The Perils of Personalized Pricing with Network Effects*

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Abstract

This research examines how network effects, a phenomenon whereby the larger the demand for a product or service the higher the value of that product or service for each consumer, influence the impact of price personalization on firms and consumers. We study the impact of network effects on price personalization in terms of price, demand, profit, and consumer surplus. One belief commonly held by companies is that price personalization increases their profit by providing means for capturing heterogeneity in consumers' willingness to pay and enticing demand from heterogeneous consumer segments. Using a game-theoretical model, we find conditions under which price personalization not only *shrinks* demand, but also *decreases* the profit of the firm and reduces surplus of all consumers. Specifically, our model shows in markets with network effects, unless when the impact of expected consumer heterogeneity is higher than the impact of network effects, personalized pricing reduces demand, lowers profit, reduces surplus of all consumers, and thus causes a deficiency in the market.

Keywords: network effects, price personalization, price discrimination, fulfilled expectations equilibrium, customer heterogeneity

1 Introduction

Price personalization is the ongoing trend of charging a different price for the same good or service based on the available data about consumers. The abundance of consumer data has enabled firms to personalize the price for each individual consumer separately (Mohammed (2017)), a practice that is becoming more and more widespread (Farnham (2013)). For example, companies that personalize prices based on analysis of customer data include online retailers such as Staples (Hardawar (2012)), financial companies such as Discover Financial Services (Valentino-Devries et al. (2012)), service providers such as Apple, Adobe, Microsoft (Rimmer (2013)), travel sites Orbitz and CheapTickets (Valentino-Devries et al. (2012)) and even grocery stores such as Safeway (Farnham (2013)). In this research, we examine how network effects impact the profitability of personalized pricing.

We consider *network effects* in which explicit benefits are generated or gained when an individual aligns his behavior with the behavior of others (Easley and Kleinberg (2010)). For example, consider video games with an online component allowing a player to play with other users. This is a product with a positive network effect if players find it more enjoyable to play with other users instead of playing against a computer. In this case, the utility one player gets from the game is dependent on whether other players also own the game. The estimated revenue of the video game industry was $30.7 billion in 2017 (Hatamoto (2014)). As video game companies such as Nintendo Switch are selling multiplayer online games (Buckley and Little (2018)), the industry has the consumer data and technology to implement personalized pricing. Our research aims to identify how network effects affect the incentives to adopt personalized pricing. Though we use video games as a motivating example, personalized pricing is increasingly possible for many industries in which a consumer gains greater value from aligning behavior with other consumers. To inform these decisions on whether to adopt or resist personalized pricing, we address the following three research questions.

- How do network effects influence the profitability of price personalization relative to the profitability of uniform pricing?
- How does price personalization affect the expected customer demand in the presence of network effects?
- How do network effects change the impact of price personalization on the expected surplus of

consumers?

To address these research questions, we develop an analytical model with a monopoly firm selling to heterogeneous customers. A customer receives greater utility from purchasing the good if another customer also purchases the good. Consumer heterogeneity is modeled by defining low-type (high-type) consumers as customers for whom the product has a lower (higher) expected stand-alone value. Stand-alone value of the product for each consumer is derived from the product independent of the network effects. We compare the equilibrium profit, demand, and customer surplus with price personalization to when the price is uniform across customers. Given that the joint consideration of price personalization and network effects is new to the literature, adopting a monopoly model allows us to isolate the impact of network effects in a parsimonious model, highlighting a novel mechanism in the process.

As we preview the novel findings of this research, consider the positive direct effect of price personalization: by charging different prices to different customers a firm can achieve margins on the high-type consumers without sacrificing volume from low-type consumers who require a lower price. One might think that the presence of network effects would magnify the positive direct effect of price personalization given that the firm can attract heterogeneous customers at different prices, allowing each to obtain the additional value associated with the network effect. However, we develop an analytical model that shows when and how network effects can mitigate the positive direct effect of personalized pricing. In fact, despite the common intuition that price personalization increases profit, we show that sufficiently strong network effects can make personalized pricing unprofitable relative to uniform pricing. The result implies that firms selling products with network effects, such as video games, should proceed with caution before deciding to utilize the wealth of customer data to personalize prices.

Note also that price personalization should intuitively increase expected demand since it allows the firm to adjust its price to the utility of the customer. However, this research illustrates a new mechanism that can result in price personalization decreasing the expected demand. The model shows when and why this can occur.

As for the third research question, there is debate about the effect of price personalization on consumer surplus. On one hand, it has been posited that price personalization is bad for consumers and should be objected (Howe (2017)). On the other hand, Wallheimer (2018) suggests that price personalization can benefit consumers because the firm can serve customers who would otherwise be unable to afford the high uniform price. In this paper, we elaborate on the conditions where price personalization results in a higher, lower, or equal expected consumer surplus. A novel contribution is that we find conditions under which price personalization creates a social deficiency hurting the firm and all, *not only some types*, of the consumers. The magnitude of the network effect plays a pivotal role in determining how total consumer surplus and the surplus of the low-type customer are affected by price personalization. Based on these results, we show when and why firms and consumer advocates can be aligned in resisting price personalization.

In summary, our research produces three key findings. First, network effects may mitigate firm gains from price personalization and even make price personalization unprofitable relative to uniform pricing. Second, expected demand can decrease with personalized pricing. Third, network effects play a critical role in determining whether price personalization benefits low-type consumers and overall consumer surplus. The research has implications for managers deciding whether or not to personalize pricing and for consumers and consumer advocates who might consider pushing to prevent this practice as the technological advances make it easier to implement.

The rest of this paper is organized in the following order. In section 2, we provide a brief review of the related literature. In section 3, we describe the model setup and in section 4, we present the analysis and derive the results. In sections 5 and 6, we study model extensions to assess robustness of the key findings. The summary of findings and discussion are in section 7. Tables of results and proofs of the statements are relegated to the Appendices A and B respectively.

2 Literature Review

Price personalization and network effects have each separately been of interest of researchers in economics and marketing. However, to the best of our knowledge, this paper is the first work that looks at the joint impact of the two by analytically studying impact of network effects on price personalization.

There is a large body of literature in economics and marketing studying the impact of price discrimination in different market structures and on different players. We separate research in markets with and without competition to describe the state of the literature on price personalization and price discrimination.

In monopoly markets, researchers have underscored the heterogeneity-capturing impact of price discrimination such that consumers with a higher willingness-to-pay get a higher price than consumers with a lower willingness-to-pay (Kennan, 2001; Hart and Tirole, 1988; Acquisti and Varian, 2005; Villas-Boas, 2004; Stokey, 1979, etc.). Schmalensee (1981) and Varian (1985) focus on the welfare implications of price discrimination by a monopolist. Price discrimination generally increases profits for a monopolist seller. However Villas-Boas (2004) and Acquisti and Varian (2005) find that a dynamic game in which prices are conditioned on the previous period's purchase can lead to diminished firm profit due to rational customers anticipating future prices. Specifically, Villas-Boas (2004) show that in a game with infinite periods in which at each period a new generation of horizontally differentiated customers arrive to the market and stay there for two periods, price discrimination makes customers wait for their second-period lowered price and decreases the profit of the firm. Similarly, Acquisti and Varian (2005) study profitability of price discrimination in a dynamic market with myopic versus rational customers. Myopic customers do not realize that their future prices depends on their current behavior, high-type customers are worse off due to receiving a higher prices in the future and low-type customers are no worse off due to receiving a lower price in the future. Depending on the benefit of selling to high-type customers and versus the loss of selling to low-type customers, the firm decides whether to price discriminate or not. However, when customers are rational, they know if they wait they will get lower prices in future. In this case uniform pricing dominates personalized pricing from the monopolist's perspective because high-type customers can resist purchasing in early stages and wait for their personalized price to fall. In contrast, we show that, in presence of network effects, price personalization can reduce firm profit in a static model.

There is also a line of research focusing on the consequences of price discrimination in competitive markets. Research has explored a variety of price discrimination mechanisms in identifying when price discrimination will increase profit and when it can lead to decreased profitability (e.g. Shaffer and Zhang, 1995; Corts, 1998; Fudenberg and Tirole, 2000; Villas-Boas, 1999; Fudenberg and Tirole, 2000; Chen et al., 2001; Chen and Iyer, 2002; Pazgal and Soberman, 2008; Shin and Sudhir, 2010; Zhang, 2011). On the one hand, consistent with common wisdom, in a competitive setting price discrimination can intensify competition among firms. It can free up both competitors to fiercely compete for each customer type, thereby reducing prices, increasing consumer surplus, and decreasing firm profitability. With endogenous product design, Zhang (2011) underscore another reason why price personalization can hurt competing firm. It happens when firms design less differentiated products to serve more customers and suppress the value of consumers' purchase history, thereby softening competition in future periods. On the other hand, past research has also identified conditions when price discrimination can lead to increased profit. Chen and Zhang (2009) show that in a dynamic game where consumers are in the market for two periods, in the first period firms set prices high enough to recognize their loyal consumers which softens the competition. In the second period, firms can benefit from selling to their loyal vs. price sensitive consumers. Thus, price personalization can increase the profit of firms in both periods. Shin and Sudhir (2010) show that price personalization can lead to a higher profit for firms in both periods if consumers have stochastic preferences and are heterogeneous in the quantity of their purchase. In contrast, and contrary to the common wisdom, our model uniquely shows that in a static and monopoly market, price personalization can simultaneously decrease customer surplus and firm profit if there exist network effects.

In summary, our paper is the first to explore how network effects influence the impact of price personalization on a market. In a static model with a monopoly firm, we counter-intuitively find that network effects can reduce a monopolist's benefit from price personalization. Relative to existing literature, we uniquely find that price personalization can simultaneously decrease customer surplus and firm profit. Furthermore, we show that price personalization can reduce demand from both low- and high-type consumers and increase prices for both consumer types.

3 Model Description

In this section, we build a game-theoretical model to predict how network effects and price personalization interact to affect prices, demand and consequently profit of the firm and consumer surplus. To this end, we consider two market possibilities (network effects versus no network effects) and two firm strategies (personalized pricing versus uniform pricing). The key insights are derived by comparing how network effects versus no network effects affect the difference in market outcomes between personalized pricing and uniform pricing.

We consider a game with three players: a firm and two consumers, consumer H and consumer L. Following the previous literature on network effects (e.g., Katz and Shapiro, 1985), we assume that a consumer's valuation for the product has two components, the intrinsic value for the product, and the utility generated from network effects. The intrinsic value represents the stand-alone value of the product (e.g., the value that the consumer derives from playing a video game individually) and is denoted by variable s_i, $i \in \{L, H\}$. Values of s_i are drawn from uniform distributions $s_H \sim U[0,1]$ and $s_L \sim U[0,a]$, where $0 < a < 1$ is an exogenous parameter. Variable a captures the heterogeneity in consumers' intrinsic valuation for the product, where a lower a implies greater heterogeneity.

In presence of network effects, the consumers have an additional value b for the product if the other consumer also purchases the product. For example, in the context of video games, parameter b represents the additional value that a consumer gets from being able to play the game online with other friends. Therefore, in presence of network effects, the reservation price of consumer i, denoted by r_i, is $r_i = s_i + b$ if the other consumer purchases the product, and $r_i = s_i$ otherwise.[1] Without network effects, the reservation price of consumer i is always $r_i = s_i$. We assume that customer type (e.g., H or$L$) and parameters a and b are common knowledge; however, the realization of each s_i is private information, and only known to consumer i. A consumer decides whether to buy the product or to obtain the value from the outside option, which we normalize to zero.

In the main model, we assume $b < 1$, i.e., the network effect is not too large compared to consumer H's expected intrinsic value for the product. The case of $b \geq 1$ leads to a corner solution where both consumers always buy the product, regardless of their intrinsic values s_i. To focus on the key insights, we relegate the analysis for when $b \geq 1$ to the Appendix.

We consider a two-stage model. In the first stage, the firm sets prices. Under uniform pricing, the firm sets a single price p that applies to both consumers. Under personalized pricing, the firm sets prices p_i for consumer i, where $i \in \{L, H\}$. Note that in the first stage, when setting prices, the firm only knows the consumer type (i.e., the distribution of each consumer's intrinsic valuations), and does not know the realizations of s_L and s_H.

In the second stage, the consumers simultaneously decide whether to buy the product or not. In this stage, if the firm is using uniform pricing, both consumers observe the price p before making their decisions. Under personalized pricing, consumer i only observes his own price p_i and does not know anything about the price of the other consumer. The firm's pricing strategy, i.e., whether the firm uses personalized or uniform pricing, is common knowledge. The timeline of the game is summarized in

[1] In the Online Appendix, we show that our results are robust to assuming a multiplicative network effect in which $r_i = s_i b$.

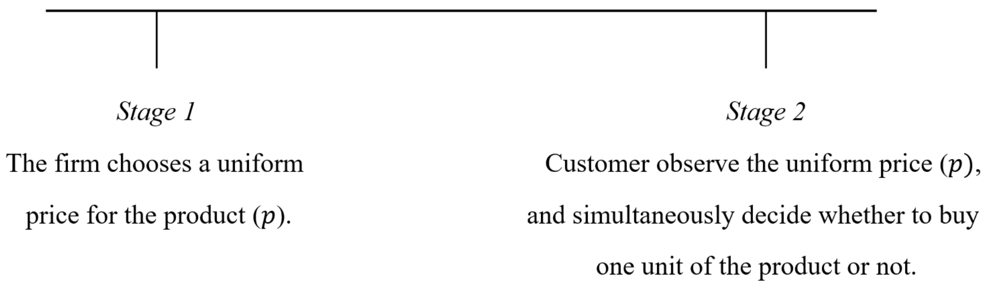


Figure 1: Timeline of the game under uniform pricing

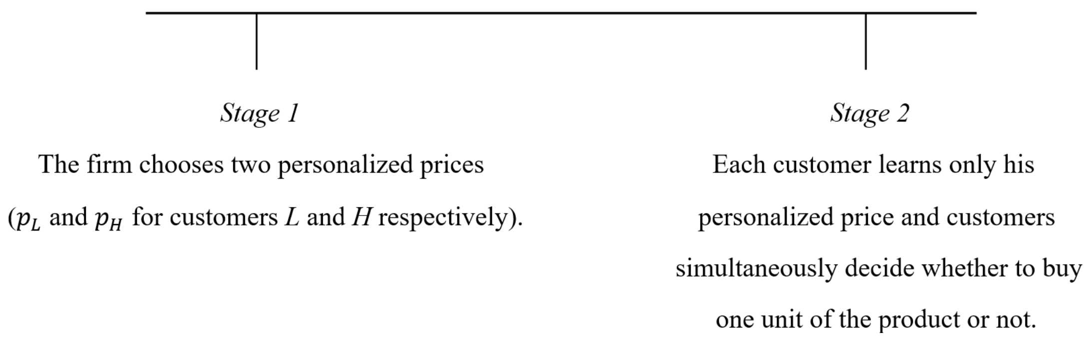


Figure 2: Timeline of the game under personalized pricing

Figures 1 and 2.

Let x_i be the probability that consumer i makes a purchase, and let j represent the index of the other consumer, i.e., $j \in \{L, H\}$ and $j \neq i$. In presence of network effects, the reservation price of consumer i is $r_i = bx_j + s_i$, and when there are no network effects, the reservation price is $r_i = s_i$. We normalize the marginal cost of production to zero. As such, the expected profit of the firm under personalized pricing is $\pi = x_L p_L + x_H p_H$, and the expected profit under uniform pricing is $\pi = x_L p + x_H p$. In presence of network effects, the decision of consumer i depends on the value of x_j that consumer i cannot directly observe. Following the previous literature on network effects (e.g., Katz and Shapiro, 1985; Navon et al., 1995; Easley and Kleinberg, 2010) we find the fulfilled expectation equilibrium of the game where, given the prices, consumers form beliefs about the purchase probability of the other consumers. In equilibrium, the belief of consumer i regarding the purchase probability of consumer j, denoted by $\hat{X}_j$, is the same as consumer j's actual probability of purchase, x_j. Finally, we use CS_i to denote the unconditional expected surplus of consumer i, before his intrinsic valuation and purchase decisions are realized. A summary of the notation is provided in Table 1.

Note that we assume that a consumer knows if a firm has adopted a uniform price. A firm may make its

Symbol	Description
s_i	consumer i's intrinsic interest in the product or the stand alone value of the product for consumer i
r_i	consumer i's reservation price for the product
b	relative importance (degree) of network effects compared to stand alone value of product
a	the expected consumer heterogeneity: consumer L's relative expected intrinsic interest in the product as compared with consumer H's
p	price offered to both consumers under uniform pricing
p_i	price offered to consumer i under price personalization
π	expected profit of the firm
CS_i	(unconditional) expected surplus of consumer i
x_i	purchase probability of consumer i: ex-ante probability that $s_i + b\hat{X}_j \geq p_i$
$\hat{X}_i$	consumer j's belief regarding the purchase probability of consumer i, x_i

Table 1: Summary of notations

uniform pricing policy common knowledge by promoting no-haggle pricing,[2] or by offering a guarantee to match its lowest price.[3]

4 Analysis

In this section, we first solve the model without network effects and then with network effects. By comparing the difference between the impact of price personalization with and without network effects, we will isolate the role network effects play in changing the impact of personalized pricing.

4.1 Markets without network effects

When there are no network effects, the reservation price of each consumer is only affected by his intrinsic interest in the product. In other words $r_i = s_i$, for $i \in \{L, H\}$, and each consumer buys the product if and only if his reservation price is higher than or equal to the price offered to him.

When the firm adopts a uniform pricing strategy, consumer i buys the product if and only if $r_i \geq p$.

[2]See for instance no-haggle pricing policies in the car industry.
[3]See for example the self-matching guarantee policies of Target and Best Buy.

Notice that customer i knows his intrinsic value for the product, s_i, and buys if and only if $s_i \geq p$. However, from the perspective of the firm and the other consumer who only know the distribution of s_i, purchase decisions appears probabilistic. The purchase probabilities of consumers are:

$$x_H(p) = \Pr(r_H \geq p) = \Pr(s_H \geq p) = 1 - p \tag{4.1}$$

$$x_L(p) = \Pr(r_L \geq p) = \Pr(s_L \geq p) = 1 - \frac{p}{a} \tag{4.2}$$

The firm earns zero profit from a consumer who does not buy the product and earns a profit equal to p if the consumer buys the product. Thus, the expected profit of the firm is:

$$\pi(p) = p\left(x_H(p) + x_L(p)\right) \tag{4.3}$$

We solve the game using backward induction. In the second stage, given the uniform price p, consumers H and L have the purchase probabilities in equations (4.1) and (4.2) respectively. In the first stage, given the purchase probabilities, the firm sets a price which maximizes her expected profit.

Lemma 1. *In the absence of network effects, when price is uniform, the equilibrium price, profit, probabilities of purchase, and consumer surplus are as follows.*

$$p^* = \frac{a}{a+1}, \pi^* = \frac{a}{a+1}, x_H^* = \frac{1}{a+1}, x_L^* = \frac{a}{a+1}, CS_H^* = \frac{1}{2(a+1)^2}, CS_L^* = \frac{a^3}{2(a+1)^2} \tag{4.4}$$

Similarly, when the firm adopts a personalized pricing strategy, we have:

$$x_H(p_H) = \Pr(r_H \geq p_H) = 1 - p_H \tag{4.5}$$

$$x_L(p_L) = \Pr(r_L \geq p_L) = 1 - \frac{p_L}{a} \tag{4.6}$$

$$\pi(p_H, p_L) = p_H x_H(p_H) + p_L x_L(p_L) \tag{4.7}$$

By solving the game using backward induction, we can obtain the equilibrium in Lemma 2.

Lemma 2. *In the absence of network effects, when prices are personalized, the equilibrium prices, profit, probabilities of purchase, and consumer surplus are as follows.*

$$p_H^* = \frac{1}{2}, p_L^* = \frac{a}{2}, \pi^* = \frac{a+1}{4}, x_H^* = x_L^* = \frac{1}{2}, CS_H^* = \frac{1}{8}, CS_L^* = \frac{a}{8} \tag{4.8}$$

Next, in Lemma 3, we compare the results of Lemmas 1 and 2 to analyze how price personalization affects the firm and the consumers when there is no network effect.

Lemma 3. *In the absence of network effects, price personalization affects the firm and the consumers as follows.*

(a) The equilibrium personalized price offered to consumer H (consumer L) is more (less) than the equilibrium uniform price.

(b) The equilibrium purchase probability of consumer H (consumer L) with personalized pricing is less (more) than the equilibrium purchase probability when prices are uniform.

(c) Personalized pricing leads to a greater expected profit for the firm.

(d) Personalized pricing leads to a smaller (greater) expected surplus for consumer H (L).

Based on Lemma 3, we can conclude that from the perspective of the firm and in the absence of network effects, a price personalization strategy dominates a uniform pricing strategy. The equilibrium uniform price is between the equilibrium personalized prices, and the expected surplus of consumer L (H) under personalized pricing is greater (smaller) than when prices are uniform.

In section 4.2, we will derive the equilibrium properties with network effects to compare the results in Lemma 3 with those in section 4.2 and identify the impact of price personalization with and without network effects in terms of price, purchase probability, profit and consumer surplus.

4.2 Markets with network effects

In presence of network effects, consumer i's reservation price is $r_i = s_i + b$ if consumer j buys the product, and $r_i = s_i$ if consumer j does not buy, where $i, j \in \{H, L\}$ and $j \neq i$. Since consumers make their decisions simultaneously in the second stage, and s_j is unknown to consumer i, consumer i's decision on whether or not to buy the product depends on his belief about x_j, consumer j's probability of purchase. Specifically, consumer i believes that consumer j purchases the product with probability $\hat{X}_j$, and buys the product if and only if the reservation price $r_i = s_i + b\hat{X}_j$ is greater than or equal to the price. In equilibrium, the beliefs should be consistent with the actual probability of purchase, i.e., $\hat{X}_j = x_j$ for $j \in \{L, H\}$.

Equilibrium Under Uniform Pricing

Under uniform pricing, a rational consumer makes a belief about the purchase probability of the other consumer and this belief naturally depends on the observed uniform price. If a fulfilled expectations equilibrium exists, beliefs are consistent with the actual realization of purchase probabilities. To narrow down the set of equilibria, we rule out beliefs which are arbitrary off the equilibrium path and solve for equilibria in which beliefs are consistent on and out of the equilibrium path (e.g., see Katz and Shapiro, 1985; Grilo et al., 2001; Griva and Vettas, 2011). In other words, we assume that the beliefs are consistent with purchase probabilities even for an off-equilibrium price p.

We use backward induction to solve the game. In the second stage, each consumer makes a belief about the purchase probability of the other consumer, which depends on p. Consumer i buys if $s_i + b\hat{X}_j(p) \geq p$. Thus, the purchase probability of consumer i is $x_i = Pr(s_i \geq p - b\hat{X}_j(p))$.

$$x_H(p) = b\hat{X}_L(p) - p + 1 \tag{4.9}$$

$$x_L(p) = \frac{b\hat{X}_H(p)}{a} - \frac{p}{a} + 1 \tag{4.10}$$

We solve for $\hat{X}_i = x_i$ to find the price-dependent beliefs which are consistent on and out of the equilibrium, and obtain:

$$\hat{X}_H(p) = x_H(p) = \frac{ab - ap + a - bp}{a - b^2} \tag{4.11}$$

$$\hat{X}_L(p) = x_L(p) = \frac{a - bp + b - p}{a - b^2} \tag{4.12}$$

In the first stage, given the purchase probabilities in equations (4.11) and (4.12), the firm sets a price to maximize expected profit. As the firm earns zero profit when consumer i does not buy the product and earns p when consumer i makes a purchase, the expected profit of the firm is:

$$\pi(p) = p(x_H(p) + x_L(p)) \tag{4.13}$$

In equilibrium, the firm chooses a price that maximizes her expected profit. Substituting (4.11) and (4.12) into (4.13) yields:

$$\pi(p) = \frac{p(-p(a + 2b + 1) + ab + 2a + b)}{a - b^2} \tag{4.14}$$

For now, assume $a \neq b^2$. We will solve for the special case of $a = b^2$ later. Purchase probabilities and price-dependent beliefs, which are consistent on and off the equilibrium ((4.11) and (4.12)), must be between 0 and 1. At an interior solution, expected profit is concave in price. However, there are parameters that lead to a corner solution in which all customers buy. Thus, there are two conditions and the resulting equilibrium is proven in the appendix and presented in the following lemma.

Lemma 4. *In presence of network effect, under uniform pricing, the equilibrium price, profit, probabilities of purchase, and consumer surplus are presented in Table 2.*

Lemma 4 presents the equilibrium strategies, when there are network effects and pricing is uniform, as functions of the exogenous variables a (i.e., the expected consumer heterogeneity) and b (i.e., the degree of network effects). The lemma shows that, depending on the values of a and b, there are two types of equilibrium. If the network effect is sufficiently large (i.e., condition A.2), the firm sets the price such that both consumers always purchase the product, regardless of their intrinsic valuation. Otherwise, when condition A.1 holds, we get an interior solution where the probabilities of purchase, consumer surplus, and the equilibrium price vary as functions of a and b. Next, we derive the equilibrium strategies under price personalization, and compare the results to those of Lemmas 3 and 4 to analyze the impact of network effects on price personalization.

Equilibrium Under Price Personalization

If the price offered to one consumer is not observable by the other consumer, a rational consumer makes a belief about the purchase probability of the other consumer independent of price. Specifically, as the price offered to consumer j is not observable to consumer i, consumer i's belief about the purchase probability of consumer j, or $\hat{X}_j$, is independent of the unobserved price p_j and independent of p_i as well, since the latter is uninformative.

We solve the game using backward induction. In the second stage, consumers make expectations about the purchase probability of each other and simultaneously make purchase decisions. The purchase probability of consumer i given his price-independent belief $\hat{X}_j$ is $x_i = Pr(s_i \geq p_i - b\hat{X}_j)$.

$$x_H(p_H) = b\hat{X}_L - p_H + 1 \tag{4.15}$$

$$x_L(p_L) = \frac{b\hat{X}_H}{a} - \frac{p_L}{a} + 1 \tag{4.16}$$

We solve for the fulfilled expectations equilibrium, in which the beliefs are consistent with the actual realization of purchase probabilities. Given Equations 4.15 and 4.16, by setting $\hat{X}_i = x_i$, the firm chooses prices to maximize expected profit:

$$\pi(p_H, p_L) = p_L x_L(p_L) + p_H x_H(p_H) = \frac{p_L\left(a + b\hat{X}_H - p_L\right)}{a} + p_H\left(b\hat{X}_L - p_H + 1\right) \tag{4.17}$$

Note, we constrain both beliefs and actual purchase probabilities between 0 and 1. The analysis yields an interior solution and a corner solution in which the purchase probability for the low type consumer is one. The equilibrium is summarized in the following lemma.

Lemma 5. *When there are network effects and prices are personalized, the equilibrium prices, profit, probabilities of purchase, and consumer surplus are presented in Table 3.*

Next, in Propositions 1 to 4, we use Lemmas 4 and 5 to compare prices, purchase probabilities (demand), profits and consumer surplus of when prices are personalized to when they are not. This allows us to analyze how price personalization affects the firm and the consumers when there are network effects. Figure 3 depicts the results of Propositions 1 to 4.

Proposition 1 (Price Comparison)**.** *When the expected consumer heterogeneity is not too high and the degree of network effects is medium, price personalization increases both prices; i.e., there exists* a_p, $\underline{b}_p(a)$, *and* $\bar{b}_p(a)$ *such that for any* $a > a_p$ *and* $b \in (\underline{b}_p(a), \bar{b}_p(a))$, *we have* $p_H^* > p^*$ *and* $p_L^* > p^*$.

Conventional wisdom suggests that price personalization should lead to a lower equilibrium price for consumer L, which occurs in our model when network effects are absent. Interestingly, Proposition 1 shows that, in presence of network effects, the opposite can happen. To understand this, first consider uniform pricing. When a firm lowers its price, it not only makes the product more affordable to consumer i, it also boosts the reservation value of consumer i via the network effect because consumer i knows the product is also more affordable to consumer j who is therefore more likely to buy it. In contrast, the latter effect of lowering one of its personalized prices p_i is absent. As such, the firm faces a flatter demand curves and dampened incentive to lower prices under price personalization than uniform pricing.

Note this finding occurs when the network effect is not too small or too large. If the network effect is too small, the marginal effect of a price decrease on the reservation price is minimal. If the network effect is very large, the firm does not set a low uniform price because the consumers buy the product in equilibrium with high probabilities, even without a low uniform price. The shaded area (Region B) in Figure 3(a) depicts the result of Proposition 1.

Proposition 2 (Demand Comparison)**.** *When the degree of network effect is medium, the equilibrium purchase probability of consumer L is lower under personalized pricing than under uniform pricing; i.e., there exist $\underline{b}_x(a)$ and $\bar{b}_x(a)$ such that for any $b \in (\underline{b}_x(a), \bar{b}_x(a))$, the purchase probability x_L is smaller when prices are personalized than when price is uniform. The equilibrium purchase probability of consumer H is lower with personalized pricing than uniform pricing.*

Proposition 2 shows that the purchase probability of consumer L can be lower under personalized pricing than under uniform pricing. This result is unique to settings with network effects; i.e., if network effects did not exist, the purchase probability of consumer L under personalized pricing would be higher than under personalized pricing. There are two effects of personalized pricing (in the presence of network effects) that lead to this result. First, as discussed in Proposition 1, the firm sometimes sets a higher price for consumer L under personalized pricing than under uniform pricing. The higher price leads to consumer L's lower probability of purchase. However, this is not the only reason; in fact, as shown in Figure 3(b), there are cases in which consumer L's price under personalized pricing is lower than under personalized pricing, but he still purchases the product with a lower probability. Intuitively, the equilibrium price that consumer L observes under uniform pricing is lower than the price that he rationally expects for consumer H under personalized pricing. As such, consumer L believes consumer H would purchase the product with a lower probability under personalized pricing than under uniform

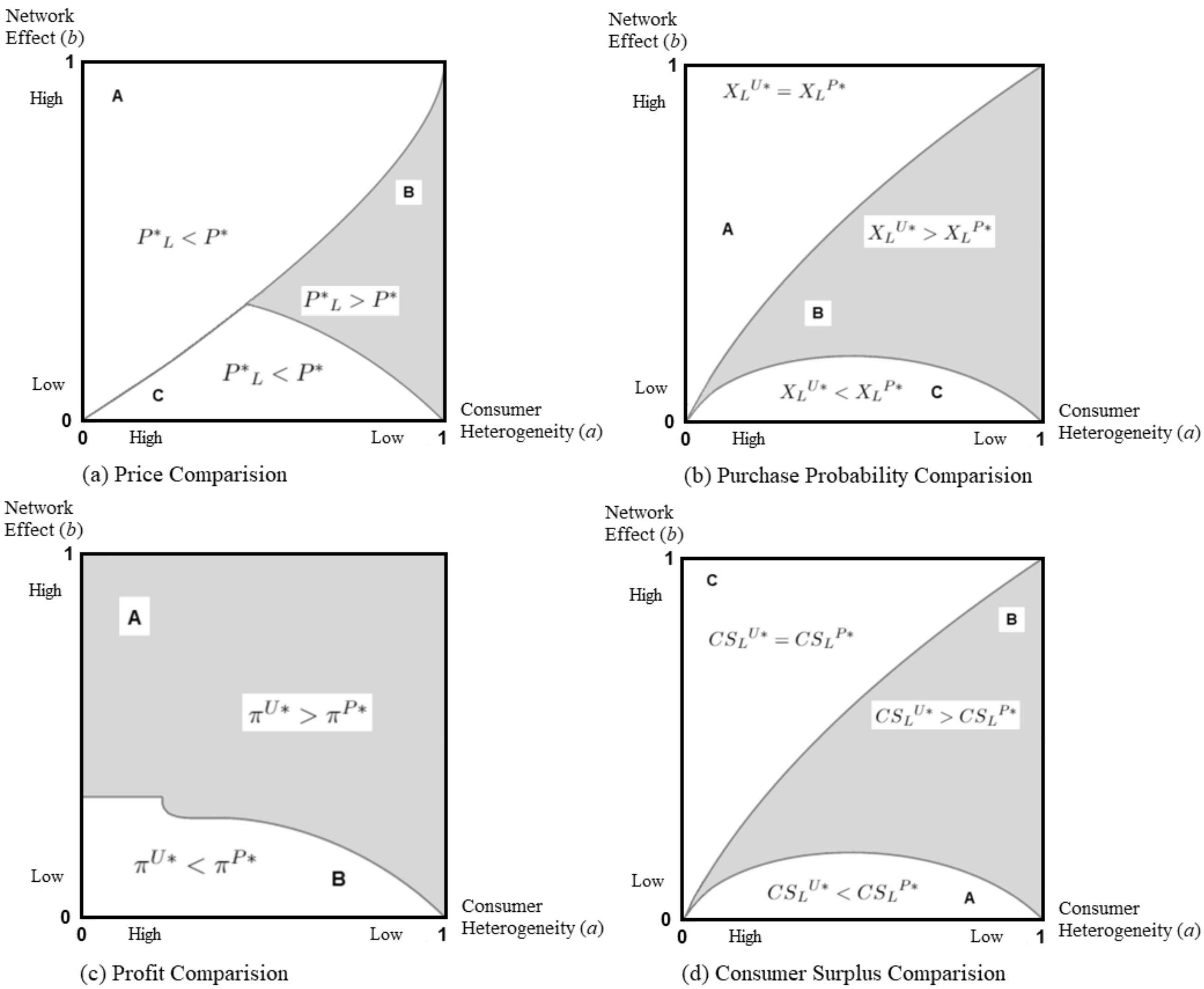


Figure 3: Equilibrium comparison (consumer L)

pricing; this lower belief leads to a lower willingness-to-pay, and a lower probability of purchase for consumer L as well, under personalized pricing than under uniform pricing.

Note that, as in Proposition 1, the purchase probability of consumer L under uniform pricing is higher than under personalized pricing only when the strength of the network effect is medium. When the network effect is small, the purchase probability of consumer L is not significantly affected by that of consumer H, and, therefore, personalized pricing (compared to uniform pricing) does not decrease the purchase probability of consumer L. Similarly, when the network effect is sufficiently strong, consumer L rationally expects that consumer H buys the product with a high probability under personalized pricing; therefore, the purchase probability of consumer L is not significantly affected by the firm's ability to communicate the price of consumer H to consumer L. Finally, for consumer H, the equilibrium personalized price is higher than the equilibrium uniform price. Intuitively, the negative impact of the higher equilibrium personalized price together with the negative impact of personalized pricing on beliefs causes a lower purchase probability for consumer H when prices are personalized.

Proposition 3 (Firm's Profit Comparison)**.** *When the network effect is sufficiently large, the firm's equilibrium profit under uniform pricing is higher than under personalized pricing. Conversely, when the network effect is sufficiently small, the firm's equilibrium profit under uniform pricing is lower than under personalized pricing. Specifically, for any* $a \in (0, 1)$*, there exist* $\underline{b}_\pi(a)$ *and* $\bar{b}_\pi(a)$ *such that when* $b \in (0, \underline{b}_\pi(a))$*, the expected profit of the firm is greater when prices are personalized and when* $b \in (\bar{b}_\pi(a), 1)$*, the expected profit of the firm is smaller when prices are personalized.*

In presence of network effects, price personalization has two opposing effects on the firm's expected profit. First, it allows the firm to charge different prices to different consumers based on their expected intrinsic interests in the product. This *heterogeneity capturing* effect always has a positive effect on firm's expected profit. Second, price personalization has a negative effect on firm's profit by causing an underlying *decision alignment failure.* By *decision alignment failure* we refer to the mechanism by which consumer i cannot observe the price charged to consumer j under personalized pricing, thereby distorting how a customer reacts to marginal changes in price. Naturally, the *heterogeity capturing* effect is stronger with greater heterogeniety and the *decision alignment failure* effect is stronger with a greater network effect.

Region A of Figure 3(c), depicts when the negative decision alignment failure effect dominates its positive heterogeneity capturing effect. Region B of Figure 3(c), depicts when the positive heterogeneity capturing effect of price personalization outweighs the negative decision alignment failure effect and personalized pricing bestows a higher expected profit upon the firm.

Next, we compare the unconditional expected consumer surplus with uniform and personalized pricing. Recall that the unconditional expected surplus of customer i is the expected surplus of consumer i unconditional of his intrinsic interest s_i. We derive the unconditional expected consumer surplus of customer i by integrating over all values of s_i at which customer i makes a purchase.

Proposition 4 (Consumer Surplus Comparison)**.** *When network effects is sufficiently small (i.e., there exist $\underline{b}_{cs}(a)$ and $b \in (0, \underline{b}_{cs}(a))$), price personalization increases the unconditional expected surplus of consumer L. When network effect is medium (i.e., there exist $\tilde{b}_{cs}(a)$ and $\bar{b}_{cs}(a)$ and $b \in (\tilde{b}_{cs}(a), \bar{b}_{cs}(a))$), price personalization decreases the unconditional expected surplus of consumer L. When network effects is sufficiently high (i.e., $b \in (\bar{b}_{cs}(a), 1)$), price personalization and uniform pricing lead to the same unconditional expected surplus for consumer L.*

Proposition 4 shows that, contrary to the common belief that price personalization weakly increases consumer surplus of the low-type consumer, price personalization can decrease the unconditional expected surplus of consumer L. This happens when the impact of network effects is high enough such that the negative decision alignment failure effect of personalized pricing and the subsequent lower purchase probabilities is more intense than the positive effect of a lower-than-the-uniform personalized price on the unconditional expected surplus of consumer L (Region B of Figure 3(d)).

When the network effect is sufficiently large (Region C of Figure 3(d)), consumer L always buys the product under both personalized and uniform pricing (i.e., $x_L^* = 1$). Consumer H, on the other hand, purchases the product with a higher probability under uniform pricing than under personalized pricing. Consumer L benefits from the higher probability of purchase of consumer H under uniform pricing; however, the firm extracts all of that extra surplus by setting a higher price for consumer L under uniform pricing than under personalized pricing. As such, when the network effect is sufficiently large, the unconditional expected surplus of consumer L is the same under uniform and personalized pricing. Finally, as one would expect, when the network effect is sufficiently small (Region A of Figure 3(d)), price personalization makes consumer L better off by allowing the consumer to purchase the product at a lower price.

5 The Role of Price Transparency

In this section, we develop two extensions to examine how price transparency influences the results. To this end, we consider two scenarios. In the first extension, we examine what happens if the high price, p_H, is observed by both consumers and the firm personalizes a discount for consumer L that is unobserved to consumer H. Through this first extension, we aim to find whether our key insights survive in this context. In the second extension, we examine what happens if there is some probability that prices are observable by all consumers. Through this second extension, we aim to show how the level of price transparency impacts the results.

5.1 Unobserved Discounts

In our main model, when prices are personalized, consumers cannot observe each others' prices. In this extension, we consider a scenario where, under personalized pricing, consumer H cannot observe the price of consumer L, but the price offered to consumer H is observable by customer L. This is motivated by situations in which the firm offers a high price at which the product is available to everyone, but also sends a price promotion to consumers who have a lower willingness to pay (i.e., the low-type customers). Note that, in this setting, the high-type consumer still forms rational expectation about the price offered to the low-type consumer.

In this model, consumer H only observe p_H whereas consumer L observes both p_H and p_L. Therefore, consumer L's belief about the purchase probability of consumer H, $\hat{X}_H(p_H, \hat{X}_L)$, depends on p_H, but not on p_L. Similarly, since consumer H knows that consumer L observes p_H when making a purchase decision, consumer H's belief about consumer L's probability of purchase, $\hat{X}_L(p_H)$, is also a function of p_H. Note that neither of these beliefs can be a function of p_L. As for the actual probabilities of purchase, consumer H's probability of purchase, $x_H(p_H, \hat{X}_L)$, is a function of the price p_H and the belief of consumer H on purchase probability of consumer L, $\hat{X}_L$. On the other hand, the actual probability of purchase of consumer L, $x_L(p_L, p_H, \hat{X}_H)$, is a function of both prices and the belief.

Since x_H and $\hat{X}_H$ are both direct functions of p_H, we have to let $x_H = \hat{X}_H$ *before* optimizing the prices. For x_L and $\hat{X}_L$, on the other hand, since $\hat{X}_L$ is not a direct function of p_L (while x_L is), we have to solve for $x_L = \hat{X}_L$ *after* we optimize the prices. In other words, since a marginal change in p_L affects

x_L but does not affect $\hat{X}_L$, the equality $x_L = \hat{X}_L$ should be imposed after the prices are optimized. However, since a marginal change in p_H directly affects both x_H and $\hat{X}_H$, equality $x_H = \hat{X}_H$ should be imposed before the prices are optimized.

Recall that:

$$x_H = 1 + b\hat{X}_L - p_H$$

$$x_L = \frac{a + b\hat{X}_H - p_L}{a}$$

By replacing $\hat{X}_H = x_H$ we get:

$$x_H = 1 + b\hat{X}_L - p_H$$

$$x_L = \frac{a + b(1 + b\hat{X}_L - p_H) - p_L}{a}$$

Next, we find the optimal prices p_L and p_H, as functions of $\hat{X}_L$, a and b, to maximize the seller's revenue:

$$\pi(p_L, p_H) = p_H x_H + p_L x_L = p_H(1 + b\hat{X}_L - p_H) + p_L \frac{a + b(1 + b\hat{X}_L - p_H) - p_L}{a}$$

Solving for the first order conditions, we get the interior solution:

$$p_L = \frac{a\left(2a + b^2\hat{X}_L + b\right)}{4a - b^2}$$

$$p_H = \frac{a(b(2\hat{X}_L - 1) + 2) - b^2(b\hat{X}_L + 1)}{4a - b^2}$$

Now, by solving for $x_L = \hat{X}_L$ we get:

$$x_L = \hat{X}_L = \frac{2a + b}{4a - 2b^2}$$

And, by replacing this into the expressions for p_L, p_H and π, we get:

$$p_L^* = \frac{a(2a + b)}{4a - 2b^2}$$

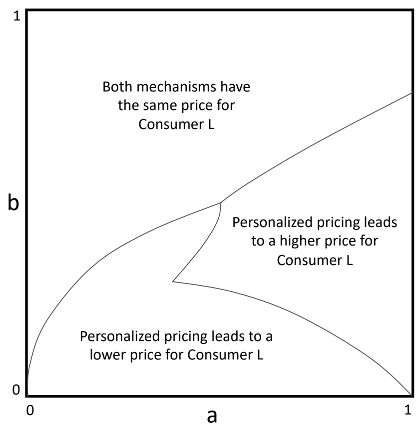
$$p_H^* = \frac{1}{2}$$

$$\pi^* = \frac{a\left(8ab + 4a(a + 1) - 2b^3 - b^2\right)}{4\left(b^2 - 2a\right)^2}$$

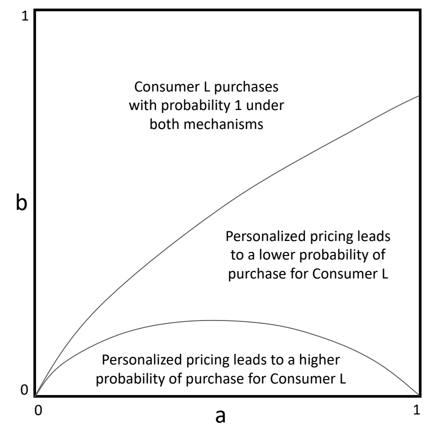
The above solution, i.e., the interior solution, is only valid if $0 \leq x_L, x_H \leq 1$, which is satisfied if:

$$b \leq \min(\frac{\sqrt{16a+1}-1}{4}, \frac{\sqrt{a^2+4a}-a}{2})$$

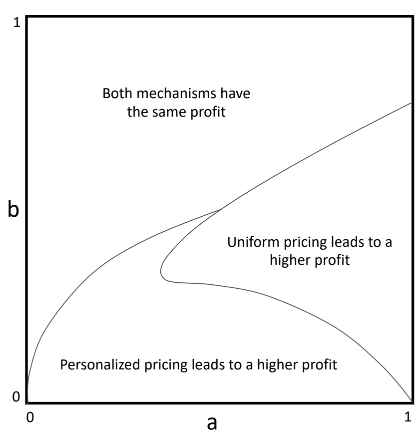
After solving for the corner solutions, we get four regions where $(x_L < 1, x_H < 1)$, $(x_L = 1, x_H < 1)$, $(x_L < 1, x_H = 1)$, and $(x_L = 1, x_H = 1)$. The summary of the results for these four regions are presented in Table 4. The regions in Table 4 are depicted in Figure 4d.



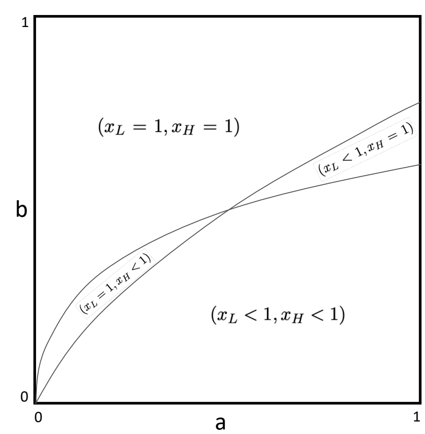
(a) Price of Consumer L



(b) Consumer L's Probability of Purchase



(c) Firm's Profit



(d) Regions

Figure 4: Effects of Personalized Pricing on Equilibrium Outcomes

Next, we compare the results of Table 4 to those of uniform pricing, Table 2, to analyze how personalized pricing affects the firm and the consumers. Figure 4c shows that our main result regarding the firm's profitability, from Proposition 3, continues to hold in this new setting where consumer L knows about both p_L and p_H and consumer H only knows p_H. As we can see, when b is large, such that $x_L = x_H = 1$, both mechanisms lead to the same equilibrium profit. When b is sufficiently small, i.e., the network effect is weak, personalized pricing leads to a higher profit. Finally, when a is sufficiently large, i.e., the consumers are not too asymmetric in terms of the intrinsic valuations, for a medium value of b, uniform pricing leads to a higher profit than personalized pricing. These results are consistent with our findings in Proposition 3 where, after excluding the corner solution where $x_L = x_H = 1$, we show that when the asymmetry in the consumers' intrinsic valuations is sufficiently low or the network effect is sufficiently strong, personalized pricing lowers the firm's equilibrium profit.

Similarly, Figure 4a compares the equilibrium price of consumer L, p_L, with and without personalized pricing. As we can see, the results are consistent with our findings in Proposition 1. In particular, our counter-intuitive finding, that when a is sufficiently large and b is medium, consumer L gets a higher price under personalized pricing than under uniform pricing, continues to hold in this extension.

Finally, Figure 4b shows how personalized pricing affects the purchase probability of consumer L, x_L. Similar to our finding in Proposition 2, we show that for any level of asymmetry between the consumers' intrinsic valuations, a, when the degree of network effect is medium, the equilibrium purchase probability of consumer L is lower under personalized pricing than under uniform pricing. Overall, this extension shows that the insights from our main model continue to hold even when consumer L can observe p_H.

5.2 Probability of Transparent Prices

In this extension, we allow for a probability of $(1-\gamma)$ that customers, independent of consumer types, know both p_H and p_L prior to purchase. In the interest of parsimony, we assume that with probability $(1-\gamma)$ both consumers are informed of both prices and with probability γ consumer i is only informed about p_i and is uninformed about p_j. Furthermore, γ is common knowledge, but the consumers' information state is unobserved by the firm.[4] In what follows, we derive the personalized pricing equilibrium and compare it to the uniform pricing equilibrium, which is the same as the one presented in Table 2.

[4]Recall that the previous extension verified that our qualitative insights persist when there is asymmetry in information across consumers. As such, we focus this extension on the case in which both consumers share the same information state. This allows us to parsimoniously examine the effect of increasing the likelihood that consumers observe both prices on our results.

Consumer i buys if $s_i + b\hat{X}_j^k \geq p_i$ where $k \in \{I, U\}$ indicates whether consumers are informed (I) or uninformed (U) about prices. The purchase probabilities as functions of beliefs are:

$$x_H(p_H) = b\hat{X}_L^k - p_H + 1 \tag{5.1}$$

$$x_L(p_L) = \frac{b\hat{X}_H^k}{a} - \frac{p_L}{a} + 1 \tag{5.2}$$

With probability $(1-\gamma)$, prices are observed and we can solve $\hat{X}_j^I = x_j$ for $j \in \{L, H\}$ to find the beliefs that are consistent on and off the equilibrium path:

$$\hat{X}_H^I(p_H, p_L) = x_H(p_H) = \frac{ab - ap_H + a - bp_L}{a - b^2} \tag{5.3}$$

$$\hat{X}_L^I(p_H, p_L) = x_L(p_L) = \frac{a - bp_H + b - p_L}{a - b^2} \tag{5.4}$$

With probability γ, prices are unobserved and as such $\hat{X}_j^U$ for $j \in \{L, H\}$ are independent of prices. The firm chooses prices that maximize her expected profit:

$$\pi(p_H, p_L) = \gamma[p_H(b\hat{X}_L^U - p_H + 1) + p_L(\frac{b\hat{X}_H^U}{a} - \frac{p_L}{a} + 1)] + (1-\gamma)[p_H(b\hat{X}_L^I - p_H + 1) + p_L(\frac{b\hat{X}_H^I}{a} - \frac{p_L}{a} + 1)] \tag{5.5}$$

Substituting (5.3) and (5.4) into (5.5) leads to:

$$\pi(p_H, p_L) = \frac{(1-\gamma)\left(p_H(ab + a - 2bp_L) + p_L(a + b - p_L) - ap_H^2\right)}{a - b^2} + \gamma\left(\frac{p_L\left(a + b\hat{X}_H - p_L\right)}{a} + p_H\left(b\hat{X}_L - p_H + 1\right)\right)$$

Constraining beliefs and purchase probabilities to be between 0 and 1, the analysis yields an interior solution and several corner solutions in which the purchase probability for consumer $i \in \{L, H\}$ is 1. To derive observations about how γ impacts our main insights, we consider the interior solution in which the network effect b is low enough such that the purchase probabilities are strictly between 0 and 1.

We first compare personalized prices to the uniform price. It is straightforward to show that p_H^* is greater than p^* for all γ. Next we consider the low price. At $\gamma = 0$, $p_L^* - p^* = \frac{(a-1)(a+b)}{2(a+2b+1)} < 0$. However, $\frac{d(p_L^* - p^*)}{d\gamma} = \frac{a\left(4ab(b\gamma+1) + b^3\gamma^2\right)}{\left(b^2\gamma^2 - 4a\right)^2} > 0$. From the proof of Proposition 1, we know that p_L^* can be greater than p^* when $\gamma = 1$. Note also that the range of a and b that will lead to an interior solution for personalized pricing in which the purchase probability is less than 1 weakly increases with γ as evidenced by the fact that $\frac{dx_H^*}{d\gamma} = -\frac{ab\left(b^2\gamma^2(a+b)+4ab(b+1)\gamma+4a(a+b)\right)}{(a-b^2)(b^2\gamma^2-4a)^2} < 0$ and $\frac{dx_L^*}{d\gamma} = -\frac{ab\left(4a(b\gamma+b+1)+b^2\gamma(b\gamma+\gamma+4)\right)}{(a-b^2)(b^2\gamma^2-4a)^2} < 0$. From these facts, we can draw the following observation analogous to Proposition 1.

Remark 1. *There exists $\gamma^* < 1$ such that price personalization can increase both prices when $\gamma \geq \gamma^*$.*

We next compare the purchase probability of consumer L with personalized pricing to that from uniform pricing. At $\gamma = 0$, the uniform x_L^* subtracted from the personalized x_L^* is equal to $\frac{1-a}{2a+4b+2} > 0$. From above, the personalized x_L^* is decreasing in γ. From the proof of Proposition 2, we know that at $\gamma = 1$, the purchase probability of consumer L can be lower in personalized pricing than in uniform pricing. From these facts, we can draw the following observation analogous to Proposition 2.

Remark 2. *There exists $\gamma' < 1$ such that the equilibrium purchase probability of consumer L can be lower under personalized pricing than under uniform pricing if $\gamma \geq \gamma'$.*

Finally, we compare the firm's profit with personalized pricing relative to uniform pricing. At $\gamma = 0$, personalized pricing generates $\frac{(a-1)^2}{4(a+2b+1)}$ more profit than uniform pricing. The derivative of the profit from personalized pricing with respect to γ is:

$$\frac{d\pi^*}{d\gamma} = -\frac{ab^2\gamma\left(8a^3+2a^2\left(3b^2\gamma(\gamma+2)+4b(3\gamma+2)+4\right)+ab^2\gamma\left(b(b+2)\gamma^2+6(2b+1)\gamma+12\right)+b^4\gamma^3\right)}{(a-b^2)(4a-b^2\gamma^2)^3} < 0$$

From the proof of Proposition 3, we know that at $\gamma = 1$, the profit from uniform pricing can be greater than the profit from personalized pricing. From these facts, we can draw the following observation analogous to Proposition 3.

Remark 3. *There exists $\gamma'' < 1$ such that firm profit can be lower under personalized pricing than under uniform pricing if $\gamma \geq \gamma''$.*

In summary, this extension shows that our counter-intuitive findings will persist as long as there is a sufficient probability that consumers do not observe each other's prices before purchase. If this happens, then personalized pricing can decrease the firm's profit, decrease the purchase probability of customer L, and lead to personalized prices that are higher than the uniform price.

In practice, there are several factors that can influence the number of consumers who do not observe personalized pricing and account for existence of a sufficiently high γ. First, in our model, for the sake of simplicity, we assumed that there are only two customers. If there are many consumers, there are difficulties for both the firm and consumers. The firm may find it cumbersome or off-message to announce all the personalized prices being charged to each individual customer. In this case, each individual customer has a personalized price which is hard for the firm to publish. Individuals may find it time-consuming or challenging to collect and analyze all the personalized prices. The firm can also update personalized prices based on customer data at each point of time which adds to the difficulty of publicizing prices on the firm side and analyzing and using prices on the customer side.

Second, consumer fairness concerns may lead to a consumer backlash and diminished profitability associated with price transparency, thereby incentivizing the firm not to announce personalized prices. Related literature has also documented that personalized pricing has adverse effect on certain customers (Dong et al. (2020)) and its impact on upsetting customers is a concern for firms (Wallheimer (2018)). Haws and Bearden (2006) use fairness heuristic theory and empirically show that price differences can lead to strong perceptions of unfairness among customers and decrease overall customer satisfaction. Research also suggests that avoiding peer-induced fairness concerns among customer can make firms avoid personalized pricing and charge a uniform price (Ho and Su (2009)). Allender et al. (2021) find that price obfuscation can effectively eliminate peer-induced fairness concerns, illustrating why a firm may choose to avoid full transparency in prices.

Third, prospect theory (KAI-INEMAN and Tversky (1979)) suggests that communicating the low prices can serve as a referent that makes consumers unwilling to pay higher prices. Thus, the lack of price transparency for personalized prices can be owing to factors that make it optimal for the firm to avoid price transparency and suboptimal for the consumer to exert the extensive effort to become informed of the prices offered to other individual customers.

6 The Role of Purchase Timing

In this extension, we allow for customers to make decisions sequentially rather than simultaneously. The goal is to check whether the key findings of the main model depend on the order by which customers make their purchase decisions. We will solve for the equilibria under uniform and personalized pricing to compare the equilibrium outcomes. For either uniform or personalized pricing, we will consider two cases. In the first (second) case, we assume that customer H (L) decides whether to buy or not.

Customer L (H) observes customer H's (L's) purchase decision and then makes his purchase decision. Assume customer i is making his purchase decision first and denote by $\hat{X}_j^B$, customer i's belief about the purchase probability of customer j knowing that he himself (i.e., customer i) is going to buy the product. Also, denote by x_j^B and x_j^{NB}, the purchase probability of customer j if customer i has bought or has not bought the product respectively.

Uniform Pricing

We solve a model with three stages to find the equilibrium outcomes. At the first stage, the firm sets the uniform price p. At the second stage, customer i observes the uniform price p and makes a belief $\hat{X}_j^B$ about the purchase probability of customer j. Similar to the main model, we assume that when price is uniform, $\hat{X}_j^B$ depends on the observable price p as it is fully informative of the price customer j is offered. Customer i buys the product if $s_i + b\hat{X}_j^B(p) > p$. At the final stage, customer j observes the purchase decision of customer i and the uniform price and makes his purchase decision. We use backward induction to solve the game.

Case 1: Customer H makes purchase decision before customer L does

Given the uniform price p, customer H buys the product if $s_H + b\hat{X}_L^B(p) > p$. So, $x_H(p) = 1 - p + b\hat{X}_L^B(p)$. After realization of customer H's decision, the purchase probability of customer L if customer H purchase or do not purchase the product is $x_L^B(p) = Pr(s_L + b > p) = \frac{1}{a}(a - p + b)$ and $x_L^{NB}(p) = Pr(s_L > p_L) = \frac{1}{a}(a - p)$ respectively. We set $\hat{X}_L^B(p) = x_L^B(p)$ since the belief depends on price and should be consistent with $x_L^B(p)$ on and off the equilibrium path. This leads to:

$$x_H(p) = \frac{b(a + b - p)}{a} - p + 1 \tag{6.1}$$

Before realization of customer H's purchase decision, the expected purchase probability of customer L is:

$$x_L(p) = x_L^B(p)x_H(p) + x_L^{NB}(p)(1 - x_H(p)) = \frac{a^2 + a(b + 1)(b - p) + b^2(b - p)}{a^2} \tag{6.2}$$

Therefore, the profit of the firm, as a function of the uniform price, is:

$$\pi(p) = p(x_H(p) + x_L(p)) = \frac{p\left(a^2(b-p+2) + a(2b+1)(b-p) + b^2(b-p)\right)}{a^2} \tag{6.3}$$

Case 2: Customer L makes purchase decision before customer H does

In this case, customer L purchases if $s_L + b\hat{X}_H^B(p) > p$. So, $x_L(p) = \frac{1}{a}(a - p + b\hat{X}_H^B(p))$. Similar to the previous case, we have $x_H^B = 1 + b - p$ and $x_H^{NB} = 1 - p$ and we set $\hat{X}_H^B(p) = x_H^B(p)$ to find the price-dependent belief. This leads to:

$$x_L(p) = \frac{(b+1)(b-p)}{a} + 1 \tag{6.4}$$

So, before realization of customer L's purchase decision, the purchase probability of customer H and the expected profit of the firm will be:

$$x_H(p) = x_H^B(p)x_L(p) + x_H^{NB}(p)(1 - x_L(p)) = \frac{(b+1)b(b-p)}{a} + b - p + 1 \tag{6.5}$$

$$\pi(p) = p(x_H(p) + x_L(p)) = p\left(\frac{(b+1)^2(b-p)}{a} + b - p + 2\right) \tag{6.6}$$

Given the profit expressions in (6.3) and (6.6), we can find the uniform price that maximizes firm's expected profit. This analysis leads to an interior solution for each case where both purchase probabilities are strictly smaller that 1 and one corner solution where both purchase probabilities are equal to 1. The equilibrium properties are presented in Tables 6 and 7.

Personalized Pricing

As the firm can set personalized prices, she can take the advantage of observing the purchase decision of the first customer and then setting the personalized price for the second customer. Note that the firm does not have this advantage in the main model where customers simultaneously make their purchase decisions. We use a model with four stages to find the equilibrium outcomes. At the first stage, the firm offers a personalized price p_i to customer i. At the second stage, customer i makes a belief (i.e., $\hat{X}_j^B$)

about the purchase probability of customer j conditional on he himself buying the product. Similar to the main model, when prices are personalized, $\hat{X}_j^B$ does not depend on p_i as it is uninformative. Customer i buys the product if $s_i + b\hat{X}_j^B > p_i$. After realization of customer i's decision, at stage three, the firm observes this decision and offers a personalized price to customer j. The price offered to customer j depends on the decision of costumer i. Denote by p_j^B and p_j^{NB} the personalized price offered to customer j conditional on customer i buying or not buying the product respectively. At stage four, customer j observes the decision of customer i, and the personalized price p_j^B or p_j^{NB} offered to him (depending on the earlier decision of customer i) and makes his purchase decision. We use backward induction for solving this game.

Case 1: Customer H makes purchase decision before customer L

At stage four, knowing the purchase decision of customer H and the price (p_L^B or p_L^{NB}), the purchase probability of customer L is $x_L^B(p_L^B) = \frac{1}{a}(a - p_L^B + b)$ or $x_L^{NB}(p_L^{NB}) = \frac{1}{a}(a - p_L^{NB})$. At stage three, the firm sets prices p_L^B or p_L^{NB} such that it maximizes her expected profit. The profit that the firm obtains from selling to customer L if customer H has bought or has not bought the product is:

$$\pi(p_L^B) = p_L^B * x_L^B(p_L^B) = p_L^B(\frac{1}{a}(a - p_L^B + b)) \tag{6.7}$$

$$\pi(p_L^{NB}) = p_L^{NB} * x_L^{NB}(p_L^{NB}) = p_L^{NB}(\frac{1}{a}(a - p_L^{NB})) \tag{6.8}$$

By maximizing the profit expressions in (6.7) and (6.8), we can find the optimal personalized prices the firm should offer customer L and the purchase probabilities of customer L.

At the second stage, customer H buys the product if $s_H + b\hat{X}_L^B > p_H$. Thus:

$$x_H(p_H) = 1 - p_H + b\hat{X}_L^B \tag{6.9}$$

Therefore, at stage one, the firm should set the price p_H that maximizes her expected profit from selling to both customers:

$$\pi(p_H) = x_H(p_H)(p_H + \pi(p_L^B)) + (1 - x_H(p_H))(\pi(p_L^{NB})) \tag{6.10}$$

Given the profit expression in (6.10), the firm sets the price p_H to maximize her expected profit. The belief $\hat{X}_L^B$ should be consistent with x_L^B on the equilibrium path. So after finding the optimal price p_H, we solve for $\hat{X}_L^B = x_L^B$ to find the consistent price-independent belief. This leads to:

$$\hat{X}_L^B = \frac{a + b - p_L^B}{a}$$

Finding the optimal prices that maximize the profit expressions in (6.7), (6.8) and (6.10) leads to an interior solution and two corner solutions. We summarized the equilibrium properties in Table 8.

Case 2: Customer L makes purchase decision before customer H

At stage four, the purchase probability of customer H knowing the decision of customer L and the personalized price (p_H^B or p_H^{NB}) is $x_H^B(p_H^B) = 1 - p_H^B + b$ or $x_H^{NB}(p^{NB}) = 1 - p_H^{NB}$ depending on customer L's earlier decision. At stage three, the firm sets p_H^B or p_H^{NB} to maximize her expected profit from selling to customer H conditional on the realized decision of customer L:

$$\pi(p_H^B) = p_H^B * x_H^B(p_H^B) = p_H^B(1 - p_H^B + b) \tag{6.11}$$

$$\pi(p_H^{NB}) = p_H^{NB} * x_H^{NB}(p_H^{NB}) = p_H^{NB}(1 - p_H^{NB}) \tag{6.12}$$

Maximizing the profit expressions in (6.11) and (6.12), we can find the optimal personalized prices and purchase probabilities for customer H.

At the second stage, the probability that customer L buys is $x_L(p_L) = Pr(s_L + b\hat{X}_H^B > p_L)$. Thus:

$$x_L(p_L) = \frac{1}{a}(a - p_L + b\hat{X}_H^B) \tag{6.13}$$

At the first stage, the firm sets the price p_L that maximizes her expected profit:

$$\pi(p_L) = x_L(p_L)(p_L + \pi(p_H^B)) + (1 - x_L(p_L))(\pi(p_H^{NB})) \tag{6.14}$$

Given the profit expression in (6.14), the firm sets the price p_L to maximize her expected profit. The belief $\hat{X}_H^B$ should be consistent with x_L^B in the equilibrium (if any). To reflect this belief consistency constraint, after finding the optimal price p_L, we set $\hat{X}_H^B = x_H^B$. This leads to:

$$\hat{X}_H^B = 1 - p_H^B + b$$

Finding the optimal prices that maximize the profit expressions in (6.11), (6.12) and (6.14) leads to an interior solution and three corner solutions. The summary of the equilibrium outcomes are presented in Table 9.

Now that we have derived the equilibrium properties of each case, we compare the outcomes. Specifically, we compare the results of Table 6 with Table 8 for the first case (i.e., customer H decides first), and we compare the results of Table 7 with Table 9 for the second case (i.e., customer L decides first).

Remark 4. *When customer heterogeneity is sufficiently small, there exists network effects such that personalized pricing leads to lower firm's profit, lower probability of purchase, and higher prices for both consumers when consumers make purchase decisions sequentially.*

Remark 4 demonstrates that the key findings of our main model regarding the impact of personalized pricing on price, demand and firm profitability continue to hold when customers make their purchase decisions sequentially. Therefore, sequential pricing is another mechanism by which the personalized prices may not be observable by all consumers and generate our counter-intuitive findings.

7 Concluding Remarks

With recent advances in information technology, collecting and analyzing consumer data is getting easier and more frequent. This has increased the use of data for personalized pricing based on consumers' willingness to pay. Network effects exist in lucrative industries such as gaming and this research identifies a decision alignment effect of price personalization that creates unique prescriptions for managers when operating within such an industry. Comparing the model predictions with and without network effects, we showed that while the *heterogeneity capturing effect* of price personalization exists in both types of markets, the *decision alignment failure effect* can lead to novel insights.

First, we showed that without price personalization, consumers know that the other consumers are also seeing the uniform price and form expectations about the purchase probability of other consumers depending on the uniform price. The firm can then lower the uniform price to increase purchase probabilities and consequently boost the reservation value for consumers who experience a gain from the network effect. Although price personalization enables the firm to capture consumer heterogeneity in willingness to pay, our model uniquely predicts that with a network effect, personalized pricing deprives the firm from boosting the reservation value (and consequently demand) through lowering the price. In other words, our interesting result is that price personalization flattens the demand curve. When the firm lowers a personalized price for a consumer, that consumer does not change his belief about the purchase probability of other consumers since he knows that prices are not the same across all. This result shows that lowering personalized prices is a less effective tool for increasing demand compared to lowering a uniform price.

Second, the results contradict the common belief that price personalization increases firm profit through the *heterogeneity capturing* effect. We showed there are situations when demand shrinks and expected profit decreases with personalized pricing. When expected customer heterogeneity is not high, price personalization decreases expected profit due to a fall in demand and the benefit gained from capturing customer heterogeneity cannot overturn the loss caused by the *decision alignment failure* effect. The negative impact of price personalization on network size can be so high so as to make the firm and the consumers worse off.

Third, there are circumstances that due to network effects, price personalization increases personalized prices offered to all consumers. This is in contrast to the common intuition that there are necessarily some consumers who receive a lower-than-the-uniform personalized price. Thus, we showed that perils of personalized pricing may adversely affect all customers not only the high-type customers.

Fourth, another belief commonly held is that expected consumer surplus for a low-type consumer will weakly increase with price personalization. However, we showed that when network effects are strong enough, price personalization can decrease the surplus of low-type consumers.

We also studied the influence of price announcements, price transparency and purchase timing on our findings by examining three more scenarios. First, we showed that our findings hold when both customer types know a regular price but the firm offers a discount only to the low-type consumers such that only the low-type customers know the promotional discount. Second, we showed that our findings continue to hold when both customer types observe the personalized price of the other customer-type with a sufficiently low probability. Thus, some level of price non-transparency is sufficient for our findings to happen. Third, we showed that our key findings continue to hold when customers do not purchase at the same time and the decision of the customer type that purchases first is visible to the other customer type and the firm.

Our findings inform firms that before initiating personalized pricing algorithms, they should weigh the costs and benefits of price personalization against each other and make sure that they are not gaining less (capturing consumer heterogeneity) by paying more (defecting their network and loosing some proportion of their demand). Our message is that price personalization does not necessarily increase the demand. Thus, the takeaway for managers is that acquiring adequate insights about the degree of heterogeneity in consumers willingness to pay and the strength of network effects in the market are necessary steps before switching to price personalization. The takeaway for consumers is that price personalization may make them better off or worse off. High-type consumers are worse off but low-type consumers should be aware that price personalization may increase or decrease their surplus. Whether price personalization is to their benefit or detriment depends on the strength of network effects and expected consumer heterogeneity.

We encourage future research to extend our work and explore the impact of network effects on price personalization in markets with different structures. Exploring the impact of network effects on personalized pricing in markets with competing firms is an interesting avenue for future research. Another research direction we recommend is empirically testing the findings of our paper in a market with sufficiently high network effects and providing evidence that customers and the firm may worse off if a firm practices personalized pricing and customer heterogeneity is not strong.

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A Tables of Results

In this Appendix, we present Tables 2 to 9.

	If $\sqrt{a(a+34)+1} \geq a+8b+1$ (A.1)	If $\sqrt{a(a+34)+1} < a+8b+1$ (A.2)
p^*	$\frac{ab+2a+b}{2a+4b+2}$	b
π^*	$\frac{(ab+2a+b)^2}{4(a+2b+1)(a-b^2)}$	$2b$
$\hat{X}_H^*, x_H^*$	$\frac{a^2b+3ab^2+3ab+2a-b^2}{2(a+2b+1)(a-b^2)}$	1
$\hat{X}_L^*, x_L^*$	$\frac{2a^2-ab^2+3ab+3b^2+b}{2(a+2b+1)(a-b^2)}$	1
CS_H^*	$\frac{\left(a^2b+3ab^2+3ab+2a-b^2\right)^2}{8(a+2b+1)^2\left(a-b^2\right)^2}$	$\frac{1}{2}$
CS_L^*	$\frac{a\left(2a^2-ab^2+3ab+3b^2+b\right)^2}{8(a+2b+1)^2\left(a-b^2\right)^2}$	$\frac{a}{2}$

Table 2: Equilibrium under uniform pricing

	If $\sqrt{8a+1} \geq 2b+1$ (A.3)	If $\sqrt{8a+1} < 2b+1$ (A.4)
p_H^*	$\frac{1}{2}\left(\frac{2ab+b^2}{4a-b^2}+1\right)$	$\frac{b+1}{2}$
p_L^*	$\frac{1}{2}\left(\frac{ab^2+2ab}{4a-b^2}+a\right)$	$\frac{1}{2}b(b+1)$
π^*	$\frac{a\left(4a^2+ab^2+8ab+4a+b^2\right)}{\left(b^2-4a\right)^2}$	$\frac{3b^2}{4}+b+\frac{1}{4}$
$\hat{X}_H^*, x_H^*$	$\frac{a(b+2)}{4a-b^2}$	$\frac{b+1}{2}$
$\hat{X}_L^*, x_L^*$	$\frac{2a+b}{4a-b^2}$	1
CS_H^*	$\frac{a^2(b+2)^2}{2\left(b^2-4a\right)^2}$	$\frac{1}{8}(b+1)^2$
CS_L^*	$\frac{a(2a+b)^2}{2\left(b^2-4a\right)^2}$	$\frac{a}{2}$

Table 3: Equilibrium under personalized pricing

	$x_L < 1$ and $x_H < 1$	$x_L = 1$ and $x_H < 1$	$x_L < 1$ and $x_H = 1$	$x_L = 1$ and $x_H = 1$
Conditions	$b < \frac{\sqrt{16a+1}-1}{4}$ and $b < \frac{\sqrt{a^2+4a}-a}{2}$	$b \geq \frac{\sqrt{16a+1}-1}{4}$ and $b < \frac{\sqrt{a^2+4a}-a}{2}$	$b < \frac{\sqrt{16a+1}-1}{4}$ and $b \geq \frac{\sqrt{a^2+4a}-a}{2}$	$b \geq \frac{\sqrt{16a+1}-1}{4}$ and $b \geq \frac{\sqrt{a^2+4a}-a}{2}$
p_H^*	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{b(a+b)}{2a}$	b
p_L^*	$\frac{a(2a+b)}{4a-2b^2}$	$b^2 + \frac{b}{2}$	$\frac{a+b}{2}$	b
π^*	$\frac{a\left(4a^2+8ab+4a-2b^3-b^2\right)}{4(b^2-2a)^2}$	$b^2 + b + \frac{1}{4}$	$\frac{(a+b)(a+3b)}{4a}$	$2b$
$\hat{X}_H^*, x_H^*$	$\frac{a(b+1)}{2a-b^2}$	$b + \frac{1}{2}$	1	1
$\hat{X}_L^*, x_L^*$	$\frac{2a+b}{4a-2b^2}$	1	$\frac{a+b}{2a}$	1

Table 4: Equilibrium under personalized pricing with unobserved discounts

Outcome	Equilibrium Value
p_H^*	$\frac{a(b\gamma+2)}{4a-b^2\gamma^2}$
p_L^*	$\frac{a(2a+b\gamma)}{4a-b^2\gamma^2}$
π^*	$\frac{a\left(4a^3+4a^2-3a^2b^2\gamma^2+8a^2b-ab^2\gamma^2\left(b^2\gamma+2b\gamma+6b+3\right)-b^4\gamma^3\right)}{(a-b^2)(b^2\gamma^2-4a)^2}$
$\hat{X}_H^*, x_H^*$	$\frac{a\left(2ab-ab\gamma+2a-b^2\gamma(b\gamma+\gamma+1)\right)}{(a-b^2)(4a-b^2\gamma^2)}$
$\hat{X}_L^*, x_L^*$	$\frac{2a^2-ab\left(b\gamma^2+b\gamma+\gamma-2\right)-b^3\gamma^2}{(a-b^2)(4a-b^2\gamma^2)}$

Table 5: Interior equilibrium under personalized pricing with $(1-\gamma)$ probability of informedness

Conditions	$x_L < 1$ and $x_L < 1$	$x_L = 1$ and $x_H = 1$
p^*	$\frac{a^2(b+2)+ab(2b+1)+b^3}{2(a^2+2ab+a+b^2)}$	b
π^*	$\frac{\left(a^2(b+2)+ab(2b+1)+b^3\right)^2}{4a^2(a^2+2ab+a+b^2)}$	$2b$
x_H^*	$\frac{a^2\left(3b^2+3b+2\right)+a^3b+3ab^2(b+1)+b^4}{2a(a^2+2ab+a+b^2)}$	1
x_L^*	$\frac{2a^4+a^3b(b+3)+a^2\left(3b^2+3b+1\right)b+ab^3(3b+2)+b^5}{2a^2(a^2+2ab+a+b^2)}$	1

Table 6: Uniform pricing equilibrium of the sequential model (customer H decides first)

Conditions	$x_L < 1$ and $x_L < 1$	$x_L = 1$ and $x_H = 1$
p^*	$\frac{ab+2a+b(b+1)^2}{2(a+(b+1)^2)}$	b
π^*	$\frac{\left(ab+2a+b(b+1)^2\right)^2}{4a(a+(b+1)^2)}$	$2b$
x_H^*	$\frac{a^2b+a\left(2b^3+3b^2+3b+2\right)+b^2(b+1)^3}{2a(a+(b+1)^2)}$	1
x_L^*	$\frac{2a^2+3ab^2+3ab+b(b+1)^3}{2a(a+(b+1)^2)}$	1

Table 7: Uniform pricing equilibrium of the sequential model (customer L decides first)

Conditions	$x_H < 1$ and $x_L^B < 1$	$x_H < 1$ and $x_L^B = 1$	$x_H = 1$ and $x_L^B = 1$
p_H^*	$\frac{1}{2} + \frac{b^2}{8a}$	$\frac{1}{2} + \frac{a}{8}$	b
p_L^{B*}	$\frac{a+b}{2}$	b	b
p_L^{NB*}	$\frac{a}{2}$	$\frac{a}{2}$	$\frac{a}{2}$
π^*	$\frac{a}{4} + \frac{9b^4}{64a^2} + \frac{24b^2(b+1)}{64a} + \frac{(b+1)^2}{4}$	$\frac{a^2}{64} - \frac{ab}{4} + \frac{a}{8} + b^2 + b + \frac{1}{4}$	$2b$
x_H^*	$\frac{3b^2}{8a} + \frac{b+1}{2}$	$b + \frac{1}{2} - \frac{a}{8}$	1
x_L^{B*}	$\frac{a+b}{2a}$	1	1
x_L^{NB*}	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

Table 8: Personalized pricing equilibrium of the sequential model (customer H decides first)

Conditions	$x_H^B < 1$ and $x_L < 1$	$x_H^B = 1$ and $x_L < 1$	$x_H^B < 1$ and $x_L = 1$	$x_H^B = 1$ and $x_L = 1$
p_L^*	$\frac{a}{2} + \frac{b^2}{8}$	$\frac{1}{8} + \frac{a}{2}$	$\frac{b}{2} + \frac{b^2}{2}$	b
p_H^{B*}	$\frac{b+1}{2}$	b	$\frac{b+1}{2}$	b
p_H^{NB*}	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
π^*	$\frac{a}{4} + \frac{b^2(3b+4)^2}{64a} + \frac{3b^2+4b+2}{8}$	$\frac{a}{4} + \frac{1+8b}{8} + \frac{(1-8b)^2}{64a}$	$\frac{3b^2}{4} + b + \frac{1}{4}$	$2b$
x_L^*	$\frac{3b^2+4b}{8a} + \frac{1}{2}$	$\frac{1}{2} - \frac{1}{8a} + \frac{b}{a}$	1	1
x_H^{B*}	$\frac{b+1}{2}$	1	$\frac{b+1}{2}$	1
x_H^{NB*}	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

Table 9: Personalized pricing equilibrium of the sequential model (customer L decides first)

B Proofs of Lemmas, Propositions and Statements

Proof of Lemma 1:

We use backward induction to solve the game. At first stage, the firm maximizes $\pi(p) = \frac{p(a(2-p)-p)}{a}$. As $\pi(p)$ is concave in p, $p^* = \frac{a}{a+1}$. Substituting p^* into (4.1), (4.2) and (4.3), we get (4.4).

The surplus of consumer i with stand-alone value s_i if he makes a purchase is $s_i - p$. Integrating over all s_i at which consumer i buys the product, we obtain the expected surplus of consumer i. Denote by CS_i, the expected surplus of consumer i.

$$CS_H(p) = \int_p^1 (s_H - p)\, ds_H = \tfrac{1}{2}(p-1)^2$$

$$CS_L(p) = \int_p^a (\tfrac{1}{a}(s_L - p))\, ds_L = \tfrac{1}{2a}(a-p)^2$$

Substituting equilibrium price leads to equilibrium unconditional expected surplus of consumers.

Proof of Lemma 2:

We use backward induction to solve the game. In the first stage, the firm maximizes her profit by setting optimal personalized prices. Substitute (4.5) and (4.6) into (4.7) to get $\pi(p_H, p_L) = -\frac{p_L^2}{a} - p_H^2 + p_H + p_L$. The expected profit function $\pi(p_H, p_L)$ is concave in p_H and p_L. So, $p_H^* = \frac{1}{2}$ and $p_L^* = \frac{a}{2}$. Substituting equilibrium prices into (4.5) and (4.6) and plugging equilibrium prices and purchase probabilities into (4.7) leads to (4.8).

The surplus of consumer i with stand-alone value s_i if he makes a purchase is $s_i - p_i$. Integrating over all s_i at which consumer i buys, we find expected surplus of consumer i denoted by CS_i.

$$CS_H(p_H) = \int_{p_H}^1 (s_H - p_H)\, ds_H = \tfrac{1}{2}(p_H - 1)^2$$

$$CS_L(p_L) = \int_{p_L}^a (\tfrac{1}{a}(s_L - p_L))\, ds_L = \tfrac{(a-p_L)^2}{2a}$$

Substituting equilibrium prices leads to equilibrium unconditional expected surplus of consumers.

Proof of Lemma 3:

Define Δp_i^* where $i \in \{L, H\}$ as the equilibrium personalized price offered to consumer i less the equilibrium uniform price. Similarly, define Δx_i^* as the equilibrium purchase probability of consumer i when prices are personalized less his equilibrium purchase probability when price is uniform. Denote by $\Delta\pi^*$, firm's expected profit with price personalization less her equilibrium expected profit with uniform pricing. Lastly, denote by ΔCS_i^*, equilibrium expected surplus of consumer i when prices are personalized less his equilibrium expected surplus when price is uniform.

$$\Delta p_H^* = \tfrac{1}{2} - \tfrac{a}{a+1},\ \Delta p_L^* = \tfrac{a}{2} - \tfrac{a}{a+1},\ \Delta x_H^* = \tfrac{1}{2} - \tfrac{1}{a+1},\ \Delta x_L^* = \tfrac{1}{2} - \tfrac{a}{a+1}$$

$$\Delta\pi^* = \tfrac{a+1}{4} - \tfrac{a}{a+1},\ \Delta CS_H^* = \tfrac{1}{8} - \tfrac{1}{2(a+1)^2},\ \Delta CS_L^* = \tfrac{a}{8} - \tfrac{a^3}{2(a+1)^2}$$

Using basic algebra it can be shown that for all $a \in (0, 1)$, we have $\Delta p_H^* > 0$, $\Delta p_L^* < 0$, $\Delta x_H^* < 0$, $\Delta x_L^* > 0$, $\Delta\pi^* > 0$, $\Delta CS_H^* < 0$ and $\Delta CS_L^* > 0$. From this comparison, we can directly obtain Lemma 3.

Proof of Lemma 4:

We solve the game using backward induction. By substituting (4.11) and (4.12) into (4.13) we obtained the profit expression in (4.14):

$$\pi(p) = \frac{p(-p(a+2b+1)+ab+2a+b)}{a-b^2}$$

$$\frac{\partial\pi(p)}{\partial p} = \frac{-p(2a+4b+2)+(ab+2a+b)}{a-b^2}$$

In first stage, the firm maximizes his expected profit. For now, assume $a \neq b^2$. We will solve for the special case of $a = b^2$ later. According to the first axiom of probability, purchase probabilities and price-dependent beliefs which are consistent on and off the equilibrium ((4.11) and (4.12)) should be between 0 and 1 which happens only when $\left(b \leq p \leq \frac{a+b}{b+1} \wedge b^2 < a\right) \vee \left(\frac{a+b}{b+1} \leq p \leq b \wedge \left(a < b^2 \vee b > 1\right)\right)$.

We will consider the special case of $b \geq 1$ later in this section. For now, assume $b < 1$.

- When $(b \leq p \leq \frac{a+b}{b+1}) \wedge (b^2 < a) \wedge (b \leq \frac{ab+2a+b}{2a+4b+2})$, $\pi(p)$ is concave in price. So, $p^* = \frac{ab+2a+b}{2a+4b+2}$. Substituting p^* into (4.11), (4.12) and (4.14) leads to the equilibrium in (A.1).

- When $(b \leq p \leq \frac{a+b}{b+1}) \wedge (b^2 < a) \wedge (b > \frac{ab+2a+b}{2a+4b+2})$, $\pi(p)$ is strictly decreasing in p over this range. So, price is set at the mininum bound within this space: $p^* = b$. Substituting p^* into (4.11), (4.12) and (4.14) leads to the equilibrium in (A.2).

- When $(\frac{a+b}{b+1} \leq p \leq b) \wedge (a < b^2)$, $\pi(p)$ is strictly increasing in p over this range. So the equilibrium price is set at the maximum bound within this space: $p^* = b$, which leads to the same equilibrium as presented in (A.2).

To derive the unconditional expected surplus of consumers notice that if consumer j ($j = H$ if $i = L$ and $j = L$ if $i = H$) does not buy the product then customer i's surplus is $s_i - p$ and if consumer j buys the product then the surplus of consumer i is $s_i + b - p$. As the purchase probability of consumer j is x_j, the expected surplus of consumer i conditional on his intrinsic interest s_i is $s_i + bx_j - p$. By integrating over all s_i at which consumer i makes a purchase, we obtain CS_i, the unconditional expected surplus of consumer i. Thus, $CS_i = E_{s_i}(s_i + bx_j - p)$.

$$CS_H(p, x_L) = \int_{p-bx_L}^{1} (s_H + bx_L - p)\, ds_H = \frac{1}{2}(1 + bx_L - p)^2 \tag{B.1}$$

$$CS_L(p, x_H) = \int_{p-bx_H}^{a} (\frac{1}{a}(s_L + bx_H - p))\, ds_L = \frac{(a + bx_H - p)^2}{2a} \tag{B.2}$$

Plugging back the equilibrium prices and purchase probabilities in (A.1), (A.2) into (B.1) and (B.2), we obtain the unconditional expected surplus of each consumer in each of these equilibria respectively.

Now, consider the case when $a = b^2$. We find the price-dependent beliefs which are consistent on and off the equilibrium path by solving for $\hat{X}_i = x_i$ given (4.9) and (4.10). This leads to:

$$\forall x_H^* \in [1-b, 1]$$
$$p^* = b,\ \hat{X}_H^* = x_H^*,\ \hat{X}_L^* = x_L^* = \tfrac{x_H^*}{b} - \tfrac{1}{b} + 1$$
$$\pi^* = bx_H^* + b + x_H^* - 1,\ CS_H^* = \tfrac{(x_H^*)^2}{2},\ CS_L^* = \tfrac{1}{2}(b + x_H^* - 1)^2$$

We use Pareto dominance refinement to tackle this equilibrium multiplicity. An equilibrium Pareto dominates another equilibrium if it makes a player strictly better off in terms of payoff without making any other player(s) worse off. As π^*, CS_H^* and CS_L^* are strictly increasing in x_H^*, the unique Pareto dominant equilibrium is the one in which $x_H^* = 1$, which is the equilibirum described in (A.2).

Proof of Lemma 5:

Substitute (4.15) and (4.16) into (4.17) to get the expected profit of the firm:

$$\pi(p_H, p_L) = p_L x_L(p_L) + p_H x_H(p_H) = \frac{p_L\left(a + b\hat{X}_H - p_L\right)}{a} + p_H\left(b\hat{X}_L - p_H + 1\right) \tag{B.3}$$

In first stage, the firm maximizes $\pi(p_H, p_L)$ knowing that $\hat{X}_i$ and x_i are between 0 and 1. As $\pi(p_H, p_L)$ is concave in both prices, $p_H^* = \frac{1}{2}\left(b\hat{X}_L + 1\right)$ and $p_L^* = \frac{1}{2}\left(a + b\hat{X}_H\right)$. Solving for $\hat{X}_i = x_i$, we find the fulfilled expectations equilibrium in (A.3) for when the parameters lead to an interior solution (i.e., $\sqrt{8a+1} \geq 2b+1$).

Also, it can be shown that maximizing (B.2) will lead to a corner solution with $\hat{X}_L = 1$ when $\sqrt{8a+1} < 2b+1$. Imposing the fulfilled expectations requirement $\hat{X}_i = x_i$ in this case, leads to the equilibrium in (A.4).

Later in this section when we consider the special case of $b \geq 1$, we consider the case when both constraints on beliefs regarding purchase probabilities are binding (i.e., $\hat{X}_H = 1$ and $\hat{X}_L = 1$).

If consumer j ($j = H$ when $i = L$ and $j = L$ when $i = H$) does not buy the product the surplus of consumer i is $s_i - p_i$. if consumer j buys the product the surplus of consumer i is $s_i + b - p_i$. As consumer j's purchase probability is x_j, the expected surplus of consumer i conditional on s_i is $s_i + bx_j - p_i$. Integrating over all s_i at which consumer i makes a purchase, we obtain the unconditional expected surplus of consumer i, or CS_i. Thus, $CS_i = E_{s_i}(s_i + bx_j - p_i)$.

$$CS_H(p_H, x_L) = \int_{p_H - bx_L}^{1} (s_H + bx_L - p_H)\, ds_H = \frac{1}{2}(bx_L - p_H + 1)^2 \tag{B.4}$$

$$CS_L(p_L, x_H) = \int_{p_L - bx_H}^{a} (\frac{1}{a}(s_L + bx_H - p_L))\, ds_L = \frac{(a + bx_H - p_L)^2}{2a} \tag{B.5}$$

Substituting equilibrium prices and purchase probabilities from (A.3) and (A.4) into (B.4) and (B.5), we get the expressions for unconditional expected consumer surplus presented in (A.3) and (A.4) respectively.

Corner Solution (Special Case of $b \geq 1$):

When price is uniform, the expected profit function (4.11) is strictly increasing in p when $b \geq 1$. So, p^* and the full equilibrium outcomes are as reported in (A.2).

When prices are personalized, in proof of Lemma 5 we solved the firm's expected profit maximization problem when none of the beliefs and purchase probability constraints are binding or only the $\hat{X}_L \leq 1$ constraint is binding. Solving firm's expected profit maximization problem (B.3) assuming both belief constraints are binding, i.e., $\hat{X}_H = 1$ and $\hat{X}_L = 1$, followed by imposing the fulfilled expectations requirement $\hat{X}_i = x_i$ leads to the equilibrium in (B.6). Notice that the case of $\hat{X}_H = 1$ and $\hat{X}_L = 1$ happens only when $b \geq 1$, otherwise the $\hat{X}_H \leq 1$ constraint is slack.

$$\forall b \geq 1 : p^* = b, \pi^* = 2b, \hat{X}_H^* = x_H^* = \hat{X}_L^* = x_L^* = 1, CS_H^* = \frac{1}{2}, CS_L^* = \frac{a}{2} \tag{B.6}$$

Thus, when $b \geq 1$, under both pricing strategies the firm sets price(s) at b and makes both customers buy the product with a purchase probability of 1. The impact of network effects is so strong so as to make the firm decide to extract only the part of consumers' surplus which is driven by network effects and shut down the part of her expected profit that is obtained by extracting the intrinsic value component of consumers' utility. The firm makes this decision for the sake of expanding the network.

Proof of Propositions 1 to 4:

The conditions and expressions for equilibrium properties are provided in Inequalities (A.1) and (A.2) in Lemma 4, and in Inequalities (A.3) and (A.4) in Lemma 5. We prove each of the Propositions 1 to 4 by iterating over all possible combinations of these conditions. Note that conditions (A.1) and (A.4) cannot hold simultaneously. So, our analysis reduces to three cases as depicted in Figure 5:

Region 1: when conditions (A.1) and (A.3) hold and the level of network effects, b, is relatively low.

Region 2: when conditions (A.2) and (A.3) hold and the level of network effects is at an intermediate level.

Region 3: when conditions (A.2) and (A.4) hold and the level of network effects is relatively high but not too high. In what follows, we drive differential equilibrium expressions in each region to prove the corresponding Propositions.

Proof of Proposition 1 (cont.):

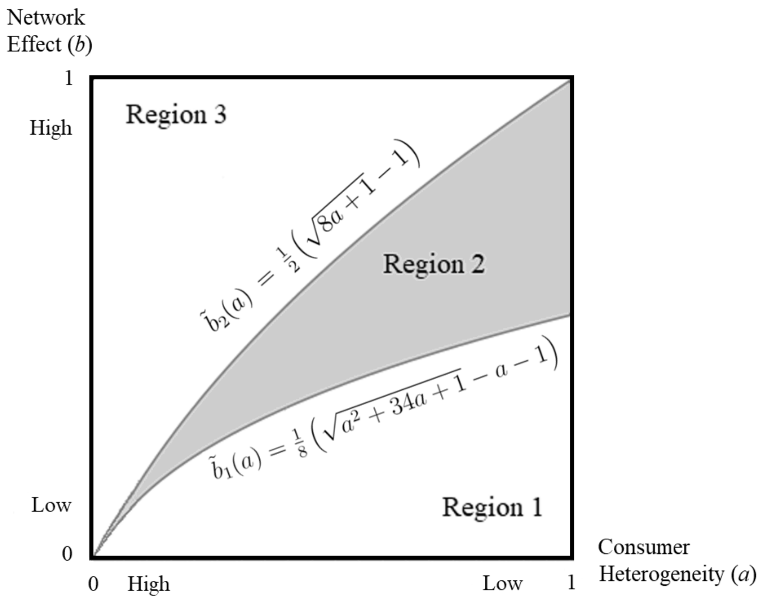

Figure 5: Regions

Define Δp_H^* (Δp_L^*) as the equilibrium price offered to consumer H (L) when prices are personalized less the equilibrium price offered to him when price is uniform.

Region 1: this region corresponds to relatively low levels of the network effect.

$$\Delta p_H^* = \tfrac{1}{2}\left(\tfrac{b(2a+b)}{4a-b^2} + 1\right) - \tfrac{ab+2a+b}{2a+4b+2},\ \Delta p_L^* = \tfrac{1}{2}\left(\tfrac{ab(b+2)}{4a-b^2} + a\right) - \tfrac{ab+2a+b}{2a+4b+2} \tag{B.7}$$

Region 2: this region corresponds to relatively medium levels of the network effect.

$$\Delta p_H^* = \tfrac{1}{2}\left(\tfrac{b(2a+b)}{4a-b^2} + 1\right) - b,\ \Delta p_L^* = \tfrac{1}{2}\left(\tfrac{ab(b+2)}{4a-b^2} + a\right) - b \tag{B.8}$$

Region 3: this region corresponds to relatively high levels of the network effect.

$$\Delta p_H^* = \tfrac{b+1}{2} - b,\ \Delta p_L^* = \tfrac{1}{2}b(b+1) - b \tag{B.9}$$

One can check that for $b < 1$ and all $a \in (0,1)$, $\Delta p_H^* > 0$, which means the equilibrium personalized price offered to consumer H is greater than the equilibrium uniform price.

Next, we compare the equilibrium uniform and personalized prices the firm sets for consumer L. We apply the intermediate value theorem to the function Δp_L^* defined in (B.7) and (B.8). Specifically, we show for any given level of customer heterogeneity a, there are two levels of network effect, $\underline{b}_p(a)$

and $\bar{b}_p(a)$, such that when b is between these two boundaries, i.e., $b \in (\underline{b}_p(a), \bar{b}_p(a))$, the equilibrium personalized price offered to consumer L is higher than the equilibrium uniform price.

First notice that for all $a \in (0, 1)$, when $b = 0$, we have $\Delta p_L^* < 0$. Thus, in a market without a network effect, in equilibrium, the firm sets a personalized price lower than the uniform price for consumer L which corroborates the result in Lemma 3. To find the appropriate $\underline{b}_p(a)$ and $\bar{b}_p(a)$, we consider Δp_L^* presented in (B.7) and (B.8) separately.

In Region 1, (B.7) holds. Define $\tilde{b}_1(a)$ as the value of b, as a function of a, for which Inequality (A.1) holds at equality; i.e., $\tilde{b}_1(a)$ defines the boundary of (A.1) and (A.2), and is given by $\tilde{b}_1(a) = \frac{1}{8}\left(\sqrt{a^2 + 34a + 1} - a - 1\right)$, see Figure 5. Also, define $\Delta p_L|_{b=\tilde{b}_1(a)}$ as the value of Δp_L^* on this boundary, i.e., the value of Δp_L^* when $b = \tilde{b}_1(a)$. Note that, for a given a, $\tilde{b}_1(a)$ also represents the highest level of b in this region. In other words, $\Delta p_L|_{b=\tilde{b}_1(a)}$ represents the price difference at the border of the region for which (B.7) is defined and b is at its maximum for each given a.

$$\Delta p_L|_{b=\tilde{b}_1(a)} =$$

$$\frac{-\sqrt{a(a+34)+1} + a\left(35\sqrt{a(a+34)+1} + a\left(3a + \sqrt{a(a+34)+1} - 248\right) - 26\right) + 1}{8(a-47)a + 8} \tag{B.10}$$

$\Delta p_L|_{b=\tilde{b}_1(a)}$ is continuous in a for all $a \in (0, 1)$, is strictly positive when $a \in [0.46, 1)$ and is strictly negative when $a \in (0, 0.45]$. Since $\Delta p_L|_{b=\tilde{b}_1(a)}$ is strictly increasing in a when $a \in [0.45, 0.46]$, according to the intermediate value theorem, there should exist a unique $a_p \in (0.45, 0.46)$ such that $\Delta p_L|_{b=\tilde{b}_1(a)} = 0$ when $a = a_p$ and:

$$\forall a \in (0, a_p)\text{: } \Delta p_L|_{b=\tilde{b}_1(a)} < 0,\ \forall a \in (a_p, 1)\text{: } \Delta p_L|_{b=\tilde{b}_1(a)} > 0$$

So, at the border of Region 1 (where b is relatively small and $\sqrt{a(a+34)+1} = a + 8b + 1$), when a is high enough (i.e., $a \in (a_p, 1)$), we have $\Delta p_L|_{b=\tilde{b}_1(a)} > 0$. In other words, the equilibrium personalized price for consumer L is higher than the equilibrium uniform price. Next, we look at how variations in b affect the sign of Δp_L^*. Note that Δp_L^* is continuous in both a and b, and recall that at $b = 0$ for all a (including $a \in (a_p, 1)$), we have $\Delta p_L^* < 0$. In other words, for all $a \in (a_p, 1)$, Δp_L^* is negative at $b = 0$ and positive at $b = \tilde{b}_1(a)$; therefore, by the intermediate value theorem, for any $a \in (a_p, 1)$, there exists $\tilde{b}_L(a)$ and $\tilde{b}_H(a)$ such that:

$$\tilde{b}_L(a) \le \tilde{b}_H(a) < \tilde{b}_1(a)$$

$$\forall b \in [0, \tilde{b}_L(a)):\ \Delta p_L^* < 0,\ \forall b \in (\tilde{b}_H(a), \tilde{b}_1(a)]:\ \Delta p_L^* > 0$$

In Region 2, (B.8) holds. This region corresponds to higher values of b as compared to Region 1 in which (B.7) holds. In this region for all $a \in (0, 0.45]$, we have $\Delta p_L^* < 0$. In other words, as in Region 1, when expected customer heterogeneity is high ($a \in (0, 0.45]$), the equilibrium personalized price for consumer L is lower than the equilibrium uniform price.

Define $\tilde{b}_2(a)$ as the value of b, as a function of a, for where Inequality (A.3) holds at equality. In other words, $\tilde{b}_2(a)$ defines the boundary of Region 2, and is given by $\tilde{b}_2(a) = \frac{1}{2}\left(\sqrt{8a+1} - 1\right)$ (see Figure 5). Note that Region 2 can be defined as $b \in (\tilde{b}_1(a), \tilde{b}_2(a)]$. Furthermore, Δp_L^* is continuous in a and b and is strictly decreasing in b. When $a \in [0.46, 1)$, we have $\Delta p_L^* > 0$ at $b = \tilde{b}_1(a)$, and $\Delta p_L^* < 0$ at $b = \tilde{b}_2(a)$. Therefore, according to the intermediate value theorem, for any $a \in [0.46, 1)$, there exists a unique $\hat{b}(a)$ such that $\tilde{b}_1(a) < \hat{b}(a) < \tilde{b}_2(a)$ and:

$$\forall b \in (\tilde{b}_1(a), \hat{b}(a)):\ \Delta p_L^* > 0,\ \forall b \in (\hat{b}(a), \tilde{b}_2(a)]:\ \Delta p_L^* < 0$$

In Region 3, (B.9) holds. Using basic algebra, it can be shown that $\Delta p_L^* < 0$ in this region.

Setting $\underline{b}_p(a) = \tilde{b}_H(a)$, and $\bar{b}_p(a) = \hat{b}(a)$ completes the proof. To sum, we proved when a is sufficiently high and b is medium, i.e., $b \in (\underline{b}_p(a), \bar{b}_p(a))$, equilibrium personalized price of consumer L is higher than the equilibrium uniform price, i.e., $\Delta p_L^* > 0$.

Proof of Propositions 2 and 3 (cont.):

To compare the purchase probability of consumers in the equilibria of Lemmas 4 and 5, denote by Δx_i^* the purchase probability of consumer i ($i \in \{H, L\}$) when prices are personalized less his purchase probability when price is uniform. Similarly, denote by $\Delta\pi^*$, the expected profit of the firm when prices are personalized less her expected profit when price is uniform.

Region 1: this region corresponds to relatively low levels of the network effect.

$$\begin{gathered}\Delta x_H^* = \tfrac{a(b+2)}{4a-b^2} - \tfrac{a^2 b + a\left(3b^2+3b+2\right) - b^2}{2(a+2b+1)(a-b^2)},\ \Delta x_L^* = \tfrac{2a+b}{4a-b^2} - \tfrac{2a^2 - a(b-3)b + (3b+1)b}{2(a+2b+1)(a-b^2)} \\ \Delta\pi^* = \tfrac{a\left(4a^2 + a\left(b^2+8b+4\right) + b^2\right)}{\left(b^2-4a\right)^2} - \tfrac{(a(b+2)+b)^2}{4(a+2b+1)(a-b^2)}\end{gathered} \tag{B.11}$$

Region 2: this region corresponds to relatively medium levels of the network effect.

$$\begin{aligned} \Delta x_H^* &= \tfrac{a(b+2)}{4a-b^2} - 1,\ \Delta x_L^* = \tfrac{2a+b}{4a-b^2} - 1 \\ \Delta\pi^* &= \tfrac{a\left(4a^2+a\left(b^2+8b+4\right)+b^2\right)}{\left(b^2-4a\right)^2} - 2b \end{aligned} \tag{B.12}$$

Region 3: this region corresponds to relatively high levels of the network effect.

$$\begin{aligned} \Delta x_H^* &= \tfrac{b+1}{2} - 1,\ \Delta x_L^* = 0 \\ \Delta\pi^* &= 2b - \tfrac{1}{4}\left(3b^2+4b+1\right) \end{aligned} \tag{B.13}$$

Next, we first prove Proposition 2 and then Proposition 3.

Using basic algebra, it can be shown that for all $b \in (0,1)$, the purchase probability expression Δx_H^* is negative in Regions 1, 2 and 3. Thus, when $b < 1$ and for all values of $a \in (0,1)$, purchase probability of customer H is smaller when prices are personalized.

Now, we compare the purchase probability of customer L under personalized and uniform pricing. We apply the intermediate value theorem to the function Δx_L^* presented in (B.11) and (B.12) and show that for any given level of expected heterogeneity in consumers' intrinsic interests in the product, a, there are two levels of network effect, $\underline{b}_x(a)$ and $\bar{b}_x(a)$, such that when b is between these two boundaries, i.e., $b \in (\underline{b}_x(a), \bar{b}_x(a))$, the equilibrium purchase probability of consumer L is lower when prices are personalized.

First notice that for all $a \in (0,1)$, when $b = 0$, we have $\Delta x_L^* < 0$. Thus, in a market without network effect, in equilibrium, purchase probability of consumer L is greater when prices are personalized and this corroborates the result in Lemma 3. To find the appropriate $\underline{b}_x(a)$ and $\bar{b}_x(a)$, we consider Δx_L^* presented in (B.11), (B.12) and (B.13) separately.

In Region 1, (B.11) holds. Define $\tilde{b}_1(a)$ as the value of b, as a function of a, for which Inequality (A.1) holds at equality; i.e., $\tilde{b}_1(a)$ defines the boundary of (A.1) and (A.2), and is given by $\tilde{b}_1(a) = \frac{1}{8}\left(\sqrt{a^2+34a+1} - a - 1\right)$, see Figure 5. Notice that for a given a, $\tilde{b}_1(a)$ represents the highest level of b in this region such that b is at its maximum for each given a.

Δx_L^* is continuous in a for all $a \in (0,1)$, is strictly positive when $b = 0$ and is strictly negative when $b = \tilde{b}_1(a)$. Since for a given a, Δx_L^* is strictly decreasing in b when $b \in [0, \tilde{b}_1(a)]$, according to the intermediate value theorem, there should exist a unique $\underline{b}_x(a)$ such that:

$$\Delta x_L^* = 0 \text{ when } b = \underline{b}_x(a)$$

$$\forall b \in [0, \underline{b}_x(a))\text{: } \Delta x_L^* > 0,\ \forall b \in (\underline{b}_x(a), \tilde{b}_1(a)]\text{: } \Delta x_L^* < 0$$

In Region 2, (B.12) holds. This region corresponds to higher values of b as compared to Region 2 in which (B.11) holds. Define $\tilde{b}_2(a)$ as the value of b, as a function of a, for where Inequality (A.3) holds at equality. In other words, $\tilde{b}_2(a)$ defines the boundary of Region 2, and is given by $\tilde{b}_2(a) = \frac{1}{2}\left(\sqrt{8a+1} - 1\right)$, see Figure 5. Also notice that Region 2 can be defined as $b \in (\tilde{b}_1(a), \tilde{b}_2(a)]$. Using basic algebra, it can be shown that when $b \in (\tilde{b}_1(a), \tilde{b}_2(a))$ we have $\Delta x_L^* < 0$ and $\Delta x_L^* = 0$ at the upper boundary of Region 2 when $b = \tilde{b}_2(a)$.

In Region 3, (B.13) holds. As presented in (B.13), $\Delta x_L^* = 0$ in this region.

Setting $\bar{b}_x(a) = \tilde{b}_2(a)$ completes proof of Proposition 2.

Next, we prove Proposition 3 given the $\Delta\pi^*$ expressions in (B.11), (B.12) and (B.13). We apply the intermediate value theorem to $\Delta\pi^*$. Notice that the piecewise-defined function $\Delta\pi^*$ is continuous in both a and b for all $a \in (0,1)$ and $b \in [0,1)$. For any $a \in (0,1)$, we see that $\Delta\pi^*$ is strictly positive when $b \leq \frac{1}{5} - \frac{4}{5}\left(a - \frac{1}{2}\right)^2$ and is strictly negative when $b \geq 0.4$. So, according to the intermediate value theorem for any given $a \in (0,1)$ there should exist $\underline{b}_\pi(a)$ and $\bar{b}_\pi(a)$ such that:

$$0.4 > \bar{b}_\pi(a) \geq \underline{b}_\pi(a) > \frac{1}{5} - \frac{4}{5}\left(a - \frac{1}{2}\right)^2$$

$$\forall b \in [0, \underline{b}_\pi(a))\text{: } \Delta\pi^* > 0,\ \forall b \in (\bar{b}_\pi(a), 1)\text{: } \Delta\pi^* < 0$$

Thus, we have shown that there exists $\underline{b}_\pi(a)$ and $\bar{b}_\pi(a)$ such that when degree of network effects is small, i.e., $b \in (0, \underline{b}_\pi(a))$, the expected profit of the firm is greater when prices are personalized and when degree of network effects is relatively high but still less than 1, i.e., $b \in (\bar{b}_\pi(a), 1)$, the expected profit of the firm is smaller when prices are personalized. This completes proof of Proposition 3.

Proof of Proposition 4 (cont.):

Denote by ΔCS_i^* ($i \in \{H, L\}$), the unconditional expected surplus of consumer i under personalized pricing less his unconditional expected surplus under uniform pricing.

Region 1: this region corresponds to relatively low levels of network effect.

$$\Delta CS_H^* = \frac{a^2(b+2)^2}{2(b^2-4a)^2} - \frac{\left(a^2b+a\left(3b^2+3b+2\right)-b^2\right)^2}{8(a+2b+1)^2(a-b^2)^2},\ \Delta CS_L^* = \frac{a(2a+b)^2}{2(b^2-4a)^2} - \frac{a\left(2a^2-a(b-3)b+b(3b+1)\right)^2}{8(a+2b+1)^2(a-b^2)^2} \tag{B.14}$$

Region 2: this region corresponds to relatively medium levels of network effect.

$$\Delta CS_H^* = \frac{a^2(b+2)^2}{2(b^2-4a)^2} - \frac{1}{2},\ \Delta CS_L^* = \frac{a(2a+b)^2}{2(b^2-4a)^2} - \frac{a}{2} \tag{B.15}$$

Region 3: this region corresponds to relatively high levels of network effect.

$$\Delta CS_H^* = \frac{1}{8}(b+1)^2 - \frac{1}{2},\ \Delta CS_L^* = 0 \tag{B.16}$$

Using basic algebra, it can be shown that for all values of $a \in (0,1)$ and $b \in (0,1)$, the unconditional expected surplus of customer H is smaller under price personalization, or $\Delta CS_H^* < 0$. Thus, when $b < 1$ price personalization makes customer H worse off.

Next, we compare the unconditional expected surplus of customer L under price personalization and uniform pricing. To this end, we apply the intermediate value theorem to the function ΔCS_L^* defined in (B.14) and (B.16). Specifically, we show that for any given level of expected heterogeneity in consumers' intrinsic interests in the product, a, there are two levels of network effect, $\underline{b}_{cs}(a)$ and $\bar{b}_{cs}(a)$, such that when b is between these two boundaries, i.e., $b \in (\underline{b}_{cs}(a), \bar{b}_{cs}(a))$, the equilibrium unconditional expected surplus of consumer L is lower under price personalization, or $\Delta CS_L^* < 0$.

First notice that for all $a \in (0,1)$, when $b = 0$, we have $\Delta CS_L^* > 0$. Thus, in a market without a network effect, in equilibrium, customer L earns a greater surplus under personalized pricing which corroborates the result in Lemma 3. To find the appropriate $\underline{b}_{cs}(a)$ and $\bar{b}_{cs}(a)$, we consider ΔCS_L^* presented in (B.14), (B.15) and (B.16) separately.

In Region 1, (B.14) holds. Define $\tilde{b}_1(a)$ as the value of b, as a function of a, for which Inequality (A.1) holds at equality; i.e., $\tilde{b}_1(a)$ defines the boundary of (A.1) and (A.2), and is given by $\tilde{b}_1(a) = \frac{1}{8}\left(\sqrt{a^2+34a+1} - a - 1\right)$, see Figure 5. Note that $b = 0$ and $\tilde{b}_1(a)$ define the lower and upper boundaries of Region 1 respectively. Thus, Region 1 can be defined as $b \in [0, \tilde{b}_1(a)]$.

At any given $a \in (0,1)$, when $b \in [0, \frac{1}{6} - \frac{2}{3}\left(a - \frac{1}{2}\right)^2]$, we see than $\Delta CS_L^* > 0$ and at $b = \tilde{b}_1(a)$, we see $\Delta CS_L^* < 0$. Noting that ΔCS_L^* is continious in both a and b in this region, according to the intermediate value theorem for each given $a \in (0,1)$, there should exist two levels of network effect, $\dot{b}_{cs}(a)$ and $\hat{b}_{cs}(a)$, such that $\frac{1}{6} - \frac{2}{3}\left(a - \frac{1}{2}\right)^2 < \hat{b}_{cs}(a) \leq \dot{b}_{cs}(a) < \tilde{b}_1(a)$ and:

$$\forall b \in [0, \hat{b}_{cs}(a))\colon\ \Delta CS_L^* > 0,\ \forall b \in (\dot{b}_{cs}(a), \tilde{b}_1(a)]\colon\ \Delta CS_L^* < 0$$

In Region 2, (B.15) holds. This region corresponds to higher values of b as compared to Region 1 in which (B.14) holds. Define $\tilde{b}_2(a)$ as the value of b, as a function of a, for which Inequality (A.3) holds at equality, it is given by $\tilde{b}_2(a) = \frac{1}{2}\left(\sqrt{8a+1} - 1\right)$. Note that $\tilde{b}_1(a)$ and $\tilde{b}_2(a)$ define the lower and upper boundaries of Region 2 respectively, see Figure 5. Thus, Region 2 can be defined as $b \in (\tilde{b}_1(a), \tilde{b}_2(a)]$. Using basic algebra it can be shown that for all values of $a \in (0,1)$ when b is between the boundaries of Region 2, $b \in (\tilde{b}_1(a), \tilde{b}_2(a))$, $\Delta CS_L^* < 0$ and at the upper boundary of this region when $b = \tilde{b}_2(a)$, $\Delta CS_L^* = 0$. Thus, in Region 2, which corresponds to relatively medium levels of network effect, unless on the upper boundary when $b = \tilde{b}_2(a)$ and $\Delta CS_L^* = 0$, we see that $\Delta CS_L^* < 0$.

In Region 3, (B.16) holds. As presented in (B.16), $\Delta CS_L^* = 0$ in this region.

To conclude, we proved that for a given value of $a \in (0,1)$: 1) there exists $\hat{b}_{cs}(a)$ such that $\Delta CS_L^* > 0$ when $b \in [0, \hat{b}_{cs}(a))$, 2) there exists $\dot{b}_{cs}(a)$ such that $\Delta CS_L^* < 0$ when $b \in (\dot{b}_{cs}(a), \tilde{b}_2(a))$, and 3) $\Delta CS_L^* = 0$ when $b \in [\tilde{b}_2(a), 1)$. Setting $\underline{b}_{cs}(a) = \hat{b}_{cs}(a)$, $\tilde{b}_{cs}(a) = \dot{b}_{cs}(a)$, and $\bar{b}_{cs}(a) = \tilde{b}_2(a)$ completes this proof.