveling argument would apply (though starting from firms with low rather high earnings).

Separate from the investor-antagonist setting of the above cases, a distinct source of firm uncertainty about audience preferences arises if investors also care about non-financial outcomes. For example, if investors care about both financial and ESG performance; the two are at least partially in conflict; and the firm is uncertain about the strength of investors' ESG preferences, then our analysis predicts that some firms stay silent about their ESG performance. Likewise, if firms become more confident that they know investors' ESG preferences, then our analysis predicts that more firms reveal ESG performance. This provides one possible explanation of the empirically-observed increase in the number of firms reporting ESG scores (e.g., Amel-Zadeh and Serafeim, 2018).

7.3 Disclosure of imperfect signals of the underlying attribute

The above applications of our model are ones in which audiences directly care about the information the firm discloses. But in many cases, the information that a firm considers disclosing is instead valuable because it is correlated with what investors and other audiences members ultimately care about. For example, investors may be interested in CEO compensation or ESG performance primarily because it represents a signal about, among other things, the corporate governance of the firm, which in turn affect future cash flows. Importantly, in these cases investors may disagree about the correlation between the object being disclosed and future cash flows. For example, some investors may believe the correlation between CEO pay and future cash flows is positive, while others may believe just the opposite. The same is true for the correlation between ESG performance and future cash flows.

In this subsection we extend our model to analyze the disclosure of imperfect signals of an underlying attribute. By doing so, we offer another explanation of why some firms refrain from disclosing items such CEO compensation packages or ESG ratings (see preceding subsection).

Formally, let y be the future cash flow—or, more generally, some other underlying attribute that audiences care about. The firm cannot disclose y, but can disclose some other quantity x—e.g., CEO pay, or ESG performance—that is potentially correlated with y. Audiences care about cash flows y, but do not have direct preferences over x. For simplicity, audiences are risk neutral over y.

Although all audiences have the same preferences, they differ in what they believe

x reveals about y. Specifically, all audiences have the same prior of the distribution of y, with support $[0, 1]$. However, they differ in their assessment of the distribution of the signal x conditional on y. For simplicity, we focus on a stark case to illustrate our results. Each audience believes that x is either perfectly correlated with y, and specifically equals y; or that x is perfectly negatively correlated with y, and specifically equals $1-y$. Audience i attaches probabilities λ_i and $1-\lambda_i$ to these two possibilities.

Consequently, audience i's conditional expectation of y after observing x is[19]

$$E[y|x] = \lambda_i x + (1-\lambda_i)(1-x). \tag{12}$$

From (12), one can see that if an audience i believes that the signal is sufficiently likely to be positively (negatively) correlated with the underlying attribute, that is, $\lambda_i > (<) 1/2$, the conditional expectation is increasing (decreasing) in x. This setting is thus covered by our analysis, with $p_i(x) = E[y|x]$.

Importantly, in this setting differences between audiences arise even though all audiences have the same preferences over the underlying attribute (e.g., they all prefer higher cash flows to lower cash flows), but differ in other information, which leads them to form different beliefs after disclosure.[20]

As a potential application and empirical prediction: in practice, investor beliefs that items such as CEO pay and ESG ratings are negatively correlated with future cash flows are likely to stem from concerns about firm governance. As such, we predict that firms are more likely to stay silent about such items when there is substantial uncertainty about governance quality.

[19] In expression (12), an audience does not update its beliefs about whether x and y are positively or negatively correlated based on the observation of x. One interpretation is simply that different audiences have heterogenous prior beliefs about these possibilities. Alternatively, if y is symmetrically distributed over $[0, 1]$, then the observation of x does not generate any updating; in this case, (12) is consistent with audiences starting from a common prior, but different audiences subsequently observing different pieces of information that lead to different posteriors on whether x and y are positively or negatively correlated.

[20] Note that the heterogeneity in audience information is independent of the information the firm is disclosing, in contrast to Harbaugh and To (2020) and Quigley and Walther (2020). Related, the forces behind silence in our paper are very different from in these papers, as evidenced by the fact that firm risk-aversion plays a critical role in our results (see Proposition 3), while coarse disclosure and disclosure costs respectively play a critical role in Harbaugh and To (2020) and Quigley and Walther (2020).

7.4 Targeted disclosure and Regulation Fair Disclosure

As we noted, the main empirical prediction of our analysis is that silence is related to firm uncertainty about what it would be most beneficial to communicate to its audience. An immediate implication is that if a firm can cheaply *target* disclose to just a subset of audiences for which this uncertainty does not arise, then it will do so. As a leading example: in cases in which firms can talk privately to sophisticated institutional investors, without fear of information leaking, then they are likely to do so; and to be much more transparent in these conversations than in announcements to the broader public.

More formally, suppose that there is a subset of economic agents such that ordinal equivalence of preferences (Proposition 2) holds for all possible audiences drawn from this subset; and moreover, that it is common knowledge both that the firm is able to disclose solely to this subset, at zero cost, and that it can prevent all leakage of information beyond this subset. Under these conditions, the standard unraveling conclusion holds (again, Proposition 2), and any equilibrium entails full disclosure to this subset of agents.

A closely related implication is that laws and technological improvements that make targeted disclosure harder will—somewhat paradoxically—decrease rather than increase firm disclosures. Specifically, as just noted, when targeted disclosure is easy and feasible, equilibria feature full disclosure to groups for which the firm is certain about preference orderings. If instead targeted disclosure is impossible, then under the conditions that our analysis characterizes there are equilibria in which some firms stay silent and do not disclose to anyone.

A leading application is to U.S. Regulation Fair Disclosure (Reg FD), which mandates that any disclosure by a public firm must be fully public, and eliminates a firm's ability to target its disclosures.[21] In particular, we interpret the impact of Reg FD to be that once a firm discloses to all investors, it is also de facto disclosing to antagonists of the various types discussed above. A significant literature has studied the effects of Reg FD, and is surveyed by Koch et al (2013). As these authors note, "Many analysts expressed concerns that FD would inhibit disclosures because companies would

[21] Related but different from us, Guembel and Rossetto (2009) also argue that Reg FD may lead to less disclosure. In their model, unsophisticated audiences may misunderstand complex messages, and thus the firm prefer to disclose to sophisticated audiences only. Under Reg FD, therefore, the firm may prefer not to say anything rather than risk being misunderstood.

withhold information that had been previously selectively disclosed," often referred to as a "chilling effect." Koch et al summarize the evidence as "generally support[ing] a chilling effect for small or high-technology firms."

Similarly, technological change that reduces frictions in sharing information may result in less firm disclosure, because it undercuts a firm's ability engage in targeted disclosure. Indeed, anecdotal evidence suggests that firms and CEOs have been increasingly reluctant to make public remarks and "acting like a politician" due to the increasing use of digital communication and recordings, which implies that any gaffes may go viral and trigger backlash from unfavorable audiences.

7.5 Which firms remain silent?

In addition to predicting that firms are more likely to remain silent when uncertainty about audience preferences is greater, and when targeted disclosure is infeasible, our analysis makes a specific prediction on which firms remain silent—namely those with "extreme" information (Corollary 1). In many settings, this prediction is challenging to assess, since an econometrician does not observe the information possessed by firms that stay silent. But it could be potentially tested in settings in which a new mandatory disclosure requirement is introduced, and in which the information being disclosed is persistent over time. In such cases, the econometrician is effectively able to observe the information of firms who stayed silent in the voluntary disclosure regime.

8 Conclusion

There are many settings in which voluntary disclosure is possible, but in which disclosure occurs with probabilities below 1, despite classic unraveling arguments. In this paper we explore a possible explanation, which is new to the literature, namely that potential disclosers do not know the preference ordering of the audience to whom they are disclosing, and because of risk-aversion they dislike the risk that this imposes. We show how these two features together naturally deliver equilibrium silence.

In contrast to existing leading explanations of silence, our explanation does not require disclosure to be either costly, or impossible for some (unobservable) subset of would-be disclosers. As such, our paper can explain silence even in settings where disclosure is costless, and there is no uncertainty about whether disclosure is possible.

Our explanation captures the intuitive notion that a firm may prefer to stay silent because anything that it says will make some audiences very unhappy, while staying silent avoids this extreme outcome. That is, silence is safest. Specifically, silence reduces the risk borne by potential disclosers with extreme information. Consequently, disclosure decreases when potential disclosers grow more risk-averse, in a sense we make precise. On the other hand, silence reduces the information available to the audience for disclosures, thereby increasing the risk borne by the audience. Because of this, potential disclosers benefit more from disclosing when audiences grow more risk-averse, leading to increased equilibrium disclosure.

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A Direct benefits to silence

A subset of our results are predicated on the weak concavity of the payoff functions p_i. As discussed in Section 4, this condition has a natural economic interpretation. Moreover, concavity is also satisfied in the imperfect signal disclosure application in subsection 7.3 (see (12)).

Here, we briefly explore the opposite case in which the payoff functions are strictly convex. As noted in the main text, convexity of p_i introduces a direct gain to silence. Here we illustrate this point in more detail. Although this is not uninteresting, this force is separate from the effects due to firm uncertainty about the audience's type, and firm risk-aversion, both of which are necessary for silence, and so are central effects we wish to study.

We focus on the specific case in which all audiences are risk-neutral, and for all audiences i, there is a constant α_i such that $p_i(x) = v^{-1}(\alpha_i x)$. Since v is strictly concave, this implies that p_i is strictly convex. In this analytically very tractable case we show how the convexity of p_i generates a direct gain to silence, and in turn leads to an equilibrium with full silence.

In this case, the firm's expected value after disclosure, $J(x)$, is linear. Assuming that α_i does not have the same sign for all audiences (see Proposition 2), we can choose probabilities $\{\Pr(i)\}$ such that $J(x)$ has a slope arbitrarily close to 0. And whenever the slope is sufficiently close to 0, there is an equilibrium in which no one discloses, as we next show.

If all firms are silent, the firm's expected value after silence is

$$E\left[v\left(E\left[p_i(x)\,|\mu_0\right]\right)\right],$$

because audiences are risk-neutral ((6) at equality). Hence the expected gain from silence relative to disclosure for firm $\hat{x}$ is

$$\begin{aligned} E\left[v\left(E\left[p_i(x)\,|\mu_0\right]\right)\right] - J(\hat{x}) &= E\left[v\left(E\left[p_i(x)\,|\mu_0\right]\right)\right] - E\left[v\left(p_i\left(E\left[x|\mu_0\right]\right)\right)\right] \\ &+ J\left(E\left[x|\mu_0\right]\right) - J(\hat{x}). \end{aligned} \tag{A-1}$$

The sense in which convexity of p_i generates a direct benefit to silence is then that, since p_i is strictly convex, for any audience,

$$E\left[p_i\left(x\right)|\mu_0\right] - p_i\left(E\left[x|\mu_0\right]\right) > 0.$$

Thus, the first difference in (A-1) is the direct benefit to silence induced by the convexity of p_i, which is bounded away from 0. The second term in (A-1) approaches 0 as the slope of $J\left(x\right)$ approaches 0. So provided probabilities $\{\Pr(i)\}$ are chosen so that $J\left(x\right)$ has a slope sufficiently close to 0, there is indeed an equilibrium in which no one discloses. As discussed, this equilibrium outcome is driven by the fact that silence generates a direct benefit.

B Micro-foundation for audience risk-aversion

We give a micro-foundation for the firm's payoff p_i from an audience i. Consider the case in which the audience is buying something from the firm; for example, a product, service, or financial security. Let $p_i\left(x\right)$ be the amount that an audience would pay the firm if it knew the firm's type is x. Then for any audience beliefs μ about the firm type, let $p_i\left(\mu\right)$ be determined by

$$E\left[u\left(p_i(x) - p_i\right)|\mu\right] = u_i\left(0\right), \tag{A-2}$$

where u_i is continuous, strictly increasing and weakly concave, reflecting (weak) audience risk aversion. That is, (A-2) maps the primitive of an audience's willingness-to-pay given known type x to the audience's willingness-to-pay given beliefs μ. Inequality (6) in the main text (weak audience risk aversion) follows directly from (A-2).

Under the above micro-foundation for $p_i\left(\mu\right)$, it further follows that an increase in audience i's risk-aversion in the sense of Pratt (i.e., a concave transformation of u_i) corresponds to a decrease in $p_i\left(\mu\right)$, and hence an increase in $E\left[p_i\left(x\right)|\mu\right] - p_i\left(\mu\right)$, as stated in the main text prior to Proposition 10.

C Generalized disclosure

Thus far, we have considered the case in which the firm either discloses that its type is in the singleton set $\{x\}$, or else discloses nothing. Here we consider instead the case in which the firm can disclose any member A of some family of sets $\mathcal{X}$, provided that $x \in A$. We assume that, at a minimum, $\mathcal{X}$ contains all singletons, all closed subintervals of the interval X, and all binary unions of closed subintervals of X. To avoid economically uninteresting mathematical complications, we assume that all members of $\mathcal{X}$ are closed. Note that silence simply corresponds to disclosing X.

This enlarged set of disclosure possibilities is most likely to be relevant if disclosure takes the form of a trustworthy auditor reporting a firm's type x to audiences; or alternatively, if severe ex-post penalties can be inflicted on firms who are found to have lied (see discussion in Glode et al (2018)). If instead disclosure takes the form of simply displaying some attribute to audiences, then our benchmark analysis so far covers the relevant case.[22]

Note that this expansion of the firm's disclosure possibilities does not affect standard unraveling results. Indeed, it is straightforward to adapt the proofs of Propositions 2 and 3 to show that, under the conditions stated in these results, in any equilibrium a firm discloses $\{x\}$ with probability one.

Our main result in this section is that, given the expanded set of disclosure possibilities, an equilibrium with less than full disclosure—"silence" in the sense that the firm does not fully disclose its type—exists under a very wide range of circumstances. This is true if the key conditions we identify in this paper are satisfied, namely, firm risk-aversion, differences in audience preferences, and audiences who are not too risk-averse. In particular, we are able to establish existence of an equilibrium with less than full disclosure without imposing the sufficient condition that $J(0)$ is sufficiently close to $J(1)$, which we used to establish Proposition 6.

Proposition 11 *If (A) there exist $\underline{\xi}, \bar{\xi} \in (0,1)$ and a pair of some audiences i, j such that $\underline{\xi} \neq \bar{\xi}$, $J\left(\underline{\xi}\right) = J\left(\bar{\xi}\right)$, and $p_i(x) \neq p_j(x)$ for $x = \underline{\xi}, \bar{\xi}$, and (B) all audiences are sufficiently close to risk neutral, then there is an equilibrium with less than full disclosure, i.e., there is a positive probability of a firm disclosing a signal other than $\{x\}$.*

[22]Specifically, Glode et al (2018) analyze a setting in which the sender can disclose any subset of the type space that includes its own type. Their analysis also differs from ours in two other important respects. First, the receiver has all the bargaining power, which implies that any sender obtains zero surplus if it fully discloses its type. Second, their paper is primarily concerned with the case in which the sender can commit to a disclosure rule before seeing its type. As an extension, they also consider the non-commitment case, and show that partial disclosure survives as an equilibrium, since given the bargaining power assumption the sender prefers to preserve some uncertainty about its type in order to obtain at least some informational rent.

It is worth stressing that the condition (A) is satisfied whenever audiences have different preferences, and these different preferences generate non-monotonicity of the expected utility from disclosing $\{x\}$, as given by the function J.

The proof of Proposition 11 is very close to previous analysis, and we give it here. We establish the existence of an equilibrium characterized by $\underline{x}, \bar{x} \in (\underline{\xi}, \bar{\xi})$, in which firms with $x \in (\underline{x}, \bar{x})$ and $x \in X \backslash [\underline{\xi}, \bar{\xi}]$ disclose their exact type $\{x\}$; while the remaining firms with $x \in [\underline{\xi}, \underline{x}] \cup [\bar{x}, \bar{\xi}]$ disclose simply $[\underline{\xi}, \underline{x}] \cup [\bar{x}, \bar{\xi}]$.

The proof of Proposition 11 builds on the proof of Proposition 6. First, if one restricts firms to disclose either $\{x\}$ or $[\underline{\xi}, \underline{x}] \cup [\bar{x}, \bar{\xi}]$, the proof is the same as that of Proposition 6.[23]

It then remains to ensure that firms do not deviate to other disclosures. The equilibrium is supported by the following off-equilibrium beliefs: If the firm discloses $A \in \mathcal{X}$, and $A \neq [\underline{\xi}, \underline{x}] \cup [\bar{x}, \bar{\xi}]$, off-equilibrium beliefs place full mass on the firm's type being in $\arg\min_{\tilde{x} \in A} J(\tilde{x})$. These off-equilibrium beliefs immediately imply that firms with $x \in X \backslash ([\underline{\xi}, \underline{x}] \cup [\bar{x}, \bar{\xi}])$ do not have a profitable deviation. For firms with $x \in [\underline{\xi}, \underline{x}] \cup [\bar{x}, \bar{\xi}]$, note that these off-equilibrium beliefs ensure that any deviation is at least weakly worse than the deviation of disclosing $\{x\}$—which has already been established to be an unprofitable deviation, by the first step of the proof.

D Proofs of results stated in main text

Proof of Proposition 2: Let S be a non-null set. Write $N = \{1, 2, ..., |N|\}$. For use below, note that ordinal equivalence of the functions $p_i(x)$ and Assumption 1 imply that, for each i, there exists $x \in S$ such that $p_i(x) > E[p_i(x) | \mu^S]$.

We recursively define $x_1, \ldots, x_{|N|} \in S$ as follows. First, define $x_1 \in S$ such that $p_1(x_1) > E[p_1(x) | \mu^S]$. Next, suppose that $x_1, \ldots, x_{k-1}$ are defined, with the properties that $x_{k-1} \in S$, and $p_i(x_{k-1}) > E[p_i(x) | \mu^S]$ for all audiences $i = 1, 2, ..., k-1$. Then, define $x_k \in S$ such that $p_k(x_k) \geq p_k(x_{k-1})$ and $p_k(x_k) > E[p_k(x) | \mu^S]$. To see that such a choice is possible, note that if $p_k(x_{k-1}) > E[p_k(x) | \mu^S]$ then one can simply set $x_k = x_{k-1}$; while if instead $E[p_k(x) | \mu^S] \geq p_k(x_{k-1})$, let $x_k \in S$ be such that $p_k(x_k) > E[p_k(x) | \mu^S] \geq p_k(x_{k-1})$. Since $p_k(x_k) \geq p_k(x_{k-1})$, by ordinal

[23] Indeed, the fact that $\underline{\xi}, \bar{\xi} \in (0, 1)$ means that the proof avoids the complications of what happens to utility and density functions as $x \to 0, 1$, which is what allows use to dispense with the regularity conditions contained in Assumptions 2 and 3.

equivalence $p_i(x_k) \geq p_i(x_{k-1})$ for any audience i, and hence $p_i(x_k) > E\left[p_i(x)|\mu^S\right]$ for all audiences $i = 1, 2, ..., k$, establishing the recursive step.

So in particular, $v\left(p_i\left(x_{|N|}\right)\right) > v\left(E\left[p_i(x)|\mu^S\right]\right)$ for all audiences $i \in N$. By (6), $E\left[p_i(x)|\mu^S\right] \geq p_i\left(\mu^S\right)$. Hence $v\left(p_i\left(x_{|N|}\right)\right) > v\left(p_i\left(\mu^S\right)\right)$ for all audiences $i \in N$, implying that there exists $x_{|N|} \in S$ such that $J\left(x_{|N|}\right) > J\left(\mu^S\right)$, establishing Condition 1 and completing the proof.

Proof of Proposition 3: We establish that Condition 1 holds. Suppose to the contrary that there exists a non-null set S such that $J(\tilde{x}) \leq J\left(\mu^S\right)$ for all $\tilde{x} \in S$. Expanding $J\left(\mu^S\right)$, and using (6), for all $\tilde{x} \in S$,

$$J(\tilde{x}) \leq E\left[v\left(p_i\left(\mu^S\right)\right)\right] \leq E\left[v\left(E\left[p_i(x)|\mu^S\right]\right)\right].$$

Since v is weakly convex,

$$E\left[v\left(E\left[p_i(x)|\mu^S\right]\right)\right] \leq E\left[E\left[v\left(p_i(x)\right)|\mu^S\right]\right] = E\left[E\left[v\left(p_i(x)\right)\right]|\mu^S\right] = E\left[J(x)|\mu^S\right].$$

It follows that, for any $\tilde{x} \in S$,

$$J(\tilde{x}) \leq E\left[J(x)|\mu^S\right].$$

If v is strictly convex, the above inequality is strict, giving a contradiction. If instead v is linear, then the above inequality holds with equality, that is, $J(\tilde{x}) = E\left[J(x)|\mu^S\right]$ for almost all $\tilde{x} \in S$, which contradicts Assumption 1, completing the proof.

Proof of Proposition 4: Let S be a non-null set, and write $\bar{S}$ for the closure of S. By Assumption 1, there must exist an audience i and an $x \in S$ such that $\inf_{\tilde{x}\in\bar{S}} p_i(\tilde{x}) < p_i(x)$. For all audiences $j \neq i$, $\inf_{\tilde{x}\in\bar{S}} p_j(\tilde{x}) \leq p_j(x)$. Hence

$$J\left(\mu^S\right) = E\left[v\left(\inf_{\tilde{x}\in supp(\mu)} p_i(\tilde{x})\right)\right] < E\left[v(x)\right] = J(x),$$

establishing Condition 1 and completing the proof.

Proof of Proposition 5: By Assumption 1, $J(x)$ is either strictly increasing or strictly decreasing. We give the proof for the former case; the proof of the latter case is parallel. Let S be a non-null set of firms S. By property (AB), $J\left(\mu^S\right) \leq J\left(E\left[x|\mu^S\right]\right)$.

Hence $J\left(\mu^S\right) < J(x)$ for any $x \in S$ such that $x > E\left[x|\mu^S\right]$. So Condition 1 holds, completing the proof.

Proof of Lemma 1: Note that

$$(qv(p_i(x)) + (1-q)v(p_j(x))) - (qv(p_i(\tilde{x})) + (1-q)v(p_j(\tilde{x})))$$

is strictly positive at $q = 0$ and strictly negative at $q = 1$. Hence there exists $\hat{q} \in (0,1)$ at which this expression is 0. So if audience probabilities are given by $\Pr(i) = \hat{q}$, $\Pr(j) = 1 - \hat{q}$, with all other audiences having zero probability, then $J(x) = J(\tilde{x})$. Moreover, $J(x)$ is strictly concave by the concavity of $p_i(x)$ and firm risk-aversion. Hence J is non-monotone at this probability distribution, and by continuity, is likewise non-monotone in the neighborhood around this probability distribution.

Proof of Proposition 6: Under the stated conditions, there exists some distribution of audiences $\{\Pr(i)\}_{i\in N}$ such that $J(0) = J(1)$. We establish the existence of a silence equilibrium for this distribution, and for the case in which all audiences are risk neutral. The general result then follows by continuity.

Because audiences are risk neutral, silence payoffs are simply given by $p_i\left(\mu^S\right) = E\left[p_i(x)|\mu^S\right]$.

Note that the strict concavity of v and weak concavity of $p_i(x)$ implies that $J(x)$ is strictly concave. Define $x_{\max} = \arg\max_{\tilde{x}} J(\tilde{x})$.

If $J(x_{\max}) \leq E[v(E[p_i(x)|\mu_0])]$ then there is an equilibrium in which no firm discloses, and the proof is complete. So for the remainder of the proof, we consider the case in which

$$J(x_{\max}) > E[v(E[p_i(x)|\mu_0])]. \tag{A-3}$$

For any $\underline{x} \in (0, x_{\max})$, define $\eta(\underline{x}) \in (x_{\max}, 1)$ by $J(\eta(\underline{x})) = J(\underline{x})$. Note that $\eta(\underline{x})$ exists and is unique, since $J(0) = J(1)$ and $J(x)$ is strictly concave. Moreover, η is continuous, with $\eta(\underline{x}) \to 1$ as $\underline{x} \to 0$, and

$$\frac{\partial}{\partial \underline{x}}\eta(\underline{x}) = \frac{\frac{\partial}{\partial x}J(x)\big|_{x=\underline{x}}}{\frac{\partial}{\partial x}J(x)\big|_{x=\eta(\underline{x})}}.$$

Since $J(0) = J(1)$, and $J(x)$ is strictly concave, $\frac{\partial}{\partial x}J(x)$ remains bounded away from 0 as $x \to 0, 1$. Assumption 2 then implies that $\frac{\partial}{\partial \underline{x}}\eta(\underline{x})$ remains bounded away from both 0 and $-\infty$ as $\underline{x} \to 0$. Assumption 3 and l'Hôpital's rule then imply that the following limit exists, and is bounded away from 0:

$$\lim_{\underline{x}\to 0} \frac{\int_0^{\underline{x}} f(x)\,dx}{\int_{\eta(\underline{x})}^1 f(x)\,dx} = -\lim_{\underline{x}\to 0} \frac{f(\underline{x})}{f(\eta(\underline{x}))\frac{\partial}{\partial \underline{x}}\eta(\underline{x})}.$$

Strict concavity of v and the condition that there are audiences $i, j \in N$ such that $p_i(0) < p_i(1)$ and $p_j(0) > p_j(1)$ then implies that

$$\lim_{\underline{x}\to 0} E\left[v\left(E\left[p_i(x)\,|\mu^{X\setminus[\underline{x},\eta(\underline{x})]}\right]\right)\right] - E\left[E\left[v(p_i(x))\,|\mu^{X\setminus[\underline{x},\eta(\underline{x})]}\right]\right] > 0. \tag{A-4}$$

Also note that

$$E\left[E\left[v(p_i(x))\,|\mu^{X\setminus[\underline{x},\eta(\underline{x})]}\right]\right] = E\left[E\left[v(p_i(x))\right]|\mu^{X\setminus[\underline{x},\eta(\underline{x})]}\right] = E\left[J(x)\,|\mu^{X\setminus[\underline{x},\eta(\underline{x})]}\right].$$

Hence, and using $J(0) = J(1)$,

$$\lim_{\underline{x}\to 0}\left(E\left[E\left[v(p_i(x))\,|\mu^{X\setminus[\underline{x},\eta(\underline{x})]}\right]\right] - J(\underline{x})\right) = 0. \tag{A-5}$$

It follows by (A-4) that

$$J(\underline{x}) - E\left[v\left(E\left[p_i(x)\,|\mu^{X\setminus[\underline{x},\eta(\underline{x})]}\right]\right)\right] < 0$$

for all $\underline{x}$ sufficiently close to 0.

Combined with (A-3), continuity then implies that there exists some $\underline{x} \in (0, x_{\max})$ such that

$$J(\underline{x}) = J(\eta(\underline{x})) = E\left[v\left(E\left[p_i(x)\,|\mu^{X\setminus[\underline{x},\eta(\underline{x})]}\right]\right)\right] = J\left(\mu^{X\setminus[\underline{x},\eta(\underline{x})]}\right).$$

Hence there is an equilibrium in which firms $[\underline{x}, \eta(\underline{x})]$ disclose, while firms $X\setminus[\underline{x}, \eta(\underline{x})]$ remain silent and do not disclose, completing the proof.

Proof of Proposition 7: We prove the result for $J(1) > J(0)$; the case $J(1) < J(0)$ is parallel. Note that, because $v(x)$ is strictly concave and property (AB) holds, $J(x)$ is strictly concave as well.

Define $x_{\max} = \arg\max_x J(x)$. By supposition $0 < x_{\max} \leq E[x|\mu_0] < 1$. Define $h(x) : [x_{\max}, 1] \to [0, x_{\max}]$ by $J(h(x)) = J(x)$. Since $J(0) < J(1)$ and $J(x)$ is strictly concave, the function h is well-defined. Moreover, h is continuous and strictly decreasing.

On the one hand, $h(x_{\max}) = x_{\max}$, and so

$$E\left[x|\mu^{[0,x_{\max}]\cup\left[h^{-1}(x_{\max}),1\right]}\right] - x_{\max} = E[x|\mu_0] - x_{\max} \geq 0.$$

On the other hand, consider $\tilde{x} = h(1) < x_{\max}$, and so

$$E\left[x|\mu^{[0,\tilde{x}]\cup\left[h^{-1}(\tilde{x}),1\right]}\right] - \tilde{x} = E\left[x|\mu^{[0,\tilde{x}]}\right] - \tilde{x} < 0.$$

So by continuity, there exists $\underline{x} \in (\tilde{x}, x_{\max}]$ such that

$$E\left[x|\mu^{[0,\underline{x}]\cup\left[h^{-1}(\underline{x}),1\right]}\right] = \underline{x}.$$

Define $S = [0,\underline{x}] \cup [h^{-1}(\underline{x}), 1]$. Since property (AB) holds with equality, it follows that

$$J\left(\mu^S\right) = J\left(E\left[x|\mu^S\right]\right) = J\left(\underline{x}\right) = J\left(h^{-1}(\underline{x})\right).$$

Hence there is an equilibrium in which firms S stay silent, completing the proof.

Proof of Corollary 1: If silence is partial, the result is immediate from (AB) and the strict concavity of $J(x)$.

In the case of full silence, Proposition 1 implies that type $E[x|\mu_0]$ is indifferent between disclosure and silence, i.e., $J(\mu_0) = J(E[x|\mu_0])$. By the strict concavity of $J(x)$, it then follows that $J(x) < J(\mu_0)$ for $x \neq E[x|\mu_0]$. Setting $\underline{x} = \bar{x} = E[x|\mu_0]$ completes the proof.

Proof of Corollary 2: If silence is full, then from Corollary 1, $E\left[x|\mu^S\right] = \underline{x} = \bar{x}$. Inequality (11) then follows immediately from (7).

The remainder of the proof deals with partial silence. From Corollary 1, $S = [0,\underline{x}) \cup (\bar{x}, 1]$, with $\underline{x} < \bar{x}$. There are two cases. If $E[p_i(x)]$ is (weakly) monotone over $[\underline{x}, \bar{x}]$ then, by Corollary 1,

$$E\left[p_i\left(E\left[x|\mu^S\right]\right)\right] \leq \max_{x_m=\underline{x},\bar{x}} E\left[p_i\left(x_m\right)\right],$$

and (11) is immediate from (7).

If instead $E[p_i(x)]$ is strictly non-monotone over $[\underline{x}, \bar{x}]$, note first that Assumption

5 implies that $E[p_i(x)]$ is strictly single-peaked over X, with the peak lying in the interval $[\underline{x}, \bar{x}]$. Moreover, weak audience risk-aversion (6) implies

$$E\left[p_i\left(\mu^S\right)\right] \leq E\left[E\left[p_i(x)|\mu^S\right]\right] = E\left[E\left[p_i(x)\right]|\mu^S\right], \tag{A-6}$$

and so there exists $\hat{x}$ in the interior of S such that

$$E\left[p_i\left(\mu^S\right)\right] < E\left[p_i(\hat{x})\right]. \tag{A-7}$$

Hence either $\hat{x} < \underline{x}$ and $E\left[p_i\left(\mu^S\right)\right] < E\left[p_i(\underline{x})\right]$ or $\hat{x} > \bar{x}$ and $E\left[p_i\left(\mu^S\right)\right] < E\left[p_i(\bar{x})\right]$, completing the proof.

Proof of Proposition 8: Consider any partial silence equilibrium, with a silence set $[0, \underline{x}) \cup (\bar{x}, 1]$.

Claim A: For each audience i, $p_i\left(\mu^S\right) \leq \max\{p_i(\underline{x}), p_i(\bar{x})\}$.

Proof of claim: If p_i is monotone over $[\underline{x}, \bar{x}]$, then

$$p_i\left(\mu^S\right) \leq p_i(E[x|\mu^S]) \leq \max\{p_i(\underline{x}), p_i(\bar{x})\},$$

where the first inequality follows from (7), and the second inequality follows from Corollary 2 and the monotonicity of p_i over $[\underline{x}, \bar{x}]$.

If instead p_i is non-monotone over $[\underline{x}, \bar{x}]$, then by concavity, it is strictly increasing over $[0, \underline{x})$ and strictly decreasing over $(\bar{x}, 1]$. Hence $p_i(x) < \max\{p_i(\underline{x}), p_i(\bar{x})\}$ for all $x \in [0, \underline{x}) \cup (\bar{x}, 1]$. So by (6),

$$p_i\left(\mu^S\right) \leq E[p_i(x)|\mu^S] < \max\{p_i(\underline{x}), p_i(\bar{x})\}.$$

Claim B: For some $x \in \{\underline{x}, \bar{x}\}$, $p_i\left(\mu^S\right), p_j\left(\mu^S\right) \in \left[\min\left\{p_i(x), p_j(x)\right\}, \max\left\{p_i(x), p_j(x)\right\}\right]$.

Proof of Claim: Now consider any silence equilibrium in which the silence set is $[0, \underline{x}) \cup (\bar{x}, 1]$. The equilibrium condition implies that $p_i(\bar{x}) - p_i(\underline{x})$ and $p_j(\bar{x}) - p_j(\underline{x})$ have opposite signs. Without loss, assume $p_i(\underline{x}) \leq p_i(\bar{x})$ and $p_j(\bar{x}) \leq p_j(\underline{x})$. So Claim A implies $p_i\left(\mu^S\right) \leq p_i(\bar{x})$ and $p_j\left(\mu^S\right) \leq p_j(\underline{x})$. The equilibrium condition then implies $p_i\left(\mu^S\right) \geq p_i(\underline{x})$ and $p_j\left(\mu^S\right) \geq p_j(\bar{x})$, and so $p_i\left(\mu^S\right) \in \left[p_i(\underline{x}), p_i(\bar{x})\right]$ and $p_j\left(\mu^S\right) \in [p_j(\bar{x}), p_j(\underline{x})]$.

If the sets $[p_i(\underline{x}), p_i(\bar{x})]$ and $[p_j(\bar{x}), p_j(\underline{x})]$ are ranked by the strong set order (Veinott, 1989) then the result is straightforward: If $[p_i(\underline{x}), p_i(\bar{x})] \preceq [p_j(\bar{x}), p_j(\underline{x})]$ under this order, then $p_i\left(\mu^S\right), p_j\left(\mu^S\right) \in [p_i\left(\underline{x}\right), p_j\left(\underline{x}\right)]$; while if instead $[p_j(\bar{x}), p_j\left(\underline{x}\right)] \preceq [p_i\left(\underline{x}\right), p_i(\bar{x})]$, then $p_i\left(\mu^S\right)_i, p_j\left(\mu^S\right) \in [p_j\left(\bar{x}\right), p_i\left(\bar{x}\right)]$.

Next, consider the cases where the two sets $[p_i\left(\underline{x}\right), p_i(\bar{x})]$ and $[p_j(\bar{x}), p_j\left(\underline{x}\right)]$ are not ranked by the strong set order. There are two sub-cases. In the first sub-case, $[p_i\left(\underline{x}\right), p_i(\bar{x})] \subset [p_j(\bar{x}), p_j\left(\underline{x}\right)]$, and so either $p_i\left(\mu^S\right) \in [p_j\left(\bar{x}\right), p_i\left(\bar{x}\right)]$ or $p_j\left(\mu^S\right) \in [p_i\left(\underline{x}\right), p_j\left(\underline{x}\right)]$ (or both), while both $p_i\left(\mu^S\right) \in [p_j\left(\bar{x}\right), p_i\left(\bar{x}\right)]$ and $p_j\left(\mu^S\right) \in [p_i\left(\underline{x}\right), p_j\left(\underline{x}\right)]$. In the second sub-case, $[p_j(\bar{x}), p_j\left(\underline{x}\right)] \subset [p_i\left(\underline{x}\right), p_i(\bar{x})]$, and so either $p_i\left(\mu^S\right) \in [p_i\left(\underline{x}\right), p_j\left(\underline{x}\right)]$ or $p_i\left(\mu^S\right) \in [p_j\left(\bar{x}\right), p_i\left(\bar{x}\right)]$ (or both), while both $p_j\left(\mu^S\right) \in [p_i\left(\underline{x}\right), p_j\left(\underline{x}\right)]$ and $p_j\left(\mu^S\right) \in [p_j\left(\bar{x}\right), p_i\left(\bar{x}\right)]$.

Claim C: If $x_m \in \{\underline{x}, \bar{x}\}$ and $p_i\left(\mu^S\right), p_j\left(\mu^S\right) \in [\min\{p_i\left(x_m\right), p_j\left(x_m\right)\}, \max\{p_i\left(x_m\right), p_j\left(x_m\right)\}]$ then $E\left[p_i\left(\mu^S\right)\right] \leq E\left[p_i\left(x_m\right)\right]$.

Proof of Claim: If instead $E\left[p_i\left(\mu^S\right)\right] > E\left[p_i\left(x_m\right)\right]$ then Theorem 3 of Hammond (1974) implies that $E\left[v\left(p_i\left(\mu^S\right)\right)\right] > E\left[v\left(p_i\left(x_m\right)\right)\right]$, contradicting the equilibrium condition.

Completing the proof: From above, for at least one $x_m \in \{\underline{x}, \bar{x}\}$, we know $p_i\left(\mu^S\right), p_j\left(\mu^S\right) \in [\min\{p_i\left(x_m\right), p_j\left(x_m\right)\}, \max\{p_i\left(x_m\right), p_j\left(x_m\right)\}]$ and $E\left[p_i\left(\mu^S\right)\right] \leq E\left[p_i\left(x_m\right)\right]$, along with the equilibrium condition $E\left[v\left(p_i\left(\mu^S\right)\right)\right] = E\left[v\left(p_i\left(x_m\right)\right)\right]$. So for any increasing and strictly concave function ϕ, Theorem 3 of Hammond (1974) implies that

$$E\left[\phi\left(v\left(p_i\left(\mu^S\right)\right)\right)\right] \geq E\left[\phi\left(v\left(p_i\left(x_m\right)\right)\right)\right]. \tag{A-8}$$

Moreover, under Condition 2, Claim A holds strictly (by Corollary 2), and hence Claims B and C hold strictly also, and so (A-8) likewise holds strictly.

Given inequality (A-8), a straightforward modification of the argument in the proof of equilibrium existence in Proposition 6 implies that, for preferences $\tilde{v}$, there exists an equilibrium in which firms $[0, \underset{\sim}{x}) \cup (\tilde{x}, 1]$ do not disclose, where if $x_m = \underline{x}$ then $\underset{\sim}{x} > \underline{x}$, and if $x_m = \bar{x}$ then $\tilde{x} < \bar{x}$. This completes the proof.

Proof of Proposition 9: Given Corollary 1, when the firm's preferences are given by v, consider an equilibrium in which firms in $[0, \underline{x}) \cup (\bar{x}, 1]$ do not disclose. By Corollary 2, for some $x_m \in \{\underline{x}, \bar{x}\}$,

$$E\left[p_i\left(\mu^S\right)\right] < E\left[p_i\left(x_m\right)\right]. \tag{A-9}$$

It follows that

$$E\left[\tilde{v}\left(p_i\left(\mu^S\right)\right)\right] > E\left[\tilde{v}\left(p_i\left(x_m\right)\right)\right], \tag{A-10}$$

since otherwise (A-9) and the definition that $v(x) = \alpha\tilde{v}(x) + x$ at all $x \in X$ implies that

$$E\left[v\left(p_i\left(\mu^S\right)\right)\right] < E\left[v\left(p_i\left(x_m\right)\right)\right],$$

contradicting the equilibrium condition when the firm's preferences are given by v. Given (A-10), the result follows as in the last step of the proof of Proposition 8.

Proof of Proposition 10: Consider the equilibrium with the least amount of disclosure. For any marginal discloser x_m the equilibrium condition $E\left[v\left(p_i\left(\mu^S\right)\right)\right] = E\left[v\left(p_i\left(x_m\right)\right)\right]$ holds. Following the increase in audience j's risk-aversion, if the silence set stays unchanged then $p_j\left(\mu^S\right)$ strictly decreases (whereas $p_i(x_m)$ stays unchanged for any $i \in N$). Hence, for both marginal disclosers $x_m \in \{\underline{x}, \bar{x}\}$ we have $E\left[v\left(p_i\left(\mu^S\right)\right)\right] < E\left[v\left(p_i\left(x_m\right)\right)\right]$. The result follows as in the last step of the proof of Proposition 8.

Online Appendix

Silence is safest: information disclosure when the audience's preferences are uncertain

Not for publication

Analysis for the example in Section 2

Given the symmetry of the disclosure value $J(x)$, the silence set must take the form $S = [0, s) \cup (1 - s]$. Hence

$$E\left[x|\mu^S\right] = \frac{\int_0^s xf(x)\,dx + \int_{1-s}^1 xf(x)\,dx}{\int_0^s f(x)\,dx + \int_{1-s}^1 f(x)\,dx}.$$

Noting that $\int f(x)\,dx = (1-a)\,x + ax^2$ and $\int xf(x)\,dx = \frac{1}{2}(1-a)\,x^2 + \frac{2}{3}ax^3$,

$$\begin{aligned} E\left[x|\mu^S\right] &= \frac{\frac{1}{2}(1-a)\left(s^2 + 1 - (1-s)^2\right) + \frac{2}{3}a\left(s^3 + 1 - (1-s)^3\right)}{(1-a)\left(s + 1 - (1-s)\right) + a\left(s^2 + 1 - (1-s)^2\right)} \\ &= \frac{(1-a)\,s + \frac{2}{3}a\left(2s^3 - 3s^2 + 3s\right)}{2s} \\ &= \frac{2}{3}as^2 - as^2 + \frac{1}{2}(1+a)\,. \end{aligned}$$

For the remainder, we focus on $a \geq 0$; the case $a \leq 0$ follows by symmetry.

Since $E\left[x|\mu^S\right] \geq s$, the equilibrium condition is

$$\frac{2}{3}as^2 - as + \frac{1}{2}(1+a) = 1 - s,$$

i.e., firm type $1-s$ is indifferent between silence, given audience beliefs $E\left[x|\mu^S\right]$, and disclosure. Hence the equilibrium value of s must solve

$$\frac{2}{3}as^2 + (1-a)\,s - \frac{1}{2}(1-a) = 0.$$

If $a = 0$ this immediately implies $s = \frac{1}{2}$. For $a \in (0, 1]$, note that the quadratic is negative when evaluated at $s = 0$ and strictly positive when evaluated at $s = \frac{1}{2}$, so the equilibrium is given by the upper root,

$$\begin{aligned} s &= \frac{-(1-a) + \sqrt{(1-a)^2 + \frac{4}{3}a(1-a)}}{\frac{4}{3}a} \\ &= \frac{-(1-a) + \sqrt{(1-a)\left(1 + \frac{1}{3}a\right)}}{\frac{4}{3}a}. \end{aligned}$$

Proof of no updating in correlation discussion in subsection 7.3

Suppose x and y are both distributed over $[0, 1]$; y has a fixed distribution; x is either perfectly correlated, and equals y, or is perfectly negatively correlated, and equals $1 - y$.

By Bayes rule:

$$\begin{aligned} \Pr(+\text{ve corr}|x) &= \frac{\Pr(+\text{ve corr and } x)}{\Pr(+\text{ve corr and } x) + \Pr(-\text{ve corr and } x)} \\ &= \frac{\Pr(x| + \text{ve corr})\Pr(+\text{ve corr })}{\Pr(x| + \text{ve corr})\Pr(+\text{ve corr }) + \Pr(x| - \text{ve corr})\Pr(-\text{ve corr })}. \end{aligned}$$

Hence there is no updating after seeing x if

$$\Pr(x| + \text{ve corr}) = \Pr(x| - \text{ve corr}),$$

which is equivalent to

$$\Pr(y) = \Pr(1 - y).$$