Failing to forecast rare events

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March 26, 2021

Do more talented traders prefer to bet on and against rare events or common events? Bets on rare events include out of the money options. Bets against rare events include the carry trade and investment grade bonds. In a model where traders specialize, equilibrium pricing reflects trading ability: a market with more skilled traders has a larger bid ask spread. We show that lower skill traders bet on and against rare events, while higher skill traders bet on and against frequent events, leading to higher bid-ask spreads in common event assets, and reducing financial markets' ability to predict rare events.

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We thank Christopher Hennessy, Igor Makarov, Stavros Panageas, Anna Pavlova, Edward van Wesep and seminar audiences at Arizona State University, CUHK, the Frankfurt School of Finance and Management, HEC Paris, HKU, London Business School, Northeastern University, the University of Washington, the Jackson Hole Finance Conference, the Financial Intermediation Research Society, the Western Finance Association, and the Paul Woolley Centre for the Study of Capital Market Dysfunctionality, for helpful comments. We are especially grateful to Lukas Kremens for sharing data on CDS prices. Any errors are our own. Declarations of interest: none.

1 Introduction

One of the main functions of financial markets is forecasting. However, many observers have expressed concern that, as they perceive it, most forecasting activity is devoted to forecasting frequent but unimportant events. The financial system has been criticized for its failure to predict the financial crisis of 2007-08.[1] Taleb (2007) asks "[w]hy do we keep focusing on the minutiae, not the possible significant large events, in spite of the obvious evidence of their huge influence?" Relatedly, many commentators have criticized the "quarterly earnings cycle" and the amount of effort spent forecasting firms' next earnings announcements (see, e.g., Kay (2012)).[2] Relatedly also, others have voiced concerns that the risk-management departments of financial institutions—whose job is to predict and mitigate infrequent but critical events—have trouble recruiting and retaining high-quality employees (e.g., Palm, 2014).[3]

In this paper we analyze the economic incentives for forecasting events of different frequencies. Are there systematic economic forces that make agents focus on predicting everyday events as opposed to rare events? Specifically, since trading is the main way that agents in financial markets profit from information, are there forces that favor trading securities whose payoffs depend on frequent events? Do traders of different skills trade different kinds of securities? Does the aggregate amount of skill dedicated to predicting rare and frequent events differ? And are rare events more or less likely to be predicted as a result?

Despite the importance of information production to financial economics, these questions have received little research attention, whether theoretical or empirical. As a first step, we consider a model with ingredients that are both standard and minimal—specifically, a Glosten-Milgrom (1985) model of equilibrium in the financial market under asymmetric information and a Roy (1951) model of equilibrium in the labor market. Individuals choose

[1] *Financial Times*, November 25 2008, "The economic forecasters' failing vision."
[2] *Financial Times*, February 29, 2012, "Investors should ignore the rustles in the undergrowth."
[3] *American Banker*, September 9, 2014, "Why Banks Face a Risk Management Talent Shortage."

between the two "occupations" of trading a "common event" asset, a security that pays off on an event that is reasonably likely, and trading an alternative "rare event" asset that pays off on an event that is highly unlikely.

Our main results are as follows. First, we identify a strong economic force that sorts agents into trading different assets depending on their skill. Traders sort into three groups: those with high skill trade on the common event, those with less skill trade on the rare event, and those with the lowest skill levels don't trade at all. Second, the common event asset has a higher bid-ask spread. Third, trading data is more informative about the common event and so financial markets have lower ability to predict the rare event.

Our analysis of trading on rare events applies equally to "safe" trades that pay off with very high probability, such as long positions in highly-rated bonds, the carry trade, and "selling volatility" (selling out of the money puts). This is because a long position in a high-payoff-probability assets (e.g., a highly rated bond) has identical cash flows to the combination of a long position in a risk-free bond and a short position in a rare event asset (e.g., selling a CDS on a highly rated bond).

What drives these results? A starting point is to observe that position limits play a part. An attraction of the rare event asset is that its price is low, allowing large long positions. But this reasoning alone is too simplistic: if many informed traders buy the asset, this is reflected in a higher ask price, reducing the size of the positions they can take. A full equilibrium analysis must consider this effect.

Strikingly, our main results hold for any reasonable position limits. Traders are subject to position limits that potentially depend on price, ability to repo, margin requirements, risk management rules, etc. We do not assume any specific functional form for position limits, and our main results hold for any position limits in a very general class. In addition, we also present some stronger results for the natural special case in which position limits are set to eliminate the possibility of default.

To convey the intuition for our results, first imagine that traders could buy and sell assets with zero bid ask spread at their expected values (in fact, a zero bid-ask spread cannot arise in equilibrium, because informed traders would make large profits at the expense of other traders). The trading patterns in the two assets would be very different: in the common event market, skilled traders would take moderately sized positions, either long or short; in the rare event market they would occasionally predict that the rare event rare event asset will pay off, taking very large long positions, but most of the time they would hold small short positions. Next, consider how this changes when prices are determined in equilibrium.

Our simple yet central observation is that the bid-ask spread on the rare event asset must be bounded away from zero as the probability of the rare event approaches zero. This is because if, instead, the bid-ask spread converged to zero, trading the rare event asset would be profitable for even the lowest-skilled traders. But then, non-negligible trading skill would be devoted to the rare event asset, leading to a non-negligible bid-ask spread.

Because the bid-ask spread on the rare event asset is bounded away from zero, traders are unable to adopt very large long positions in the rare event asset. Also, while most of the time traders in the rare event asset take short positions, these are not particularly profitable since the bid price of the asset is negligible. Hence prediction skill has low value when it is devoted to the rare event, which leads to our result that the highest skill traders choose to trade the common event asset. Moreover: because higher talent is allocated to the common event asset, it has a higher bid-ask spread.

Next consider financial markets'—as opposed to individuals'—ability to predict rare events. How much can one learn about the likelihood of an event by observing market data such as order flow or average transaction price? Since the most skilled agents trade the common event asset, while the rare event asset is traded by only relatively unskilled traders, then unless only a very few highly skilled traders specialise in the common event asset, and a great many lower skill traders specialise in the rare event asset, there is more total skill at work predicting the common event. So normally, aggregate trading activity contains more information about common events than about rare events.

Our model omits many additional and plausibly important forces, for example, execution skill and agency problems. However, in terms of empirical predictions, we emphasize three points.

First, our prediction on the allocation of skill matches informal perceptions that a lot of forecasting "talent" is devoted to forecasting frequent events, and separately, that "nickels in front of a steamroller" strategies #(e.g., the carry trade, selling out-of-the-money put options) are often implemented by people with mediocre talents. Second, our analysis predicts that low-rated bonds (short positions in common event assets) have larger bid-ask spreads than high-rated bonds. This is consistent with empirical evidence, including evidence that controls for other factors (see Section 6). Third, our analysis predicts that professional investors who specialize in common event assets earn higher returns than those who specialize in rare event assets. #We review cross-sectional evidence on performance within broad classes of funds, which is consistent with this prediction (Section 6). But this prediction would be better evaluated using trader-specific data, and among other things, we hope that our paper encourages greater attention—both theoretical and empirical—on the internal division of labor within investment firms.

Related literature:

Existing literature on information acquisition mostly deals with ex ante homogeneous investors dividing their information acquisition efforts across different assets. In contrast, we study matching between heterogeneous investors (different skills) and heterogeneous assets (different payoff frequencies). That is, we study the inter-personal division of labor in information acquisition, while the existing literature focuses on the intra-personal issues.

Van Niewerburgh and Veldkamp (2010) analyze investors who choose which assets to acquire information about in order to improve portfolio allocation, and establish conditions for specialization in acquiring information about just one asset. An important difference with our paper is that in our analysis asset prices, including the bid-ask spread, are determined endogenously. Veldkamp (2006) analyzes a multi-asset REE model in which traders buy information from information providers who enjoy economies of scale. Information delivers trading profits at the expense of less informed agents, and improves portfolio allocation. She shows that different traders choose the same signals, increasing asset price comovement. In Peng and Xiong (2006) the representative investor has a cognitive constraint (which could alternatively be interpreted as a cost of information production), and chooses signals that are informative about many assets. Information improves the consumption-savings decision, while in our paper information helps traders make money from uninformed traders.

Buffa and Javadekar (2019) study the allocation of managers of different skills across mutual fund strategies. They argue that stock picking generates a career track record with many observations that are informative about skill, and therefore attracts highly-skilled traders since they want their skill to become known. In contrast, market timing strategies are less informative about skill and are chosen by lower skilled managers in equilibrium.

Gandhi and Serrano-Padial (2015) also study pricing of rare and common event assets. They argue belief heterogeneity can explain the favorite-longshot bias in sporting bets (empirically, bets on competitors with a low chance of winning are overpriced). In their model, a small (but fixed) fraction of gamblers who are overoptimistic about longshots bet all their money on them as their probability of winning converges to zero, stopping the price of longshot bets converging to zero. They are the marginal buyers of the longshot bets, as short sales are assumed impossible. In contrast, in our model different trader valuations result from different private signals with the same prior beliefs, and there is a bid-ask spread because uninformed agents learn from the demands of informed agents.

2 Model

2.1 Assets

There are two financial assets, the r-asset and the c-asset ("rare" and "common"). Each asset pays either 0 or 1 (in other words, the asset price should be understood as the price per unit of payoff if the asset pays off). We model the assets as associated with two underlying independent random variables, ψ^r and ψ^c, each distributed uniformly over $[0,1]$, with each asset $j = r, c$ paying off in the event $\psi^j \leq q^j$, for parameters $q^r, q^c \in (0,1)$ that equal the payoff probabilities of the two assets. This formulation allows us to study comparative statics as q^r grows small—i.e., the r-asset pays off rarely—in a natural way. As will be seen below, the properties of informed traders' signals and the behavior of liquidity traders respond in economically consistent ways.

We focus on the case in which q^r approaches zero, i.e., the r-asset only pays off rarely. This allows us to obtain results with only very mild assumptions on position limits. However, in Online Appendix D we also show that in the economically natural case of default-free position limits (see (3)), our main results hold for *any* pair of asset payoff probabilities with $\min\{q^r, 1-q^r\} < \min\{q^c, 1-q^c\}$.

There is a single period in which the assets trade, after which payoffs are realized. Traders who take long (short) positions in the j-asset buy (sell) at the ask (bid) price price P_L^j (respectively, P_S^j). The following subsection covers price determination.

2.2 Financial market structure

The r- and c-assets are traded by a mixture of skilled traders, who receive informative signals about one of ψ^r and ψ^c , and liquidity traders. Both groups are described further below.

Long and short trades are executed, respectively, at ask and bid prices P_L^j and P_S^j, which are set by the zero profit condition of a competitive market maker. Traders arrive simultaneously and fulfill their orders at these prices. The expected quantity of buys and sells for the j-asset depends on the realization of the state ψ^j. Notationally, we write $E\left[\text{buys}|\psi^j \leq q^j\right]$ for the expected measure of buy orders stemming from skilled traders and liquidity traders, conditional on the event $\psi^j \leq q^j$, with analogous notation for other cases.

Each market maker takes into account the equilibrium skill and behavior of skilled and liquidity traders when posting prices:

$$\begin{aligned} E\left[\text{buys}|\psi^j \leq q^j\right] \Pr\left(\psi^j \leq q^j\right)\left(P_L^j - 1\right) + E\left[\text{buys}|\psi^j > q^j\right] \Pr\left(\psi^j > q^j\right) P_L^j &= 0 \\ E\left[\text{sales}|\psi^j \leq q^j\right] \Pr\left(\psi^j \leq q^j\right)\left(1 - P_S^j\right) + E\left[\text{sales}|\psi^j > q^j\right] \Pr\left(\psi^j > q^j\right)\left(-P_S^j\right) &= 0. \end{aligned}$$

Rearranging and simplifying gives

$$P_L^j = q^j \frac{E\left[\text{buys}|\psi^j \leq q^j\right]}{E\left[\text{buys}\right]} \tag{1}$$

$$P_S^j = q^j \frac{E\left[\text{sales}|\psi^j \leq q^j\right]}{E\left[\text{sales}\right]}. \tag{2}$$

This price setting mechanism is similar to that in Glosten and Milgrom (1985). The interpretation of the zero profit condition is that there are many market makers each posting binding quotes for bid and ask prices.

2.3 Skilled traders

There is a unit continuum of risk-neutral skilled traders. Each trader observes either an informative signal or noise. When a trader observes a signal $s^j \in [0,1]$ no-one, including the trader, knows whether the signal is informative or not. However, there is heterogeneity in traders' chances of getting an informative signal: each trader knows their probability α of receiving an informative signal. We refer to α as the trader's "skill." The population distribution of α is given by probability measure μ, defined over the Borel sets $\mathcal{B}$ of $[0,1]$, which admits a density g. The support of μ is a lower interval, i.e., takes the form $[0,\bar{\alpha}]$ for some $\bar{\alpha} > 0$. A trader with skill α who chooses to observe a signal about ψ^j observes the true realization with probability α, and otherwise observes the realization of a noise term uniformly distributed over $[0,1]$. This assumption has the natural property that the unconditional probability distribution of signals is the same for all α.

Collecting information takes time. To capture this, we assume that signals have an opportunity cost: each trader must choose between receiving signals about ψ^r or signals about ψ^c. They also have the option of not trading. After observing their signals, traders choose whether to trade (at this stage, we allow for the possibility they might choose an asset but only trade sometimes, although this is not an equilibrium outcome). They can take either long or short positions. Let $V^j(\alpha)$ denote the expected payoff of a skilled trader with skill α who specializes in the j-asset for $j \in \{r, c, 0\}$. The payoff of a trader who chooses not to trade either asset is $V^0(\alpha) = 0$.

Instead of assuming traders are risk neutral, we equivalently could assume traders are risk-averse but are insured by risk-neutral employers. In subsection 8.1 we extend our analysis to cover risk aversion. Also, while we focus on a single period model, in Online Appendix I we show that our main results continue to hold in a dynamic setting in which agents learn about their skill from their past trading outcomes.

2.4 Position limits

Traders face position limits, corresponding to margin constraints and limits on borrowing. We denote the long and short position limits in asset j by $h_L^j\left(P_L^j\right)$ and $h_S^j\left(P_S^j\right)$ respectively, where the notation reflects the natural property that position limits may depend on prices. Position limits are important because a potential attraction of the r-asset is that its price is low, so a trader can scale up profits by buying large amounts; position limits determine how much. In practice position limits are determined by traders' budget constraints, by margin requirements, and by the ability to use purchased securities or sales proceeds as collateral; consequently, low prices typically allow larger long positions but smaller short positions.

We do not restrict position limits to be any particular function. Indeed, we allow for nearly any possible specification of position limits, making only the following pair of mild assumptions:

Assumption 1 h_L^j *and* h_S^j *are continuous functions over* $(0, \infty)$ *and take strictly positive values.*

Assumption 2 $\lim_{P \to 0} P h_S^j (P) = 0$.

Assumption 1 is very weak. It includes the case in which h_L^j and/or h_S^j is close to 0 for all prices, corresponding to long and/or short positions being almost impossible. (The only reason that we do not simply allow $h_L^j \equiv 0$ or $h_S^j \equiv 0$ is that in these cases ask and bid prices respectively are undefined in equilibrium, complicating the statements of many results.)

Assumption 2 requires that short positions do not grow too fast as the rare event becomes rarer and its price (presumably) falls. This is a very weak assumption because lower prices mean lower short proceeds to collateralize future obligations, which would normally lead to *lower* position limits.[4] There is no need for an analogous assumption on long position limits $h_L^j (P)$ because, in equilibrium, ask prices P_L^j are bounded away from zero (Lemma 2).

As a concrete example, an economically natural case is default-free position limits, where traders can take the largest positions that allow them to meet their obligations in all states. In this case, position limits for a trader with initial wealth W are[5]

$$h_L^j \left(P_L^j \right) = \frac{W}{P_L^j} \text{ and } h_S^j = \frac{W}{1 - P_S^j}. \tag{3}$$

A second concrete example is price-invariant position limits, e.g., traders can trade one unit, regardless of the price. Formally, price-invariant position limits are simply $h_L^j (\cdot)$ and $h_S^j (\cdot)$ both constant. Although this assumption is commonly made in the literature, it is important to consider the (realistic) possibility that traders can buy larger quantities of cheaper assets, since this is one the main potential attractions of the r-asset.

Both default-free (3) and price-invariant position limits satisfy Assumptions 1 and 2.

[4]FINRA rule 4210 requires $h_S^j (P) = W/2.5$ as $P \to 0$, which satisfies this requirement. (For larger P, minimum required margin is a percentage of position value, rather than a fixed \$2.50 amount per share.)

[5]The long position limit follows from the fact that, since the asset may pay 0, leveraged positions are impossible. The short position limit arises as follows. A trader who short sells x units has total wealth $W + xP_S^j$, which is sufficient collateral for $W + xP_S^j$ short positions. So the largest feasible short position is given by the solution to $x = W + xP_S^j$.

2.5 Liquidity traders

In addition to skilled traders, there is a continuum of traders with no information about asset realizations. We assume these "liquidity traders" trade for hedging purposes (Diamond and Verrecchia (1981), Dow and Gorton (2008)). Each liquidity trader receives an endowment shock that gives them a strong desire for resources in a particular state. A measure λ^r of liquidity traders are r-liquidity traders, and each receives a shock $\chi^r \sim U[0,1]$, meaning that they want resources in state $\psi^r = \chi^r$. Similarly, a measure λ^c of liquidity traders are c-liquidity traders, and each receives a shock $\chi^c \sim U[0,1]$, meaning they want resources in state $\psi^c = \chi^c$. Except in Section 7, we make no assumption on whether and how liquidity shocks are correlated across liquidity traders. We assume that j-liquidity trader preferences for resources in state χ^j are lexicographic, so that each j-liquidity trader takes as large a long position as possible in the j-asset as possible if $\chi^j \leq q^j$, and as large a short position as possible if $\chi^j > q^j$.[6] The long and short position limits for j-liquidity traders are the same as for skilled traders, namely h_L^j and h_S^j. Given this, j-liquidity traders each buy $h_L^j(P_L^j)$ units of the j-asset if they experience a shock $\chi^j \leq q^j$, and short sell $h_S^j(P_S^j)$ units of the j-asset if they experience a shock $\chi^j > q^j$.

Consequently, the expected number of buy orders for the j-asset from liquidity traders is $q^j \lambda^j h_L^j(P_L^j)$, and the expected number of sell orders is $(1-q^j)\,\lambda^j h_S^j(P_S^j)$. In particular, as the probability q^r that the r-asset pays off approaches 0, the expected number of liquidity traders who place buy orders approaches 0.

The following concrete interpretation may be helpful. A natural interpretation of the r- and c-assets is as CDS contracts on high- and low-rated borrowers. The CDS on the high-rated borrower pays off only in a few states of the world, so only a few liquidity traders need insurance against one of these small number of states and buy it. The CDS on the

[6] Since liquidity traders trade to the position limit, their trades are price elastic. Our main results also hold if instead liquidity traders are price inelastic (see discussion in subsection 8.2), or if they are more price elastic than skilled traders (this latter case would strengthen our results, akin to other perturbations discussed in subsection 8.2).

low-rated borrower is bought by more liquidity traders.

Some readers may prefer an alternative interpretation of our formal assumptions in which "liquidity" traders are instead overconfident traders. Traders who are unskilled ($\alpha = 0$) but who mistakenly believe they are highly skilled[7] behave exactly as just described.

The volume of liquidity trade affects equilibrium prices and hence the trading decisions of skilled traders. Our analysis requires assumptions on how the volume of liquidity trade behaves as the probability q^r of the rare event gets smaller. Our specification of liquidity trade has the attractive feature that if skilled traders were randomly allocated (without regard to talent, but proportionally to λ^r and λ^c) between the r- and c-asset, then an individual skilled trader would find the r- and c-assets equally attractive to trade, independent of the probabilities q^r and q^c, and the bid-ask spread would be the same. In this sense, our liquidity trade assumptions represent a natural case. They ensure we are not making an assumption that directly implies the r-asset has a vanishingly small bid-ask spread, which would make it easy for the least-skilled among the skilled traders to profit by trading it. See also subsection 8.2 for a discussion of alternative specifications.

3 Equilibrium in financial and labour markets

Consider a skilled trader of skill α who specializes in the j-asset. The trader receives a "buy" signal $s^j \leq q^j$ with probability q^j. Conditional on this the expected payoff on the asset is $\alpha + (1-\alpha) q^j$: here, α is the probability the signal was informative, $(1-\alpha) q^j$ is the probability the signal was uninformative but correct anyway. Profits are expected value, minus price, multiplied by size of the position: $h_L^j \left(P_L^j\right) \left(\alpha + (1-\alpha) q^j - P_L^j\right)$. Similarly if the trader receives a "sell" signal $s^j > q^j$ then profits are $h_S^j \left(P_S^j\right) \left(P_S^j - (1-\alpha) q^j\right)$. Given that traders may choose not to trade if their skill α is too low, a trader's expected payoff

[7] Specifically: They believe their skill levels are high enough to profitably trade. Lemma 1 characterizes minimum skill levels for profitable trading.

from trading the j-asset, $V^j(\alpha)$, is given by[8]

$$\begin{aligned} V^j(\alpha) &= q^j \max\left\{0, h_L^j\left(P_L^j\right)\left(\alpha + (1-\alpha)\, q^j - P_L^j\right)\right\} \\ &+ \left(1-q^j\right) \max\left\{0, h_S^j\left(P_S^j\right)\left(P_S^j - (1-\alpha)\, q^j\right)\right\}. \end{aligned} \tag{4}$$

3.1 Benchmark: No financial market equilibrium

We start by considering how traders would allocate themselves if assets were simply priced at their expected values (with no bid-ask spread), i.e.,

$$P_L^j = P_S^j = q^j, \tag{5}$$

instead of satisfying the equilibrium property that prices reflect informed trading. For this exercise, we assume default-free position limits (3). A trader of skill α specializing in the j-asset has expected profits (4), which reduces to $V^j(\alpha) = \alpha W$. Importantly, this is the same for both assets; traders are indifferent between the two assets, regardless of skill level. So this benchmark illustrates that our model treats the two assets neutrally, and is not based on biased assumptions that automatically imply that one asset is easier to profitably trade than the other. This conclusion depends on the availability of both long and short positions. As the asset becomes rare, long positions grow more profitable while short positions become less profitable. If only long positions are possible, the expected payoff to specializing in the j-asset is $\alpha W(1-q^j)$, so trading the r-asset is more profitable. In line with this, one might suppose that the r-asset is attractive because it is so cheap that very large positions are possible. As we argue below, this supposition is fallacious because it fails to recognize that equilibrium bid and ask prices respond to the level of informed trading activity.

[8] It is straightforward to verify that if $P_S^j \leq q^r$ then a skilled trader would never sell after observing $s^j \leq q^j$. Similarly, if $P_L^j \geq q^r$ then a skilled trader would never buy after observing $s^j > q^j$. We verify below that $P_L^j \geq q^j \geq P_S^j$ indeed holds in equilibrium.

3.2 Equilibrium in both financial and labor markets

With skilled traders present, assets do not trade at their unconditional expected values; they trade at prices that reflect the incidence of informed trading. Our analysis jointly characterizes asset prices and traders' choices of which asset to trade. Equilibrium in financial markets requires that prices reflect the level of informed trade. Equilibrium in labor markets requires that, given asset prices, traders optimally choose which asset to specialize in.

As (4) makes clear, some traders have too little skill to profitably trade either asset, and prefer to do nothing. Others specialize in trading the r- or c-asset. In cases in which a trader is indifferent between specializing and doing nothing, we assume the latter.[9]

For the following equilibrium definition, let $M(\mu)$ denote the support of μ.

Definition 1 *An equilibrium consists of prices* $(P_L^r, P_S^r, P_L^c, P_S^c)$ *and an allocation of skilled traders* (μ^r, μ^c, μ^0) *across the* r*-asset, the* c*-asset and doing nothing, such that:*

1. Labor market equilibrium:

(a) Traders optimize over choice of asset: $V^r(\alpha) \geq V^c(\alpha)$ *and* $V^r(\alpha) > 0$ *for almost all* $\alpha \in M(\mu^r)$; $V^c(\alpha) \geq V^r(\alpha)$ *and* $V^c(\alpha) > 0$ *for almost all* $\alpha \in M(\mu^c)$; *and* $V^r(\alpha) = V^c(\alpha) = 0$ *for almost all* $\alpha \notin M(\mu^r) \cup M(\mu^c)$.

(b) Labour markets clear: $\mu^r(B) + \mu^c(B) + \mu^0(B) = \mu(B)$ *for all Borel sets* $B \in \mathcal{B}$.

2. Financial market equilibrium: Given profit-maximizing trading by skilled traders, prices satisfy (1) and (2).

[9] From (4), there is at most a single skill level at which an agent is indifferent between trading and doing nothing. So nothing is at stake in what indifferent agents do.

4 Prices conditional on skill allocation

We first solve for the financial market equilibrium given the allocation of skill. Given a labour market allocation (μ^r, μ^c, μ^0), write A^j for the aggregate skill in asset j, i.e.,

$$A^j \equiv \int \alpha \mu^j (d\alpha), \tag{6}$$

and N^j for the mass ("number") of skilled traders in asset j, i.e.,

$$N^j \equiv \int \mu^j (d\alpha). \tag{7}$$

Define

$$X^j \equiv \frac{A^j}{\lambda^j + N^j}. \tag{8}$$

A market maker who fills a buy or sell order is concerned about the informational advantage of the counterparty, which in our setting is the probability the order comes from a skilled trader, multiplied by the expected skill conditional on the trader being skilled. This is

$$\frac{N^j}{\lambda^j + N^j}.\frac{A^j}{N^j} = \frac{A^j}{\lambda^j + N^j} = X^j,$$

so bid and ask prices for the asset should reflect X^j. Similarly, traders also care about X^j; if it is too high relative to their skill then trading is unprofitable. The intuition (familiar from Glosten and Milgrom (1985) and the microstructure literature) is that the bid-ask spread is a measure of the amount of skilled trading. The more skilled trading there is, the larger the bid-ask spread, the harder it is for low-skill traders to make profits, and the higher the threshold level of skill required to trade profitably. Hence X^j is related to both the bid-ask spread and the minimum skill required to profitably trade the asset:

Lemma 1 *Given (A^r, N^r, A^c, N^c), for $j = r, c$ the bid-ask spread of the j-asset is X^j, and* *ask and bid prices are*

$$
\begin{aligned}
P_L^j &= q^j + \left(1 - q^j\right) X^j && (9) \\
P_S^j &= q^j - q^j X^j. && (10)
\end{aligned}
$$

Moreover, the minimum skill required both to profitably buy the j-asset after observing signal $s^j \leq q^j$ and to profitably sell the j-asset after observing signal $s^j > q^j$ is X^j.

5 Equilibrium analysis

5.1 Equilibrium existence

To establish existence, we construct a correspondence that maps skill distributions into themselves. We sketch the approach here. By Lemma 1, the effect of skill distributions on asset prices in the financial market equilibrium is fully captured by (N^r, N^c, A^r, A^c), the mass of skilled traders who trade each asset and the aggregate skill devoted to each asset. Given a candidate skill distribution, we use Lemma 1 to construct candidate ask and bid prices. We then use the labor market equilibrium condition to determine which asset a trader with skill α specializes in. Kakutani's fixed point theorem implies that this correspondence has a fixed point, at which both labor and financial markets are in equilibrium.

Proposition 1 *An equilibrium exists.*

5.2 The bid-ask spread in the r-asset is bounded away from zero

We start by showing that the combination of equilibrium in financial and labor markets implies that both the bid-ask spread (X^r) in the r-asset, and the minimum skill level required to trade it (also X^r), are bounded away from 0, even as the r-event grows rare ($q^r \to 0$). This result is central to our analysis.

To build intuition, suppose there is just one asset in the economy, and let the probability that it pays off approach zero, so its expected payoff also approaches zero. Is it possible that its ask price also approaches zero? For example, a fixed percentage markup over expected value implies an ask price approaching zero. If it does approach zero, then *all* agents, however low their chance of receiving an informative signal, will buy the asset when they receive a buy signal ($s^r \leq q^r$). But this implies that a positive measure of skilled traders buy the asset after observing a buy signal, so a buy trade is informative about asset value, implying that the ask price cannot be close to zero. This is a contradiction. A zero ask price in the limit would violate a basic equilibrium condition.

More constructively, we can see what does happen in the limit: as the payoff probability approaches zero, the price approaches a limit that is higher than zero. At this price, higher-skilled traders trade while lower-skilled traders do not trade. In between, there is a marginal trader whose skill is just high enough to be indifferent between trading and not trading. Given this, the ask price is higher than the expected value by a premium that reflects the average informativeness of signals of all types above this marginal type. Informally, this premium reflects the cumulative "brainpower" of traders who buy when they receive a positive signal. In equilibrium, the premium in turn implies that the marginal type is indeed indifferent between trading and not trading.

The above intuition is for an economy where the r-asset is the only asset. The argument for two assets is only slightly more involved, and is formalized by Lemma 2.

Lemma 2 *Both the bid-ask spread for the r-asset and the minimum skill level required to trade the r-asset remain bounded away from 0 as $q^r \to 0$, i.e., there exists $\delta > 0$ such that $X^r \geq \delta$ for all q^r small.*

Note that Proposition 1 does not establish equilibrium uniqueness.[10] However, Lemma 2

[10] In brief, the main obstacle to establishing uniqueness is that as the bid-ask spread in the r-asset falls, the average skill of skilled traders trading the r-asset falls, which potentially leads to a further drop in the bid-ask spread (depending on the number of liquidity traders and the specific distribution of skill). This is similar to issues encountered for liquidity traders in Admati and Pfleiderer (1988) and Dow (2004).

and all similar results below cover *any* possible sequence of equilibria, and so the possibility of multiple equilibria does not affect any of our conclusions.

An immediate but important consequence is:

Corollary 1 *The ask price P_L^r is bounded away from 0 as the unconditional expected value of the r-asset $q^r \to 0$.*

Moreover:

Corollary 2 *Aggregate skill in the r-asset, A^r, is bounded away from 0 as $q^r \to 0$.*

5.3 Skill allocation across assets

Given equilibrium prices (Lemma 1), a skilled trader's profits from asset j are (from (4))

$$V^j(\alpha) = q^j \left(1-q^j\right) \left(h_L^j \left(q^j + \left(1-q^j\right) X^j\right) + h_S^j \left(q^j - q^j X^j\right)\right) \max\left\{0, \left(\alpha - X^j\right)\right\}. \quad (11)$$

Hence the marginal value of skill in trading asset j is, for $\alpha > X^j$,

$$\frac{\partial V^j(\alpha)}{\partial \alpha} = q^j \left(1-q^j\right) \left(h_L^j \left(q^j + \left(1-q^j\right) X^j\right) + h_S^j \left(q^j - q^j X^j\right)\right). \quad (12)$$

From this expression, and using Corollary 1 and Assumption 2, the marginal value of skill is low in the r-asset because q^j is low. Formally:

Lemma 3 *As $q^r \to 0$, the marginal value of skill in the r-asset (12) approaches 0.*

To understand Lemma 3, notice from (11) that for a skilled trader who chooses to trade in one of the assets, profits as a function of α are a straight line. The slope of this line is the marginal value of skill. Therefore, to show the marginal value of skill goes to zero as $q^r \to 0$, we can show that trading profits go to zero. There are two economic effects underlying this. First, as $q^r \to 0$, traders only rarely buy the r-asset. Consequently, the expected profit from long positions also becomes small unless traders are able to make enormous profits from long positions—which could only happen if they took enormous long positions, as they do in the benchmark model of Section 3 without financial market equilibrium. But by Corollary 1, the dual requirement of equilibrium in financial and labor markets means that the ask price of the r-asset stays bounded away from 0. The lower bound on the price implies an upper bound on the size of the positions, so traders' long positions cannot grow arbitrarily large, implying that the expected profit from long positions indeed approaches 0.

Second, turning to short positions, as $q^r \to 0$ traders specializing in the r-asset nearly always adopt short positions. Traders with skill α have an expected profit on each short position of $P_S^j - (1-\alpha) q^j = q^j (\alpha - X^j)$, which converges to 0 as $q^r \to 0$. So it would only be possible for traders to make non-negligible expected profits on the short position if they could take large enough short positions, but Assumption 2 stops the short position from growing large (as noted above, it is natural for position limits on short positions to *de*crease as price falls, so this is a very weak assumption).

In contrast, the marginal value of skill in the common asset does not go to zero (see the proof of Proposition 2). The higher marginal value of skill in the common asset implies that high skill traders have a comparative advantage in the common asset and hence specialize in that asset. High skill traders are better at trading both assets, i.e., they have an absolute advantage compared to low skill traders. But an additional unit of skill is more valuable in the common asset, giving higher skilled traders a comparative advantage in that asset. Hence in equilibrium there is a threshold skill level so that traders with skill below the threshold choose the r-asset while those with skill above the threshold choose the c-asset.

Proposition 2 *For all q^r sufficiently small, the minimum skill required to profitably trade the r-asset is below the minimum skill to profitably trade the c-asset, i.e., $X^r < X^c$. Moreover, there exists $\hat{\alpha} \in (X^c, \bar{\alpha})$ such that traders with skill $\alpha \in (X^r, \hat{\alpha})$ trade the r-asset and traders with skill $\alpha \in (\hat{\alpha}, \bar{\alpha})$ trade the c-asset.*

Proposition 2 is illustrated by Figure 1, which shows how expected profits from special-

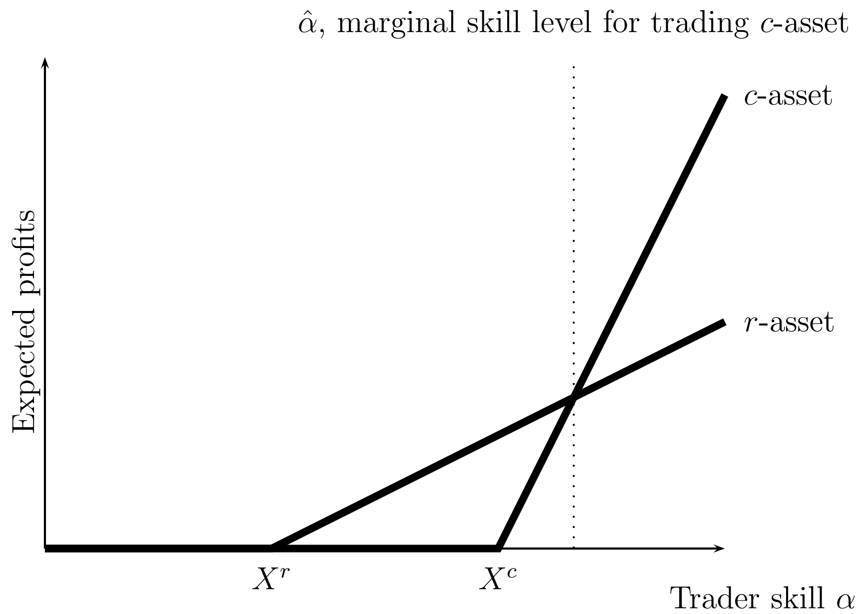

Figure 1: Equilibrium allocation of traders

izing in each asset depend on the trader's skill level.

Proposition 2 predicts that (among active traders) the least-skilled traders specialize in the r-asset. As noted, most of the time, they take a short position in this asset. The short position nets a small immediate profit, but exposes the trader to a small risk of a much larger loss in the future if the rare event is realized. Hence, our model predicts that the least skilled traders pursue what are often described as "picking-up nickels in front of a steamroller" strategies, such as the carry trade in currency markets, or writing out-of-the-money puts. See Online Appendix C for a numerical solution to a parameterized example.

Although we state Proposition 2 as a limiting result, it is worth highlighting that under default-free position limits (3) it in fact holds for any payoff probabilities such that $\min\{q^r, 1-q^r\} < \min\{q^c, 1-q^c\}$; see Online Appendix D.

Because traders in the r-asset are relatively unskilled, only a few of them manage to successfully predict the rare event when it actually occurs. Hence our model rationalizes the fact that rare events are foreseen by few people. Nonetheless, for those few traders, the posterior estimate of their skill is high.[11] In Online Appendix I we explore these issues

[11] If the prior belief that a trader receives an accurate signal is α, then the posterior belief after a successful

further in an extended version of our model in which agents learn about their skill over time.

5.4 Bid-ask spreads

Since more skill is needed to trade common events than to trade rare events (Proposition 2), and bid-ask spreads coincide with minimum required skilled levels (Lemma 1), Proposition 2 implies that the bid-ask spread is larger for the c-asset. As we discuss in Section 6, this prediction can be tested empirically.

Corollary 3 *For all q^r sufficiently small, the bid-ask price is smaller for the r-asset than the c-asset, $P_L^r - P_S^r < P_L^c - P_S^c$.*

6 Empirical predictions

6.1 The cross-section of skill

Proposition 2 predicts that the highest skill traders trade the c-asset, intermediate skill traders trade the r-asset, and the lowest skill traders cannot trade profitably in either asset. Trading profits are increasing in skill (traders with higher α could always add noise to their signals to effectively lower α). Therefore, traders who trade the c-asset, being higher skilled, make higher trading profits than traders who trade the r-asset. To interpret this prediction recall that "picking up pennies" trades, yielding frequent small profits and occasional significant losses, correspond to short positions on the r-asset.

This prediction applies both for traders trading on their own account, and for intermediated investments such as hedge funds, private equity, and mutual funds. Because of data availability, we focus on intermediated investments. We also focus on comparisons within broad classes of intermediated investment, since across these classes there are many differences that are unrelated to our analysis.

long trade of the r-asset is $\frac{\alpha}{q^r+\alpha(1-q^r)}$, which approaches 1 as $q^r \to 0$ for any $\alpha > 0$. By Lemma 2, the skill of any trader trading the r-asset is indeed bounded away from 0.

For hedge funds, there is a long-standing argument that a significant number of funds engage in low-skill "picking up pennies" trades. Lo (2001) gives the colorful example of a fictional fund, "Capital Decimation Partners," and shows how a simple strategy of selling options can yield hedge-fund like returns. As he points out, this strategy could be implemented in a less obvious fashion by dynamic trading that replicates the option payoffs. More recently, Jurek and Stafford (2015) show empirically that the hedge fund return index can be replicated using this kind of strategy, strengthening Lo's argument.

Although there is considerable cross-sectional variation across different hedge funds in the types of trades they undertake, the opaque nature of many funds means that there is relatively little research relating hedge fund returns to style of trading. One exception is Gibson and Gyger (2007), who use a standard classification of hedge funds into four broad groups: Tactical Trading, Long and Short Market Hedged, Event Driven, Relative Value. Of these, the first two correspond roughly to trading the c-asset, the fourth one to trading the r-asset, while the remaining "Event Driven" category falls outside our analysis.[12]As shown in Gibson and Gyger's Table 1, and consistent with our analysis, hedge funds in the first two categories have experienced higher average returns than those in the fourth category (with the Event Driven category falling in between).

Turning to private equity funds, among venture capital (VC) investments the style split that maps most easily to our model is between early- and late-stage VCs, corresponding to the r- and c-asset respectively. Early stage investments have a low probability of paying off. Korteweg and Sorensen (2017) study the performance of different types of VC funds, using performance persistence as their preferred empirical measure of outperformance. They report greater persistence of returns among late-stage funds, consistent with Proposition 2's prediction that these are higher-skill funds.

[12]Tactical trading and Long and Short Market Hedged are funds that carry out standard timing and stock selection strategies that either win or lose quite frequently. Relative Value funds typically concentrate on trades such as anomalies in the yield curve, in Libor/bond spreads, or in yield spreads between similar bonds, that pay off small amount most of the time but occasionally lose large amounts of money. Event Driven trades comprise distressed investing (common events); merger arbitrage (rare events); and special situations (for example, negotiating restructuring of distressed securities) that are outside our model.

Among other (non-VC) private equity investment, it is hard to identify a division of styles that match our model.

Finally, within mutual funds, we consider the best mapping between style classifications and our model to be that income funds correspond to the r-asset, while growth funds correspond to c-asset. Income funds have a high probability of modest return, with a low probability of losses, akin to shorting the r-asset. Growth funds are likely to give both high and low returns. Grinblatt and Titman (1993) and Kosowski et al. (2006) both find evidence of higher returns among growth funds than income funds, consistent with Proposition 2.[13]

6.2 The cross section of bid-ask spreads

Our model predicts that the r-asset has a smaller bid ask spread than the c-asset (Corollary 3). This prediction is most easily applied to bonds and CDS securities because their payoffs are well approximated by the binary payoff of our assets. A bond is equivalent to a long position in the risk free asset combined with a short position in the j-asset (Appendix B shows this in detail). So the r-asset corresponds to a short position in a highly-rated bond, which pays off if the bond defaults (a rare event). Similarly, the c-asset corresponds to a short position in a lower-rated bond, for which default is more likely. So our model predicts lower bid-ask spreads for higher-rated bonds and their CDS contracts.

These predictions are consistent with evidence from both corporate and sovereign debt markets. Edwards, Harris, and Piwowar (2007) study all transactions in US corporate bonds for 2003 and 2004 to estimate the bid ask spread implied by transaction prices (even for infrequently traded bonds). They find that bonds with higher credit risk have higher bid ask spreads, controlling for other factors (including issue size). The positive relationship between default risk and bid ask spread is also found in Goldstein and Hotchkiss' (2020) study over a longer period, and by Benmelech and Bergman (2019) and Feldhütter and Poulsen (2018). Calice et al (2013) study the sovereign debt CDS market for the Eurozone and also show a

[13]These are risk adjusted returns; and, in the case of Kosowski et al. (2006), adjusted for the value premium as they use a multifactor model.

positive relationship between default risk (credit spreads) and bid ask spreads. Indeed, this relationship can be seen very clearly by direct inspection of the data on credit spreads and bid ask spreads (see Table 1 below), where one can see that it is not driven by individual outliers.

6.3 Proportional bid-ask spreads

Data on bid ask spreads is often expressed in proportional terms (absolute bid ask spread divided by the mid price). For most bonds, the distinction between proportional and absolute spreads is of minor importance since prices are close to 1, and it does not affect the cross-sectional relationship between bid ask spreads and default risk.[14] In contrast, for CDS contracts it matters whether we consider absolute or proportional bid ask spreads. Our main result on bid-ask spreads, Corollary 3, concerns absolute spreads. Proportional spreads instead allow us to empirically assess Corollary 1—which, recall, is a key step towards our main results.

Corollary 1 says that the ask price of the r-asset is bounded away from 0. The bid price of the r-asset converges to zero, since it is below the expected value. The proportional bid-ask spread on the j-asset is $\frac{P_L^j - P_S^j}{\frac{1}{2}\left(P_L^j + P_S^j\right)}$, and its value ranges from 0 up to 2. Hence Corollary 1 implies that the proportional bid-ask spread approaches its upper bound of 2 as q^r approaches 0; and hence implies that the proportional bid-ask spread is larger for the r-asset than for the c-asset. It is this last prediction that we examine.

CDS contracts on high- and low-rated bonds correspond to the r-asset and the c-asset, respectively. Table 1 presents data for CDS contracts on Euro sovereigns,[15] and exhibits

[14]Moreover, the cross-sectional relationship would remain even if the bond prices on lower rated bonds were sharply lower than 1. Since Corollary 3 predicts that the absolute bid-ask spread is lower for higher-rated bonds, and the bond price is 1 minus the asset price, it follows that proportional spreads should also be lower for higher-rated bonds. Most of the bond market studies cited in subsection 6.2 use the relative bid ask spread.

[15]Table 1 reports averages computed over the period 10/2014 to 6/2018, as reported by Bloomberg. We thank Lukas Kremens for generously sharing the data used to construct this table. While the tables in Calice et al (2013) report CDS bid ask spreads, they do not report the levels of bid and ask prices, and hence do not allow the computation of proportional bid ask spreads. We focus on CDS contracts on government bonds

Table 1: Average CDS prices and Bid-Ask Spreads, 10/2014 to 06/2018

	GER	NED	AUT	FRA	BEL	IRE	ESP	ITA	POR
CDS	15.95	20.17	23.79	32.69	34.52	48.6	81.03	132.26	195.36
Bid-Ask Spread	3.6	4.68	4.64	4.29	4.3	9.82	7.12	6.97	15.5
(Bid-Ask Spread)/CDS	23%	23%	20%	13%	12%	20%	9%	5%	8%
Source: Bloomberg									

the predicted pattern: higher-rated bonds have lower proportional bid-ask spreads. (Table 1 also shows the positive predicted relation between credit risk and absolute bid-ask spreads, as previously discussed in 6.2.)

A closely related observation is that the unconditional expected return on a long position in the r-asset is very low; it is given by $\frac{q^r}{P_L^r}$, and since the ask price remains bounded away from 0, it approaches 0 as q^r approaches 0. This is consistent with empirical evidence on low returns to wagers on extreme underdogs in betting markets (the "longshot-favorite bias"), and with low returns to buying out-of-money puts and calls in option markets (the "smile" in implied volatilities).

7 Predictions from the market

Thus far, we have focused on the ability of individual traders to forecast rare events. In this section, we instead consider the information content of aggregate trading activity.

Bid and ask prices are set before the arrival of orders, so they are independent of the true state and hence uninformative. In contrast, the aggregate order flow is informative. We consider the inferences about the likelihood of the j-event ($\psi^j \leq q^j$) made by an outside observer who sees the total numbers of buy and sell orders for asset j. The total numbers of buy and sell orders can, alternatively, be inferred from observing a combination of any two of: (i) the average transaction price, (ii) aggregate volume (unsigned; this is the total number of units of the asset bought or sold), and (iii) order flow imbalance (signed; this is

because, other than Calice et al (2013), to our knowledge there are no published empirical studies on the cross-sectional relation between CDS price levels and bid-ask spreads for corporate bonds.

the difference between the total number of units bought and total number of units sold).

Write L^j and S^j for total buy (long) and sell (short) orders for asset j. Write λ_L^j and λ_S^j for the mass of liquidity traders who buy and sell asset j. Write N_L^j and N_S^j for the mass of skilled traders who buy and sell asset j. Hence

$$\begin{aligned} L^j &= \left(\lambda_L^j + N_L^j\right) h_L^j\left(P_L^j\right) \\ S^j &= \left(\lambda_S^j + N_S^j\right) h_S^j\left(P_S^j\right). \end{aligned}$$

Since both liquidity traders and active skilled traders always trade in one direction or the other, $\lambda_L^j + \lambda_S^j = \lambda^j$ and $N_L^j + N_S^j = N^j$. Hence observing the total number of buy and sell orders (L^j, S^j) has the same information content as simply observing the total number of buy orders, L^j.

The information content of the aggregate order flow depends critically on the correlation among liquidity traders, and similarly, on the correlation among skilled traders. For example, if skilled trades are uncorrelated conditional on the realization of ψ^j (a natural assumption), and liquidity trades are uncorrelated, then by the law of large numbers L^j perfectly reveals whether or not $\psi^j \leq q^j$. In the literature, it is assumed that liquidity trades are correlated so as to prevent full revelation (e.g., Grossman and Stiglitz 1980, Hellwig 1980, Kyle 1985). We follow the literature and assume that liquidity trades are correlated, so that λ_L^j is a non-degenerate random variable. For simplicity, we assume that skilled trades are uncorrelated conditional on ψ^j (see, e.g., Grossman 1976, Hellwig 1980),[16] so that

$$N_L^j = A^j \mathbf{1}_{\psi^j \leq q^j} + \left(N^j - A^j\right) q^j. \tag{13}$$

Given (13), the information content of the aggregate order flows in asset j is the same as

[16] We obtain similar results in the case in which the assumption of conditional independence is relaxed.

the information content of

$$\tilde{L}^j \equiv \frac{L^j}{h_L^j\left(P_L^j\right)} - \left(N^j - A^j\right) q^j = A^j \mathbf{1}_{\psi^j \leq q^j} + \lambda_L^j. \tag{14}$$

From (14), one can see that the aggregate skill A^j deployed to asset j is the key factor that determines the information content of aggregate order flow.

So far we have shown (Proposition 2) that all traders in the r-asset have skill below a certain threshold $\hat{\alpha}$ while all traders in the c-asset have skill higher than that threshold. This implies that the average skill of traders in the r-asset is lower than that of traders in the c-asset, i.e., $\frac{A^r}{N^r} < \frac{A^c}{N^c}$, and relatedly, that the bid-ask spread is smaller for the r-asset than for the c-asset, i.e., $\frac{A^r}{\lambda^r + N^r} < \frac{A^c}{\lambda^c + N^c}$ (Corollary 3). We now investigate whether aggregate skill is likewise lower, i.e., $A^r < A^c$.

7.1 Lower aggregate skill in the r-asset

Clearly, a sufficient condition for aggregate skill devoted to the r-asset to be lower is that fewer people trade it, $N^r \leq N^c$: we already know traders in this asset are less skilled, so if there are fewer of them, the total skill must be low. More generally, aggregate skill devoted to the r-asset is lower provided that N^r does not exceed N^c by too much. But if there are a very large number of low-skill traders in the r-asset, and not many high-skill traders in the c-asset, it appears that aggregate skill in the r-asset could be higher.

Intuitively, N^r can only exceed N^c by a large amount if the density function g of the skill distribution declines rapidly in skill α. Our next result formalizes this intuition, and gives a simple sufficient condition on the slope of the density function g that guarantees that N^r is not too large relative to N^c, and hence in turn that less aggregate skill is indeed deployed to the r-asset than to the c-asset.

Proposition 3 *If there are equal numbers of liquidity traders in the two assets,* $\lambda^r = \lambda^c$, *and the density of skill g satisfies*

$$x\int_{z}^{x}\alpha g\left(\alpha\right)d\alpha > z^{2}g\left(z\right)\left(x-z\right) \text{ for all } z < x \leq \overline{\alpha}, \tag{15}$$

then for any q^r sufficiently small, less aggregate skill is deployed to the r-asset, i.e., $A^r < A^c$.

Condition (15) holds trivially if the density function is weakly increasing in skill. In particular, condition (15) holds if skill α is distributed uniformly over $[0, \bar{\alpha}]$. Moreover, even when (15) is violated, the conclusion that $A^r < A^c$ still holds for a very wide class of parameters. For example, in Online Appendix E we analyze the case in which there are equal numbers of liquidity traders in the two assets ($\lambda^r = \lambda^c$) and the distribution of skill is left triangular, i.e., $g\left(\alpha\right) = \frac{2}{\bar{\alpha}^2}\left(\overline{\alpha}-\alpha\right)$. This class of distributions captures the plausible idea that the skill density tapers to zero as the upper bound $\bar{\alpha}$ is approached, and allows for a rapid decline in the density g, which as discussed above, is the case in which the conclusion $A^r < A^c$ is least likely to obtain. We first show that the ratio A^c/A^r that obtains in the limit as $q^r \to 0$ is a function only of the number of liquidity traders $\lambda^r = \lambda^c$, and is independent of all other model parameters, including $\bar{\alpha}$. We then calculate the ratio A^c/A^r numerically, and show that $A^r < A^c$ holds except for cases in which $\lambda^r = \lambda^c$ is extremely small (i.e., below 0.01, corresponding to the total number of liquidity traders being below 2% of the population of skilled traders).

The conclusion of Proposition 3 also holds for *any* distribution of skill provided that the mass of liquidity traders is sufficiently large. In brief, the argument is as follows. As in Proposition 3, we assume that $\lambda^c = \lambda^r$. As $\lambda^c = \lambda^r \to \infty$, it is straightforward to show that $N^c + N^r$ is bounded away from 0. (Intuitively, if there are many liquidity traders then it is easy for skilled traders to make profits.) If $N^r \to 0$ but $N^c \not\to 0$, it is immediate that $A^r < A^c$. If instead $N^r \not\to 0$, then $\frac{X^r}{X^c}$ is bounded away from 1 (from above). We know $\frac{A^r}{A^c} = \frac{X^r}{X^c}\frac{\lambda^c+N^c}{\lambda^r+N^r}$. Since N^c and N^r are both bounded, we know $\frac{\lambda^c+N^c}{\lambda^r+N^r} \to 1$ as $\lambda^c = \lambda^r \to \infty$. It follows that $\frac{A^r}{A^c} < 1$ for $\lambda^c = \lambda^r$ large enough.

7.2 Market predictions from the r-asset are less informative

Skilled traders in our model can work at either one of two tasks, producing information about the r-asset or producing information about the c-asset.

Our main result of this section uses results from the theory of information orderings (see Blackwell 1953, Lehmann 1988). It requires the mild assumption that the density of λ_L^r is log-concave. Recall that, as discussed in subsection 7.1, the condition $A^c > A^r$ is typically satisfied in equilibrium: there is more aggregate skill deployed in the c-asset. We can use this to compare the accuracy of learning in the two assets. We consider the impact of exogenously interchanging the sets of traders who trade the two asset types, i.e., A^r trade the c-asset while A^c trade the r-asset. We show that this switch increases the informativeness of the aggregate order flow in the r-asset.

To say that one information structure is more Blackwell-informative than another is a strong statement. It means that any agent who needs to take any decision would prefer to have the former information structure. It is only a partial ordering of information structures. However in this case the event agents are trying to predict (the asset pays off) is binary, which as Jewitt (2007) observes, simplifies the application of Blackwell's theorem.

Proposition 4 *Suppose the density of λ_L^r is log-concave. If there are equal numbers of liquidity traders in the two assets ($\lambda^r = \lambda^c$), and $A^c > A^r$, then the aggregate order flow of the r-asset would be more Blackwell informative if the sets of people trading the r-asset and c-asset were exogenously switched.*

By exogenously switching the sets of people who trade the r-asset and c-asset, we mean that everyone who used to trade the c-asset (i.e., with skill α exceeding the threshold level $\hat{\alpha}$) is now restricted to either trading the r-asset or doing nothing, and similarly, that everyone who used to trade the r-asset (skill $\alpha \in [X^r, \hat{\alpha}]$) is now restricted to either trading the c-asset or doing nothing. The option of doing nothing potentially matters because after the people trading the two assets are switched, asset prices change, and consequently it is possible that not everyone who previously traded the c-asset wants to trade the r-asset at its new equilibrium prices. The role of the condition $\lambda^r = \lambda^c$ is to ensure that profitably trading the r-asset is not much more difficult than trading the c-asset solely because of a lack of liquidity traders; if instead λ^r were much lower than λ^c, it is possible that many traders who used to trade the c-asset drop out of trading after they are exogenously switched to the r-asset.

Proposition 4 suggests that, unless the social value of forecasting common events is significantly greater than that of forecasting rare events, there is a basic force leading to a socially suboptimal undersupply of resources to forecasting rare events. In particular, if it is socially more important to predict the rare event than the common event then information is under-produced.

8 Generalizations

A natural question concerning our model is the extent to which the results are robust to generalization. We have assumed very general forms for position limits. Here we investigate two further generalizations, to risk aversion and alternative specifications of liquidity trade, and show that suitably modified versions of our main results continue to hold. As mentioned earlier, we also show in Online Appendix I that the results on trader allocation generalize to a repeated setting with career concerns.

8.1 Trader risk aversion

Thus far, we have assumed that traders are risk neutral, or equivalently are insured by a risk neutral employer. Here, we consider the case in which traders are instead risk averse and uninsured. A significant difference relative to the case of risk neutral traders is that risk averse traders may find it worthwhile to take positions in only one direction.

We focus on the highly tractable case in which a trader's initial wealth is W, and utility over final wealth Y is given by Y if $Y \geq W$ and $W - (1+\kappa)(W-Y)$ if $Y \leq W$, for some

$\kappa \geq 0$. That is, utility is a piecewise linear concave function, and κ measures the degree of risk aversion, with $\kappa = 0$ representing risk neutrality (Dow (1998) and Carlin and Gervais (2009), for example, also use this specification of risk aversion). Prices now take the form $P_L^j = q^j + (1 - q^j) X_L^j$, and $P_S^j = q^j - q^j X_S^j$, where X_L^j and X_S^j are defined analogously to X^j (see (8)), but depend on the aggregate skill and number of skilled traders in long and short positions, respectively.

Online Appendix F analyzes this case, and includes numerical simulations to evaluate equilibrium outcomes. The most important conclusion is that the main qualitative features of our analysis remain. The lowest skilled traders trade nothing, medium skilled traders trade the r-asset, and high-skilled traders trade the c-asset.

In addition, this material demonstrates a sense in which, for the r-asset, long positions are more attractive than short positions. Consider first the benchmark case in which bid and ask prices of the r-asset both equal the unconditional expected payoff q^r. Then an unskilled trader ($\alpha = 0$) makes zero expected profits from both long and short trades, and risk-aversion implies that both trades are unattractive. Indeed, the two trades are equally unattractive, both yielding expected utility $-\kappa q^r (1 - q^r)$.[17] It follows that skill ($\alpha > 0$) is more valuable in avoiding losses for long positions in the r-asset than for "picking-up-pennies" short positions. This can be seen for the extreme case of perfect skill ($\alpha = 1$): for long positions, the probability of loss is reduced from $1 - q^r$ to 0, while for short positions the probability of loss is reduced from q^r from 0. Numerical simulations illustrate that these insights extend to equilibrium prices, and are not limited to the case of $P_L^r = P_S^r = q^r$. In particular, short "picking-up-pennies" trades in the r-asset are unattractive for unskilled risk-averse agents, and considerable skill is required to render such trades attractive. Accordingly, the minimum skill required for a risk-averse agent to benefit from a long position in the r-asset is lower than the minimum skill required to benefit from a short position. The c-asset exhibits a smaller difference between long and short positions; indeed, at $q^c = \frac{1}{2}$ and under

[17]The long trade has expected utility of $q^r (1 - P_L^r) - (1 - q^r)(1 + \kappa) P_L^r$ and the short trade has expected utility of $(1 - q^r) P_S^r - q^r (1 + \kappa)(1 - P_S^r)$.

microfounded position limits such as (3), long and short positions are symmetric.

8.2 Alternative formulations of liquidity trade

Our assumptions on liquidity trade are economically natural in the sense that as we vary the probability q^r of the rare event, which coincides with the probability of an informed trader getting a positive signal about this event, the probability of a liquidity trader buying the asset also moves in exactly the same way. Intuitively, this can be motivated by the liquidity traders wanting to insure against the event, and our construction of ψ^j is designed as a natural way to model this. Because of this link between liquidity trade and the probability of the rare event, our results on skill allocation are by no means immediate.

In contrast, in the literature, liquidity trade (also known as noise trade) is often modelled as exogenous for simplicity. This presents problems for comparative statics because by assumption, liquidity trade then cannot respond to variation in the exogenous parameters even though the informal motivations for this trade, such as liquidity or hedging, suggest that it should respond (Dow and Gorton, 2008). Nevertheless, we can consider how our model behaves under the simpler assumption that liquidity trade is exogenous and does not respond to changes in the probability of the rare event. In this alternative case, our results that low-skill traders prefer the r-asset, and that the r-asset has a smaller absolute bid-ask spread, follow almost immediately from assumptions. If the number of liquidity traders buying the r-asset is independent of the probability q^r, then as the rare event becomes rare and there are very few informed buy orders, a market-maker interprets a buy order as being very likely to stem from a liquidity trader. So the ask price P_L^r of the r-asset is very close to the "fair" price q^r, and even skilled traders with low skill can make profits. See Online Appendix G for a formal analysis of this case.

A second alternative case is that liquidity trade responds to neither changes in the probability of the rare event, nor changes in the price (Online Appendix H). In this case, the ask price of the r-asset also converges to the fair price because the relative frequency of liquidity trades to informed trades becomes very large, although this convergence is partially mitigated because informed trades grow larger as the price falls while liquidity trades are assumed constant. Our results on skill allocation and on the comparing the absolute bid ask spread across the r-asset and the c-asset remain. Finally, we note for completeness that if liquidity trade depends on the probability of the rare event, but not on the asset prices, the bid-ask spread on the r-asset does not go to zero, so our results on skill allocation remain.

9 Conclusion

One of the main functions performed by the financial sector is forecasting future events. Many observers have expressed concern that, as they perceive it, the majority of forecasting activity is devoted to forecasting frequent but relatively unimportant events. In this paper we analyze a simple equilibrium model of the number and skill of financial sector participants who are predict different types of events. The key feature of our model is that it combines equilibrium analysis of the financial market, using a standard Glosten and Milgrom (1985) model, with equilibrium analysis of the labor market, using a standard Roy (1951) model.

Our main result is the following prediction: individuals with higher skill trade the common event asset, while individuals with less skill trade the rare event asset. Moreover, because this leads to more informed trading in the common event asset, the bid-ask spread for this asset is higher. In other words, trades on the frequent event are more informative. Our prediction on the allocation of skill matches perceptions that a lot of forecasting "talent" is devoted to forecasting frequent events. It is also consistent with the view that many standard trading strategies (e.g., the carry trade, selling out-of-the-money put options) are "nickels in front of steamroller strategies" that are typically carried out by people with mediocre talents.

The bid-ask spread prediction is easiest to apply to bonds (or corresponding CDS), as the binary payoff assumption of our model is a good approximation of their payoff structure.

Our model predicts that low-rated bonds have larger bid-ask spreads than high-rated bonds. This is consistent with evidence from both sovereign and corporate bond markets.

Finally, we show that the endogenous distribution of talent across different types of assets reduces financial markets' ability to predict future rare events. Specifically, we show that financial markets produce less information about rare events compared to a counterfactual in which the people trading rare and common assets are exogenously interchanged.

References

Admati, A., Pfleiderer P., 1988. A theory of intraday patterns: volume and price variability. Review of Financial Studies 1, 3-40.

Benmelech, E. and Bergman, N., 2019. Debt, information, and illiquidity. Unpublished working paper. Northwestern University.

Blackwell, D., 1953. Equivalent comparisons of experiments. Annals of Mathematical Statistics 24, 265-272..

Border, K., 1985. Fixed Point Theorems with Applications to Economics and Game Theory. Cambridge University Press, Cambridge.

Buffa, A., Javadekar, A, 2019. The allocation of talent across mutual fund strategies. Unpublished working paper. University of Colorado, Boulder.

Calice, G., Chen, J., Williams, J., 2013. Liquidity spillovers in sovereign bond and CDS markets: An analysis of the eurozone sovereign debt crisis. Journal of Economic Behavior & Organization 85, 122-143.

Carlin, B., Gervais, S., 2009. Work ethic, employment contracts, and firm value. Journal of Finance 64, 785-821.

Diamond, D., Verrecchia, R., 1981. Information aggregation in a noisy rational expectations economy. Journal of Financial Economics 9, 221-235.

Dow, J., 1998. Arbitrage, hedging, and financial innovation. Review of Financial Studies

11, 739–755.

Dow, J., 2004. Is Liquidity self-fulfilling? Journal of Business 77, 895-908.

Dow, J., Gorton, G., 2008. Noise trade. In: Durlauf, S., Blume, L. (Ed.), New Palgrave Dictionary of Economics, 2nd Edition. Palgrave Macmillan, London.

Edwards, A., Harris, L., Piwowar, M., 2007. Corporate bond market transaction costs and transparency. Journal of Finance 62, 1421-1451.

Feldhütter, P., Poulsen, T., 2018, What determines bid-ask spreads in over-the-counter markets? Unpublished working paper. Copenhagen Business School.

Gandhi, A., Serrano-Padial, R., 2015. Does belief heterogeneity explain asset prices: the case of the longshot bias. Review of Economic Studies 82, 156–186.

Gibson, R., Gyger, S., 2007. The style consistency of hedge funds. European Financial Management 13, 287-308.

Glosten, L., Milgrom, P., 1985. Bid, ask, and transaction prices in a specialist market with heterogeneously informed traders. Journal of Financial Economics 14, 71-100.

Goldstein, M. and Hotchkiss, E., 2020. Providing liquidity in an illiquid market: dealer behavior in U.S. corporate bonds. Journal of Financial Economics 135, 16-40.

Grinblatt, M. Titman, S., 1993. Performance measurement without benchmarks: an examination of mutual fund returns. Journal of Business 66, 47-68.

Grossman, S., 1976. On the efficiency of competitive stock markets where traders have diverse information. Journal of Finance 31, 573-585.

Grossman, S., Stiglitz, J., 1980. On the impossibility of informationally efficient markets. American Economic Review 70, 393-408.

Hellwig, M., 1980. On the aggregation of information in competitive markets. Journal of Economic Theory 22, 477-498.

Jewitt, I., 2007. Information order in decision and agency problems. Unpublished working paper. University of Oxford.

Jurek, J., Stafford, E., 2015. The cost of capital for alternative investments. Journal of

Finance 70, 2185-2226.

Korteweg, A., Sorensen, M., 2017. Skill and luck in private equity performance. Journal of Financial Economics 124, 535-562.

Kosowski R., Timmermann, A., Wermers, R., White, H., 2006. Can mutual fund "stars" really pick stocks? New evidence from a bootstrap analysis. Journal of Finance 61, 2551-2595.

Kyle, A., 1985. Continuous auctions and insider trading. Econometrica 53, 1315-1335.

Lehmann, E., 1988. Comparing location experiments. Annals of Statistics 16, 521-533.

Lo, A., 2001. Risk management for hedge funds: introduction and overview. Financial Analysts Journal 57(6), 16-33.

Peng, L., Xiong, W., 2006. Investor attention, overconfidence and category learning. Journal of Financial Economics 80, 563-602.

Roy, A., 1951. Some thoughts on the distribution of earnings. Oxford Economic Papers 3, 135-146.

Taleb, N., 2007. The Black Swan: the Impact of the Highly Improbable. Random House, New York.

Van Nieuwerburgh, S., Veldkamp, L., 2010. Information acquisition and under-diversification. Review of Economic Studies 77, 779-805.

Veldkamp, L., 2006. Information markets and the comovement of asset prices. Review of Economic Studies 73, 823-845.

A Proofs of results stated in main text

Proof of Lemma 1: We first compute prices under the conjecture that any skilled trader who trades j-asset takes both long and short positions; and then confirm this conjecture.

Under this conjecture:

$$\begin{aligned}
E\left[\text{buys}|\psi^j\right] &= q^j \lambda^j h_L^j\left(P_L^j\right) + \int \left(\alpha \mathbf{1}_{\psi^j \le q^j} + (1-\alpha) q^j\right) \mu^j (d\alpha)\, h_L^j\left(P_L^j\right) \\
&= \left(q^j \lambda^j + A^j\left(\mathbf{1}_{\psi^j \le q^j} - q^j\right) + q^j N^j\right) h_L^j\left(P_L^j\right) \\
E\left[\text{sells}|\psi^j\right] &= \left(1-q^j\right) \lambda^j h_S^j\left(P_L^j\right) + \int \left(\alpha \mathbf{1}_{\psi^j > q^j} + (1-\alpha)\left(1-q^j\right)\right) \mu^j (d\alpha)\, h_S^j\left(P_S^j\right). \\
&= \left(\left(1-q^j\right)\lambda^j + A^j\left(\mathbf{1}_{\psi^j > q^j} - \left(1-q^j\right)\right) + \left(1-q^j\right) N^j\right) h_S^j\left(P_S^j\right).
\end{aligned}$$

Hence from (1) and (2),

$$\begin{aligned}
P_L^j &= q^j \frac{q^j \lambda^j + A^j (1-q^j) + q^j N^j}{q^j \lambda^j + q^j N^j} = q^j \left(1 + \frac{A^j}{\lambda^j + N^j} \frac{1-q^j}{q^j}\right) && \text{(A-1)} \\
P_S^j &= q^j \frac{(1-q^j) \lambda^j - A^j (1-q^j) + (1-q^j) N^j}{(1-q^j) \lambda^j + (1-q^j) N^j} = q^j \left(1 - \frac{A^j}{\lambda^j + N^j}\right). && \text{(A-2)}
\end{aligned}$$

Substituting for X^j in expressions (A-1) and (A-2) yields prices (9) and (10).

Note that substituting for prices (9) and (10) in the profit expression (4), profits per unit conditional on receiving buy and sell signals rewrite as follows:

$$\begin{aligned}
\alpha + (1-\alpha) q^j - P_L^j &= \left(1-q^j\right)\left(\alpha - \frac{A^j}{\lambda^j + N^j}\right) = \left(1-q^j\right)\left(\alpha - X^j\right) \\
P_S^j - (1-\alpha) q^j &= q^j \left(\alpha - \frac{A^j}{\lambda^j + N^j}\right) = q^j \left(\alpha - X^j\right).
\end{aligned}$$

Hence the minimum skill level required to profitably buy the j-asset after observing signal $s^j \le q^j$ is X^j, and similarly, the minimum skill level required to profitably sell the j-asset after observing signal $s^j > q^j$ is X^j. Hence any skilled trader who trades the j-asset takes both long and short positions. **QED**

Proof of Proposition 1: We sketch the approach in the main text preceding Proposition 1. Formally, we construct a correspondence $\xi : [0,1]^4 \to [0,1]^4$ as follows. For any $(N^r, N^c, A^r, A^c) \in [0,1]^4$, define $x^j (N^r, N^c, A^r, A^c) \equiv \min\left\{\frac{A^j}{\lambda^j + N^j}, 1\right\}$ for $j = r, c$. Let M be the set of measures on $[0,1]$. Then define

$$\begin{aligned}\xi\left(N^r,N^c,A^r,A^c\right) \equiv & \left\{\left(n^r,n^c,a^r,a^c\right): \exists\left(\mu^r,\mu^c,\mu^0\right)\in M^3 \text{ such that } \left(n^j,a^j\right)=\left(\int\mu^j\left(d\alpha\right),\int\alpha\mu^j\left(d\alpha\right)\right)\right.\\ & \text{and } \mu^r,\mu^c,\mu^0 \text{ satisfy the equilibrium conditions 1(a) and 1(b) if for } j=r,c\\ & \text{ask and bid prices are } P_L^j{=}q^j{+}\left(1-q^j\right)x^j\left(N^r,N^c,A^r,A^c\right)\\ & \left.\text{and } P_S^j=q^j-q^jx^j\left(N^r,N^c,A^r,A^c\right)\right\}.\end{aligned}$$

To establish equilibrium existence we apply Kakutani's fixed point theorem. To do so, we need to verify that ξ is closed,[18] with non-empty convex compact values.

For prices $P_L^j{=}q^j{+}(1-q^j)X^j$ and $P_S^j=q^j-q^jX^j$, a trader α makes profits (11) from the j-asset. In particular, profits are 0 for skill $\alpha\leq X^j$, corresponding to doing nothing; and linear and strictly increasing in skill $\alpha\geq X^j$. So it is immediate that either (i) other than at at most a single skill level, traders have a strict preference over the choices of trading the r- versus c-asset, or (ii) for all α above some critical value X, traders are indifferent between trading the r- and c-asset, and strictly prefer doing so to doing nothing. For use below, note that a necessary (but not sufficient) condition to be in case (ii) is that $x^r\left(N^r,N^c,A^r,A^c\right)=x^c\left(N^r,N^c,A^r,A^c\right)$.

In case (i), $\xi\left(N^r,N^c,A^r,A^c\right)$ is a singleton, and hence trivially non-empty, convex, and compact valued. For use below, note that there exist cutoffs β_1 , β_2, β_3 with $(\beta_1,\beta_2)\cap(\beta_2,\beta_3)=\emptyset$ and $[\beta_1,\beta_2]\cup[\beta_2,\beta_3]=\left[\min\left\{x^r\left(N^r,N^c,A^r,A^c\right),x\left(N^r,N^c,A^r,A^c\right)\right\},\bar{\alpha}\right]$ such that

$$\xi\left(N^r,N^c,A^r,A^c\right)=\left\{\left(\int_{\beta_1}^{\beta_2}\mu\left(d\alpha\right),\int_{\beta_2}^{\beta_3}\mu\left(d\alpha\right),\int_{\beta_1}^{\beta_2}\alpha\mu\left(d\alpha\right),\int_{\beta_2}^{\beta_3}\alpha\mu\left(d\alpha\right)\right)\right\}. \tag{A-3}$$

In words: all traders with skill above $\min\left\{x^r\left(N^r,N^c,A^r,A^c\right),x\left(N^r,N^c,A^r,A^c\right)\right\}$ trade one of the two assets, and for each asset, the set of traders who trade it is a (possibly degenerate)

[18] If a correspondence maps into a compact set, and is closed-valued (as is the case here, see proof below), then it is closed if and only if it is upper hemi-continuous; see Border (1985).

interval.

In case (ii), $\xi\left(N^{r},N^{c},A^{r},A^{c}\right)$ is not a singleton. Convexity follows straightforwardly from the possibility of taking the convex combination of different allocations of traders across the r- and c-assets. To establish compactness, for any $X\in[0,1]$ and $n^{r}\in\left[0,\int_{X}\mu\left(d\alpha\right)\right]$ define (uniquely) $\hat{\alpha}_{1}\left(n^{r},X\right)$ and $\hat{\alpha}_{2}\left(n^{r},X\right)$ by $\int_{X}^{\hat{\alpha}_{1}\left(n^{r},X\right)}\mu\left(d\alpha\right)=\int_{\hat{\alpha}_{2}\left(n^{r},X\right)}\mu\left(d\alpha\right)=n^{r}$, and $\mathcal{A}\left(n^{r},X\right)=\left[\int_{X}^{\hat{\alpha}_{1}\left(n^{r},X\right)}\alpha\mu\left(d\alpha\right),\int_{\hat{\alpha}_{2}\left(n^{r},X\right)}\alpha\mu\left(d\alpha\right)\right]$. In words: $\hat{\alpha}_{1}\left(n^{r},X\right)$ corresponds to achieving a target number of traders n^{r} trading the r-asset by choosing the least skilled group possible in $[X,\bar{\alpha}]$, while $\hat{\alpha}_{2}\left(n^{r},X\right)$ corresponds to choosing the most skilled group. Compactness follows from the following claim, which we prove further below.

Claim: If $\xi\left(N^{r},N^{c},A^{r},A^{c}\right)$ is not a singleton, then there exists $X=x^{r}\left(N^{r},N^{c},A^{r},A^{c}\right)=x^{c}\left(N^{r},N^{c},A^{r},A^{c}\right)$ such that

$$\begin{aligned}\xi\left(N^{r},N^{c},A^{r},A^{c}\right) &= \{\left(n^{r},n^{c},a^{r},a^{c}\right):n^{r}\in\left[0,\int_{X}\mu\left(d\alpha\right)\right],a^{r}\in\mathcal{A}\left(n^{r},X\right),\\ &\qquad n^{c}=\int_{X}\mu\left(d\alpha\right)-n^{r},a^{c}=\int_{X}\alpha\mu\left(d\alpha\right)-a^{r}\}.\end{aligned}\qquad\text{(A-4)}$$

Finally, closedness of ξ follows from (A-3) and (A-4), combined with the fact that β_{1} ,β_{2}, β_{3} in (A-3) and $\min\left\{x^{r}\left(N^{r},N^{c},A^{r},A^{c}\right),x\left(N^{r},N^{c},A^{r},A^{c}\right)\right\}$ are all continuous functions of $\left(N^{r},N^{c},A^{r},A^{c}\right)$.

Proof of claim: As noted, $\xi\left(N^{r},N^{c},A^{r},A^{c}\right)$ is not a singleton if and only if $x^{r}\left(N^{r},N^{c},A^{r},A^{c}\right)=x^{c}\left(N^{r},N^{c},A^{r},A^{c}\right)$, and moreover, the expected profits (11) from trading the two assets exactly coincide for all skill levels α when evaluated at $X^{r}=X^{c}=x^{r}\left(N^{r},N^{c},A^{r},A^{c}\right)$. For the remainder of the proof of the claim, let $X=x^{r}\left(N^{r},N^{c},A^{r},A^{c}\right)$. Hence the number of traders trading either the r-asset or the c-asset, and the aggregate skill of these traders, are respectively

$$n^{r}+n^{c} = \int_{X}\mu\left(d\alpha\right) \qquad\text{(A-5)}$$

$$a^{r}+a^{c} = \int_{X}\alpha\mu\left(d\alpha\right). \qquad\text{(A-6)}$$

Consequently, $(n^r, n^c, a^r, a^c) \in \xi(N^r, N^c, A^r, A^c)$ only if $n^r \in \left[0, \int_X \mu(d\alpha)\right]$ and (A-5) and (A-6) both hold, and moreover, $a^r \in \mathcal{A}(n^r, X)$, establishing that $\xi(N^r, N^c, A^r, A^c)$ is a subset of the RHS of (A-4). To establish that (A-4) holds with equality, simply note that for any $n^r \in \left[0, \int_X \mu(d\alpha)\right]$ one can vary α_1 continuously from X to $\hat{\alpha}_1(n^r, X)$ while simultaneously continuously increasing α_2 from $\hat{\alpha}_2(n^r, X)$ to satisfy

$$n^r = \int_X^{\alpha_1} \mu(d\alpha) + \int_{\alpha_2} \mu(d\alpha)$$

and that by doing so the quantity

$$\int_X^{\alpha_1} \alpha\mu(d\alpha) + \int_{\alpha_2} \alpha\mu(d\alpha)$$

varies continuously from $\int_{\hat{\alpha}_2(n^r,X)} \mu(d\alpha)$ to $\int_X^{\hat{\alpha}_1(n^r,X)} \alpha\mu(d\alpha)$, i.e., covers all values in $\hat{A}(n^r, X)$, completing the proof of the claim. **QED**

Proof of Lemma 2: Suppose to the contrary that there exists some sequence $\{q^r\}$ such that $q^r \to 0$ and the associated $X^r \to 0$.

First, consider the case in which X^c stays bounded away from 0, by $\underline{x}^c$ say. But then for any $X^r < \underline{x}^c$, traders in the skill interval $[X^r, \underline{x}^c]$ certainly trade the rare asset. It follows that as $X^r \to 0$, the mass of skilled traders N^r trading the r-asset is bounded below by $\mu\left(\left[\frac{1}{2}\underline{x}^c, \underline{x}^c\right]\right)$, which in turn (using (8)) implies that X^r is bounded away from 0, a contradiction.

Second, consider the case in which $X^c \to 0$ for some subsequence. So along this subsequence, the mass of skilled traders who trade one of the assets approaches 1, i.e., $N^r + N^c \to 1$. Hence there exists a subsequence in along which at least one of N^r and N^c is bounded away from 0, which in turn implies (by (8)) that at least one of X^r and X^c is bounded away from 0. This contradicts $X^c + X^r \to 0$, completing the proof. **QED**

Proof of Corollary 2: From Lemma 2, there exists $\underline{x} > 0$ such that $X^r \geq \underline{x}$ even as as

$q^r \to 0$. Since $A^r \leq N^r$, it follows that $\frac{A^r}{\lambda^r+A^r} \geq \frac{A^r}{\lambda^r+N^r} \geq \underline{x}$, and hence that there exists $\underline{A}$ such that $A^r \geq \underline{A}$ even as as $q^r \to 0$. **QED**

Proof of Lemma 3: By Lemma 2, as $q^r \to 0$, the term $q^r(1-q^r)\,h_L^r(q^r+(1-q^r)X^r)$ in equation (12) approaches 0. The remaining term $q^r(1-q^r)\,h_S^r(q^r-q^rX^r)$ can be written $\frac{1-q^r}{1-X^r}q^r(1-X^r)\,h_S^r(q^r(1-X^r))$, of which $q^r(1-X^r)\,h_S^r(q^r(1-X^r))$ approaches 0 as $q^r \to 0$ by Assumption 2 while $\frac{1-q^r}{1-X^r}$ is bounded above since $X^j \leq \frac{1}{1+\lambda^j}$ (because $A^j \leq N^j$). **QED**

Proof of Proposition 2: Note first that, for all q^r, by (12) the marginal value of skill in the c-asset is bounded below by

$$q^c(1-q^c)\min_{\tilde{X}\in\left[0,\frac{1}{1+\lambda^c}\right]} h_L^c\left(q^c+(1-q^c)\tilde{X}\right) > 0.$$

In contrast, from Lemma 3 we know the marginal value of skill in the r-asset approaches 0.

To establish $X^r < X^c$ when q^r is small, suppose to the contrary that $X^r \geq X^c$ even as q^r grows small. From the above comparison of the marginal value of skill, it follows that no-one trades the r-asset for q^r sufficiently small (since the payoff functions are linear and upward sloping, and the payoff for the r-asset has a larger intercept and a smaller slope). But then $X^r = 0$, which contradicts Lemma 2 and so establishes that $X^r < X^c$.

Given $X^r < X^c$ and the comparison of the marginal value of skill, the existence of a cutoff skill level $\hat{\alpha}$ is immediate.

Finally, $\hat{\alpha} > X^c$ because a trader with skill $\alpha = X^c$ has a strictly positive payoff from trading the r-asset but a zero payoff from trading the c-asset; and $\hat{\alpha} < \bar{\alpha}$ because otherwise $A^c = 0$, implying $X^c = 0$, a contradiction. **QED**

Proof of Proposition 3: Write λ for the common value of λ^r and λ^c. For any $x \in (0, \overline{\alpha}]$, define $f(x)$ as the unique solution in $(0,x)$ to

$$f(x)\left(\lambda+\int_{f(x)}^{x} g(\alpha)\,d\alpha\right)-\int_{f(x)}^{x}\alpha g(\alpha)\,d\alpha = 0.$$

The existence of $f(x)$ follows from the fact that

$$z\left(\lambda+\int_z^x g(\alpha)\,d\alpha\right)-\int_z^x \alpha g(\alpha)\,d\alpha$$

is strictly negative at $z=0$ and strictly positive at $z=x$. Uniqueness follows from the fact that differentiation implies that this same function is strictly increasing in z. Moreover, and for use below, note that

$$f'(x)\left(\lambda+\int_{f(x)}^x g(\alpha)\,d\alpha\right)+f(x)\,g(x)-f(x)\,f'(x)\,g(f(x))-xg(x)+f'(x)\,f(x)\,g(f(x))=0,$$

and hence

$$f'(x)=\frac{g(x)(x-f(x))}{\lambda+\int_{f(x)}^x g(\alpha)\,d\alpha}=\frac{f(x)\,g(x)(x-f(x))}{\int_{f(x)}^x \alpha g(\alpha)\,d\alpha}.$$

Define

$$\begin{aligned}\bar{X}^c &= f(\overline{\alpha})\\ \bar{X}^r &= f\left(\bar{X}^c\right),\end{aligned}$$

so that

$$\bar{X}^c = \frac{\int_{\bar{X}^c}^{\overline{\alpha}} \alpha g(\alpha)\,d\alpha}{\lambda+\int_{\bar{X}^c}^{\overline{\alpha}} g(\alpha)\,d\alpha} \tag{A-7}$$

$$\bar{X}^r = \frac{\int_{\bar{X}^r}^{\bar{X}^c} \alpha g(\alpha)\,d\alpha}{\lambda+\int_{\bar{X}^r}^{\bar{X}^c} g(\alpha)\,d\alpha}. \tag{A-8}$$

From the observations about the marginal value of skill in the proof of Proposition 2, we know that as $q^r \to 0$, $X^c \to \bar{X}^c$, $X^r \to \bar{X}^r$ and $\hat{\alpha} \to \bar{X}^c$. So to establish the result, we show

$$\int_{\bar{X}^c}^{\overline{\alpha}} \alpha g(\alpha)\,d\alpha > \int_{\bar{X}^r}^{\bar{X}^c} \alpha g(\alpha)\,d\alpha,$$

or equivalently,

$$\int_{f(\overline{\alpha})}^{\overline{\alpha}} \alpha g(\alpha)\, d\alpha > \int_{f(\bar{X}^c)}^{\bar{X}^c} \alpha g(\alpha)\, d\alpha.$$

Since $\overline{\alpha} > f(\overline{\alpha}) = \bar{X}^c$, it suffices to show that $\int_{f(x)}^{x} \alpha g(\alpha)\, d\alpha$ is increasing in x, or equivalently,

$$xg(x) - f'(x) f(x) g(f(x)) > 0,$$

which substituting in the earlier expression for $f'(x)$ is equivalent to

$$xg(x) > \frac{f(x) g(x) (x - f(x))}{\int_{f(x)}^{x} \alpha g(\alpha)\, d\alpha} f(x) g(f(x)),$$

i.e.,

$$x \int_{f(x)}^{x} \alpha g(\alpha)\, d\alpha > f(x)^2 g(f(x)) (x - f(x)).$$

This inequality is implied by (15), completing the proof. **QED**

Proof of Proposition 4: First note that when traders who trade the c-asset in equilibrium are exogenously reallocated to trading the r-asset, they are happy to actively trade the r-asset. This follows because (by Lemma 1), the minimum skill required to profitably trade the r-asset after the exogenous switch coincides with the minimum skill required to profitably trade the c-asset before the switch.

The result then follows from the following claim:

Claim: The Blackwell informativeness of the aggregate order flow in the r-asset is increasing in the aggregate skill A of the people actively trading the r-asset.

Proof of claim: Let ω_0^r and ω_1^r respectively denote the events that the r-asset does not pay off, $\psi^r > q^r$ and that it does pay off, $\psi^r \leq q^r$. Let H denote the distribution function of λ_L^r. As discussed in the main text, if aggregate skill A actively trades the r-asset, then the information content of the aggregate order flow of the r-asset with respect to $\omega \in \{\omega_0^r, \omega_1^r\}$ is the same as the information content of $A\mathbf{1}_{\omega=\omega_1^r} + \lambda_L^r$. Let $F(\cdot; \omega, A)$ denote the distribution function of $A\mathbf{1}_{\omega=\omega_1^r} + \lambda_L^r$.

Evaluating, $F(y;\omega,A) = H\left(y - A\mathbf{1}_{\omega=\omega_1^r}\right)$ and $F^{-1}(t;\omega,A) = H^{-1}(t) + A\mathbf{1}_{\omega=\omega_1^r}$.

Consider any pair of aggregate skill levels A and $\tilde{A} > A$. Hence

$$F^{-1}\left(F(y;\omega,A);\omega,\tilde{A}\right) = H^{-1}\left(H\left(y - A\mathbf{1}_{\omega=\omega_1^r}\right)\right) + \tilde{A}\mathbf{1}_{\omega=\omega_1^r} = y + \left(\tilde{A} - A\right)\mathbf{1}_{\omega=\omega_1^r}.$$

Consequently, for any y,

$$F^{-1}\left(F(y;\omega_1^r,A);\omega_1^r,\tilde{A}\right) \geq F^{-1}\left(F(y;\omega_0^r,A);\omega_0^r,\tilde{A}\right),$$

i.e., the r-asset order flow is more informative in the Lehmann sense (Lehmann 1988) if it is actively traded by a set of people with aggregate skill $\tilde{A}$ rather than A. Since the density of λ_L^r is log-concave, the distribution function $F(y;\omega,A)$ has the monotone likelihood ratio property. Since $\{\omega_0^r,\omega_1^r\}$ is a binary set, it follows from Proposition 1 in Jewitt (2007) that an increase in aggregate skill A makes the r-asset order flow more informative in the Blackwell sense (Blackwell 1953), completing the proof of the claim, and hence the proof. **QED**

B Explicit calculation of payoffs from trading bonds

As noted in the main text, our model applies to bonds. A bond should be viewed as a combination of a long position in a risk free asset combined with a short position in the j-asset. This assumes the approximation of zero recovery in default. To make this interpretation explicit, here we verify that profits from trading a bond coincide with profits from trading the j-asset in our analysis.

Recall that we denote the ask and the bid prices for the j-asset by P_L and P_S respectively. Viewing the bond as a long position in the risk free asset, which has a price of 1, combined with a short position in the j-asset, its ask price is $1 - P_S$ (the price to take a long position in the bond). Its bid price is $1 - P_L$. Given unlimited size positions in the risk free asset, then the binding position limits are $h_S(P_S)$ for the long position in the bond and $h_L(P_L)$ for the short position. This assumes that position limits are set so that the probability of default on the largest permissible long position in the bond is the same as the probability of default on the largest permissible short position in the the j-asset (for example, default-free position limits (3)).

A skilled trader with skill α buys the bond (i.e., shorts the underlying risky asset) after observing a signal $s > q$ and sells the bond (i.e., is long the underlying risky asset) after observing a signal $s \leq q$. The probability of buying the bond, i.e. signal $s > q$, is $(1 - q)$ and selling the bond, i.e. signal $s < q$, is q.

Conditional on signal $s > q$, the bond defaults with probability $(1 - \alpha) q$. Conditional on signal $s \leq q$, the bond defaults with probability $\alpha + (1 - \alpha) q$. A trader who buys the bond obtains the face value, pays the price, and then loses the face value in the event of default. A trader who sells it short gets the price, minus the face value, plus gains the face value in the event of default. So the trader's expected payoff is

$$
\begin{aligned}
& (1-q)\, h_S\,(P_S) \max\{0, 1-(1-P_S)-(1-\alpha)\, q\} \\
+ \quad & q h_L\,(P_L) \max\{0, (1-P_L)-1+(\alpha+(1-\alpha)\, q)\}\}
\end{aligned}
$$

This expression coincides with expression (4).