Expertise in Online Markets [1]

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Expertise in Online Markets

Abstract

We examine the effect of the presence of knowledgeable buyers (experts) in online markets where auctions with a hard close and posted prices are widely used. We model buyer expertise as the ability to accurately predict the quality, or condition, of an item. In auctions with a hard close, sniping–submitting bids in the last minute–emerges as an equilibrium strategy for experts. We show that non-experts bid more aggressively as the proportion of experts increases. As a consequence, we establish that the auction platform may obtain a higher revenue by (i) enforcing a hard close and allowing sniping, and (ii) withholding information regarding the quality of the item. Moreover, in online markets where both auctions and posted prices are available, we show that the presence of experts allows the sellers of high quality items to signal their quality by choosing to sell via auctions.

1 Introduction

The advent of online auctions such as those in eBay led to the first massive scale deployment of simple second-price auction mechanisms for consumer products. Even though eBay started as a platform for consumer-to-consumer auctions for selling items off one's garage, it is now a large selling platform enabling over $200 billion commerce volume and reaching over 200 million users annually.[1] The addition of posted-price sales has fueled this growth by allowing it to serve as a competitor to other online retail sites. The growth of this new segment of online markets that combine auctions with posted prices raises important new questions about the optimal strategies for the buyers and sellers to follow in the market as well as for the platform in its design choices.

[1] http://venturebeat.com/2013/10/16/ebay-earnings-sales-up-21-revenue-up-14-and-double-digit-paypal-user-growth/

It is well known that customers in online markets have different levels of experience and expertise in purchasing through Internet auctions. For example, while 90% of eBay customers use eBay not more than once a month, 5% use eBay everyday. Such "expert" buyers are better at evaluating the items, are more efficient in finding their desired products, and are aware of more alternatives and outside options. Many of these expert buyers use online markets as a trade platform. They use their knowledge and expertise to find and buy items at bargain prices to later sell them at a profit.[2]

The presence of such expert buyers in these online markets raises the natural question of how it affects each of the stakeholders in the market: the expert and non-expert buyers, the sellers and the platform. For example, how do expert buyers benefit from their expertise in online markets? How do non-expert buyers adjust their strategies to compete with experts? How does the presence of experts affect the platform and sellers' strategies in online markets? In this paper, we address these questions and discuss the implications of the heterogeneity in buyers' expertise in online markets.

First, we show that the phenomenon of *sniping* (submission of bids at the closing time of the auction) arises as a natural equilibrium strategy for the more experienced buyers to hide their information about the item until the last minute. This is in line with the previous proposed explanations in the literature on existence of sniping. For example, Roth and Ockenfels (2002) argue, and provide empirical evidence, that existence of sniping in online markets is partly due to buyers' heterogeneity in their experience with online markets and their expertise in the product category: *"... there may be bidders who are dealers/experts and who are better able to identify high-value antiques. These well-informed bidders ... may wish to bid late because other bidders will recognize that their bid is a signal that the object is unusually valuable."*

More interestingly, we show that the presence of experts encourages the non-experts to bid more aggressively. In particular, we show that because of the sniping strategy of the

[2] E.g., http://www.ebay.com/gds/tips-for-Selling-Full-Time-on-Ebay-/10000000000850400/g.html

expert buyers, non-expert buyers have to bid more than their expected value; otherwise they only win items of low quality value against the expert buyers (i.e., when the experts do not want the item). Quantifying this, we show in Proposition 1 that the higher the proportion of experts among the bidders, the more aggressively the non-experts bid above their expected value for the item.

Next, we consider the platform's decisions regarding the presence of experts. In particular, should the platform allow the experts to snipe? Also, if the platform knows the quality value of the item, should it commit to share that information with the buyers? We find interesting answers to these questions. Regarding the first question, at the outset, it appears that sniping may hurt platform revenue since without sniping, non-expert buyers could respond to bids of experts, and the item would sell at a higher price. Since the platform's fee is usually a fixed fraction of the selling price, the platform would then have an incentive to eliminate sniping.[3] Contrary to this expectation, we show that the aggressive bidding behavior of the non-experts that we describe above implies that the platform's overall revenue *increases* by allowing sniping for a wide range of parameter values (Proposition 2). This is a very interesting and novel explanation as to why online auction companies such as eBay retain the auction format that allows for sniping from a revenue perspective.[4] This result has another important and interesting implication regarding the second question: the platform can benefit from committing to withholding the quality information (Corollary 1). This is in contrast to the celebrated *linkage principle*[5] (Milgrom and Weber, 1982), and is driven by buyers' heterogeneity in their level of expertise.

Finally, we consider the impact of the presence of expert buyers on the sellers' strategies. In particular, we investigate the choice of selling mechanisms between the auction and a posted price sale when they are both available (as is common in most online auction-houses).

[3] In fact, some auction platforms such as the now defunct Amazon Auctions and Trademe, removed sniping by automatically extending the auction time whenever a bid is submitted.

[4] EZsniper.com provides an extensive list of auction sites with a hard close.

[5] The linkage principle argues that the auction house always benefits from committing to reveal all available information.

In the presence of expert buyers, under certain conditions, we show that by selling in an auction, a seller can credibly signal[6] the quality value of his item (Proposition 3). By selling in an auction, the seller shows that he can rely on the *market* (specifically on the expert buyers) to decide the value of the item. This is a risk that a seller with a low quality value item cannot take. Furthermore, this signaling is only possible if there are enough experts, who know the value of the item, in the market. Otherwise, the seller of a high quality-value item will not be able to separate himself from the seller of a low-value item. In other words, the existence of experts in the market allows the sellers of high quality products to separate themselves by selling in auctions. This finding is in line with auction houses' claim that selling in auction increases buyers' confidence. For example, Fraise Auction[7] argues that one of the benefits of selling in auction is that *"The competitive bidding format creates confidence among the buyers when they see other people willing to pay a similar amount for the property."* To best of our knowledge, this result is a new explanation for popularity of auctions in certain product categories.

Taken together, our comprehensive study of the effect of the presence of knowledgeable buyers in online markets establishes important and interesting phenomena that are missing in their absence: (i) sniping not only naturally arises as an equilibrium behavior, but also can help increase the revenue to the seller, (ii) the platform may benefit from withholding quality value information, and (iii) the sellers can credibly signal the quality value of the item by choosing to sell via auctions. Our results provide new insights for buyers, sellers and intermediaries in online two-sided markets.

In what follows, we review related literature. Section 2 introduces the main model and Section 3 solves the equilibria and compares them with the corresponding equilibria of the auction with a soft close, which does not allow for sniping. In Section 4, we analyze the sellers' game of choosing among selling formats. Section 5 provides robustness for our results by

[6]Note that the signal that we discuss here is the seller's choice of the selling mechanism. This is different from bids by other bidders, which can also be signals of the quality of the product.

[7]http://fraiseauction.com/why-auction/

considering some extensions under which our main results continue to hold. We conclude the paper in Section 6. All proofs are relegated to Appendices A, B, C.

Related Literature

Our work relates to the literature on informed buyers' purchase decisions, intermediaries' incentives to reveal product quality information, sellers' strategies to signal product quality, and the advantages and disadvantages of auctions versus posted prices. In the following, we review the related literature on each topic.

In our model we assume that expert buyers have more information about the value of the item. This is motivated by Roth and Ockenfels (2002) where the authors empirically argue that some bidders have more information about the value of an item, and those bidders hide that information from other bidders by last-minute bidding. Along the same lines, our paper is also related to the literature on online auctions with *common values*. Bajari and Hortacsu (2003) argue that last-minute bidding is an equilibrium in a stylized model of eBay auctions with common values. They develop and estimate a structural econometric model of bidding in eBay auctions with common value and endogenous entry. Wilcox (2000) and Rasmusen (2006) use common values to model sniping and bidders' behavior on ebay auctions. Wilcox (2000) shows that sniping increases as buyers' experience increases. Furthermore, the increase in the sniping behavior of the more experienced bidders is more pronounced for the type of items that are more likely to have a common value component. Similarly, a model with no common value as in Yoganarasimhan (2013) demonstrates no sniping behavior. Rasmusen (2006) considers a model where bidders incur a cost for learning the common value of the item. As a result, those who acquire the information snipe to hide their information from other bidders. Similar to the previous literature,[8] sniping emerges as an equilibrium strategy in our model as well. However, our focus is the effect of the presence of experts (including their sniping) on non-experts', sellers' and the platform's strategies and revenues,

[8]The literature on trying to explain sniping in online auctions is vast, other than previously mentioned papers, see also Hossain (2008), Wintr (2008), and Ely and Hossain (2009).

which is missing in the earlier literature. More specifically, we provide an explanation as to why online auction companies such as eBay retain the auction format that allow for sniping from a revenue perspective that takes into account the aggressive bidding behavior of the non-experts.

In this paper, we show that an intermediary could benefit from withholding information about the quality of the items in an auction. This is in contrast with the well-known linkage principle by Milgrom and Weber (1982). The linkage principle argues that the auction house always benefits from committing to reveal all available information. The intuition behind the principle is that revealing the information can mitigate the winner's curse, and motivates the buyers to bid more aggressively. We arrive at the contrast due to buyers' *heterogeneity* in terms of their information about the quality value of the item, as modeled by their expert status. More specifically, the result of Milgrom and Weber (1982) is established when valuation of bidders depend *symmetrically* on the unobserved signals of the other bidders, a condition that is not satisfied in our setup. Failure of the linkage principle has also been argued in a few other papers in the auction theory literature. For example, Perry and Reny (1999), Chapter 8.1 of Krishna (2002), and Fang and Parreiras (2003) show the failure in setups with multiple items, ex-ante asymmetries and budget constraints, respectively. Withholding information, under certain circumstances, has also been shown to increase social welfare, by Zhang (2013), in the context of product labeling.

Many researchers in marketing have studied signaling unobserved quality under information asymmetry. Moorthy and Srinivasan (1995) and Soberman (2003) show that sellers can use warranties such as money-back guarantees to signal the quality of their items. Bhardwaj et al. (2005) show that by letting the customers to request information about an item, rather than revealing it without solicitation, a seller can signal the quality of his item. Nelson (1974) and Milgrom and Roberts (1986) show that firms can signal their product quality by "burning money" through advertising. Mayzlin and Shin (2011) show that uninformative advertising, as an invitation for search, can be used to signal product quality. Finally, Li et al. (2009) investigate auction features such as pictures and reserve price that enable sellers to reveal more information about their credibility and product quality, and empirically examine how different types of indicators help alleviate uncertainty. We introduce a new dimension for sellers to signal the quality of their items. In particular, for product categories with a common value component where assessing the common value needs expertise (e.g., antiques category), we show that selling via auction can signal that the item has a high common value.

Finally, we review the related literature that compare auctions to posted price selling mechanisms. Einav et al. (2013) propose a model to explain the shift from Internet auctions to posted prices and consider two hypotheses: a shift in buyer demand away from auctions, and general narrowing of seller margins that favors posted prices. By using eBay data, they find that the former is more important. There is a significant economics literature that compares auctions to posted price mechanisms. Notably, Wang (1993) compares auctions with posted prices and shows that auctions become preferable when buyers valuations are more dispersed. In another important paper, Bulow and Klemperer (1996) have shown that additional revenue by attracting one more bidder is greater than setting the optimal reserve price, hence in a sense establish that "value of negotiating skills is small relative to value of additional competition." In an empirical work, Bajari et al. (2009) conclude that the choice of sales mechanism may be influenced by the characteristics of the product being sold. To the best of our knowledge, our paper is the first work that considers the signaling effects of the choice of the mechanism on buyers' beliefs.

2 Model

We consider a model with two buyers and one item.[9] We assume that there are two types of buyers, *experts* and *non-experts*, and each buyer is an expert with probability p. Due to anonymity of online marketplaces, we assume that each buyer does not know whether his

[9]In the Extensions section 5, we consider more buyers.

opponent is an expert or not.[10]

In our model, the items sold in online auctions have differing levels of "quality value," which may reflect the condition of a used good, or the relative efficacy of a product among its competitors. Note that this value is similar to a common value in that its benefit accrues equally to both expert bidders (who can accurately predict quality value) and non-expert bidders (who do not know the quality value). The total value of the item for a bidder is the sum of the quality value and an additional private value component. Since experts typically have access to more efficient search mechanisms and more channels for procuring the item, we assume that the private value, and hence, the total willingness to pay, of experts is lower than non-experts. This is motivated by Salop and Stiglitz (1977) and Varian (1980) where the authors show that *informed* buyers, who are aware of cheaper outside options, pay less for the products.[11]

We assume that the quality value, denoted by a binary random variable C with realizations 0 and c, is only known by experts and it is the same for both experts and non-experts (therefore it can be described as a common value). We *normalize* expert's private value to be 0 and let v denote the *additional* private value of the item for the non-expert.

We assume that C has a binary distribution: $\Pr(C = c) = q$ (high common value) and $\Pr(C = 0) = 1 - q$ (low common value). The value of the item for an expert is C, and the value of the item for a non-expert is $C + v$.[12] We assume that c, v, p and q are common knowledge. Moreover, buyer types are privately known by buyers and the realization of C is privately known by experts.

[10] On ebay and most other auction platforms, identities of bidders are revealed only after an auction ends. Furthermore, bidders can easily hide their type by creating and using a new account online.

[11] Salop and Stiglitz (1977) and Salop (1977) use a similar model to explain price dispersion, and Varian (1980) uses it to explain inter-temporal price discrimination as a model of sales. More recently, Kostandini et al. (2011) empirically shows more experienced buyers are able to purchase products at lower prices compared to relatively less experienced buyers.

[12] Dropping the assumption that experts have a lower willingness to pay adds two more types of buyers to our model: those who have valuation C and do not know C, and those who have valuation $C + v$ and know C. However, the former is dominated by both experts and non-experts (and will always lose), and the latter dominates both experts and non-experts (and will always win). To facilitate exposition, we exclude these two types in our analysis.

We model the online auction as a two-stage bidding game where the second stage represents the very last opportunity to submit a bid (the sniping window), while the first stage represents the whole time window preceding the close. Even though in practice the period before the sniping window is a dynamic game, we model it (Stage 1) by allowing each bidder to submit a single bid: To reconcile this with reality, we can think of the highest bid that a bidder submitted before the sniping window as the first stage bid. Bidders can observe competitors' bids of Stage 1, and respond to it in Stage 2; however, they do not have enough time to respond to competitors' bids of Stage 2. It is worth mentioning that we can derive all of our results with a more realistic dynamic game model of the first stage.[13] However, while being a bit more involved, it does not add any further insight to our analysis so we use the simpler two-stage formulation here. In case of a tie after Stage 2 between bidders of the same type, we assume that the winner is chosen randomly. For technical reasons, we assume that in case of a tie between two bidders of different types, the tie is broken in favor of the non-expert.

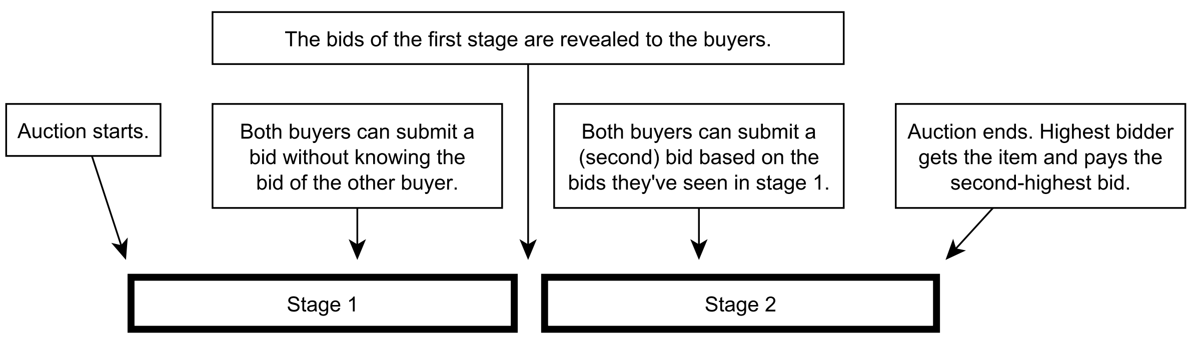

Figure 1: Timeline of the game

The timing of the model is as follows (see also Figure 1). Before Stage 1, each buyer knows his own type (expert or non-expert), but not the type of the other buyer. If a buyer is an expert, he also knows the common value (whether $C = 0$ or $C = c$). In Stage 1, both buyers simultaneously submit their bids. After Stage 1 and before Stage 2, both buyers

[13]We consider a dynamic auction in the time interval $[0, 1)$ and sniping at time 1.

observe the other buyer's bid, and may be able to infer their opponent's type. In Stage 2, both buyers simultaneously decide if they want to increase their bid from Stage 1, and if so by how much. In other words, bids of Stage 2 have to be greater than or equal to bids of Stage 1. After Stage 2, the item is given to the buyer with the highest bid at the price of the second highest bid.[14]

3 Experts and Sniping in Online Markets

In this section, we characterize *all* pure strategy equilibria of the auction game. In equilibrium, experts use sniping to protect their information about the common value of the item. We show that non-experts bid aggressively—even above their expected valuation—in order to compete with experts. Finally, we show that, under certain conditions, non-experts' aggressive behavior leads to higher revenue for the platform to the extent that the platform benefits from allowing sniping. In other words, even though experts have lower willingness to pay, their existence and their ability to hide their information force the non-experts to bid more aggressively, and ultimately lead to higher revenue for sellers and for the platform.

Experts Induce Sniping. Based on the relation of the parameters c, v, p and q, we split the set of possible values into three mutually exclusive and collectively exhaustive ranges. In the first two ranges, we have that $cq + v < c$, while in the third, we have $cq + v \geq c$. Note that $cq + v$ is the expected value of a non-expert for the item. Let $m = \frac{(1-p)(1-q)}{2pq+(1-p)} \cdot c$ and $M = (1 - q) \cdot c$. Note that one can easily verify that $m < M$. The three different cases we consider are as follows: $v \in [0, m]$, $v \in (m, M)$, and $v \in [M, +\infty)$. We describe equilibrium bidding strategy for buyers in these three cases in the following lemma, whose proof is relegated to Appendix A.

Lemma 3.1. *For the auction model described in Section 2, the buyers' equilibrium bidding strategy is given below.*

[14] In the Extensions section C, we consider the second-price auction with a reserve price.

1. *If $v < m$ then there is no symmetric pure equilibrium in the auction. There exists a $t \in [0,1]$ such that the following set of strategies form an equilibrium.*

   - *Non-Expert: He bids $cq+v$ in Stage 1. In Stage 2, he bids c with probability $1-t$.*
   - *Expert: If $C = 0$, he does not bid in any stage. If $C = c$, he bids $cq+v$ in Stage 1, and c in Stage 2.*

2. *If $m \leq v < M$, all the symmetric pure equilibria of the auction are of the following form for $x \in [0, c]$.*

   - *Non-Expert: He bids x in Stage 1. In Stage 2, he bids c.*
   - *Expert: If $C = 0$, he does not bid in any stage. If $C = c$, he bids x in Stage 1, and c in Stage 2.*

3. *If $v \geq M$, the following set of strategies form an equilibrium.*

   - *Non-Expert: He bids $cq + v$ in Stage 1. He does not increase the bid in Stage 2.*
   - *Expert: If $C = 0$, he does not bid in any stage. If $C = c$, he does not bid in Stage 1, and bids c in Stage 2.*

Lemma 3.1 characterizes the equilibria of the game in three cases. In all three cases, experts bid for the item only if the common value is high ($C = c$). If the common value is high, the expert "pretends" to be a non-expert by mimicking a non-expert's strategy in Stage 1, and then snipes with his true value in Stage 2. In other words, experts use sniping to hide their valuable information from non-experts. This is in line with the findings of Roth and Ockenfels (2002).

Impact of Experts on Non-experts' Strategy. A non-expert's optimal strategy depends on the value of v. If v is sufficiently high ($v \geq M$), a non-expert's expected value for the item is higher than c. In this case, non-experts always win the competition against experts. When v is smaller than M, the situation is more interesting. By bidding their expected value against experts, non-experts only win when the common value is low. Therefore, non-experts have to bid higher than their expected value in order to win a high-common-value item against experts. Note that bidding above the expected value does not necessarily mean that they have to pay more than their expected value because the auction format is second price. The only risk is that if two non-experts compete with each other, they may both bid above their expected value, and end up paying more than their expected value. In this case, a non-expert's payoff could be negative.

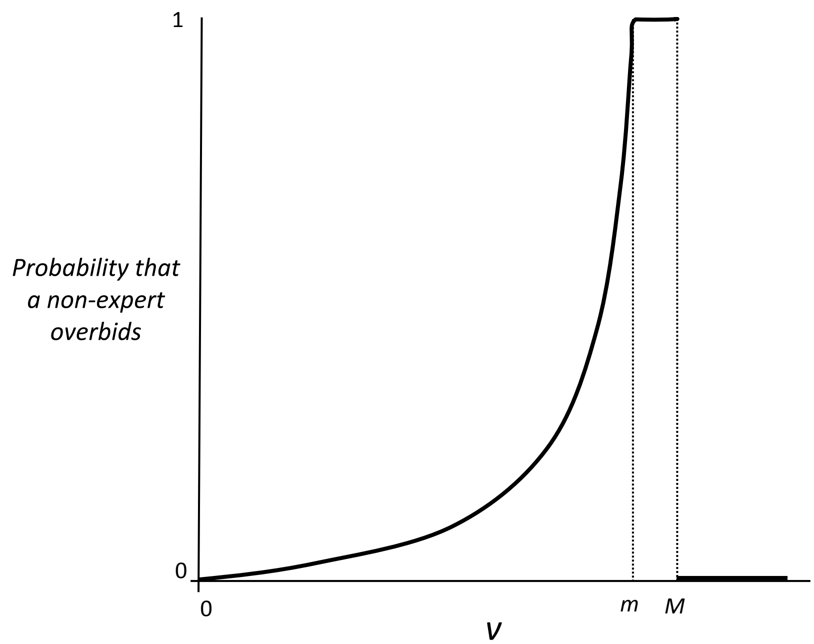

Figure 2: Probability that a non-expert overbids as v increases for $q = 0.1$, $p = 0.3$ and $c = 1$.

As depicted in Figure 2, if the value of v is high enough (i.e., $v \in (m, M)$), non-experts always take the risk of over-paying, and bid above their expected value in order to win against experts. However, if $v < m$, a non-expert over-bids only with probability $1 - t$. This mixed strategy allows the non-experts to mitigate the risk of over-paying due to competition with another non-expert. In Proposition 1, we show that as the probability that the opponent is an expert increases, a non-expert's willingness to take the risk and bid above his expected value increases.

Proposition 1. *If the expected value of a non-expert for the item is less than the common value of the item (i.e., $cq + v < c$), the non-expert may bid more than their valuation for the item in equilibrium. The probability of over-bidding increases as the fraction of experts in the market (i.e., p) increases.*

An important assumption in Proposition 1 is that experts can hide their information by sniping. The platform can eliminate sniping by extending the duration of the auction whenever a bid is submitted. In this case, non-experts always have enough time to respond to experts' bids, and therefore, do not have to bid above their expected valuation. Next, we model and analyze a platform in which sniping is not possible. Using this analysis, we can derive the conditions under which the platform benefits from allowing sniping.

An auction without sniping. We now consider a model in which sniping is not possible. One way to prevent sniping is by extending the duration of the auction by a few minutes every time there is a bid near the current end time of the auction. This auction is called an auction with a soft close and used by the now defunct Amazon Auctions. A way to model this is by starting with a game that has only one stage and every time there is a bid during the current stage, the auction extends for one more stage. In other words, every time someone makes a bid, the other buyers can see it and respond to it. First, we find an equilibrium for this new model, and characterize experts' and non-experts' strategies. Then we compare seller's revenue and the platform's revenue across the two models. The goal is to see if sniping behavior is good for the platform or if the platform will benefit from preventing it.

Lemma 3.2. *If $v > M$, the equilibrium strategy of the bidders in a platform which does not allow sniping is the same as the equilibrium strategy in a platform which allows sniping. If $v \leq M$, the following is an equilibrium for the bidders in a platform which does not allow sniping: non-experts bid $cq + v$, but when they see any bid different from $cq + v$ from their* *opponent, they increase their bid to c. Experts bid c if common value is high, and do not bid if common value is low.*

Note that when $v > M$, sniping does not happen even if it is possible. Therefore, the outcome of the model with sniping is the same as the model without sniping in this case. If $v \leq M$, when sniping is not possible, non-experts bid their expected value. If they see a bid c, they infer that the opponent is expert and the common value is high. In that case, they increase their bids to c to win the item at price c.

Effect of Experts on Platform Revenue. Using Lemma 3.2, it is easy to see that when sniping is not possible, experts always reveal the value of a high-common-value item to non-experts. This increases the non-experts willingness to pay and in some cases leads to higher revenue for the seller. However, when sniping is not possible, non-experts do not have to bid above their valuation. This reduces the competition and can hurt sellers' revenue as well as the platform's revenue. In Lemma 3.3 we derive the conditions under which sellers benefit from existence of sniping. We use this lemma to analyze the platform's incentive in allowing sniping.

Lemma 3.3. *If $v > M$, the seller's revenue is the same with and without sniping; otherwise:*

- *The seller of an item with low common value always benefits from existence of sniping.*
- *The seller of an item with high common value benefits from existence of sniping if and only if $p > \frac{(cq+v-c)^2}{c^2(1-q)^2-2c(1-q)^2v-2q(1-q)v^2+v^2}$.*

Lemma 3.3 shows that the seller of an item with low common value always benefits from existence of sniping. This is intuitively because existence of sniping prevents the flow of information from experts to non-experts. Therefore, when sniping is possible, non-experts are more likely to over-pay for an item with low common value. The interesting part is that even the seller of an item with high common value benefits from existence of sniping if p is high enough. This is because when sniping is possible, non-experts know that they will not be able to infer the common value, and therefore, have to bid more aggressively to win the item. As we saw in Proposition 1, this aggressive bidding behavior increases as p increases. If p is sufficiently large, the positive effect of this aggressive bidding behavior on seller's revenue dominates the negative effect of lack of information flow, and therefore, the revenue of the seller of a high-quality item can be higher when sniping is allowed than when it is not. Using the same argument, we can see that the platform can also benefit from allowing sniping when p is sufficiently large. This result is formalized in Proposition 2.

Proposition 2. *If the expected value of the non-experts for the item is less than the common value of the item (i.e., $cq + v < c$), and the fraction of experts in the market (i.e., p) is sufficiently large, the platform benefits (i.e. collects higher revenue) from allowing sniping.*

Figure 3 shows the regions in which sniping increases the platform's revenue. Note that the region where sniping provides higher revenue appears only when v is sufficiently large compared to c, and p is sufficiently large. This is because higher v and higher p both lead to non-experts' aggressive bidding, as we saw in Figure 2 and Proposition 1, respectively.

Experts and the Breakdown of the Linkage Principle. Finally, we discuss the connection between sniping and revelation of information in the marketplace. Note that sniping allows the experts to protect their information about the value of the item. We know that the platform sometimes benefits from allowing sniping. This could suggest that the platform may also benefit from *withholding information* about the value of the item. This is an important implication because it is in contrast with the well-known "linkage principle" in auction theory (Milgrom and Weber, 1982).

The linkage principle states that auction platforms (e.g. auction houses) benefit from committing to reveal all available information about an item, positive or negative. The platform revealing the information reduces the downside risk of winning the item, also known as the winner's curse. But we show that there is also a downside in revealing the information in the presence of heterogeneous bidders, and the platform may sometimes benefit from

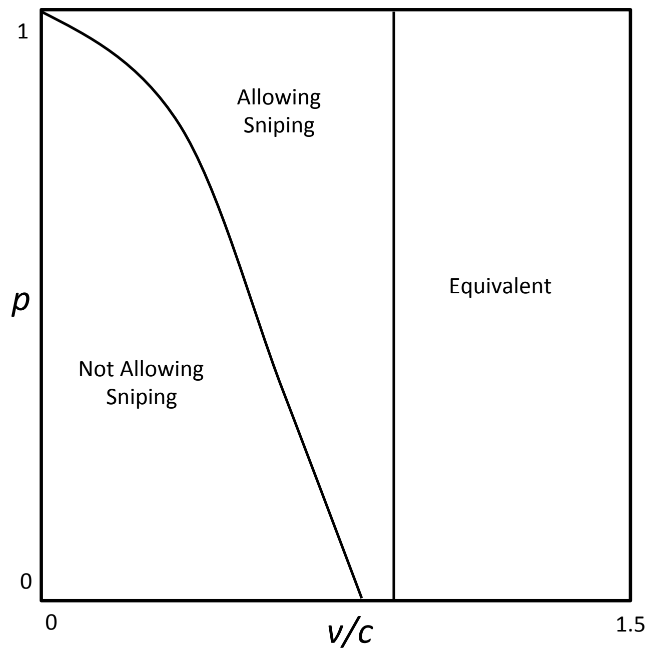

Figure 3: The regions show whether allowing sniping provides higher revenue for the platform (for $q = 0.1$).

committing to not revealing the information.

Our result shows that when bidders are *asymmetric* in terms of their information about the value of the product, bidders with less information have to bid more aggressively, otherwise, they only win the item when bidders with more information do not want the item (i.e., the common value is low). This aggressive behavior incentivizes the platform to withhold any information about the quality value of an item. This result is formalized in the following corollary.

Corollary 1. *When sniping is allowed, the platform benefits (i.e. has higher revenue) from committing to reveal the common value to the buyers if and only if the fraction of experts in the market is sufficiently small (i.e., p is sufficiently small).*

So far we discussed the effect of the existence of experts on non-experts' and the platform's decisions. In the next section, we analyze the effect on sellers' choice of selling mechanism. In particular, we show that existence of experts can help the sellers of items with high common value to signal the value of their items to non-experts.

4 Signaling via Selling Mechanism

In this section, we show that the existence of experts in the market could help the sellers to signal the quality/common value of their item to non-experts. We look at sellers' choice of selling mechanism between an auction and a posted price sale. We call the seller of an item with high common value a high-type seller, and the seller of an item with low common value a low-type seller. A seller is high-type with probability q where q is common knowledge. A seller naturally knows his own type; experts also know the seller's type (since they know the common value of items being offered). But non-experts do not know the seller's type. We investigate whether a seller can signal his type using the selling mechanism (auction versus posted price). In particular, we derive conditions for existence of a separating equilibrium. We show that existence of enough experts in the market is a necessary condition for a separating equilibrium to exist; furthermore, when the fraction of experts in the market, p, is sufficiently large, a separating equilibrium always exists. However, for intermediate value of p, depending on the values of v and c, a separating equilibrium may or may not exist.

A seller sets his selling mechanism M (posted price or auction). In case of posted price, M also includes the price. For a mechanism M, we assume that all non-experts have the same belief about a seller who sets M. In general, non-experts' belief about a mechanism is the probability that they think a seller using that mechanism is high-type. However, since we only consider pure strategy Nash equilibria of the game, non-experts' belief about a mechanism is limited to three possibilities: Low (L), High (H), and Unknown (X). In belief L, non-experts believe that a seller using mechanism M is always a low-type seller. In belief H, non-experts believe that a seller using mechanism M is always a high-type seller. Finally, in belief X, non-experts cannot infer anything about the seller's type and believe that the seller is high-type with probability q.

Non-experts have beliefs about each mechanism M. In equilibrium, the beliefs must be consistent with sellers' strategies. In particular, if both types of sellers use the same mechanism in (a pooling) equilibrium, non-experts' belief for that mechanism must be X. If the two types of sellers use different mechanisms in (a separating) equilibrium, non-experts' belief for the mechanism used by the low-type seller must be L and for the mechanism used by the high-type seller must be H. Furthermore, in an equilibrium, given the non-experts' beliefs, sellers should not be able to benefit from changing their strategies.

Separating Equilibria. Note that sniping is relevant only when buyers' belief about some mechanism M is X. Therefore, in a separating equilibrium, the platform's decision on whether to allow sniping or not does not affect buyers' equilibrium behavior or sellers' strategies. In other words, the following analysis applies to both cases where sniping is and is not allowed.

Proposition 3 below shows that when the fraction of experts in the market is sufficiently large, there exists a separating equilibrium in which a high-type seller chooses an auction and a low-type seller chooses posted price as their respective selling mechanisms. A proof and related analysis are provided in Section B in the Appendix. Figure 4 shows the region in which a separating equilibrium exists as a function of p and v/c. It is interesting to note that existence of separating equilibrium does not depend on the value of q.

Proposition 3. *When $v \leq 2c$, there exists a separating pure strategy Nash equilibrium if and only if $p \geq \frac{c}{c+2v}$. When $v > 2c$, there exists a separating pure strategy Nash equilibrium if and only if $p \in [\frac{c}{c+2v}, \frac{v-\sqrt{v(v-2c)}}{2v}]$ or $p \geq \frac{v+\sqrt{v(v-2c)}}{2v}$. In any separating equilibrium, a high-type seller uses an auction and a low-type seller uses posted price with price v.*

The intuition behind the proof of Proposition 3 is as follows. First, note that in general, an auction is more favorable to a high-type seller than a low-type seller. In particular, even if one of the two buyers in an auction is an expert, a low-type seller's revenue will be zero. A high-type seller, on the other hand, always has non-zero revenue. This could allow the high-

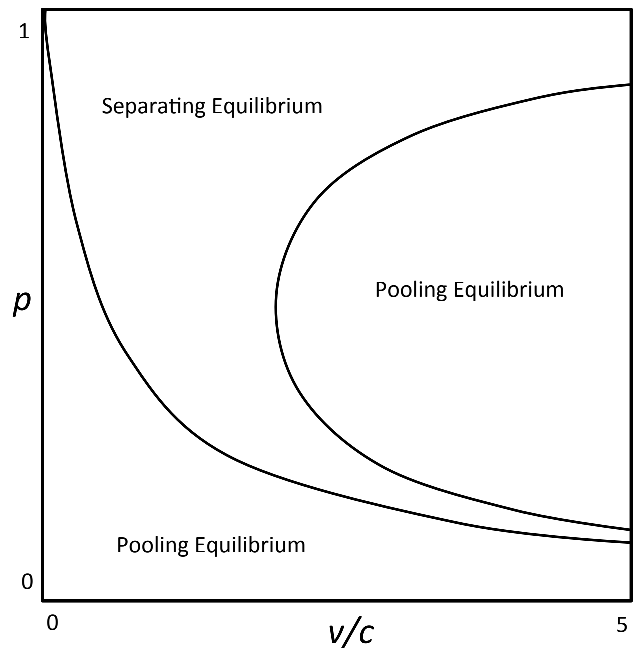

Figure 4: Existence of separating equilibrium as a function of the probability of a buyer being expert (p) and the ratio of v to c (for all values of q).

type seller to separate himself from the low-type seller by selling in an auction. But for a separating equilibrium to exist, the low-type seller's incentive to mimic has to be sufficiently low and the high-type seller's incentive to separate has to be sufficiently high. These two forces give us the conditions for existence of a separating equilibrium.

In a separating equilibrium, even non-experts know that the low-type seller is low-type. Hence, non-experts are willing to pay at most v for the item sold by the low-type seller. Therefore, the low-type seller's incentive to mimic increases as v or p decrease. If p and v are sufficiently small, since the low-type seller's incentive to mimic is sufficiently large, a separating equilibrium does not exist. This is captured by condition $p \geq \frac{c}{c+2v}$ in Proposition 3, and is represented by the left contour in Figure 4.

On the other hand, as v increases, the high-type seller's incentive to signal his type (and to separate himself) decreases. When v is large enough, we show that the high-type seller chooses to sell via an auction only if p is sufficiently small or sufficiently large. For the proof and a longer discussion of Proposition 3, please refer to Section B in the Appendix.

Pooling Equilibria. We also analyze pooling equilibria of the game when sniping is allowed. We seek to determine under what conditions sellers use auction versus posted price in a pooling equilibrium. Similar to the previous literature on signaling, there can be infinitely many pooling equilibria supported by different out-of-equilibrium beliefs in this game. Following a similar approach to Desai and Srinivasan (1995), Jiang et al. (2011), and Simester (1995), we refine the equilibrium set to get a unique equilibrium. However, we find that the common "Intuitive Criterion" refinement is not enough to find a unique pooling equilibrium in our setting.[15] Instead, we use Requirement $D1$ to refine the equilibria.[16] Requirement $D1$ requires that if message m is sent, zero weight must be put on the type t if there is another type t' such that t' always strictly benefits from the deviation whenever t benefits from the deviation.[17]

In Section B of the Appendix, we prove that Requirement $D1$ guarantees a unique equilibrium in our setting. More specifically, we show that in a pooling equilibrium that satisfies Requirement $D1$, both types of sellers use the mechanism that maximizes the high-type seller's profit. Sellers' strategies in a pooling equilibrium are depicted in Figure 5.

To understand sellers' strategies in pooling equilibrium we refer to our discussion in Section 3. The benefit of auction, compared to posted price, for a seller is that it forces the non-experts to bid more aggressively (i.e., above their expected valuation). As we saw in Section 3, this aggressive bidding happens only when v/c is large enough. Therefore, when v/c is small, if there are enough experts in the market (p is sufficiently large), a high-type seller prefers posted price, at price c, to an auction: At posted price c, a high-type seller can extract revenue c if there is at least one expert in the market. However, in an auction, since

[15] An example is available upon request.

[16] For an extended discussion of $D1$ and $D2$ criteria, and divine equilibrium, see Section 11.2 of Fudenberg and Tirole (1991).

[17] Cho and Sobel (1990) show that for monotonic signaling games (our game can easily shown to be a monotonic signaling game: all types prefer higher beliefs for being a high type), the set of D1 equilibria is the same as the set of "stable equilibria."

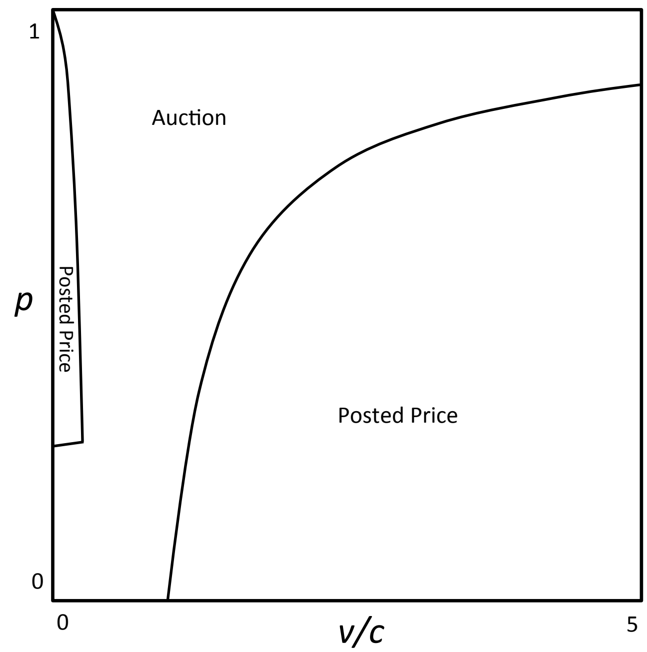

Figure 5: Sellers' strategies in pooling equilibrium (for $q = 0.1$).

non-experts bid $cq + v$, the seller only extracts revenue $cq + v$ (which is smaller than c) if at least one of the buyers is non-expert. This explains the contour in the left side of Figure 5.

If v/c is sufficiently large, c becomes smaller than $cq+v$; non-experts always bid $cq+v$ in this case. But the seller extracts revenue $cq+v$ only if *both* buyers are non-expert. However, if the seller uses posted price $cq + v$, it extract revenue $cq + v$ if *at least* one of the buyers is non-expert. Therefore, when v/c is sufficiently large, the equilibrium strategy is to use a posted price of $cq + v$. Note that experts cannot afford the item in this equilibrium, and therefore, using posted price is the equilibrium strategy only if p is small enough (i.e., there are enough non-experts in the market); this explains the contour in the right side of Figure 5.

Remark 4.1. By analyzing Figures 4 and 5 together, we can see that for low values of v/c both types of sellers choose auctions, for intermediate ranges of v/c low types choose posted prices and high types choose auctions, and for high ranges of v/c both types of sellers choose posted prices. Hence, posted prices are being used more and more as v becomes more important than c (i.e., expertise becomes less important). This finding is in line with the finding of Einav et al (2013) that "...product categories that have a higher fraction of 'new' or duplicate listings (and hence less 'idiosyncrasy') also have a higher frequency of posted price listings."

5 Extension to Reserve Prices and Multiple Bidders

Our results about the presence of sniping in equilibria for auctions with a closing time continues to hold when we add reserve prices to the analysis. Furthermore, we show that they also continue to hold in auctions with more than two bidders. This demonstrates the robustness of our key findings of increased revenues for both sellers and the platform in such auctions relative to the ones where sniping is not allowed.

Figure 6 shows the region in which allowing sniping increases the platform's revenue, when the sellers use the optimum reserve price. The regions in Figure 6 are similar to those in Figure 3. For the analysis, please refer to Appendix C.

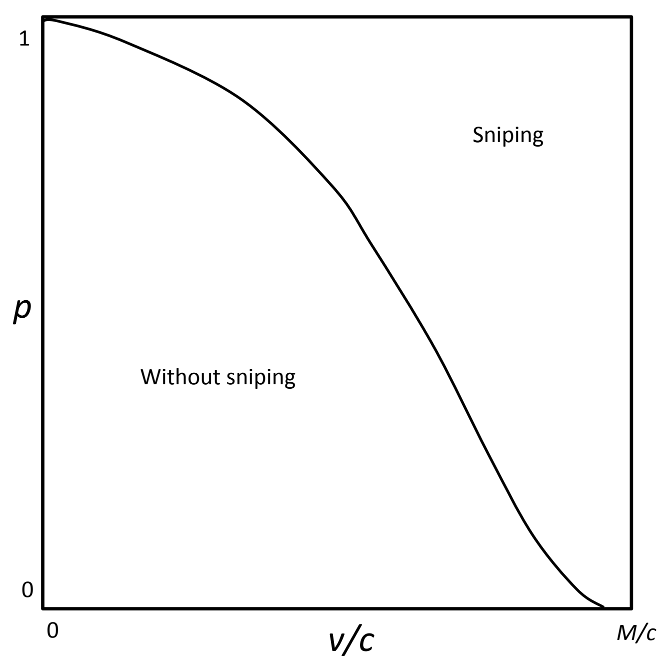

Figure 6: The region in which allowing sniping increases the platform's revenue when sellers use optimum reserve price (for $q = 0.1$).

We also consider a model with multiple buyers (but no reserve prices). In case of multiple buyers, we cannot characterize the equilibria for all values of the parameter space. Nevertheless, we show existence of regions in which allowing sniping increases the platform's revenue. Our result, for the case of 10 buyers, is shown in Figure 7. The lower left corner is the region in which we cannot find an equilibrium. Following the same intuition as before, we can see that when v is sufficiently large, allowing sniping increases the platform's revenue. Furthermore, we find that as the number of buyers increases, the region in which not allowing sniping is beneficial shrinks. In other words, in the regions where we can find an equilibrium, a larger number of buyers works in favor of allowing sniping.

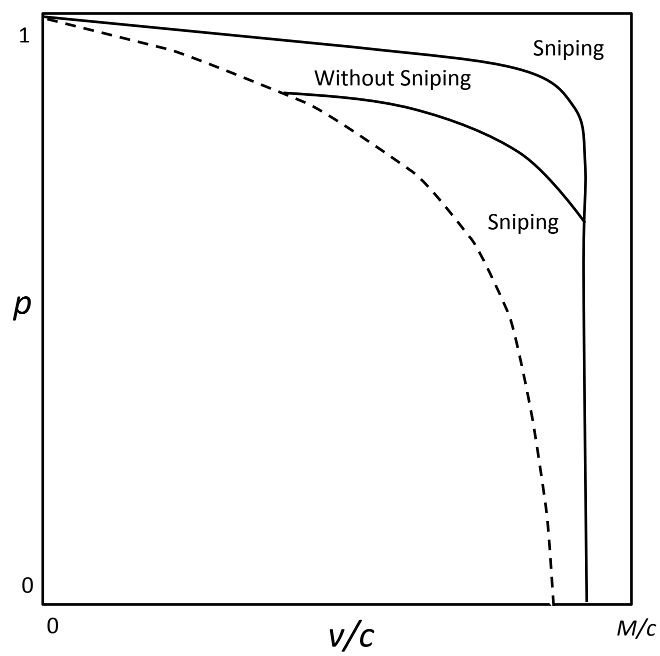

Figure 7: The regions in which allowing sniping increases the platform's revenue when there are 10 bidders (for $q = 0.1$).

The two extensions in this section show the robustness of our findings in Section 3. The details of these models are presented in Appendix C.

6 Conclusion

In this paper we examined important questions for the buyers, sellers and the platform for an online market supporting auctions and posted prices; we answered questions about optimal behavior for each of them using the well-documented presence of expertise among the bidders as the key underlying assumption. In particular, we studied the impact of the presence of expert bidders in online markets using a simple model of auctions with hard close and posted-prices. Motivated by large number of *used* items sold in online markets such as eBay.com, we supposed that items have differing levels of "quality" (which we model as common values), and different bidders have different capacities (which we model as expertise) to predict the quality. Bidders with low expertise may be affected by bids earlier in the auction, as these can be interpreted as signals for quality of the item. In our model, sniping emerges as an equilibrium strategy for experts to hide their information about the quality of the item.

Our results provide several important managerial implications.

- We show that, as a consequence of sniping behavior in equilibrium by the experts, non-expert buyers with less information have to bid aggressively, i.e. more than their expected value. This result highlights the compensatory behavior adopted by the large majority of bidders (non-experts) in typical auctions that arises endogenously in these marketplaces.

- Surprisingly, due to aggressive behavior of non-experts, the platform's revenue can be *higher* with sniping when compared to the situation when such sniping is not allowed. This is a new explanation as to why many online online auction sites use hard close rather than soft close. Another interesting implication of non-experts' aggressive behavior is that, the platform can benefit from committing to hide the information. This result has important managerial implications as it suggests that when buyers are heterogenous in terms of their information about the value of the item, the linkage principle does not always hold.

- In a platform with the choice of selling in an auction or posted price mechanism, a seller may be able to signal the quality (or authenticity) of his item to the buyers by selling in an auction and thus separate himself from low quality item sellers as long as there are enough experts (who know the quality of the item) in the market. This provides useful guidance to vendors in such markets, where the magnitude and extent of these decisions can be moderated based on the degree and extent of the presence of expert buyers in the mix.

- In the equilibrium of seller's choice among selling mechanisms, posted prices are being used more and more as the common values (and therefore, the expertise) become less important. Thus, our model provides one plausible explanation for the slow evolution in this direction seen in online markets.

Collectively, our work sheds a comprehensive light on the important differences that arise when knowledgeable or expert buyers are introduced in online marketplaces, and leads to useful guidelines for all participants in such markets.

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A Analyses and Proofs of Section 3

Proof of Lemma 3.1

Proof. **Small Private Value $v \leq m$:** Let's consider a non-expert first. If he doesn't see any bid during phase 1 from her opponent, then this means that her opponent is an expert and the common value is equal to 0. In this case, he takes the item and her utility is v, no matter what he does in the second phase, so we assume he bids c with probability $1 - t$.

If he sees a bid of $cq + v$ during phase 1, her utility by doing nothing in the second phase is

$$p_1' \cdot (t \cdot \tfrac{cq+v-(cq+v)}{2} + (1-t) \cdot 0) + p_2' \cdot (0) = 0,$$

where $p'_1 = \Pr(NE \mid cq+v) = \frac{\Pr(cq+v|NE)\Pr(NE)}{\Pr(cq+v)} = \frac{1\cdot(1-p)}{pq+(1-p)} = \frac{1-p}{1-p(1-q)}$, and similarly $p'_2 = \Pr(E \mid cq+v) = \frac{pq}{1-p(1-q)}$.

Her utility by bidding c in the second phase is

$$p'_1 \cdot (t \cdot \tfrac{cq+v-(cq+v)}{2} + (1-t) \cdot \tfrac{cq+v-c}{2}) + p'_2 \cdot (v).$$

Since he is indifferent, we want this expression to be equal to 0. Solving with respect to $1-t$, we get

$$1-t = \tfrac{2p'_2 v}{p'_1(c-(cq+v))} = \tfrac{2pqv}{(1-p)(c-(cq+v))} \in [0,1].$$

Now suppose he decides to change strategy and bid $c+\epsilon$ in the second phase for some $\epsilon > 0$. Then her utility becomes

$$p'_1 \cdot (t \cdot \tfrac{cq+v-(cq+v)}{2} + (1-t) \cdot (cq+v-c)) + p'_2 \cdot (v) \leq 0,$$

therefore he doesn't have any incentive to do that.

If he decides to bid $c-\epsilon$ in the second phase for some small $\epsilon > 0$, then her utility becomes

$$p'_1 \cdot (t \cdot \tfrac{cq+v-(cq+v)}{2} + (1-t) \cdot (0)) + p'_2 \cdot (0) = 0,$$

therefore again he doesn't have any incentive to do that.

Now let's consider an expert. If $C = 0$, her valuation is 0, therefore he doesn't expect a utility better than 0 so he doesn't participate.

If $C = c$, her valuation is c and he knows it, therefore he it is weakly dominant for her to bid c in the second phase. But he doesn't want to reveal the common value to a potential non-expert opponent, so he bids c in the second phase.

During the first phase he can make any bid below c he wants, but note that the strategy of a non-expert in phase 2 is independent of the bids of the first phase, therefore it doesn't really matter what he bids. So, we assume that he bids $cq + v$. Her utility is

$$p \cdot (t \cdot (c - (cq + v)) + (1 - t) \cdot 0) + (1 - p) \cdot (\tfrac{c-c}{2}) \geq 0.$$

Medium Private Value $m < v < M$: Let's consider a non-expert first. If he doesn't see any bid during phase 1 from her opponent, then this means that her opponent is an expert and the common value is equal to 0. In this case, he takes the item and her utility is v, no matter what he does in the second phase, so we assume he bids c.

Her utility by bidding c in the second phase is

$$p'_1 \cdot (\tfrac{cq+v-c}{2}) + p'_2 \cdot (v).$$

Now suppose he decides to change strategy and bid $c + \epsilon$ in the second phase for some $\epsilon > 0$. Then her utility becomes

$$p'_1 \cdot (cq + v - c) + p'_2 \cdot (v) \leq p'_1 \cdot (\tfrac{cq+v-c}{2}) + p'_2 \cdot (v),$$

therefore he doesn't have any incentive to do that.

If he decides to bid $c - \epsilon$ in the second phase for some small $\epsilon > 0$, then her utility becomes

$$p'_1 \cdot (0) + p'_2 \cdot (0) = 0 \leq p'_1 \cdot (\tfrac{cq+v-c}{2}) + p'_2 \cdot (v),$$

therefore again he doesn't have incentive to do that.

Now let's consider an expert. If $C = 0$, her valuation is 0, therefore he doesn't expect a utility better than 0 so he doesn't participate.

If $C = c$, her valuation is c and he knows it. Therefore, it is weakly dominant for her to bid c in the second phase. But he doesn't want to reveal the common value to a potential non-expert opponent, so he bids c in the second phase. During the first phase he can make any bid below c he wants, but note that the strategy of a non-expert in phase 2 is independent of the bids of the first phase, therefore it doesn't really matter what he bids. So, we assume that he bids x. Her utility is

$$p \cdot (0) + (1-p) \cdot (\tfrac{c-c}{2}) = 0.$$

Large Private Value $v \geq M$: It holds that $cq + v \geq c$, therefore no matter what an expert does, her utility will be 0 and he can't expect something better.

A non-expert has expected valuation $cq + v$, and he bids that. If he faces an expert, he always wins, so he doesn't need to do anything else. If he faces a non-expert, both bid $cq+v$, and her utility is 0. If he bids below $cq + v$, her utility remains 0, so he has no incentive to do that. If he bids above $cq + v$ he wins, but he pays $cq + v$, which again results in 0 utility.

Note that when $cq + v \geq c$, bidding c is a weakly dominant strategy for experts and bidding $cq + v$ is a weakly dominant strategy for non-experts. Therefore, this is the unique equilibrium in undominated strategies.

Uniqueness of Equilibria: The strategy of each player consists of two components: first a bid in the first round (which can also be 0), and second a function that takes as argument the bid of the opponent in the first round and returns the bid for the second round. Let (x, b_{NE}) be the strategy of a non-expert, (y, b_{EL}) the strategy of an expert that knows that $C = 0$, and (z, b_{EH}) the strategy of an expert who knows that $C = c$, in a symmetric pure equilibrium of the game. Since we have already proved uniqueness for the case of $v > M$, we assume that $v < M$.

The first thing we can notice is that $y = 0$ and $b_{EL} \equiv 0$. This is because the valuation of a low expert for the item is 0, and therefore the strategy to bid 0 dominates all the other strategies. If the low expert change strategy, the best he can hope for is the same utility, i.e. 0, and sometimes he can even have negative utility.

It is easy to see that z is equal to x in any pure strategy equilibrium. This is because any other bid from a high expert in the first round will give to a potential non-expert opponent the signal that he is an expert and that the common value is high. This motivates the non-expert to bid more than c in the second phase, and leads to the expert losing the auction.

Also, note that $b_{EH} \equiv c$ in any pure strategy equilibrium. The reason is that experts know that value of $C = c$. Since the auction format is second price, bidding c in the second phase is a weakly dominant strategy for them.

Next, we show that the second bid of a non-expert in a pure strategy equilibrium is always c, i.e. $b_{NE} = c$. First, we consider the case where $x = 0$. Assume for sake of contradiction that the bid is $b < c$. The utility of a non-expert is

$$q\left(p \cdot 0 + (1-p) \cdot \frac{c+v-b}{2}\right) + (1-q)\left(p \cdot v + (1-p) \cdot \frac{v-b}{2}\right).$$

If some non-expert changes the bid from b to c, then the utility increases by $qpv + (1-p) \cdot \frac{cq+v-b}{2}$, so this expression must be non-positive. But then, if some non-expert bids below b, her utility increases by $-(1-p) \cdot \frac{cq+v-b}{2} > 0$. This means that he can improve her utility, so we have a contradiction.

Similarly, assume for sake of contradiction that the bid is $b > c$. The utility is then

$$q\left(p \cdot v + (1-p) \cdot \frac{c+v-b}{2}\right) + (1-q)\left(p \cdot v + (1-p) \cdot \frac{v-b}{2}\right).$$

If some non-expert changes the bid from b to $b + \epsilon$ for some $\epsilon > 0$, then the utility increases by $(1-p) \cdot \frac{cq+v-b}{2}$, so this expression must be non-positive. But then, if some non-expert bids below b, her utility increases by $-(1-p) \cdot \frac{cq+v-b}{2} \geq 0$. Therefore, it must be that $(1-p) \cdot \frac{cq+v-b}{2} = 0$, or equivalently $b = cq + v$. But we assumed that $b > c$, thus $cq + v > c \Rightarrow v > M$, again a contradiction.

Finally, let $b = c$ be the second bid of non-experts. The utility by bidding $c + \epsilon$ increases by $(1-p) \cdot \frac{cq+v-c}{2}$, which is non-positive because $v \leq M$. The utility by bidding $c - \epsilon$ decreases by $qpv + (1-p) \cdot \frac{cq+v-c}{2}$, which must be non-negative. But this is non-negative iff $v \geq m = \frac{c(1-q)(1-p)}{2qp+1-p}$, so this is the only case where we have a pure symmetric equilibrium.

Now, let's consider the case where $x > 0$. Let p_1' be the probability that the opponent is non-expert conditional on the fact that he bids x in the first round, and let p_2' be the probability that the opponent is an expert and the common value is high conditional on the fact that he bids x in the first round. With a similar reasoning as above we can show that the second bid of the non-expert must be equal to c and this is an equilibrium iff $v \geq \frac{c(1-q)p_1'}{2p_2'+p_1'} = m$.

Therefore, for the case of $v < M$, we have a symmetric pure equilibrium iff $v \geq m$, as claimed in the proposition. □

Proof of Proposition 1

Proof. This result comes directly from Lemma 3.1. We can see that when $m < v < M$, non-experts overbid all the time, and when $v \leq m$, they overbid with probability $1 - t$. We saw in the proof of Lemma 3.1 that the probability of over-bidding is $1 - t$, where $1 - t = \frac{2pqv}{(1-p)(c-(cq+v))}$. It is easy to see that this is an increasing function on p. □

Proof of Lemma 3.2

Proof. Let's consider the case $v \leq M$ first, and prove that the given strategies form an equilibrium. If the common value is high and an expert bids anything above or below c, he either takes the item for c, or he loses. In both cases the utility is 0, so he has no incentive to do that. A non-expert always bids $cq + v$, because this is her expected valuation when he doesn't have any information. When he sees a bid of c, this means that the opponent is an expert and the common value is high, therefore the non-expert's valuation goes to $c + v$. By bidding c in this case, he gets the item and has utility v. If the non-expert sees a bid of $cq + v$, he either wins and pays $cq + v$, or he loses. In both cases the expected utility is 0 and he cannot do something better, since he faces another non-expert. If the non-expert sees a bid of 0, this means that the opponent is an expert and the value of C is 0. In this case, the non-expert wins either way, so again there is no incentive to bid something different than $cq + v$.

Now, let's consider the case $v > M$. In this case, when the platform allows sniping everyone bids her expected valuation and the timing doesn't matter (for 2 bidders). Therefore, the equilibrium remains the same even when the platform does not allow sniping. $\square$

Proof of Lemma 3.3

Proof. Let $v \leq M$. The expected revenue of the seller of an item with high common value in the non-sniping model is

$$E_H^N = (1-p)^2(cq+v) + (1-(1-p)^2)c.$$

If $m \leq v$, the revenue in the model with sniping is c, which is always better. If $v \leq m$, the expected revenue with sniping is

$$E_H^S = (1-(p+(1-p)(1-t))^2)(cq+v) + (p+(1-p)(1-t))^2 c.$$

One can verify that if $p \leq \frac{(cq+v-c)^2}{c^2(1-q)^2-2c(1-q)^2v-2q(1-q)v^2+v^2}$, then we have $E_H^N \geq E_H^S$, while if $p > \frac{(cq+v-c)^2}{c^2(1-q)^2-2c(1-q)^2v-2q(1-q)v^2+v^2}$, we get $E_H^N < E_H^S$.

For the seller of an item with low common value, the revenue with sniping is either the same as the revenue without sniping, or better in the cases that two non-experts face each other and both bid c.

When $v > M$, sniping does not happen even if it is possible. Therefore, the outcome of the model with sniping is the same as the model without sniping. $\square$

Proof of Proposition 2

Proof. Let $R = \frac{c(1-q)(1-p+pq-\sqrt{pq(2(1-p)+pq)})}{1-p}$. We will prove that if $v \leq R$, preventing sniping is better for the platform, and if $R < v \leq M$, allowing sniping is better for the platform.

If $m < v \leq M$, then $p > \frac{c-(cq+v)}{c-(cq+v)+2qv} \geq \frac{(cq+v-c)^2}{c^2(1-q)^2-2c(1-q)^2v-2q(1-q)v^2+v^2}$, thus the revenue is better with sniping by Lemma 3.3. If $v \leq m$, platform's revenue with sniping is

$$S_1 = q \cdot E_H^S + (1-q) \cdot E_L^S,$$

where E_H^S is from the proof of Lemma 3.3 and $E_L^S = (1-p)^2((1-t)^2 c + (1-(1-t)^2)(cq+v))$ is the expected revenue of the seller of an item with low common value when sniping is allowed. The platform's revenue without sniping is

$$S_2 = q \cdot E_H^N + (1-q) \cdot E_L^N,$$

where again E_H^N is from the proof of Lemma 3.3 and $E_L^N = (1-p)^2(cq+v)$ is the expected revenue of the seller of an item with low common value when sniping is not allowed.

Let $R_2 = \frac{c(1-q)(1-p+pq+\sqrt{pq(2(1-p)+pq)})}{1-p}$. If $v \in [R, R_2]$, we can verify that $S_1 \geq S_2$. Otherwise, $S_1 < S_2$. Moreover, it holds that $R \in [0, m]$ and $R_2 \geq M$.

Now, if we solve the inequality $R < v$ with respect to p, we get $p > \frac{(cq+v-c)^2}{c^2(1-q)^2 - 2c(1-q)^2 v + v^2}$. Therefore, for sufficiently large p, allowing sniping is better for the platform. □

Proof of Corollary 1

Proof. When the platform reveals the common value to everyone, all bidders bid their true valuation. The expected revenue for the platform is then equal to the platform's expected revenue in the model that prevents sniping. Therefore, the result follows from Proposition 2.

□

B Analyses and Proofs of Section 4

We use the following notation to explain the results of this section: Let $\pi_T^B(M)$, where $T \in \{L, H\}$ and $B \in \{L, H, X\}$ denote the expected profit of a seller who uses mechanism M, has type T, and non-experts believe has type B. Let M^{pool} be the mechanism that both types of sellers use in a pooling equilibrium. Let $M^{H,sep}$ and $M^{L,sep}$ be the mechanisms that high-type and low-type sellers use in a separating equilibrium, respectively. Finally, let Π_T^{pool} and Π_T^{sep}, where $T \in \{L, H\}$, be the pooling equilibrium and separating equilibrium profits of a seller with type T, respectively. By definition, we have

$$\Pi_L^{pool} = \pi_L^X(M^{pool}) \tag{1}$$

$$\Pi_H^{pool} = \pi_H^X(M^{pool}) \tag{2}$$

$$\Pi_L^{sep} = \pi_L^L(M^{L,sep}) \tag{3}$$

$$\Pi_H^{sep} = \pi_H^H(M^{H,sep}). \tag{4}$$

The revenue of a high- or low-type seller in an auction, where non-experts have belief high or low, is

$$\pi_H^H(A) = c + (1-p)^2 v$$

$$\pi_L^L(A) = (1-p)^2 v$$

$$\pi_H^L(A) = \begin{cases} p^2 c + (1-p^2) v & \text{if } v \leq c \\ (1-p)^2 v + (1-(1-p)^2) c & \text{if } v > c. \end{cases}$$

$$\pi_L^H(A) = (1-p)^2 (c+v)$$

Similarly, the revenue of a high- or low-type seller using posted price with price z, in each of the four cases, is

$$\pi_H^H(B,z) = \begin{cases} z & \text{if } z \le c \\ (1-p^2)z & \text{if } c < z \le c+v \\ 0 & \text{otherwise} \end{cases}$$

$$\pi_H^L(B,z) = \begin{cases} z & \text{if } v \le c \text{ and } z \le v \\ (1-(1-p)^2)z & \text{if } v \le c \text{ and } v < z \le c \\ z & \text{if } v > c \text{ and } z \le c \\ (1-p^2)z & \text{if } v > c \text{ and } c < z \le v \\ 0 & \text{otherwise} \end{cases}$$

$$\pi_L^H(B,z) = \begin{cases} (1-p^2)z & \text{if } z \le c+v \\ 0 & \text{otherwise} \end{cases}$$

$$\pi_L^L(B,z) = \begin{cases} (1-p^2)z & \text{if } z \le v \\ 0 & \text{otherwise} \end{cases}$$

The intuition behind the proof of Proposition 3 is as follows. First, note that in general, an auction is more favorable to a high-type seller than a low-type seller. In particular, even if one of the two buyers in an auction is an expert, a low-type seller's revenue will be zero. A high-type seller, on the other hand, always has non-zero revenue. This could allow the high-type seller to separate himself from the low-type seller by selling in an auction. But for a separating equilibrium to exist, the low-type seller's incentive to mimic has to be sufficiently low and the high-type seller's incentive to separate has to be sufficiently high. These two forces give us the conditions for existence of a separating equilibrium.

In a separating equilibrium, even non-experts know that the low-type seller is low-type. Hence, non-experts are willing to pay at most v for the item sold by the low-type seller.

Therefore, the low-type seller's incentive to mimic increases as v or p decrease. If p and v are sufficiently small, since the low-type seller's incentive to mimic is sufficiently large, a separating equilibrium does not exist. This is captured by condition $p \geq \frac{c}{c+2v}$ in Proposition 3, and is represented by the left contour in Figure 4.

On the other hand, as v increases, the high-type seller's incentive to signal his type (and to separate himself) decreases. When v is large enough, we show that the high-type seller chooses auction only if p is sufficiently small or sufficiently large.

In all three cases, assume that the posted price is $cq + v$.[18] **Case (a):** Both buyers are non-experts: in this case, the auction revenue (in a separating equilibrium) is $c + v$ whereas the posted price revenue is $cq + v$. Therefore, the high-type seller prefer auction to posted price in this case. **Case (b):** One buyer is expert and one buyer is non-expert: in this case, the revenue of the auction is c whereas the revenue of the posted price is $cq+v$. Therefore, the high-type seller prefers posted price (pooling equilibrium) when v is sufficiently large. **Case (c):** Both buyers are experts: in this case, the revenue of auction is c and the revenue of posted price is 0. Therefore, the high-type seller prefers an auction (separating equilibrium) in this case. We can see that for cases (a) and (c), the high-type seller prefers a separating equilibrium (using auction) and in case (b) he prefers posted price. When p is small enough, case (a) is more likely, and when p is large enough, case (c) is the more likely case; but for medium values of p, case (b) is more likely. Therefore, the high-type seller chooses to use auction (and to separate himself) only if p is large enough or small enough. Note that when p is small, it still has to be large enough so that the low-type seller does not want to mimic the high-type seller. These conditions for existence of a separating equilibrium are captured by $p \in [\frac{c}{c+2v}, \frac{v-\sqrt{v(v-2c)}}{2v}]$ and $p \geq \frac{v+\sqrt{v(v-2c)}}{2v}$ in Proposition 3, and are represented by the right contour in Figure 4.

[18] Note that the optimum posted price is not necessarily $cq + v$, and we take this into account in the full proof of Proposition 3 below. However, to facilitate exposition, in this example, we assume that the posted price is $cq + v$.

Proof of Proposition 3

Proof. We prove the proposition in three parts. In part A, we show that both types of sellers cannot be using posted price in a separating equilibrium (with different prices). In part B, we show that there is no separating equilibrium in which the high-type seller uses posted price and the low-type seller uses auction. Finally, in part C, we show that for $v \in [\frac{c(1-p)}{2p}, \frac{c}{2p(1-p)}]$, there exists a separating equilibrium in which the high-type seller uses auction and the low-type seller uses posted price.

Part A: We show that both types of sellers cannot be using posted price in a separating equilibrium. Suppose that this is the case. The price that the low-type seller sets will be v. Suppose the the price that the high-type seller sets is ξ. If $\xi > v$, then the low-type seller benefits from mimicking. Therefore $\xi \leq v$. Now, if $v < c$, the high-type seller benefits from deviating to auction ($\pi_H^L(A)$); therefore, we must have $c \leq v$. Using the same argument, we also get $\xi \geq c$. But $\xi \geq c$ means that the high-type seller's revenue is $(1-p^2)\xi$ (according to the expression for $\pi_H^H(B,z)$). Note that if the high-type seller increases ξ up to v, even though non-experts' belief may change from H to L, the high-type seller's revenue increases to $(1-p^2)v$. Therefore, high-type seller's optimum posted price price would also be v (the same price as the low-type seller), leading to a pooling equilibrium.

Part B: Next, we show that there is no equilibrium in which the high-type seller uses posted price and the low-type seller uses an auction. Assume for the sake of contradiction that there is some ξ such that high-type seller uses posted price with price ξ and low-type seller uses auction. Now consider the following facts. **(1)** ξ **must be less than or equal to** c, otherwise low-type seller benefits from mimicking. Note that for $\xi > c$, high-type and low-type sellers have the same revenue in posted price ($\pi_L^H(B,\xi) = \pi_H^H(B,\xi)$). On the other hand, $\pi_H^L(A) > \pi_L^L(A)$. Therefore, either the high-type seller benefits from deviating to auction, or low-type seller benefits from deviating to posted price (mimicking). **(2) We must have** $v \leq c$, otherwise, the high-type seller benefits from deviating to auction. Note that using (1), high-type seller's revenue in posted price is ξ which is less than or equal to c.

On the other hand, $\pi_H^L(A)$ is strictly greater than c if $v > c$. Therefore, for the deviation to be unprofitable, we must have $v \leq c$. **(3) We must have** $\xi < v$, otherwise the low-type seller benefits from mimicking. Note that low-type seller's revenue in the auction is $(1-p)^2 v$ and by mimicking, it would be $(1-p^2)\xi$. Therefore, since $p > 0$, for the mimicking to be unprofitable we need $\xi < v$. So, we know that $c \geq v > \xi$, but at the same time, since deviating to auction must be unprofitable for the high-type seller we must have $\xi \geq p^2 c + (1-p^2)v$, which is a contradiction.

Part C: We show that a separating equilibrium in which the high-type seller uses an auction and the low-type seller uses posted prices exists if and only if $v \in [\frac{c(1-p)}{2p}, \frac{c}{2p(1-p)}]$. First note that the low-type seller's optimal price is v, and the revenue at this price is $(1-p^2)v$. We know that the low-type seller should not benefit from deviating to auction. This means $(1-p^2)v \geq \pi_L^H(A) = (1-p)^2(c+v)$. This gives us $v \geq \frac{c(1-p)}{2p}$. Similarly, the high-type seller should not benefit from deviating to posted price. This means $\pi_H^H(A) \geq \pi_H^L(B, z)$ for any z. If $c \geq v$, this inequality is automatically satisfied. Otherwise, we need $v \leq \frac{c}{2p(1-p)}$ for this inequality to hold. Since $v \leq \frac{c}{2p(1-p)}$ is weaker than $v \leq c$, it suffices to have $v \leq \frac{c}{2p(1-p)}$ as the necessary condition. Finally, it is easy to check that the condition $v \in [\frac{c(1-p)}{2p}, \frac{c}{2p(1-p)}]$ is equivalent to $p \geq \frac{c}{c+2v}$ when $v \leq 2c$, and equivalent to $p \in [\frac{c}{c+2v}, \frac{v-\sqrt{v(v-2c)}}{2v}] \cup [\frac{v+\sqrt{v(v-2c)}}{2v}, +\infty)$ when $v > 2c$. $\square$

Existence and Uniqueness of Pooling Equilibrium Under $D1$ Criterion

Using our analysis in Section 3, we can show that a high-type seller's revenue in an auction pooling equilibrium is

$$\pi_H^X(A) = \begin{cases} (p + (1-p)(1-t))^2 c + (1 - (p + (1-p)(1-t))^2)(cq + v) & \text{if } v \leq m \\ c & \text{if } m < v < M \\ (1-(1-p)^2)c + (1-p)^2(cq+v) & \text{if } v \geq M. \end{cases}$$

Similarly, a high-type seller's revenue in a posted price (at a price where high-type seller's revenue is maximized) is

$$\pi_H^X(B) = \begin{cases} \max\{cq+v, (1-(1-p)^2)c\} & \text{if } cq+v \leq c \\ \max\{c, (1-p^2)\,(cq+v)\} & \text{otherwise.} \end{cases}$$

Let E be a pooling equilibrium of the game defined as follows. Non-experts have belief X for this mechanism M^E, where

$$M^E = \arg\max_M \pi_H^X(M),$$

and belief L for any other mechanism. Intuitively, when non-expert have belief X about all mechanisms M then M^E is the one that maximizes the high-type seller's profit.

It is easy to see that the high-type seller never wants to deviate in M^E. For any mechanism $M' \neq M$, we know by definition of M^E that $\pi_H^X(M^E) \geq \pi_H^X(M')$. We also know that $\pi_H^X(M') \geq \pi_H^L(M')$. Therefore, a high-type seller cannot benefit from deviating to mechanism M'. For the low-type seller, it is easy to show that he may only want to deviate from M^E if M^E is an auction.[19] Therefore, in all regions where the pooling equilibrium is (B,B), we know that the equilibrium exists. In the top region, there is only the (A,B) equilibrium. In the middle region, we have both the separating (A,B) and the pooling (B,B) equilibrium. In the lower left region with (A,A), the pooling equilibrium exists if and only if $\pi_L^X(A) \geq \pi_L^L(B)$, meaning that the low-type seller does not benefit from deviating to posted price.

Next, we show that E is the only pooling equilibrium that satisfies Requirement $D1$.

Theorem B.1. *In a pooling that satisfies Requirement $D1$, both types of sellers use the mechanism that maximizes the high-type seller's profit.*

[19]Essentially, deviating from posted price to an auction is always dominated for the low-type seller. Also, in the posted price sale, the low-type seller cannot benefit from setting a higher or lower price than what is set in M^E.

Proof. We argue by method of contradiction and consider two (exhaustive) cases:

- $\pi_H^X(A) > \pi_H^X(B)$, $\pi_L^X(A) < \pi_L^X(B)$ and (B, B) is a pooling equilibrium.
- $\pi_H^X(A) < \pi_H^X(B)$, $\pi_L^X(A) > \pi_L^X(B)$ and (A, A) is a pooling equilibrium.

For $\{C, D\} = \{A, B\}$, we get a contradiction as follows:

Suppose $\pi_H^X(C) > \pi_H^X(D)$, $\pi_L^X(C) < \pi_L^X(D)$ and (D, D) is a pooling equilibrium. We know that the belief when D is observed is X. Let us denote the (off-equilibrium) belief when C is observed by $\widetilde{q}$: non-expert assigns the probability $\widetilde{q}$ for being the high type when C is observed. We can immediately see that $\widetilde{q} < q$, as otherwise the high type sellers would deviate. Hence, non-experts has to assign positive probability for the seller to be a low type.

Next, let us denote the threshold beliefs (of non-experts) for high type sellers and low type sellers in order not to deviate to C by $\widetilde{q}^H$ and $\widetilde{q}^L$, respectively. That is, a high (low) type would deviate to C if and only if $\widetilde{q} > \widetilde{q}^H$ ($\widetilde{q} > \widetilde{q}^H$). Since $\pi_H^X(C) > \pi_H^X(D)$, $\pi_L^X(C) < \pi_L^X(D)$, we can see that $\widetilde{q}^H < q$ and $\widetilde{q}^L > q$, hence we have $\widetilde{q}^L > \widetilde{q}^H$. That is, if deviation C is observed, whenever a high type benefits from this deviation, low type also benefits from this deviation. Then, Requirement $D1$ dictates that the receiver (non-expert) should put zero probability on the low type ($\widetilde{q} = 1$). A contradiction. $\square$

Therefore, both types of sellers use posted price if and only if $\pi_H^X(B) \geq \pi_H^X(A)$. In other words, if

$$v \geq \frac{c(p-2)}{2(p-1)} - cq$$

or if

$$v \leq \min\left\{ \frac{(p-1)(q-1)}{p(2q-1)+1}, \frac{c\left(\sqrt{p\left(p\left(p^2+16((p-2)p-1)q^3-8(3(p-2)p+1)q^2+8(p-1)^2q-4p+16q^4+6\right)-4\right)+1}+p\left(p\left(4q^2-6q+3\right)-2\right)+2q-1\right)}{2\left(p^2(1-2q)^2-1\right)} \right\}$$

in case of pooling equilibrium we have (posted price, posted price). Otherwise, in case of pooling equilibrium we have (Auction, Auction).

C Analyses and Proofs of Section 5

In this section, we show that our key observation about the presence of sniping in equilibria for auctions with a closing time continues to hold when we add reserve prices to the analysis; furthermore, we show that it also continues to hold in auctions with more than two bidders. This demonstrates the robustness of our findings and also supports the observation of increased revenues in such auctions relative to the ones where sniping is not allowed even in these extensions.

C.1 Equilibria in Auction with Reserve Price

Our first set of results generalize the equilibrium bidder strategies demonstrating sniping for auctions with reserve price. When we write

NE	x	b_{NE}
LE	y	b_{LE}
HE	z	b_{HE}

,

we mean that the strategy of a non-expert (represented by NE) is to bid x in the first round and b_{NE} in the second round, the strategy of an expert that knows that $C = 0$ (low-type expert or LE) is to bid y in the first round and b_{LE} in the second round, and the strategy of an expert that knows that $C = c$ (high-type expert or HE) is to bid z in the first round and b_{HE} in the second round.

Lemma C.1. *Suppose that $v \leq M$ and consider the auction with a reserve price $r \geq 0$. We have the following cases:*

1. *If $r \leq v$ and $v \leq m$, the following is an equilibrium for $t = \frac{(2pq+1-p)(m-v)}{2pqm+(1-p)(m-v)}$*

NE	$cq+v$	$\begin{cases} c & \text{, with prob. } 1-t \\ 0 & \text{, with prob. } t \end{cases}$
LE	0	0
HE	$cq+v$	c

,

NE	0	$\begin{cases} c & \text{, with prob. } 1-t \\ cq+v & \text{, with prob. } t \end{cases}$
LE	0	0
HE	0	c

2. *If $r \leq v$ and $v \geq m$, the following are equilibria for any $x \in [0, c)$*

NE	x	c
LE	0	0
HE	x	c

,

NE	c	0
LE	0	0
HE	c	0

3. *If $v \leq r \leq v + \frac{2pq+1-p}{2p(1-q)}(v-m)$, the following is an equilibrium*

NE	0	c
LE	0	0
HE	0	c

4. *If $\max\left\{v + \frac{2pq+1-p}{2p(1-q)}(v-m), v\right\} \leq r \leq \frac{cq(1-p)+v}{1-pq}$, the following is an (asymmetric) equilibrium*

NE 1	0	c
NE 2	0	0
LE	0	0
HE	0	c

5. *If $\frac{cq(1-p)+v}{1-pq} \leq r \leq c$, the following is an equilibrium*

NE	0	0
LE	0	0
HE	0	c

6. *If $r \geq c$, the following is an equilibrium*

NE	0	0
LE	0	0
HE	0	0

Proof of Lemma C.1

Proof. The proof in the first two cases is identical to that for the case $r = 0$. The only thing that changes is that the utility of a non-expert when he faces a low expert is $v - r \geq 0$ instead of just v.

Let's consider the case (3) when $v \leq r \leq v + \frac{2pq+1-p}{2p(1-q)}(v - m)$. In order for this to hold it must also be $v \geq m$. The utility of a non-expert by bidding c is

$$q\left(pv + (1-p)\frac{v}{2}\right) + (1-q)\left(p(v-r) + (1-p)\left(\frac{v-c}{2}\right)\right).$$

This must be non-negative, which is equivalent to $r \leq \frac{(1+p)v-c(1-p)(1-q)}{2p(1-q)} = v + \frac{2pq+1-p}{2p(1-q)}(v - m)$. If the non-expert changes the bid to $c - \epsilon \geq r$, the utility becomes

$$q\left(p \cdot 0 + (1-p) \cdot 0\right) + (1-q)\left(p(v-r) + (1-p) \cdot 0\right) \leq 0,$$

since $r \geq v$. If he changes the bid to $c + \epsilon$, the utility becomes

$$q\left(pv + (1-p) \cdot v\right) + (1-q)\left(p(v-r) + (1-p) \cdot (v-c)\right),$$

i.e. it increases by $q(1-p)\frac{v}{2} + (1-q)(1-p)\frac{v-c}{2} = \frac{(1-p)(v-M)}{2} \leq 0$, so it becomes smaller. The expected utility of a high expert is

$$p \cdot \frac{c-c}{2} + (1-p) \cdot 0 = 0,$$

and it remains 0 even if he bids above or below c.

Next, let's consider the case (4): $\max\left\{v + \frac{2pq+1-p}{2p(1-q)}(v - m), v\right\} \leq r \leq \frac{cq(1-p)+v}{1-pq}$. In this case there is no symmetric equilibrium (pure or mixed), but there is an asymmetric pure equilibrium. So, we give identities to the two players (before they draw their type) and if player 1 is a non-expert he follows a different strategy than player 2 if he is a non-expert. The utility of a non-expert player 1 is

$$q(pv + (1-p)(c+v-r)) + (1-q)(p(v-r) + (1-p)(v-r)),$$

which is non-negative iff $r \leq \frac{cq(1-p)+v}{1-pq}$. If he bids something below c and at least r, the utility decreases by qpv. If he doesn't bid, the utility becomes 0. The utility of a non-expert player 2 is 0. If he bids something below c and at least r, it becomes $(1-q)p(v-r) \leq 0$. If he bids c, it becomes

$$q\left(pv + (1-p)\tfrac{v}{2}\right) + (1-q)\left(p(v-r) + (1-p)\tfrac{v-r}{2}\right),$$

which is non-positive iff $r \geq \frac{(1+p)v - c(1-p)(1-q)}{2p(1-q)} = v + \frac{2pq+1-p}{2p(1-q)}(v-m)$. If he bids something above c, it becomes even worse by $\frac{(1-p)(cq+v-c)}{2} \leq 0$. The expected utility of a high expert player 2 is 0 and of a high expert player 1 is

$$p \cdot \tfrac{c-c}{2} + (1-p) \cdot (c-r) = (1-p)(c-r) \geq 0.$$

Next, let's consider the case (5): $\frac{cq(1-p)+v}{1-pq} \leq r \leq c$. The utility of a non-expert is 0. If he bids something at least c the utility becomes

$$q(pv + (1-p)(c+v-r)) + (1-q)(p(v-r) + (1-p)(v-r)),$$

which is non-positive iff $r \geq \frac{cq(1-p)+v}{1-pq}$. If he bids something below c and at east r, it becomes even worse. The utility of a high expert is

$$p \cdot \tfrac{c-c}{2} + (1-p) \cdot (c-r) = (1-p)(c-r) \geq 0.$$

Finally, let's consider the case (6): $r \geq c$. In this case the (expected) valuation of every buyer is at most the reserve price, therefore no-one has incentive to bid. $\square$

We note also that the case in which we have an asymmetric equilibrium in the lemma, can be broken in three cases, in two of which we have a symmetric mixed equilibrium. More specifically, we have the following lemma.

Lemma C.2. *Suppose that $v \leq M$.*

1. *If $v \geq m$ and $v + \frac{2pq+1-p}{2p(1-q)}(v-m) \leq r \leq \frac{cq(1-p)+v}{1-pq}$, then the following is an equilibrium for $t = \frac{c(1-p)(1-q)+2pr(1-q)-(1+p)v}{(1-p)(c(1+q)-2r+v)}$*

NE	0	$\begin{cases} c & \text{, with prob. } 1-t \\ 0 & \text{, with prob. } t \end{cases}$
LE	0	0
HE	0	c

2. *If $v \leq m$ and $v \leq r \leq \frac{c^2q(1-p)(1-q)-cv(pq^2+2q-1)-(1+pq)v^2}{c(1-q)(1-pq)-(1+pq)v}$, then the following is an (asymmetric) equilibrium*

NE 1	0	c
NE 2	0	0
LE	0	0
HE	0	c

3. *If $v \leq m$ and $\frac{c^2q(1-p)(1-q)-cv(pq^2+2q-1)-(1+pq)v^2}{c(1-q)(1-pq)-(1+pq)v} \leq r \leq \frac{cq(1-p)+v}{1-pq}$, then the following is an equilibrium for $t = \frac{c(1-p)(1-q)+2pr(1-q)-(1+p)v}{(1-p)(c(1+q)-2r+v)}$*

NE	0	$\begin{cases} c & \text{, with prob. } 1-t \\ 0 & \text{, with prob. } t \end{cases}$
LE	0	0
HE	0	c

The proof of Lemma C.2 is similar to the proof of Lemma C.1.

Now, we consider the case $v \geq M$.

Lemma C.3. *Suppose that $v \geq M$ and consider the auction with a reserve price $r \geq 0$. We have the following cases:*

1. *If $r \leq c$, the following is an equilibrium*

NE	$cq + v$	0
LE	0	0
HE	0	c

2. *If $c \leq r \leq cq + v$, the following is an equilibrium*

NE	$cq + v$	0
LE	0	0
HE	0	0

3. *If $r \geq cq + v$, the following is an equilibrium*

NE	0	0
LE	0	0
HE	0	0

Proof of Lemma C.3

Proof. The auction is reduced to a simple sealed-bid second price auction with reserve price, where the valuation of a non-expert is $cq + v$, of a high expert is c, and of a low expert is 0. The result follows. □

When we compare the average revenue for an auction with sniping and optimal reserve price and an auction without sniping and optimal reserve price, we get the plot in Figure 6.

C.2 Auction with More Bidders

In this subsection, we generalize the main results of the auction with two bidders to an arbitrary number of bidders. First, we start with a lemma for three bidders.

Lemma C.4. *Suppose we have* 3 *bidders.*

- *If* $\frac{c(1-p)(1-q)}{2-p} \leq v \leq \frac{c(1-p)(1-q)(2p+1)}{3p^2q+(1-p)(2p+1)}$*, then there exists a* $t \in [0,1]$ *such that the following is an equilibrium*

NE	0	$\begin{cases} c & \text{, with prob. } 1-t \\ cq+v & \text{, with prob. } t \end{cases}$
LE	0	0
HE	0	c

- *If* $\frac{c(1-p)(1-q)(2p+1)}{3p^2q+(1-p)(2p+1)} \leq v \leq M$*, then the following is an equilibrium*

NE	0	c
LE	0	0
HE	0	c

The proof of Lemma C.4 is very similar with the case of two bidders. The main difference is that to find the correct probability t of the mixed equilibrium, we need to solve a quadratic equation and find the conditions for which it has a root in $[0,1]$. By doing this, we derive that v must be in the interval $\left[\frac{c(1-p)(1-q)}{2-p}, \frac{c(1-p)(1-q)(2p+1)}{3p^2q+(1-p)(2p+1)}\right]$.

Next, we consider an auction with $n \geq 2$ bidders.

Lemma C.5. *Suppose we have* $n \geq 2$ *bidders. Let* $A_n = \frac{1-np^{n-1}+(n-1)p^n}{n(1-p)}$*. If* $\frac{c(1-q)A_n}{p^{n-1}q+A_n} \leq v \leq M$*, then the following is an equilibrium*

NE	0	c
LE	0	0
HE	0	c

Note that $\frac{c(1-q)A_2}{p^{n-1}q+A_2} = m$, therefore the lemma above is a generalization of the second equilibrium for the two-bidders-case, and the proof is very similar. It remains to generalize the first equilibrium, which is done by the next lemma.

Lemma C.6. *Suppose we have $n \geq 2$ bidders. If $\frac{c(1-q)(-1+n(1-p)-(n-1)(1-p)^n+p^n)}{n(1-p-(1-p)^n)} \leq v \leq \frac{c(1-q)A_n}{p^{n-1}q+A_n}$, then there exists a $t \in [0,1]$ such that the following is an equilibrium*

NE	0	$\begin{cases} c & \text{, } \textit{with prob. } 1-t \\ cq+v & \text{, } \textit{with prob. } t \end{cases}$
LE	0	0
HE	0	c

Proof of Lemma C.6

Proof. We have to find when the following is an equilibrium for $n \geq 2$ bidders

NE	0	$\begin{cases} c & \text{, with prob. } 1-t \\ cq+v & \text{, with prob. } t \end{cases}$
LE	0	0
HE	0	c

The utility of a non-expert if he bids c is

$$p^{n-1}(qv+(1-q)v) + \sum_{k=1}^{n-2}\left[\binom{n-1}{k}p^{n-1-k}(1-p)^k\left(t^k(q(cq+v-c)) + \sum_{i=1}^{k}\binom{k}{i}(1-t)^i t^{k-i}\frac{cq+v-c}{i+1}\right)\right]+$$
$$+(1-p)^{n-1}\left(t^{n-1}\cdot 0 + \sum_{i=1}^{n-1}\binom{n-1}{i}(1-t)^i t^{n-1-i}\frac{cq+v-c}{i+1}\right) =$$
$$= p^{n-1}v + (cq+v-c)\cdot\left[q\sum_{k=1}^{n-2}\binom{n-1}{k}p^{n-1-k}(1-p)^k t^k + \sum_{k=1}^{n-2}\binom{n-1}{k}p^{n-1-k}(1-p)^k\left(\frac{1-(k+1)t^k+kt^{k+1}}{(k+1)(1-t)}\right)\right]+$$
$$+(cq+v-c)\left[(1-p)^{n-1}\left(\frac{1-nt^{n-1}+(n-1)t^n}{n(1-t)}\right)\right] =$$
$$= p^{n-1}v + (cq+v-c)\cdot\left[(q-\tfrac{1}{1-t})L + \tfrac{1}{1-t}\cdot\frac{1-np^{n-1}+(n-1)p^n-(1-p)^n}{n(1-p)} + \tfrac{t}{1-t}L - \tfrac{t}{1-t}R\right]+$$
$$+(cq+v-c)\left[(1-p)^{n-1}\left(\frac{1-nt^{n-1}+(n-1)t^n}{n(1-t)}\right)\right] =$$
$$= p^{n-1}v + (cq+v-c)\cdot\left[-(1-q)L + \tfrac{1}{1-t}\cdot\frac{1-np^{n-1}+(n-1)p^n-(1-p)^n}{n(1-p)} - \tfrac{t}{1-t}R + (1-p)^{n-1}\left(\frac{1-nt^{n-1}+(n-1)t^n}{n(1-t)}\right)\right],$$

where $L = \sum_{k=1}^{n-2} \binom{n-1}{k} p^{n-1-k}(1-p)^k t^k = (p+(1-p)t)^{n-1} - p^{n-1} - (1-p)^{n-1}t^{n-1}$ and $R = \sum_{k=1}^{n-2} \binom{n-1}{k} \frac{p^{n-1-k}(1-p)^k t^k}{k+1}$.

The utility if he bids $cq+v$ is

$$(1-q)p^{n-1}v - cq(1-q)R,$$

and we want these two expressions to be equal. If we substitute t with 0 we get $v = \frac{c(1-q)A_n}{p^{n-1}q+A_n}$. If we substitute t with 1, we get $v = \frac{c(1-q)(-1+n(1-p)-(n-1)(1-p)^n+p^n)}{n(1-p-(1-p)^n)}$. □

The following lemma generalizes the equilibrium without sniping for multiple bidders.

Lemma C.7. *The following strategies form an equilibrium for the auction without sniping and $n \geq 2$ bidders.*

- *Non-expert: He starts with a bid of v. If at any point he sees a bid different than v, $cq+v$, and $c+v$, he bids $c+v$. If there is at least one buyer with no bid and the rest either have a bid of v or no bid, then he stays with the bid of v. If everyone else has a bid of v, he bids $cq+v$.*

- *Expert: If $C = 0$, he doesn't participate. If $C = c$, he bids c.*

Proof of Lemma C.7

Proof. No bidder wants to deviate from the strategy he has in the equilibrium. Non-experts bid their (expected) valuation at any point in time, and experts bid their valuation. Moreover, high experts have no incentive to bluff by pretending that they are non-experts, because they lose in case of a tie. □

As in the case of two bidders, we can see that non-experts bid more aggressively in the equilibrium with sniping. Therefore, sniping can again increase the average revenue for high values of p, or v, or q.