Exclusive Display in Sponsored Search Advertising

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Abstract

As sponsored search becomes increasingly important as an advertising medium for firms, search engines are exploring more advanced bidding and ranking mechanisms to increase their revenue from sponsored search auctions. For instance, Google, Yahoo! and Bing are investigating auction mechanisms in which each advertiser submits two bids: one bid for the standard display format in which multiple advertisers are displayed, and one bid for being shown exclusively. If the exclusive-placement bid by an advertiser is high enough then only that advertiser is displayed, otherwise multiple advertisers are displayed and ranked based on their multiple-placement bids. We call such auctions two-dimensional auctions and study the GSP_{2D} mechanism, which is an extension of the GSP mechanism and has recently been patented by Yahoo! as a key candidate for implementing two-dimensional exclusive-display auctions. In a significant advance on the existing literature on sponsored search auctions, we assume that advertisers have uncertain valuations and solve for the Bayesian Nash equilibria of the GSP and GSP_{2D} auctions.

We find that allowing advertisers to bid for exclusivity can increase the revenue of the search engine because competition is heightened—bidders compete not only for positions in the non-exclusive outcome but also compete for the outcome to be exclusive or non-exclusive. Interestingly, however, under certain conditions, the revenue of the search engine can decrease—competition between outcomes leads to bidders reducing bids for their non-preferred outcome; specifically, a bidder who values the exclusive outcome highly will bid high for exclusivity and, simultaneously, bid low for non-exclusivity to increase the chance of obtaining the exclusive outcome. We also find interesting results on the bidding strategies of advertisers in GSP_{2D}; for instance, we find that, under certain conditions, advertisers have the incentive to bid *above* their true valuations.

Keywords: sponsored search advertising, exclusive display, game theory, position auctions, two-dimensional auctions.

1 Introduction

Online advertising is fast becoming an increasingly important component of any firm's advertising mix. In turn, one of the primary forms of online advertising is sponsored search advertising on popular search engines such as Google, Yahoo! and Bing. In sponsored search advertising, advertisers pay a fee to the search engine to have links to their websites listed as relevant results in response to a keyword search. When a user submits a query on the search engine, she is presented with advertisements (henceforth, ads) that are placed into positions, usually arranged linearly down the side of the page (along with the organic search results which are not sponsored).[1] Sponsored search advertising is the primary source of revenue for search engines; for instance, Google, Yahoo! and Bing earn millions of dollars per day through this channel.[2]

Being their largest source of revenue, the pricing mechanism for sponsored search advertising is of critical importance to search engines. All the prominent search engines currently run Generalized Second Price (GSP) auctions to sell their advertising space. However, this choice of the auction mechanism was not a straightforward one, and the industry went through several phases before the GSP auction became the dominant choice. Sponsored search was introduced in 1997 by the search engine GoTo (renamed Overture in 2001, and acquired by Yahoo in 2003) which used a Generalized First Price auction (also adopted by Yahoo! and Bing) in which every advertiser submitted a bid and the advertisers were arranged in descending order of bids, with each one paying his bid. The payment mechanism was also experimented with, and while initially advertisers had to pay every time their ad was shown (pay per impression), this was changed to payment every time their ad was actually clicked (pay per click). The Generalized First Price auction, however, was soon found to be an unstable auction mechanism in which advertisers had the incentive to cyclically bid low and high amounts to game the system (Edelman and Ostrovsky 2007). In 2002, Google introduced a more stable mechanism called the Generalized Second Price (GSP) auction in which every advertiser submits his per-click bid but has to actually pay only the minimum amount necessary to keep his current position in the list of results (i.e., GSP is a "second-price" auction). GSP was gradually adopted by other prominent search engines as well, such as Yahoo! and Bing. Search engines also

[1]Throughout the paper, we refer to a user as "she," an advertiser as "he," and the search engine as "it."

[2]See http://investor.google.com/earnings.html for Google, http://yhoo.client.shareholder.com/results.cfm for Yahoo! and http://www.microsoft.com/investor/default.aspx for Bing.

continually conduct their own internal experimentation, based on which they apply slight variations (the exact details of which are often not publicly announced) to the basic GSP mechanism. For instance, advertisers are ranked based not on their submitted bids, but based on their effective bids, which are a function of the submitted bid and an advertiser-specific quality score.

The above discussion shows that the spectacular rise of sponsored search advertising in the last decade has been accompanied by constant effort from the search engines to refine their pricing mechanisms by gradually fixing the deficiencies in them, including developing new auction mechanisms to rank advertisers. As the industry matures, search engines are looking to further expand their bidding mechanisms by allowing advertisers to be more specific about their utilities and to express a richer set of preferences. For example, Google has implemented "hybrid" advertising auctions which allow advertisers to bid on a per-impression or a per-click basis for the same advertising space. Zhu and Wilbur (2011) show that such auctions can enhance both search engine revenue and the efficiency of advertisers' allocation to positions.

A recent and very interesting development in this context has been the exploration by search engines of auction mechanisms that allow advertisers to bid for *exclusive display* in response to a user search. In other words, advertisers can bid for their ad to be the only ad displayed, rather than being one of many ads displayed. Exclusive display may be an attractive option for an advertiser as it can create strong brand associations by being the only one displayed in response to certain keywords. For example, if the ad of only the manufacturer Canon gets displayed in response to the keyword "digital camera," it can be a significant branding advantage for Canon over its competitors such as Nikon and Olympus. Moreover, multiple ads shown next to each other may impose negative externalities on each other. For example, if a user who has searched for the keyword "car rental" clicks on the ad of Hertz, chances are that she will also go back to check the ads of some other companies displayed in the sponsored list, such as Avis and Budget, before finalizing the transaction with Hertz. These negative externalities, which can decrease the values of clicks to advertisers, are likely to be smaller if only one ad is shown to the user.

Exclusive display is even more valuable when a particular brand name is the keyword searched, because the brand owner would want to be the only advertiser displayed in response to prevent potential customers from being poached (Sayedi, Jerath and Srinivasan 2012). For example, in high-profile cases, Rosetta Stone and Louis Vuitton sued Google in USA and Europe, respectively

(Mullin 2010, Sterling 2010), in an attempt to have laws enacted to prevent bidding on trademarks by competitors. While these companies lost these legal battles, it does reveal the strong incentive of brand owners to be displayed exclusively, which the search engines could profit from. Desai et al. (2010) similarly argue, with experimental support, that when advertisers are listed next to each other, "context effects" influence users' perceptions of their relative qualities, which in certain cases can hurt the advertisers, especially the high-quality ones. Such effects may motivate advertisers to prefer exclusive display. In summary, exclusive display will not only increase the expected clicks on an ad if the relevant keyword is searched but, for some advertisers, may also increase the valuation per click.

Interestingly, in November 2011 Yahoo!, along with Microsoft adCenter, introduced the exclusive display option for "North" ads, i.e., ads at the top of the search results page, through the "Rich Ads in Search" program. Rich Ads in Search allows advertisers to bid higher for being shown *exclusively* at the top of search results for the associated query. Winners of Rich Ads in Search can also include videos and pictures in their otherwise text-only ads—elevating advertisers' valuation for exclusive outcome even more. Given that a majority (up to 85%) of clicks on sponsored ads are from the North ads (if available) (Reiley et al. (2010)), this adoption of exclusive display can have a significant impact on search engine's revenue as well as advertisers' strategies. While this is not a complete switch to exclusive display, it is a significant step in that direction. Google has also considered displaying exclusive ads, i.e., only one ad per page, as part of its "perfect ad" initiative (Metz 2008, 2011). However, the dilemma Google faced was that allowing exclusive display might reduce revenue because of the loss of revenue from the many advertisers who will not be displayed, which can lead to reduced clicks and less competition. Due to a lack of proper understanding of the implications, for the time being Google has chosen to stay with the status quo of displaying multiple ads in response to user queries. Our study sheds light on these issues by identifying the key driving forces at play, and how they can influence outcomes in different conditions, in exclusive display in sponsored ads.

We develop a game theory model to study exclusive display in sponsored search advertising. We start by assuming that each advertiser can have different per-click valuations for clicks obtained when it is displayed with other advertisers (multiple display) and clicks obtained when it is the only one displayed (exclusive display). Given that GSP is the auction mechanism used widely by search engines, we analyze a mechanism that allows exclusive display and is conceptually a simple extension of GSP. Specifically, we analyze GSP_{2D}, a mechanism that has been recently patented by Yahoo! as a key candidate for implementation in exclusive display auctions. In GSP_{2D}, each advertiser submits a two-dimensional bid—its maximum willingness to pay per click for multiple display, and its maximum willingness to pay per click for exclusive display. When multiple ads are displayed, the allocation and pricing rules of GSP_{2D} are defined to be exactly those of GSP. Following the idea of second price auctions, if an exclusive bid wins in GSP_{2D}, the winner pays the minimum amount that it could have bid to win the exclusive outcome. Note that when multiple ads are displayed in GSP_{2D}, advertisers see no difference at all between the new auction and the existing GSP system.

A key ingredient of our model is that bidders have uncertain valuations and competitors only know the distribution from which the valuations arise. This is unlike most of the previous work on search advertising that assumes that bidders know the valuations of other bidders. Given our assumption of uncertain valuations, we use the Bayesian equilibrium concept, in contrast with most previous work in which the ex-post equilibrium concept is used (Edelman, Ostrovsky and Schwarz 2007, Varian 2007 and Katona and Sarvary 2010). Given that we use a different framework, we not only derive new insights regarding the exclusive display GSP_{2D} auction, but we also derive new results and insights that help us to better understand the widely-studied GSP auction. To best of our knowledge, no other paper in the literature has studied bidders' strategies in the Bayesian equilibrium of GSP with uncertain bidder valuations. Our results on GSP also serve as a benchmark to compare results on GSP_{2D}.

We assume that there are two types of advertisers—those who value exclusive placement more than non-exclusive placement (type D, with "D" indicating that they have "dual" valuations), and those who do not attach extra value to exclusive placement (type S, with "S" indicating that they have "single" valuations). In the context of GSP, only the valuation for non-exclusive placement is relevant. We show that, in GSP, bidders shade their bids (i.e., bid below their valuations), that shading is more for higher-value bidders, and that shading increases as click-through-rates of the slots become more similar. Furthermore, we show that setting a higher reserve price has a cascading effect on all bidders, i.e., increases all bidders' bids and not just the bids of lower valuation bidders who are directly affected by reserve price. In fact, we show that the equilibrium bid of a higher-value bidder is affected *more* by a higher reserve price than the equilibrium bid of a lower-value bidder who is directly affected by the reserve price. This shows that that, in GSP, most of the incremental revenue from setting a reserve price optimally comes not from the direct effect on low-value bidders but rather from the cascading effect on high-value bidders.

For GSP_{2D}, we show that there are two types of equilibria: one in which type D bidders bid for exclusive placement and type S bidders bid for non-exclusive placement, and another in which both type D and type S bidders bid for exclusive placement, even though type S bidders do not attach extra valuation to exclusive placement. Regarding advertisers' strategies in GSP_{2D}, we show that low-value type S bidders, rather than shading their bids as in GSP, may have the incentive to even bid *above* their valuation. This is because in a two-dimensional auction, not only are the type S bidders competing for positions on the results page in the non-exclusive outcome, they are also trying to ensure that the outcome is actually the non-exclusive one rather than the exclusive one. The low-value type S bidders want the auction outcome to be non-exclusive display, but expect to be placed low in the resulting list and therefore expect to pay above their valuation with low probability, and therefore bid above their valuation. On the other hand, high-value type S bidders shade their bids even more than in GSP. This is because they know that if there is any type D bidder with high non-exclusive valuation, he will also have a high exclusive valuation, and will submit a high exclusive bid but a low non-exclusive bid to get the exclusive display outcome. Therefore, high-value type S bidders expect to have lesser competition in GSP_{2D} than in GSP and shade their bids significantly. Regarding the search engine's revenue, introducing the option of exclusive display can, interestingly, *increase or decrease* this revenue. In particular, as the click-through rates for different slots in non-exclusive display become similar, the revenue of the GSP becomes higher than the revenue of GSP_{2D}.

Microsoft and Yahoo have already implemented exclusive-display for North ads through "Rich Ads in Search" program. At the same time there is lack of good understanding of exclusive display, as is clear from the Google episode (Metz 2008, 2011). In this context, our study is very timely and relevant. Our work characterizes the conditions under which allowing exclusive bidding increases or decreases search engine's revenue. Furthermore, we discuss how advertisers of different types should adopt new bidding strategies in response to exclusive display in search advertising.

The rest of this paper is structured as follows. In the next section, we discuss the literature related to our work. In Section 3, we decribe our analytical model, define the GSP and GSP_{2D} auctions, and discuss the equilibrium concept we use. In Section 4, we present the results of our analysis of the two auctions and compare their key characteristics. In Section 5, we extend our analysis to the "Rich Ads in Search" setting used by Yahoo! and Microsoft adCenter. In Section 6, we conclude with a discussion. In the appendix to the paper, we provide important technical details and sketches of proofs.

2 Related Literature

Theoretical studies in the Economics and Marketing communities have significantly enhanced our understanding of position auctions used in sponsored search advertising. Edelman and Ostrovsky (2007) studied first-price auctions and established that bidding will be cyclical and unstable in these auctions. Edelman, Ostrovsky and Schwarz (2007) and Varian (2007) showed that bidding is stable but not truthful in the Generalized Second Price auction (GSP). Various other papers that consider different aspects of second-price position auctions include Athey and Ellison (2011), Edelman and Schwarz (2010), Jerath et al. (2011), Katona and Sarvary (2010), Katona and Zhu (2011), Liu et al. (2010), Desai et al. (2010) and Wilbur and Zhu (2008). Many of the above papers consider both pay-per-impression and pay-per-click payment schemes. Amaldoss, Desai and Shin (2010) study the strategic impact of uncertain reserve price on search engine's revenue. Sayedi et al. (2012) discuss the interaction between search advertising and traditional media advertising and discuss the role of trademark bidding. All of the above papers, however, study auctions that only consider displaying multiple advertisers in response to a keyword search.

To our knowledge, only two other papers (in the Computer Science community) analyze position auctions in which advertisers can express their preferences beyond simply turning in bids for a multiple-display outcome. Muthukrishnan (2009) considers a second-price auction and allows each advertiser to submit a per-click bid (its maximum willingness to pay) and specify the maximum number of other advertisers he wants to be displayed with. Note that this is a very different auction mechanism from GSP_{2D}, which we consider in this paper. Furthermore, the focus of Muthukrishnan (2009) is on developing a fast algorithm to determine the outcome of this auction (which includes deciding how many ads to display, and which advertisers to include and how to rank them), while the revenue and efficiency properties of the auction itself are not analyzed. The paper closest to our work is Ghosh and Sayedi (2010), who analyze the same GSP_{2D} auction as we do. However, they derive a very different set of results as they use ex-post equilibrium concept and their focus is on comparing the properties of the multiple equilibria that the GSP and GSP_{2D} auctions can attain. In contrast, in this paper, our aim is to understand at an intuitive level how exclusive-display auctions work, how they affect advertisers' bidding strategies, and which auction is more beneficial to the search engine and to the advertisers under different conditions. We believe that our results and insights, while being of academic interest, also speak closely to the needs of a managerial audience.

In auction theory literature, our work fits in the scope of combinatorial auctions, in which multiple items are for sale and bidders can submit bids for combinations of items. The FCC spectrum auction is a notable example of mechanism design for combinatorial auctions (Rothkopf et al. 1998). Due to the complex auction structure, there has not been much success in mechanism design for combinatorial auctions in general. Ausubel and Milgrom (2006) discuss the application of Vickrey-Clarke-Groves (VCG) mechanism to combinatorial auctions. While VCG achieves maximum welfare in the dominant-strategy equilibrium, it is not suitable for our framework as it generates low (or zero) revenue for the search engine. Cramton, Shoham and Steinberg (2006) provide a comprehensive survey of advances in combinatorial auctions.

We note that our work is distinct from what is referred to as "Multidimensional Auctions" in the literature. Multidimensional auctions usually apply to settings in which bids contain more attributes, e.g. quality, than just price (Branco 1997, Che 1993, Mori 2006, Thiel 1988). The most common application of multidimensional auctions is procurement auction in which bidders are required to specify several characteristics of the contract to be fulfilled. For example, in an auction for a contract to build an aircraft, bidders quote a price and also specify the components of the aircraft (Branco 1997). The terminology of multidimensional auctions is however sometimes used in settings related to combinatorial and multi-product auctions (Koppius et al. 2000). A notable example related to our work is Belloni, Lopomo and Wang (2009). The authors study the setting in which seller decides the *grade* of the single item to be sold while bidders have different valuations for different grades. Their work is related to our work because in our setting also the seller has to decide whether to sell exclusively or non-exclusively (the grade). However, the two models become very different if the seller decides to sell non-exclusively as the item has to be given to multiple bidders, while in their model the item must always be given to exactly one bidder. On the other hand, in the literature related to multi-product auctions (Armstrong 1996 and Manelli and Vincent 2007), the seller can always sell different items to different bidders and there is no notion of exclusivity.

Finally, note that exclusivity contracts are often negotiated between media providers and advertisers for traditional media advertising. For example, Anheuser-Busch and Volkswagen held the rights for advertising exclusively in the beer and automotive categories, respectively, during Super Bowl 2011.[3] Dukes and Gal-Or (2003) study this market. However, our work is very different from this literature stream. First, the institutional details of our setting introduce several differences (e.g., ranked outcomes, per-click bidding by advertisers, bid-weighting by the auctioneer). Second, in our specific case the auction mechanism allows multiple as well as exclusive winners and the auctioneer decides *after* the bidders have submitted their bids whether there will be multiple winners with a rank ordering or only one winner.

Notably, in the late 1990s and early 2000s, AOL, which had a large share of the online search market at the time, was using exclusive listings. For example, it signed contracts with eBay and Monster which allowed them to be the exclusive providers of certain services when accessed through AOL (Bradley 2001, Hallowell 2002, Rayport 2000). This, however, was different from the sponsored ads arrangement, and the pricing mechanism was not auction based. Similar to this practice, some firms that run their own website-specific sponsored search advertising allow exclusive display; for instance, autotrader.com allows advertisers to buy rights for being listed exclusively in specified geographical regions in response to customer searches for cars.

3 Model

In this section, we describe the general framework that we use in the paper. When a user of the search engine submits a query, she is shown two lists of results—the organic list and the sponsored list. The sponsored list is a ladder of, usually text-only, ads towards the right of the results page.

[3]http://beernews.org/2011/01/beer-playing-a-major-role-in-super-bowl-xlv-chatter;
http://media.vw.com/press_releases/volkswagen-of-america-will-be-the-super-bowls-exclusive-car-advertiser-a-first-for-the-german-automaker

Sometimes, one to four ads are also placed above the organic search results. A position that contains an ad is called a slot, and the search engine basically assigns the ads to the slots. The slots that are placed above the organic search results (usually called "North" slots) are more likely to get clicks than those placed on the right of the organic search results (usually called "East" slots) and are considered more valuable. Within the "North" or "East" slots, slots at upper positions are more valuable than those at lower positions. Therefore, we get a total ordering, and we can model the ad presentation as an array of slots where the earlier positions in the array are more valuable and more likely to get clicks than the later positions.

We assume that there are three advertisers who want to display their ads. In our context, ads can be displayed in one of two formats. In the first format, multiple ads (two ads in our simplified model) are displayed. Slot i ($1 \leq i \leq 2$) is associated with a number $0 < \theta_i \leq 1$ called the click-through rate (CTR) of the slot. The number θ_i indicates the probability of being clicked if multiple ads are displayed and an ad is placed at slot i. We follow previous literature (Edelman, Ostrovsky and Schwarz 2007, Varian 2007) and assume that CTR of a slot is independent of what advertiser is shown in that slot, and that $\theta_2 \leq \theta_1$. Furthermore, we do not model advertiser specific click-through rates and assume that the CTR of any advertiser shown in slot i is θ_i.

In the second format, only one ad is displayed exclusively. In this case, we assume that the CTR of the only slot shown is $\hat{\theta}$. We assume that $\hat{\theta} \geq \theta_1$, i.e., the only slot shown in the exclusive-display outcome gets at least as many clicks as the first slot in the multiple-display outcome. Since normalizing does not affect our results, we assume $\hat{\theta} = 1$ for simplicity. Note that the total number of clicks on all the sponsored links combined can be higher when multiple links are displayed, i.e., $\theta_1 + \theta_2$ can be higher than $\hat{\theta}$.

Each advertiser i of the three advertisers has a vector of valuations (v_i^M, v_i^E), where v_i^M is the valuation of a click when displayed with multiple other advertisers and v_i^E is the valuation of a click when displayed alone in response to a keyword search (the superscripts M and E stand for "Multiple" and "Exclusive," respectively). Note that we use "multiple" and "non-exclusive" interchangably throughout the paper. As discussed in the introduction, quality perceptions and post-click conversion rates can improve under exclusive display. For these reasons, we make the reasonable assumption that $v_i^E \geq v_i^M$. We assume that there are two types of advertisers—those who value exclusive placement more than non-exclusive placement (type D, with "D" indicating that they have "dual" valuations), and those who do not attach extra value to exclusive placement (type S, with "S" indicating that they have "single" valuations). We assume that v_i^M is drawn independently, uniformly and randomly from $[0, 1]$ for all bidders, and assume that $v_i^E = \alpha v_i^M$, where $\alpha > 1$, for D-bidders and $v_i^E = v_i^M$ for S-bidders. For simplicity, we assume that two advertisers (without loss of generality, advertisers 1 and 2) are of type S and only one advertiser (advertiser 3) is of type D. We assume that the value of α and the fact that only advertiser 3 is of type D are common knowledge. The utility of an advertiser at a position with CTR θ is defined as $\theta(v - p)$, where v is the value per click and p is the price paid per click.

3.1 The *GSP* Auction

Major search engines, such as Google, Yahoo! and Bing, use a Generalized Second Price (*GSP*) auction for allocation and pricing. Consider a keyword for which each advertiser i submits a bid b_i.[4] The search engine sorts the advertisers in descending order of their bids and allocates the first (highest, and most valuable) slot to the highest bidder and the second slot to the second-highest bidder, conditional on the bids being higher than the reserve price $r \geq 0$. A reserve price of r means that the search engine would rather leave the slot empty than to sell it at a price less than r. The payment rule is pay per click, and every bidder has to pay the minimum amount necessary to keep his position. The following is a formal definition.

Definition 1 (The *GSP* auction) *Suppose that the bids submitted by the advertisers are b_1, b_2 and b_3, and without loss of generality assume that $b_1 \geq b_2 \geq b_3$. Let r be the reserve price. Bidder i (for $1 \leq i \leq 2$), if $b_i \geq r$, is allocated to the i-th slot and pays* $\max(b_{i+1}, r)$ *per click. If $b_i < r$, the bidder is not allocated and pays nothing.*

The revenue of *GSP* is given by $\theta_1 \max(b_2, r) + \theta_2 \max(b_3, r)$ as long as $b_1 \geq b_2 \geq r$.

[4] Search engines often transform a bid b_i to an *effective bid* $\hat{b}_i = \gamma_i \times b_i$, where γ_i is a quality score which depends upon the past performance of the ad (how likely it is to generate clicks), the relevance of the ad to the keyword, the reputation of the advertiser, etc. The search engine only works with the effective bids rather than the original bids. The practice of transforming original bids into effective bids does not play an important role in the analysis presented in our paper and does not change the key insights. Therefore, for ease of understanding, we present our results without considering this transformation.

3.2 The GSP_{2D} Auction

The goal of this paper is to understand auctions which allow advertisers to bid for being shown exclusively on the results page. While search engines do not reveal the details of the exact mechanisms they use, we analyze mechanism GSP_{2D} which has been recently proposed and patented by Yahoo!.

In GSP_{2D}, each advertiser simultaneously submits two bids, b^M and b^E, where b^M indicates how much they are willing to pay per click if their ad is shown among other ads, and b^E indicates how much they are willing to pay per click if their ad is shown exclusively (as before, the superscripts M and E stand for multiple and exclusive, respectively). Similarly, the outcome of the auction can be either E or M, where M means that multiple ads are displayed and E means that only one ad is displayed. We call the non-exclusive bid of an advertiser, b^M, its M-bid, and the exclusive bid, b^E, its E-bid. If the outcome is M, we assume that two ads are shown and their click-through rates are θ_1 and θ_2. If the outcome is E, the click-through rate of the only slot shown is $\hat{\theta} = 1 \geq \theta_1$.

GSP_{2D} is an intuitive extension of GSP. If the outcome is M, it uses the same allocation and pricing rule as GSP. If the outcome is E, following the "second price" rule, the single slot is allocated to the bidder with the highest E-bid at price of the minimum amount he could have bid to win the E outcome. Finally, since the search engine's goal is to increase revenue, whether the outcome is M or E is decided by comparing the highest E-bid to the revenue term of outcome M. A formal definition of these rules is as follows.

Definition 2 (The GSP_{2D} auction) *Assume that bidders submit bids* (b_i^M, b_i^E)*, and without loss of generality assume that* $b_1^M \geq b_2^M \geq b_3^M$*. Since the ordering of* M*-bids might be different from that of* E*-bids, assume that* e_i *is the index of the* i*-th highest* E*-bid:* $b_{e_1}^E \geq b_{e_2}^E \geq b_{e_3}^E$*. The allocation and pricing under* GSP_{2D} *is:*

- *If* $b_{e_1}^E \geq \theta_1 \max(b_2^M, r) + \theta_2 \max(b_3^M, r)$ *the outcome is* E*. If* $b_{e_1}^E \geq r$*, the single slot in outcome* E *is allocated to bidder* e_1*. Bidder* e_1*'s per-click payment is* $\max(b_{e_2}^E, \theta_1 \max(b_2^M, r) + \theta_2 \max(b_3^M, r), r)$*. If* $b_{e_1}^E < r$*, no bidder wins and the revenue is zero.*

- *If* $b_{e_1}^E < \theta_1 \max(b_2^M, r) + \theta_2 \max(b_3^M, r)$*, the outcome is* M *and pricing and allocation are the same as those in* GSP*.*

3.3 Bayesian Equilibrium: Game of Incomplete Information

A key characteristic of our work is the asusmption of incomplete information about other players' valuations and the associated use of Bayesian equilibrium concept for solving the game. While the use of Bayesian equilibrium in game theory papers is very common, majority of analytical papers in the search advertising literature use ex-post Nash equilibrium (Edelman, Ostrovsky and Schwarz 2007, Varian 2007 and Katona and Sarvary 2010). The main argument for using the ex-post equilibrium concept is that, in the context of search advertising, the game is played repeatedly multiple times. Therefore, it is plausible to assume that players learn each others' values and strategies, and converge to a steady state bid profile over time (Edelman, Ostrovsky and Schwarz 2007). Furthermore, finding an ex-post equilibrium is usually easier analytically, especially in the context of search advertising due to existence of numerous ex-post equilibria and previous literature on ex-post equilibria. On the other hand, assumption of complete information and use of ex-post equilibrium concept has certain limitations. Below we compare certain relevant features of Bayesian equilibrium with uncertain valuations and ex-post Nash equilibrium with certain valuations.

Advertisers' Strategies: When using ex-post equilibrium, an advertiser's bid depends on the exact *realization* of other advertisers' bids and values. Therefore, one cannot discuss an advertiser's strategy without conditioning on other advertisers' bids and values. On the other hand, when using Bayesian equilibrium, an advertiser's strategy does not depend on the bids of the other advertisers; therefore, we can analyze the relation between an advertiser's strategy and the exogenous parameters of the auction, e.g., click-through rates of the slots.

Impact of Noise: There are hundreds of parameters involved when search engine decides how to rank the advertisers. Moreover, advertisers have budget constraints which are spread over thousands of keywords in hundreds of campaigns. Therefore, changes to demand and/or supply of one keyword can affect the budget used in the corresponding campaign and can in turn affect advertiser's bid for other keywords. In a Bayesian equilibrium, an advertiser knows other advertisers' bids only up to a distribution, which can potentially capture some of the noise in the system that is created by demand shocks and/or changes in other advertisers' strategies.

Endogenous Reserve Price: Conceptually, if we want to endogenize reserve price in GSP, we have to use Bayesian equilibrium. When using ex-post equilibrium, revenue is maximized if the reserve price is set just below the valuation for each bidder. The trade-off in setting reserve price is that low reserve price leaves more bidders in game but extracts lower expected revenue from each, while high reserve price allows fewer bidders to participate but extracts more revenue from each. This trade-off can be modeled only if there is uncertainty in bidders' valuations.

Multiplicity of Equilibria: GSP has a large number of ex-post equilibria (Varian 2007). As a result, analytical studies of GSP that use ex-post equilibrium concept have to either rely on ad-hoc refinements (Amaldoss, Desai and Shin 2012, Katona and Zhu 2011, Desai, Shin and Staelin 2010) or derive upper and lower bounds on equilibrium revenue (Varian 2007) or both (Ghosh and Sayedi (2010), Katona and Sarvary (2010)). On the other hand, we show that in our model, GSP has a unique symmetric Bayesian Nash equilibrium. In the case of GSP_{2D}, we find that the results are even more dependent on equilibrium selection criteria—one ex-post equilibrium of GSP_{2D} always generates less revenue than in GSP while another ex-post equilibrium always generates more revenue—making the comparison of the two mechanisms only about equilibrium selection.

Our work is not the first to use Bayesian equilibrium for analyzing GSP. Edelman and Schwarz (2010), Roughgarden and Sundarajan (2007), Lahaie (2006), Chen, Liu and Whinston (2009), Liu, Chen and Whinston (2010) and Paes Leme and Tardos (2010) all use Bayesian Nash equilibrium to study different aspects of GSP. In particular, Edelman and Schwarz (2010) use Bayesian equilibrium to find optimum reserve price of GSP. However, to best of our knowledge, no other paper analyzes *advertisers' strategies* in GSP in Bayesian setting. Since the main motivation of all previous work in this context is to analyze welfare and revenue, they can avoid calculating the equilibrium by using "revelation principle," a common framework for analyzing welfare and revenue without explicitly calculating equilibrium.[5] Calculating the equilibrium, however, is particularly important from a Marketing point of view because it gives us insight into advertisers' bidding strategies. Using Bayesian equilibrium, we provide new insights into how advertisers' strategies are affected by changes in click-through rates and reserve price of GSP and GSP_{2D}.

[5] Revelation principle can be briefly stated as: "For any Bayesian Nash equilibrium of a game of incomplete information, there exists a payoff-equivalent revelation mechanism that has an equilibrium where the players truthfully report their types."

4 Analysis and Results

We start with analysis of GSP, providing new insights on how advertisers bid in GSP in equilibrium. Then, we present corresponding results for GSP_{2D}, highlight the differences and compare the two mechanisms in terms of search engine's revenue and advertisers' strategies.

4.1 Equilibrium Analysis of *GSP*

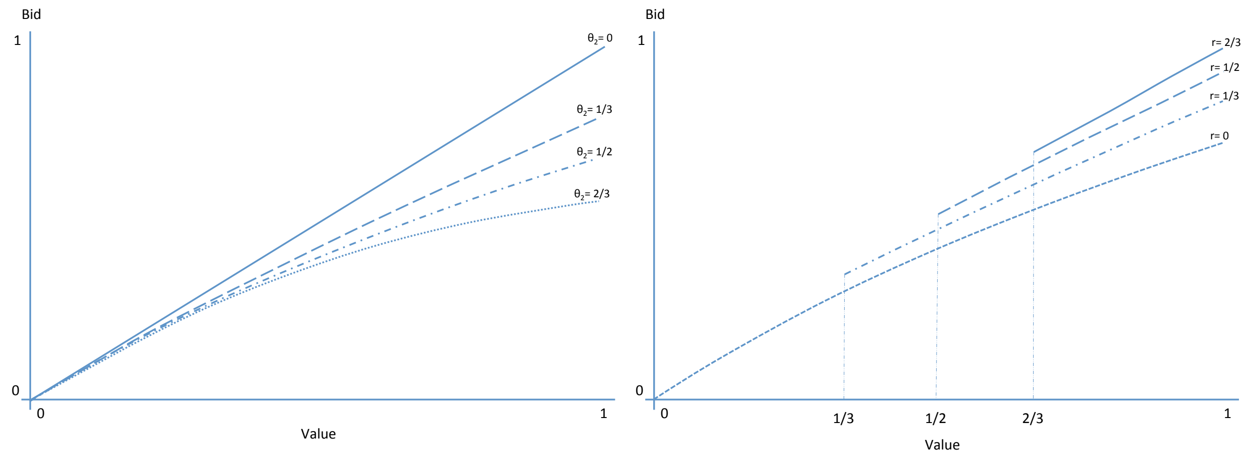
In this section, we calculate the unique symmetric Nash equilibrium of GSP. Note that, in this analysis, v_i^E is irrelevant. For given θ_1, θ_2 and reserve price r, assume that $f(v)$ is the equilibrium bid of a bidder with valuation v. Using the Monotonic Selection Theorem by Milgrom and Shannon (1994), Lahaie (2006) shows that in symmetric Bayesian Nash equilibrium of GSP, bid is an increasing function of value. In other words, we can assume that $f(v)$ is increasing in v. Using the definition of equilibrium, if we assume other bidders bid according to $f(.)$, expected utility of a bidder with valuation v must be maximized when bidding $f(v)$. The expected utility of a bidder with valuation v who bids $f(v')$ is

$$(1-v')^2 \times 0 + 2v'(1-v') \times Pr\left(v' \geq r | v' \text{is second}\right) \theta_2 \left(v - E[\max(f(x), r) | x < v']\right)$$
$$+ v'^2 \times Pr\left(v' \geq r | v' \text{is first}\right) \theta_1 \left(v - E[\max(f(x_1), f(x_2), r) | x_1, x_2 < v']\right).$$

The first term corresponds to the case where bid $f(v')$ is placed third, and therefore the bidder wins nothing. Since bids are increasing in valuation and values come from uniform distribution, probability of this case is $(1-v')^2$. The second term correspond to the case where bid $f(v')$ is the second-highest bid. Note that a bidder bids greater than or equal to reserve price r if and only if his valuation is at least r. Therefore, we can replace $Pr(f(v') \geq r)$ with $Pr(v' \geq r)$. The third term corresponds to the case where bid $f(v')$ is the highest bid, and therefore the bidder gets the first slot.

After calculating the conditional expectations and probabilities, the expected utility simplifies to

$$2v'(1-v')(1-3r^2+2r^3)\theta_2 \left(v - r^2/v' - \int_r^{v'} f(x)dx/v'\right)$$
$$+ v'^2(1-r^3)\theta_1 \left(v - (r^3 + \int_r^{v'} 2f(x)xdx)/v'^2\right).$$



(a) Bid functions for different values of θ_2 when $r = 0$ and $\theta_1 = 1$.

(b) Bid functions for different values of r when $\theta_1 = 1$ and $\theta_2 = 1/2$.

Figure 1: Equilibrium bids of GSP as function of valuation when $\theta_1 = 1$, for different values of θ_2 and r.

Since this expression must be maximized at $v' = v$, we take the derivative with respect to v', let $v' = v$, and equate the resulting expression to zero. Therefore, we have

$$\begin{aligned} &-(1-3r^2+2r^3)\theta_2\left(2v^2-2r^2-2\int_r^v f(x)dx\right)+(1-v)(1-3r^2+2r^3)\theta_2\left(2v-2f(v)\right) \\ &\quad+(1-r^3)\theta_1\left(2v^2-2vf(v)\right)=0. \end{aligned} \tag{1}$$

The closed form solution to this differential equation is in Section A1 in the appendix. We use the notation $b_{1D}(v)$ to denote this equilibrium bid, and state this result as a proposition.

Proposition 1 *In GSP, the equilibrium bid of a type S advertiser, denoted by $b_{1D}(v)$, is as given in Equation (A1) in Section A1 in the appendix.*

Figure 1(a) shows the bid for different values of the ratio θ_2/θ_1, which can be interpreted as slot CTR similarity, and Figure 1(b) shows the solution for different values of reserve price r.

We now discuss various results on advertisers' bidding strategies and the search engine's revenue in GSP. We state these results, which can be derived using the result in Proposition 1, in the following corollary. Please see Section A2 in the appendix for a sketch of the proof of the corollary.

Corollary 2

(a) For given v, r and θ_1, the bid $b_{1D}(v)$ is always decreasing in θ_2.

(b) The shading measure $\frac{v-b_{1D}(v)}{v}$ *increases with* v *in* GSP.

(c) The bid $b_{1D}(v)$ *increases with* r *for any* $v \geq r$*. Moreover,* $\Delta_r b(v)$*, which denotes the change in the equilibrium bid of a bidder with valuation* v *when changing the reserve price from* 0 *to* r*, is an increasing function of* v*. In other words, the effect of reserve price on bids of high-value bidders is larger than on bids of low-value bidders.*

(d) For given r *and* θ_1*, search engine's revenue in* GSP *increases with* θ_2.

Before proceeding, we note that advertisers' strategies in GSP do not change with changes in θ_1 and θ_2 as long as θ_2/θ_1 remains unchanged. For example, advertisers' strategies when $\theta_1 = 1$ and $\theta_2 = 1/2$ are the same as when $\theta_1 = 0.6$ and $\theta_2 = 0.3$.

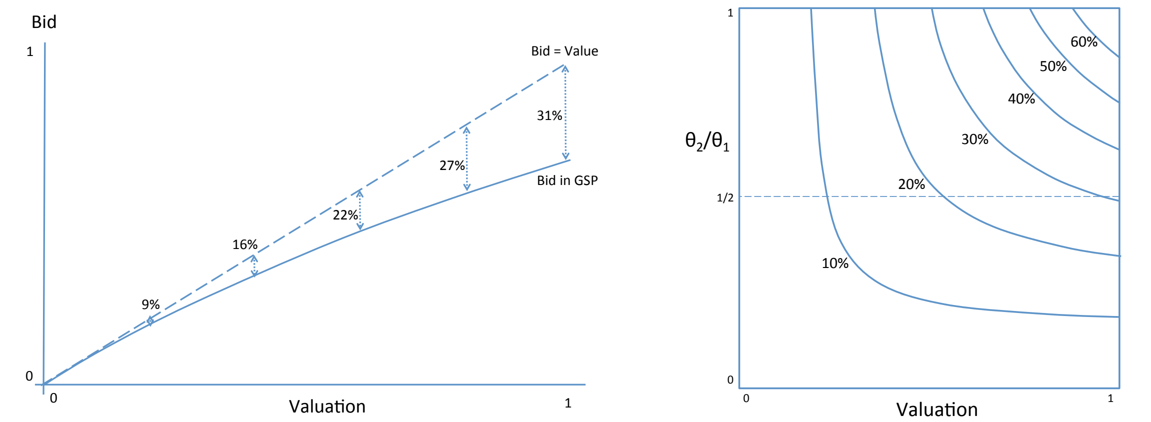
Parts (a), (b) and (c) of Corollary 2 are related to the shading behavior of bidders (i.e., bidding below valuation) in GSP. Previous papers on sponsored search advertising have already shown that bidders always bid less than or equal to their valuation in GSP (Edelman, Ostrovsky and Schwarz 2007). Our model also confirms the fact that bidders "shade" their bids in GSP. Furthermore, we take a step forward in better understanding the shading behavior of the bidders.

Corollary 2(a) shows that as the two slots become more similar in terms of click-through rate, bidders decrease their bids. This is intuitively because bidders have less incentive to win the first slot. In other words, they prefer to win the second slot at a lower payment as the two slots become more similar.[6]

Corollary 2(b) shows that bidders with higher valuation, even percentage-wise, shade their bids more than bidders with lower valuation. Intuitively, this happens because low-value bidders cannot afford to shade too much because they may lose to both other bidders. However, as the valuation increases, the bidder knows that the probability of not winning anything is low. Therefore, the bidder compares only the first and the second slot when deciding how much to shade. In other words, high-value bidders have lower chance of not winning anything and, therefore, can afford to shade their bids more than low-value bidders.[7] Figure 2(a) shows the percentage-wise shading in equilibrium bids when reserve price $r = 0$ and $\theta_2/\theta_1 = 1/2$. Figure 2(b) shows contours representing the shading percentage in equilibrium bids as function of valuation and θ_2/θ_1 when $r = 0$. The

[6]Figure 6(a) shows that the GSP bid of a bidder with valuation 1/3 decreases as θ_2/θ_1 increases.

[7]If there are only two bidders (still with two slots) in GSP, every bidder gets positive allocation (at least θ_2). In this situation, we still see that bidders shade their bids, but percentage-wise shading is the same for high- and low-value bidders.



(a) Percentage of shading of bids for $v = 0.2, 0.4, 0.6, 0.8$ and 1 when $r = 0$, $\theta_1 = 1$ and $\theta_2 = 1/2$.

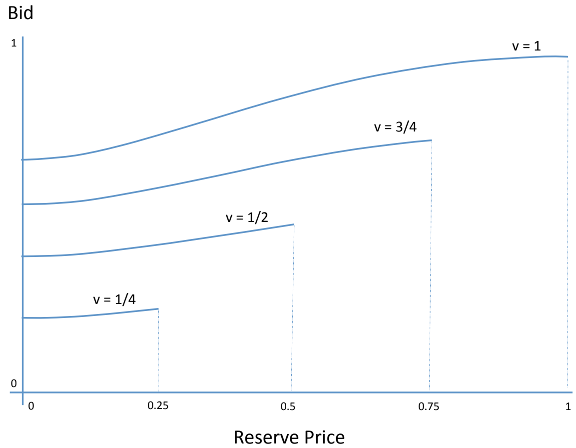
(b) Contours representing shading percentage when $r = 0$.

Figure 2: In GSP, bidders with higher valuations shade their bids more.

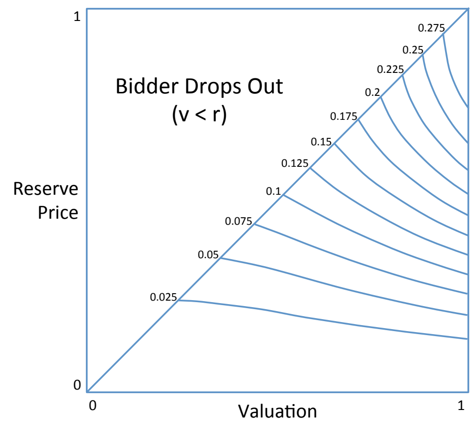
dashed line in Figure 2(b) corresponds to the percentages in Figure 2(a).

Corollary 2(c) shows an interesting effect of reserve price in GSP bids which we call *cascading*. Note that the direct effect of reserve price is only on low-value bidders who have to bid high enough to meet the reserve price. But the increase in low-value bidders' bids *cascades* and causes the increase in higher-value bidders' bids. It is even more interesting to see that the increase in bids of high-value bidders, who are only affected through cascading, is larger than the increase in bids of low-value bidders who are directly affected by reserve price.

Edelman and Schwarz (2010) also observe the direct and indirect effects of reserve price on GSP bids. They analyze the effect in lowest-revenue ex-post equilibrium of GSP. In their model, increasing the reserve price increases every bidder's total payment by the same amount. However, bids of high-value bidders increase less than bids of low-value bidders. Higher click-through rates in higher slots balances off the smaller increase in per-click payment, and in the end, the increment in every bidder's total payment is the same. Since low-value bidders may drop out of the auction by the increase in reserve price, they argue that, in expectation, high value bidders contribute more to the incremental revenue than low-value bidders. Our findings in a game of incomplete information are partially similar to and partially different from the results in Edelman and Schwarz (2010). We also show that most of the incremental revenue by setting an optimal reserve price is generated by high-value bidders. However, in our model, bids of high-value bidders increase more than bids



(a) Bids as function of reserve price when $\theta_2/\theta_1 = 1/2$ and for four valuations $v = 1/4, 1/2, 3/4$ and 1. The curve corresponding to value v is drawn only for the part that $r \leq v$.



(b) Contours representing $\Delta_r b(v)$ as function of r and v assuming $\theta_2/\theta_1 = 1/2$.

Figure 3: Reserve price affects the bids of high-value bidders more than low-value bidders.

of low-value bidders, and as a result, even per-click payments of high-value bidders increase more than low-value bidders.

Figure 3 shows the effect of reserve price on advertisers' bids for different levels of valuation. As it can be seen in Figure 3(a), even for high-value bidders who are not directly affected by reserve price, increasing the reserve price indirectly increases the bids. Figure 3(b) shows how much the bid of a bidder with specific valuation and reserve price increases compared to the case when there is no reserve price ($\Delta_r b(v)$). Note that for the same level of reserve price, the increase in the bid of high-value bidders is more than the increase in the bid of low-value bidders.

According to Corollary 2(a), advertisers decrease their bids in GSP as θ_2 increases. On the other hand, total number of clicks increase as θ_2 increases. Therefore, it is interesting to know whether search engine's revenue increases or decreases with θ_2. Corollary 2(d) states that the search engine's revenue in GSP is an increasing function of θ_2.

4.2 Equilibrium Analysis of GSP_{2D}

In this section, we calculate Bayesian Nash equilibria of GSP_{2D}. Interestingly, we find two distinct classes of equilibria which we call All-Exclusive equilibrium and Differentiating equilibrium. In All-Exclusive equilibrium, all bidders bid only for exclusive outcome. This is a counter-intuitive and interesting consequence of introducing exclusivity. Intuitively, bidders of type S can win non-exclusive outcome only if they can *coordinate* and both bid for non-exclusivity. In other words, for any given bidder, if no other bidder bids for non-exclusive outcome, bidding for non-exclusivity is dominated. In the following, we analyze equilibrium bids in each of the two classes of equilibria.

In Differentiating equilibrium, bidders of type D (those who have extra value for exclusivity) bid only for exclusive outcome and bidders of type S bid only for non-exclusive outcome. Intuitively, bidders of type D do not bid for non-exclusive outcome because it may increase their own price for exclusivity, and bidders of type S do not bid for exclusive outcome because, given the equilibrium structure, they know that if they cannot win the non-exclusive outcome they cannot win the exclusive outcome either. We formally discuss the conditions necessary for existence of Differentiating equilibrium in Section A3 in the appendix.

All-Exclusive Equilibrium

The following proposition shows that All-Exclusive equilibrium always exists, in which all bidders bid truthfully for the exclusive outcome. To emphasize the robustness of this result, we provide a proof that applies to a more general framework. In fact, the given proof is independent of the number of bidders, the number of slots and the underlying distribution of valuations. The only important assumption for this proof to hold is that $\hat{\theta} \geq \theta_1$, i.e., the click-through rate of the slot in exclusive outcome is at least equal to the click-through rate of the highest slot in non-exclusive outcome.

Proposition 3 *All bidders bidding truthfully and only for exclusive outcome is an equilibrium.*

Proof: Consider bidder i and assume that every other bidder bids only for exclusivity. Assume (to prove by contradiction) that bidder i benefits from bidding for non-exclusivity. Bidding for non-exclusivity could be beneficial only if bidder i wins non-exclusive outcome with positive probability. Consider a profile in which bidder i wins non-exclusive outcome. Using definition of GSP_{2D}, this means that $\theta_1 r$ is larger than every other bidders' bid for exclusivity. Bidder i's utility is $\theta_1(v_i - r)$. In the same profile, bidder i could also win exclusive outcome with utility at least $v_i - r$. Since $\theta_1 \leq 1$, bidder i could not be benefitting from bidding for non-exclusivity. On the other hand, submitting a non-zero bid for non-exclusive outcome, while also bidding for exclusivity, could increase bidder i's own payment if he wins the exclusive outcome. Bidding for non-exclusivity is, therefore, weakly dominated if every other bidder bids only for exclusivity. Therefore, all bidders bidding only for exclusivity is an equilibrium. □

Next, we analyze the revenue of GSP_{2D} in All-Exclusive equilibrium. For this, note that we use the endogenously maximized reserve price r^* for each set of values of the exogenous parameters. The following proposition characterizes the revenue. Please see Section A2 in the appendix for a sketch of the proof.

Proposition 4 *In the All-Exclusive equilibrium of GSP_{2D}, if $\alpha > 2.70$, then search engine's revenue is $\frac{\alpha}{4}$, maximized at $r^* = \frac{\alpha}{2}$; if $\alpha \leq 2.70$, then search engine's revenue is $\frac{-416+1760\alpha+24\alpha^2+8\alpha^3+\alpha^4}{2592\alpha}$, maximized at $r = \frac{2+\alpha}{6}$.*

Differentiating Equilibrium

We use the same method as in Section 4.1 to calculate symmetric Bayesian Nash equilibria of GSP_{2D}. We make two assumptions before calculating the equilibrium. First, as in Section 4.1, we assume that the equilibrium bid is an increasing function of valuation. Second, we assume that bidders of type D only bid for exclusive outcome and bidders of type S only bid for non-exclusive outcome. We calculate the equilibrium assuming that these assumptions hold. In Section A3 in the appendix, we discuss the conditions needed for them to hold and calculate the region in which Differentiating equilibrium exists.

We start by analyzing D bidders' bidding strategy. The following proposition shows that that D-bidders bid truthfully in Differentiating equilibrium.

Proposition 5 *If a bidder only bids for exclusive outcome, bidding truthfully is a weakly dominant strategy.*

Proof: The proof follows from truthfulness of second price auction. □

Given that bidder 3, the only D-bidder in our model, bids truthfully, we calculate S-bidders bidding strategy. Assume that an S-bidder with valuation v bids $f(v)$ in equilibrium. Our goal is to calculate the bidding function $f(.)$. Expected utility of an S-bidder with valuation v who bids $f(v')$, assuming $f(v') \geq r$, is

$$\int_0^r \frac{(\theta_1 r + \theta_2 r)}{\alpha} \theta_1 (v - r) dx + \int_r^{v'} \frac{(\theta_1 f(x) + \theta_2 r)}{\alpha} \theta_1 (v - f(x)) dx + \int_{v'}^1 \frac{(\theta_1 f(v') + \theta_2 r)}{\alpha} \theta_2 (v - r) dx.$$

The first two terms in the above expression correspond to the case where the highest bid is $f(v')$ and the second highest bid is less than r (in the first term) and more than r (in the second term). The third term corresponds to the case where $f(v')$ is the second highest bid. Note that we have to assume that $\alpha \geq \theta_1 + \theta_2 r$ to make sure that the support of D-bidder's valuation distribution ($[0, \alpha]$) always covers $\theta_1 f(x) + \theta_2 r$, for any $0 \leq x \leq 1$. In other words, exclusive outcome should always happen with positive probability.[8]

After taking derivative with respect to v' and letting $v' = v$ we get the following differential equation

$$(\theta_1 f(v) + \theta_2 r)\,\theta_1(v - f(v)) + \theta_2(v - r)\big((1 - v)\theta_1 f'(v) - (\theta_1 f(v) + \theta_2 r)\big) = 0. \tag{2}$$

The closed form solution to this differential equation is in Section A1 in the appendix.

Proposition 6 *In the Differentiating equilibrium of GSP_{2D}, the equilibrium bid of an advertiser, denoted by $b_{2D}(v)$, is as given in Equation (A2) in Section A1 in the appendix.*

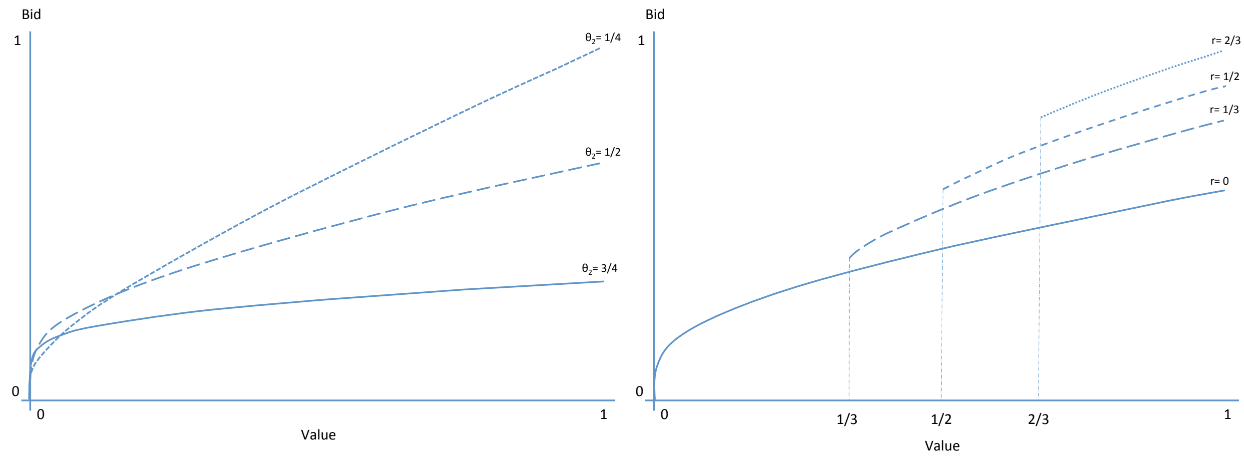
Figure 4(a) shows the bid for different values of θ_2, and Figure 4(b) shows the solution for different values of reserve price r.

We now present results on bidding strategies in GSP_{2D}, and compare the bidding strategies to those in GSP. We state these results, which can be derived using the result in Proposition 6, in the corollary that follows. We focus on the Differentiating equilibrium bids of the advertisers because there are interesting dynamics here; as Proposition 3 shows, in the All-Exclusive equilibrium, all advertisers simply bid truthfully for the exclusive outcome. We also note that the S-bidders' bids for non-exclusivity in GSP_{2D}, as long as Differentiating equilibrium exists, do not change with changes in θ_1 and θ_2 as long as θ_2/θ_1 remains unchanged. Please see Section A2 in the appendix for a sketch of the proof of the corollary.

Corollary 7

(a) For given r, θ_1 and θ_2, there exists $\nu \in [0, 1]$ such that bid $b_{2D}(v) > v$ if and only if $v < \nu$. In other words, bidders with low valuation may bid more than their valuation in equilibrium.

[8] Without this assumption, $(\theta_1 f(x) + \theta_2 r)$ and $(\theta_1 f(v') + \theta_2 r)$ in the expression should be replaced with $\min((\theta_1 f(x) + \theta_2 r), 1)$ and $\min((\theta_1 f(v') + \theta_2 r), 1)$, respectively. The derivatives and solutions will be piecewise functions, but our results hold as long as $\alpha \geq 1$.



(a) Bid functions for different values of θ_2 when $r = 0$ and $\theta_1 = 1$.

(b) Bid functions for different values of r when $\theta_1 = 1$ and $\theta_2 = 1/2$.

Figure 4: Equilibrium bids of GSP_{2D} as function of valuation when $\theta_1 = 1$, for different values of θ_2 and r.

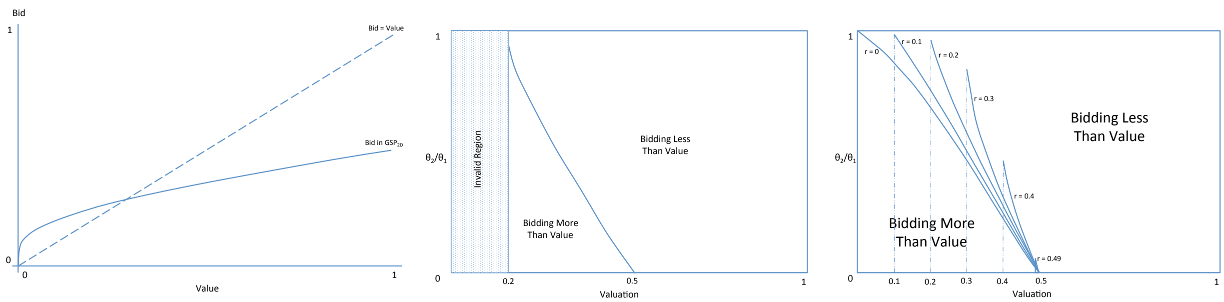
(b) For given v, r and θ_1, there exists $\tau \in [0, 1]$ such that bid $b_{2D}(v)$ increases with θ_2 if and only if $\theta_2 < \tau$.

(c) The shading measure $\frac{v - b_{2D}(v)}{v}$ increase with v in GSP_{2D}.

(d) For given r, θ_1 and θ_2, there exists $\hat{\nu} \in [0, 1]$ such that $b_{2D}(v) < b_{1D}(v)$ if and only if $v > \hat{\nu}$. In other words, high-value bidders shade their bids more in GSP_{2D} than in GSP.

Corollary 7(a) shows that, interestingly, in GSP_{2D}, bidders may benefit from bidding *more* than their valuation. There are two forces involved when bidders decide how much to bid in GSP_{2D}. First, bidders do not want to risk a high payment, and therefore have the tendency to bid lower and get a lower slot. Note that this force becomes stronger as θ_2/θ_1 increases. In other words, as the two slots become more similar, bidders are more willing to lose the first slot if it leads to lower payment. This force also exists in GSP and is the main deriver of Corollary 2(a). In GSP_{2D}, there is also a second force which acts in the opposite direction. Since S-bidders want the outcome to be M, they have to bid high enough so that the non-exclusive outcome materializes. Note that this force affects low-value bidders more because low-value bidders are *price-setters*, and by bidding higher they can increase search engine's revenue in non-exclusive outcome, and consequently make the non-exclusive outcome more likely. Therefore, if v is low and θ_2/θ_1 is low, the second force becomes dominant and causes the bidders to bid even more than their valuation in equilibrium.

The interaction of these two forces can be observed in Figure 5. Figure 5(a) shows the equilib-



(a) Equilibrium bid in GSP_{2D} compared to truthful bidding when $r = 0$, $\theta_1 = 1$ and $\theta_2 = 1/2$.

(b) The region in which bidders bid more than their valuation when $r = 0.2$.

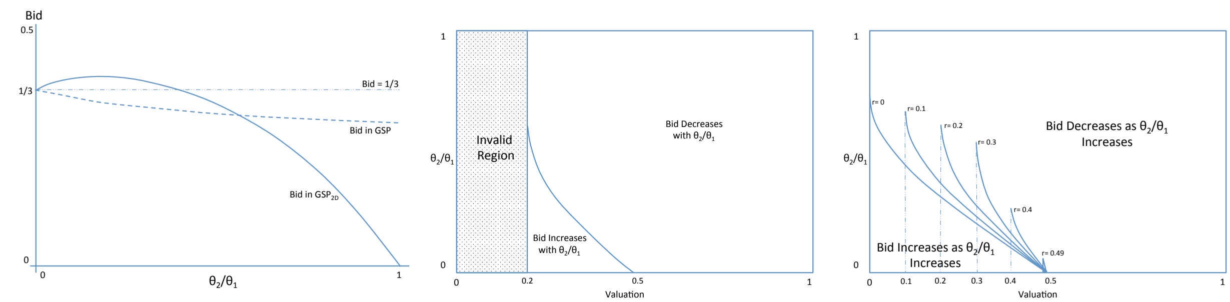
(c) The regions in which bidders bid more than their valuation for different values of r.

Figure 5: Bidders may bid more than their valuation in GSP_{2D}.

rium bids of GSP_{2D} when $\theta_2/\theta_1 = 1/2$ and reserve price $r = 0$. As discussed above, the second force is the dominant force for bidders with low valuation. As a result, we see that bidders with low valuation bid more than their valuation in equilibrium. Figure 5(b) show the region in which bidders bid more than their valuation in equilibrium when reserve price $r = 0.2$. As discussed above, we see that bidding over value happens when θ_2/θ_1 and v are both small enough. Note that since $r = 0.2$, the region in which $v < 0.2$ is invalid in this discussion as the bidders in that region do not win anything in equilibrium and may bid anything below r. Finally, Figure 5(c) shows the same graph of (b) but for several values of r. The graph shows that bidding more than valuation may only happen if $v < 1/2$ and $r < 1/2$ because the corresponding region shrinks to zero as either v or r approach $1/2$.

Corollary 7(b) shows that bids may *increase* with θ_2 for low values of θ_2/θ_1 and v. The interaction of the same two forces as in the discussion of Corollary 7(a) can also explain the intuition behind Corollary 7(b). When θ_2/θ_1 and v are small enough, the second force (S-bidders trying to obtain the M outcome) dominates. As a result, low-value bidders increase their bids with θ_2 to increase the probability of non-exclusive outcome. However, when v is large or θ_2/θ_1 is large, the first force dominates and, therefore, bidders shade their bids more as θ_2 increases.

Figure 6(a) shows equilibrium bids in GSP_{2D} and GSP as a function of θ_2. When $\theta_2 = 0$, the bidders bid equal to their valuation in both mechanisms. As θ_2 increases, the bid in GSP_{2D} increases while the bid in GSP decreases. However, as θ_2 becomes larger, GSP_{2D} bid also starts to decline. It is interesting to see that GSP_{2D} bids decline faster and eventually become lower than GSP bids. The main reason for this is that there is less competition in non-exclusive dimension



(a) Equilibrium bids, as a function of θ_2/θ_1, in GSP_{2D} and GSP for a bidder with valuation $1/3$ when $r = 0$.

(b) The region in which bid increases as a function of θ_2/θ_1, assuming $r = 0.2$.

(c) The regions in which bid increases as a function of θ_2/θ_1, for different values of r.

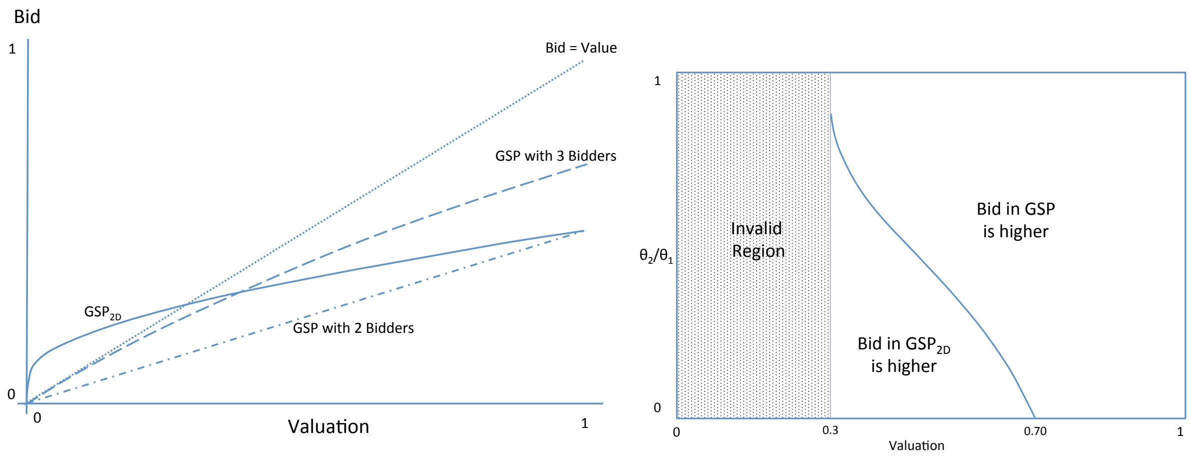
Figure 6: Bid is a non-monotone function of θ_2/θ_1 in GSP_{2D}.

of GSP_{2D}, with only two bidders, than in GSP with three bidders. We explain this effect in more details when discussing Corollary 7(d).

Figure 6(b) shows the region in which bids increase with θ_2 as a function of valuation and θ_2/θ_1, assuming that reserve price $r = 0.2$. As before, the region in which valuation is less than reserve price is invalid in this discussion. Finally, Figure 6(c) shows the same region as in Figure 6(b) for different values of reserve price r. As can be seen, the region shrinks as r increases, and converges to zero as r approaches $1/2$.

Corollary 7(c) shows that shading increases, even percentage-wise, as valuation increases. This is consistent with the corresponding result in Corollary 2(b) of GSP, the intuition is however slightly different. Remember that if there are two bidders in GSP, shading percentage will be the same for all (low-value and high-value) bidders. In GSP_{2D}, however, although there are two S-bidders interested in non-exclusive outcome, shading percentage is more for high-value bidders. The reason is that low-value bidders shade their bids less to increase the probability of non-exclusive outcome. High-value bidders, however, do not have that incentive, and therefore, shade their bids more.

Corollary 7(d) shows that high-value bidders shade their bid more in GSP_{2D} than in GSP. The main driver for this result is that there is *less* competition in the non-exclusive dimension of GSP_{2D} than in GSP. Since there are only two bidders interested in the non-exclusive outcome in GSP_{2D}, they know that as long as the non-exclusive outcome is obtained, they get at least the second slot. Therefore, they do not need to bid as aggressively as they do in GSP with three bidders. Note that high-value bidders are not price-setters and therefore cannot affect the search



(a) Bids as function of value in GSP_{2D}, GSP with two bidders and GSP with three bidders, assuming $\theta_2/\theta_1 = 1/2$ and $r = 0$.

(b) Comparing bids in GSP_{2D} and GSP, assuming $r = 0.3$.

Figure 7: Comparing bids in GSP_{2D} and GSP.

engine's decision regarding the exclusivity/non-exclusivity of the outcome.

To better illustrate the effect of less competition in non-exclusive outcome of GSP_{2D}, we also compare the revenue of GSP_{2D} to GSP with two bidders (instead of three). Figure 7(a) shows bids in GSP_{2D} and two versions of GSP, one with two bidders and one with three bidders. As stated above, bids in GSP_{2D} are higher than in GSP with three bidders for low-value bidders, and lower for high-value bidders. On the other hand, bids in GSP_{2D} are always greater than or equal to bids in GSP with two bidders. This is because GSP_{2D} and GSP with two bidder both have the same level of competition, however, bidders in GSP_{2D} also want to bid high to increase the probability of non-exclusive outcome. The lower the valuation is, the higher is the probability of being price-setter, and the larger is the difference in bids of GSP_{2D} and GSP with two bidders. As the valuation increases, the difference diminishes, and in fact, when the value is 1, we see that the bid in GSP_{2D} is exactly the same as the bid in GSP with two bidders. This is because a bidder with valuation 1 in GSP_{2D} has no chance of being price-setter, and therefore has no chance of increasing the probability of non-exclusive outcome. The bid, therefore, is exactly the same as the bid in GSP with two bidders. Figure 7(b) shows the region in which bids in GSP_{2D} are higher than bids in GSP, assuming the reserve price is $r = 0.3$. The pattern is the same for other values of r.

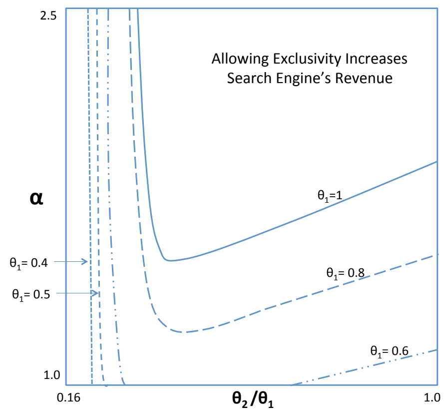
4.3 Comparison of search engine revenue from GSP and GSP_{2D}

In this section, we compare when the two equilibria of GSP_{2D} give higher or lower revenue than GSP. To do this, given bidders' bidding strategies, we calculate search engine's revenues in each of the equilibria of GSP_{2D} and GSP. In this calculation, we use the endogenously maximized reserve price for each set of values of the exogenous parameters. The details regarding the optimum reserve price are available in Section A4 in the appendix.

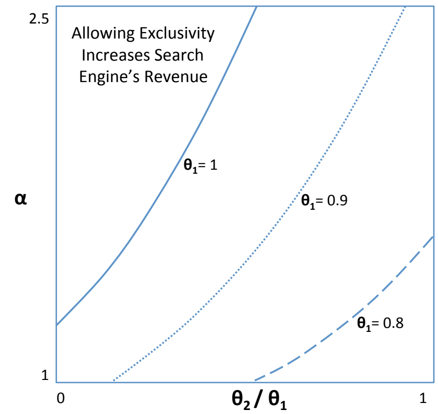
First, we consider the Differentiating equilibrium of GSP_{2D}. Figure 8(a) shows the region in which search engine's revenue in Differentiating equilibrium of GSP_{2D} is more than the revenue in GSP. Note that we have conducted a numerical analysis to plot Figure 8(a).[9] Given a fixed value of θ_1, the profits in both auctions depend only on the ratio θ_2/θ_1, which lies between 0 and 1, and the value of α, which is always > 1. Figure 8(a) represents all possible values of θ_2/θ_1 and large enough values of α (for $\alpha > 2.5$, the nature of the plot remains the same), and therefore gives a complete characterization of the results. We see from Figure 8(a) that the revenue of the Differentiating equilibrium of GSP_{2D} dominates the revenue of GSP as α increases for any fixed value of θ_2/θ_1. This is an intuitive result because, as α increases, the D-bidder's expected value for the exclusive outcome increases. Consequently, using the optimum reserve price, the search engine can extract more revenue from the D-bidder and GSP_{2D} outperforms GSP.

Next, notice that for the same level of α and θ_1, GSP can generate more revenue than GSP_{2D} under two conditions: first, if θ_2/θ_1 is large enough, and second, if θ_2/θ_1 is small enough. The reasoning behind this result is as follows. As θ_2/θ_1 increases, the two slots become more similar. As a result, bidders start to shade their bids. But according to Corollary 7(d), bidders shade their bids more in GSP_{2D} than in GSP. Therefore, although the revenue grows in both auctions as θ_2 increases, it grows more slowly in GSP_{2D} than in GSP. As a result, search engine's revenue in GSP may become larger than that in Differentiating equilibrium of GSP_{2D} when θ_2/θ_1 becomes large enough. Regarding the reason why the revenue of the Differentiating equilibrium of GSP_{2D} is lower than the revenue of GSP when θ_2/θ_1 is small, this is essentially because of the condition imposed by the existence of Differentiating equilibrium. When θ_2/θ_1 is small, Differentiating equilibrium exists only if reserve price r is small. Therefore, reserve price r has to be set sub-optimally low

[9] Our numerical analysis involves numerical integration to calculate search engine's revenue and to find optimum reserve price.



(a) The region above each curve is where search engine's revenue in Differentiating equilibrium of GSP_{2D} is more than the revenue in GSP. Each curve represents a given value of θ_1.



(b) The region above each curve is where search engine's revenue in All-Exclusive equilibrium of GSP_{2D} is more than the revenue in GSP. Each curve represents a given value of θ_1.

Figure 8: Comparing the revenue of GSP_{2D} and GSP.

for Differentiating equilibrium to exist. Consequently, search engine's revenue in Differentiating equilibrium of GSP_{2D} may become lower than the revenue of GSP due to sub-optimality of reserve price when θ_2/θ_1 is small. Note that when θ_2/θ_1 is smaller than ≈ 0.16, Differentiating equilibrium does not exist for *any* value of r.

Next, we consider the revenue of the All-Exclusive equilibrium of GSP_{2D} (derived in Proposition 4), and compare its revenue with that of GSP. Figure 8(b) shows the region in the parameter space in which All-Exclusive equilibrium of GSP_{2D} has higher revenue than GSP. The figure shows that the revenue of GSP_{2D} is larger than the revenue of GSP if α is large enough for fixed θ_2/θ_1, and if θ_2/θ_1 is small enough for fixed α. Both results are expected and are based on the insights discussed previously.

To highlight the above results, we state them briefly in the following proposition.

Proposition 8 *The Differentiating equilibrium of GSP_{2D} generates more expected revenue than GSP for the search engine as shown in Figure 8(a), and the All-Exclusive equilibrium of GSP_{2D} generates more expected revenue than GSP for the search engine as shown in Figure 8(b).*

5 Application to Rich Ads in Search

Rich Ads in Search (RAIS) is a service in Yahoo and Microsoft search advertising that allows the advertisers to be placed exclusively and prominently on top of search results page. Using RAIS, advertisers can include videos, images and multiple links in their ads. Furthermore, their ads are displayed exclusively on top of search results page—displacing all other ads to the right section.[10] Figure 9 shows an example of RAIS in Yahoo search results page. In this example, Staples is being displayed exclusively on top for keyword "`staples`" while other bidders like Walmart and Amazon are displayed in the right section.

RAIS was introduced by Yahoo in 2009 and was initially offered to a small subset of advertisers.[11] The service was suspended shortly after Yahoo search advertising was transferred to Microsoft. In 2011, the service became available again in both Yahoo and Microsoft. As of now, RAIS is available to all advertisers with a premium account with Yahoo and Microsoft search advertising. While the details of the auction used are not publicly available, advertisers who participate submit two bids, one bid for the usual multiple display format and a second bid, called RAIS bid, to display their RAIS ad exclusively on top. Yahoo suggests that RAIS bids should be at least 50% higher than standard non-exclusive bids, and marketing agencies suggest that the second bid should be two to three times higher.[12] RAIS is still in its beta stage trial version and, currently, only brand owners can bid for RAIS ads and only on their trademarked keywords.

The model described in Section 3 captures certain features of RAIS. Advertisers of type D have extra value for showing images or videos in their ads or for being displayed exclusively on top (the "North" area of the page). Similar to our model in Section 3, RAIS also allocates the top section of search results page exclusively. However, in RAIS, some ads may still be placed on the right-hand side of the page (the "East" area of the page). In other words, only the top section may be sold exclusively. This slight difference might potentially affect advertisers' strategies as well search engine's revenue. In this section, we show that this is not the case, and our results for GSP_{2D} in Section 4 do not qualitatively change even if some ads are displayed in the right section in addition to the exclusive ad at the top of the page.

[10] See `http://advertising.yahoo.com/article/rich-ads-in-search.html` for more details.
[11] See `http://www.rimmkaufman.com/blog/tag/rich-ads-in-search/` for more details.
[12] `http://www.location3.com/yahoo-bing-rich-ads-in-search/`

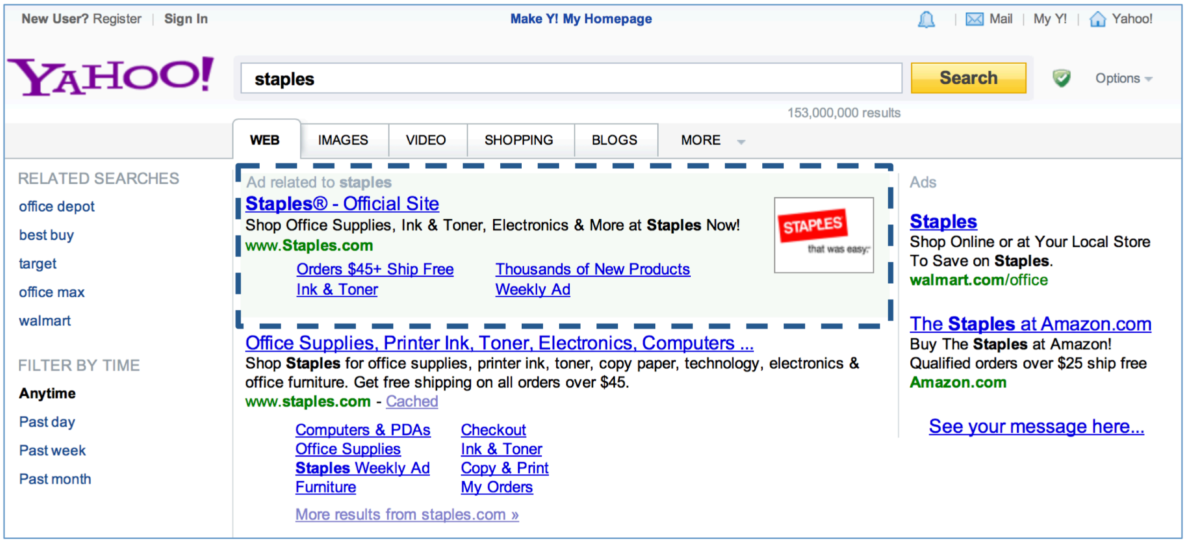
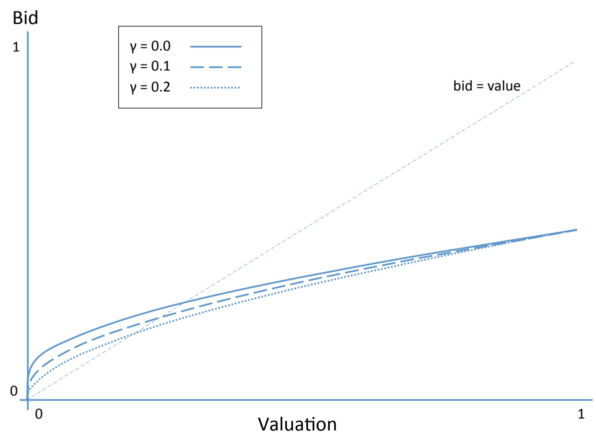


Figure 9: In RAIS, ad can include image, video and multiple links and is displayed exclusively on top.

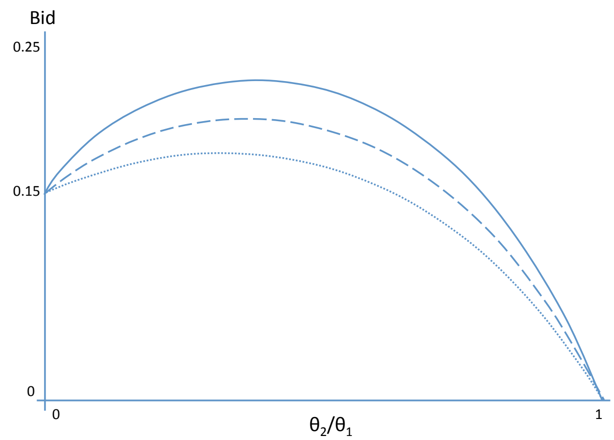
Due to space constraints, we are unable to repeat all the analysis from Section 4 in this new setting. However, to support the argument that our results do not change qualitatively, we show the outline of the analysis of the most involved part of the previous analysis, which is the derivation of the bids in the Differentiating equilibrium of GSP_{2D}. We use the same model as in Section 3. However, we assume that if the search engine chooses the exclusive outcome, the other two advertisers are shown in the right-hand section of the page using GSP allocation and pricing for the right-hand section. As in Section 3, we assume that θ_i is the click-through rate of the i-th slot on top. We assume that the click-through rate of the i-th slot on the right-hand side of the page is $\gamma\theta_i$ where $\gamma < 1$. Ads on the right-hand side usually get much fewer clicks than those on the top. Reiley et al. (2010) shows that up to 85% of total clicks on ads are on ads on top. Furthermore, according to their results, an ad on right-hand side usually generates 10%-20% as many clicks as one on top. Therefore, we expect γ to be between 0.1 and 0.2. According to Yahoo,[13] RAIS increases the click-through rate of the ad by an average of 40%-50%.

The analysis is very similar to that in Section 4. The utility of an S-bidder with valuation v

[13] http://advertising.yahoo.com/article/best-practices-for-rich-ads-in-search.html



(a) Bid functions for three values of $\gamma = 0, 1/10, 1/5$ are compared with each other and with the 45 degree line representing truthful bidding. Top, middle and bottom curves represent $\gamma = 0$, $\gamma = 1/10$ and $\gamma = 1/5$, respectively. Reserve price $r = 0$ and $\theta_2 = 0.5$. As can be seen, low value bidder bid more than their valuation.



(b) Bid as function of θ_2/θ_1 for 3 values of $\gamma = 0, 1/10, 1/5$ are compared with each other. Top, middle and bottom curves represent $\gamma = 0$, $\gamma = 1/10$ and $\gamma = 1/5$, respectively. Reserve price $r = 0$ and $v = 0.15$. As can be seen, bids increase with θ_2/θ_1 for low values of θ_2/θ_1.

Figure 10: GSP_{2D} bids in RAIS.

and bid $f(v')$ simplifies to

$$(1-\gamma)\left(\int_0^r \frac{(\theta_1 r + \theta_2 r)}{\alpha}\theta_1(v-r)dx + \int_r^{v'} \frac{(\theta_1 f(x) + \theta_2 r)}{\alpha}\theta_1(v - f(x))dx \right.$$
$$\left. + \int_{v'}^1 \frac{(\theta_1 f(v') + \theta_2 r)}{\alpha}\theta_2(v-r)dx.\right) + \gamma r\theta_1(v-r) + \gamma(1-v')\theta_2(v-r) + \gamma\int_r^{v'} \theta_1(v-f(x))dx.$$

We take derivative of the above expression with respect to v' and let the expression be equal to zero at $v' = v$, and obtain the following differential equation:

$$\begin{aligned}(1-\gamma)\left(\theta_1(v-f(v))(\theta_2 r + \theta_1 f(v)) + \theta_2(r-v)\left(\theta_2 r + \theta_1 f(v) + \theta_1(-1+v)f'(v)\right)\right) \\ -\gamma\theta_2(v-r) + \gamma\theta_1(v-f(v)) = 0.\end{aligned} \tag{3}$$

The solution to this equation for different values of γ is given Section A1 in the appendix. The results are qualitatively the same as those in Section 4, and the results in Corollary 7 regarding advertisers' strategies in GSP_{2D} still hold as long as γ is small enough. Figure 10(a) shows equilibrium bid as a function of value for 3 levels of $\gamma = 0, 0.1$ and 0.2. As in Corollary 7(a), low-value bidders bid above their valuation in equilibrium. The effect, however, decreases as γ increases. Similarly, Figure 10(b) shows how equilibrium bids are affected by variations in θ_2/θ_1. As in Corollary 7(b), we see that bids increase with θ_2/θ_1 when θ_2/θ_1 is small enough and decrease otherwise. Note that this effect also decreases as γ increases. Intuitively, as γ increases, S-bidders have less incentive to win the non-exclusive outcome because the slots on the right-hand side would also have high click-through rate. Therefore, as γ increases the bidding behaviors specific to GSP_{2D} disappear, and bidding pattern becomes more similar to that in GSP.

The results regarding search engine's revenue also stay qualitatively the same. The region in which the All-Exclusive equilibrium exists, however, shrinks as γ increases. In other words, All-Exclusive equilibrium, which always exists in the model of Section 4, now only exists if γ is small enough. As γ increases, S-bidders would get better allocation in exclusive outcome. Therefore, a low-value S-bidder bids for non-exclusivity (to be placed on the right-hand side) even if he knows that the probability of non-exclusive outcome is zero. As a result, All-Exclusive equilibrium may not exist anymore if γ is not small enough.

6 Conclusions

Popular Internet search engines run auctions to price the ranked list of ads presented to a user in response to a keyword search. In the last decade, the type of auction used has evolved from a Generalized First Price auction to a Generalized Second Price (GSP) auction with numerous small adjustments, and search engines continue to explore new auction mechanisms that can improve revenue. Recently, the three largest search engines, namely Google, Yahoo! and Bing, have been considering the idea of allowing an advertiser to bid to display his ad exclusively rather than in a list of multiple ads, as shown by the "Rich Ads in Search" service launched by Yahoo! and Microsoft adCenter, and the "perfect ad" initiative of Google. Advertisers typically have a higher willingness to pay for exclusive display than for multiple display because exclusively displayed ads can have a stronger impact on the user, which can allow the search engine to make larger revenue from exclusive display. However, exclusively displayed ads are expected lead to fewer total clicks, and could also lead to bidding patterns of advertisers that could reduce the search engine's revenue. Overall, there is a lack of clarity regarding the different effects of introducing exclusive display is search advertising, which has led to search engines adopting a cautious approach on this front. For instance, the "Rich Ads in Search" service was launched by Yahoo! in beta stage,[14] while

[14] http://www.location3.com/yahoo-bing-rich-ads-in-search/

Google postponed the plans for exclusive display after considering them at the highest levels of management (Metz 2008, 2011).

In this paper, we study exclusive display in search advertising using a game theory model. Our work sheds light on the different forces at play in exclusive display auctions and how they interact with each other. We charcaterize the conditions under which it is beneficial for the search engine to switch to a format that gives the advertisers the option to bid for being displayed exclusively. We analyze the GSP_{2D} auction, which is an extension of GSP to two dimensions, and was recently patented by Yahoo! as a primary candidate for implementation as an exclusive-display auction. In addition, a major advancement that we introduce over existing theoretical work on sponsored search advertising is that we allow advertisers to have uncertain valuations and characterize their bidding strategies in the Bayesian Nash equilibria of the game. This, in fact, is an advancement in the study of the GSP auction as well. We therefore also obtain novel results that help to develop a deeper understanding of the already widely-studied GSP auction.

For the GSP auction, we find that bidders shade their bids, and the extent of shading increases with the valuation. We also find that increasing the reserve price of the auction not only leads to an increase in the bids of the low-value bidders, but also has a strong cascading effect on the bids of the high-value bidders who are not directly affected by the higher reserve price. In fact, the increase in the revenue of the search engine from a higher reserve price is attributable more to the increase in the bids of the high-value bidders than to the bids of the low-value bidders.

To analyze the GSP_{2D} auction, we consider two types of bidders: those who value exclusivity more than non-exclusivity (D-type), and those who do not value exclusivity more (S-type). In GSP_{2D}, each advertiser submits a two-dimensional bid, one for the exclusive-display format and another for the non-exclusive-display format. Like GSP, GSP_{2D} is a second-price auction in which if an advertiser's bid for exclusive display is larger than the revenue from multiple display given the multiple display bids, then the search engine will choose exclusive display and the advertiser has to pay the minimum amount needed to maintain the exclusive display outcome (i.e., the "second" price). We find that there are two equilibria in GSP_{2D}. The first equilibrium is an All-Exclusive equilibrium in which, surprisingly, all bidders, even those who do not value exclusivity higher than non-exclusivity, bid for exclusivity. In this equilibrium, all bidders bid their truthful exclusivity valuations. The second equilibrium is a Differentiating equilibrium in which the D-type bidders bid for exclusivity and the S-type bidders bid for non-exclusivity. We find that while the D-type bidders bid truthfully, the S-type bidders distort their bids. Interestingly, the low-valuation S-type bidders may bid *above* their valuation. This is because these bidders want to increase the chance of obtaining the non-exclusive outcome but do not expect to pay their full bid in this outcome, so they bid high in an attempt to beat the exclusivity bid and obtain the non-exclusive outcome. However, high-valuation S-type bidders shade their bids.

Given that GSP_{2D} has two equilibria, one may come up with refinements to exclude one equilibrium. We find both equilibria to be interesting and choose to analyze both in detail rather than refining one away. This, of course, does not resolve the problem of which equilibrium will appear in reality. We choose to stay agnostic on this point. However, we note an interesting observation regarding the "Rich Ads in Search" service by Yahoo!. In this RAIS service, Yahoo! seems to be restricting bidder strategies so that the All-Exclusive equilibrium may not appear. Specifically, at the time of writing of this manuscript, Yahoo! allows bidding for exclusive display only for trademarked keywords and only by the trademark owners. In other words, only one bidder per keyword can bid for exclusivity, which indicates that Yahoo! seems to prefer that the Differentiating equilibrium arise. In spite of this observation, we believe that in our theoretical study, it is useful to characterize both equilibria.

In terms of search engine revenue, we find that GSP_{2D} may lead to higher or lower expected revenue than GSP. A priori, it may seem that revenue in GSP_{2D} should always be higher than GSP–first, competition is heightened because bidders compete not only for positions in the non-exclusive outcome but also compete for the outcome to be exclusive or non-exclusive and, second, the search engine is offering more "options" to the advertisers so should be able to extract more surplus from them. While this true, this is not the complete picture. We find that there are forces at play which can reduce revenue—first, the total number of clicks may be smaller and, second, competition between outcomes also leads to bidders reducing bids for their non-preferred outcome (specifically, a D-type bidder who wants the exclusive outcome will bid high for exclusivity and, simultaneously, bid low for non-exclusivity to increase the chance of obtaining the exclusive outcome). Overall, we find that both equilibria of GSP_{2D} can lead to higher or lower revenue than GSP, and we characterize the conditions under which this happens.

Our work is one of the first to model exclusive display in sponsored search advertising, and there are numerous avenues for future research. First, we analyze the GSP_{2D} auction that has been proposed as a candidate for implementation at Yahoo! (and is possibly being used, albeit in a slightly varied form, for the "Rich Ads in Search" system, though we note that the details of the auction used are not publicly declared). There can, of course, be various other auction mechanisms that can be used for exclusive display. We expect the basic forces that we identify to be in play in other mechanisms as well, and future research can explore this. Furthermore, future research can explore what the optimal mechanism for an exclusive-display auction is, which is a very challenging question.

Second, we make the assumption that every advertiser values exclusive display at least as much as multiple display, which is a very reasonable assumption. However, these valuations are assumed to be independent across competitors to keep the model simple. Explicitly modeling the effect of one advertiser on another advertiser is an interesting direction for future work. For example, a luxury car manufacturer such as Lexus may want to be listed exclusively if the competitive advertiser is another luxury car manufacturer such as Acura, but may care less about being listed next to a lower-quality manufacturer such as Kia. In other words, in the spirit of Jerath et al. (2011), the competitive environment of a firm may significantly influence its valuation for exclusive display and therefore its bidding strategy. Desai et al. (2010) study such context effects in a one-dimensional multiple-display auction. Future work can explicitly model these phenomena with the exclusive-display option also available to advertisers.

Finally, allowing each bidder to submit bids for multiple and exclusive display is simply one way to make the currently-prevailing auction format more expressive. However, there may be various other formats in which advertisers can reveal their preferences in more detail (e.g., Muthukrishnan (2009) discussed earlier). Future research can work towards a general theory of "expressive sponsored advertising auctions." This theory should also consider practical limitations such as ease of bidding by advertisers and the real-time calculation and implementation of auction outcomes by the search engine.

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Appendix

A1 Bid Functions

The solution to the differential equation (1) is

$$b_{1D}(v) = \Bigg(\left(1+r-2r^2\right)^4 \theta_2^4 + 2v^3 \left(\left(1+r+r^2\right)\theta_1 + (-1+r)(1+2r)\theta_2\right)^2$$

$$\left(\left(1+r+r^2\right)\theta_1 + 2(-1+r)(1+2r)\theta_2\right)^2 + 2v\theta_2^2 \left(\left(-1-2r+r^3+2r^4\right)^2 \theta_1^2 - 3\left(1+r-2r^2\right)^3 \left(1+r+r^2\right)\theta_1\theta_2 + 3\left(1+r-2r^2\right)^4 \theta_2^2\right)$$

$$+ v^2\theta_2 \left(-4(-1+r)(1+2r)\left(1+r+r^2\right)^3 \theta_1^3 - 19\left(-1-2r+r^3+2r^4\right)^2 \theta_1^2\theta_2 + 30\left(1+r-2r^2\right)^3 \left(1+r+r^2\right)\theta_1\theta_2^2 - 15\left(1+r-2r^2\right)^4 \theta_2^3\right)$$

$$-(-1+r)(1+2r)\theta_2 \left(\left(1+r+r^2\right)v\theta_1 + (-1+r)(1+2r)(-1+v)\theta_2\right)^{\frac{\left(1+r+r^2\right)\theta_1}{\left(1+r+r^2\right)\theta_1+(-1+r)(1+2r)\theta_2}}$$

$$\left(\left(r+r^2+r^3\right)^2 \theta_1^2 + 3r^2\left(-1-2r+r^3+2r^4\right)\theta_1\theta_2 - \left(1+r-2r^2\right)^3 \theta_2^2\right)$$

$$\left(\theta_2 + r\left(\left(1+r+r^2\right)\theta_1 + r(-3+2r)\theta_2\right)\right)^{\frac{(-1+r)(1+2r)\theta_2}{\left(1+r+r^2\right)\theta_1+(-1+r)(1+2r)\theta_2}} \Bigg)$$

$$/\Bigg(\left(\left(1+r+r^2\right)\theta_1 + 2(-1+r)(1+2r)\theta_2\right)\left(2\left(1+r+r^2\right)\theta_1 + 3(-1+r)(1+2r)\theta_2\right)$$

$$\left(\left(1+r+r^2\right)v\theta_1 + (-1+r)(1+2r)(-1+v)\theta_2\right)^2 \Bigg). \tag{A1}$$

The solution to the differential equation (2) is

$$b_{2D}(v) = -\Bigg(\theta_2 r(-1+v)^{\frac{2}{-1+r}} \Bigg((\theta_1+\theta_2)(-1+v)^{-\frac{2}{-1+r}} \left(\frac{-1+v}{-1+r}\right)^{\frac{2}{-1+r}}$$

$$\left(-(-1+v)^{\frac{\theta_1-\theta_2+2\theta_2 r}{\theta_2-\theta_2 r}}(-r+v)^{\frac{(\theta_1+\theta_2)r}{\theta_2(-1+r)}} + rC\right)$$

$$+\theta_2(-1+r)(-1+v)^{\frac{\theta_1+3\theta_2 r}{\theta_2-\theta_2 r}}\left(\frac{-1+v}{-1+r}\right)^{\frac{1+r}{-1+r}}(-r+v)^{\frac{(\theta_1+\theta_2)r}{\theta_2(-1+r)}}$$

$$\text{Hypergeometric2F1}\left[1,-1+\frac{\theta_1}{\theta_2},1+\frac{(\theta_1+\theta_2)r}{\theta_2(-1+r)},\frac{r-v}{-1+r}\right]\Bigg)\Bigg) \tag{A2}$$

$$/\Bigg(\theta_1\Bigg((\theta_1+\theta_2)r\left(\frac{-1+v}{-1+r}\right)^{\frac{2}{-1+r}}C + \theta_2(-1+r)$$

$$(-1+v)^{\frac{\theta_1-2\theta_2+3\theta_2 r}{\theta_2-\theta_2 r}}\left(\frac{-1+v}{-1+r}\right)^{\frac{1+r}{-1+r}}(-r+v)^{\frac{(\theta_1+\theta_2)r}{\theta_2(-1+r)}}$$

$$\text{Hypergeometric2F1}\left[1,-1+\frac{\theta_1}{\theta_2},1+\frac{(\theta_1+\theta_2)r}{\theta_2(-1+r)},\frac{r-v}{-1+r}\right]\Bigg)\Bigg)$$

where Hypergeometric2F1$(a, b, c, z) = \sum_{k=0}^{\infty} \frac{(a)_k(b)_k}{(c)_k} \frac{z^k}{k!}$ and $(a)_k = a(a+1)(a+2)\ldots(a+k-1)$. The solution to the differential equation (3) for $\gamma = \frac{1}{10}$ is:

$$-\Bigg(\theta_2(r-v)(-1+v) \tag{A3}$$

$$\Bigg(\frac{2ie^{\frac{2i\pi}{-1+r}}\pi(1-r)^{-2+\frac{\theta_1}{\theta_2}}(\theta_1+9\theta_2 r)\text{Gamma}\left[\frac{\theta_1+9(\theta_1+\theta_2)r}{9\theta_2(-1+r)}\right]}{9\left(e^{\frac{2i\pi}{-1+r}} - e^{\frac{4i\pi(5\theta_1+9\theta_2 r)}{9\theta_2(-1+r)}}\right)\theta_2(r-v)(-1+v)\text{Gamma}\left[-1+\frac{\theta_1}{\theta_2}\right]\text{Gamma}\left[3-\frac{10\theta_1+9\theta_2}{9\theta_2-9\theta_2 r}\right]}$$

$$+\Bigg((-1+v)^{\frac{10\theta_1-18\theta_2+27\theta_2 r}{9\theta_2-9\theta_2 r}}(-r+v)^{-\frac{\theta_1+9\theta_2+9\theta_1 r}{9\theta_2-9\theta_2 r}}\Bigg(\theta_1+9(\theta_1+\theta_2)r-(\theta_1+9\theta_2 r)$$

$$\text{Hypergeometric2F1}\left[1,-1+\frac{\theta_1}{\theta_2},\frac{\theta_1-9\theta_2+9(\theta_1+2\theta_2)r}{9\theta_2(-1+r)},\frac{r-v}{-1+r}\right]\Bigg)\Bigg)/(\theta_1+9(\theta_1+\theta_2)r)\Bigg)\Bigg)$$

$$/\Bigg(\theta_1\text{Gamma}\left[\frac{\theta_1+9(\theta_1+\theta_2)r}{9\theta_2(-1+r)}\right]\Bigg(\frac{2ie^{\frac{2i\pi}{-1+r}}\pi(1-r)^{-2+\frac{\theta_1}{\theta_2}}}{\left(e^{\frac{2i\pi}{-1+r}} - e^{\frac{4i\pi(5\theta_1+9\theta_2 r)}{9\theta_2(-1+r)}}\right)\text{Gamma}\left[-1+\frac{\theta_1}{\theta_2}\right]\text{Gamma}\left[3-\frac{10\theta_1+9\theta_2}{9\theta_2-9\theta_2 r}\right]}$$

$$+\Bigg((-1+v)^{\frac{10\theta_1-9\theta_2+18\theta_2 r}{9\theta_2-9\theta_2 r}}(-r+v)^{\frac{\theta_1+9(\theta_1+\theta_2)r}{9\theta_2(-1+r)}}$$

$$\text{Hypergeometric2F1regularized}\left[1,-1+\frac{\theta_1}{\theta_2},\frac{\theta_1-9\theta_2+9(\theta_1+2\theta_2)r}{9\theta_2(-1+r)},\frac{r-v}{-1+r}\right]\Bigg)/(-1+r)\Bigg)\Bigg).$$

where Hypergeometric2F1regularized$(a, b, c, z) = \frac{\text{Hypergeometric2F1}(a,b,c,z)}{\text{Gamma}(c)}$ and Gamma$(x) = \Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$. Due to space constraints, we do not present the solution for other values of γ.

A2 Proofs

Before presenting the proofs of Corollary 2 and Corollary 7, we mention that several parts of proofs involve showing that an expression is positive/negative. We confirm the sign (positive or negative) of the expression numerically for the whole domain and the expressions themselves are derived analytically. In other words, while we are able to analytically calculate the derivative, due to complexity of the structure, we are not able to analytically prove that it is always negative/positive. Fortunately, the expressions are all smooth so that numerical comparisons are easy. Details on numerical comparisons and analytical expressions for the derivatives are available upon request. Finally, since advertisers' strategies do not depend on θ_1 and θ_2 as long as θ_2/θ_1 is unchanged, without loss of generality, we assume throughout the proofs that $\theta_1 = 1$.

Proof of Corollary 2: To Prove part (a), we take derivative of $b_{1D}(v)$ with respect to θ_2. We confirm that the expression $\frac{\partial b_{1D}(v)}{\partial \theta_2}$ is negative for any value of $0 \le r < 1$, $r < v < 1$ and $0 < \theta_2 < 1$. To prove part (b), we show that $\frac{\partial(\frac{v-b_{1D}(v)}{v})}{\partial v}$ is always positive for any value of $0 \le r < 1$, $r \le v < 1$

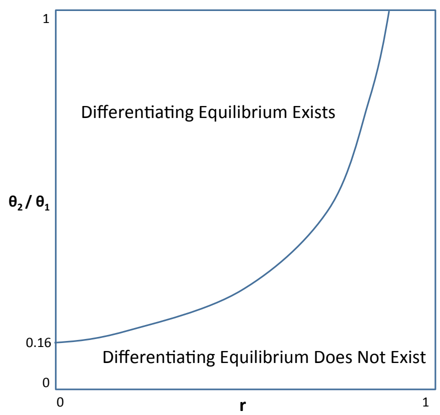

Figure A1: Existence of separating equilibrium as a function of r and $\frac{\theta_2}{\theta_1}$.

and $0 < \theta_2 < 1$. For part (c), we confirm that $\frac{\partial b_{1D}(v)}{\partial r}$ is positive for any value of $0 \leq r < 1$, $r < v \leq 1$ and $0 < \theta_2 < 1$. Furthermore, for the same domain, we have $\frac{\partial \Delta_r b(v)}{\partial v} > 0$. Finally, for part (d), we take derivative of revenue of GSP with respect to θ_2. The derivative is positive for any value of $r \geq 0$ and $\theta_2 \geq 0$.

Proof of Proposition 4: We first show that search engine's revenue in the All-Exclusive equilibrium with reserve price r is $\frac{-1+4r^3-9r^4+2\alpha\left(2+r^3\right)}{6\alpha}$ if $r \leq 1$ and is $(1-\frac{r}{\alpha})r$ if $r > 1$. In pooling equilibrium, bidders are participating in a second-price auction. If all bidders bid less than r, revenue is 0, otherwise, the revenue is maximum of r and the second highest bid. Let F and f be the CDF and PDF of the second highest bid. We have $F(x) = \frac{x^3}{\alpha} + x^2(1-\frac{x}{\alpha}) + 2\frac{x^2}{\alpha}(1-x)$ and $f(x) = F'(x) = \frac{2(2+\alpha-3x)x}{\alpha}$. The search engine's revenue is $(F(r) - \frac{r^3}{\alpha})r + \int_r^1 f(x)dx$ if $r \leq 1$ and $(1-\frac{r}{\alpha})r$ otherwise. The integration simplifies to $\frac{-1+4r^3-9r^4+2\alpha\left(2+r^3\right)}{6\alpha}$. If we optimize the expression with respect to r for each interval $r < 1$ and $r \geq 1$ we get the results in Proposition 4.

Proof of Corollary 7: To prove part (a), we show that $\frac{\partial(v-b_{2D}(v))}{\partial v}$ is positive for any $r \leq v \leq 1$, $0 \leq r < 1$ and $0 < \theta_2 < 1$. If we let ν be such that $b_{2D}(\nu) = \nu$, we would have $b_{2D}(v) > v$ if and only if $v < \nu$. Value ν is positive if r and θ_2 are small enough as depicted in Figure 5(c). For part (b), we verify that $\frac{\partial^2 b_{2D}(v)}{\partial \theta_2^2}$ is negative for any $r \leq v \leq 1$, $0 \leq r < 1$ and $0 < \theta_2 < 1$. This means that $\frac{\partial b_{2D}(v)}{\partial \theta_2}$ is decreasing in θ_2. Therefore, if we let τ be such that $\frac{\partial b_{2D}(v)}{\partial \theta_2} = 0$ for $\theta_2 = \tau$, bid $b_{2D}(v)$ would be increasing in θ_2 when $\theta_2 < \tau$. Value τ is positive if v and r are small enough as depicted in Figure 6(c). For part (c), we show that $\frac{\partial(\frac{v-b_{2D}(v)}{v})}{\partial v}$ is positive for any $r \leq v \leq 1$, $0 \leq r < 1$ and $0 < \theta_2 < 1$. Finally, to prove part (d), we show that $\frac{\partial(b_{2D}(v)-b_{1D}(v))}{\partial v}$ is negative for any $r \leq v \leq 1$, $0 \leq r < 1$ and $0 < \theta_2 < 1$. If we let $\hat{\nu}$ be such that $b_{2D}(\hat{\nu}) = b_{1D}(\hat{\nu})$, we would have $b_{2D}(v) < b_{1D}(v)$ if and only if $v > \hat{\nu}$.

A3 Existence of Differentiating Equilibrium

In this section, we discuss the conditions needed for the existence of Differentiating equilibrium in GSP_{2D}. The first condition is that the solution obtained in Equation (A2) must be an increasing function of v. Note that when deriving the utility function for the differential equation (2) we assumed that the bid function is an increasing function of v. Hence, the solution is valid only if this

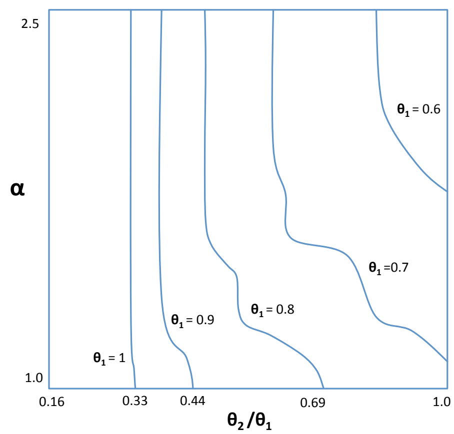

Figure A2: Each curve represents a value of θ_1; the region above (and to the right of) each curve is where Differentiating equilibrium exists. As can be seen in the figure, the region in which Differentiating equilibrium exists shrinks as θ_1 becomes smaller.

condition is satisfied. Since the solution to the differential equation (2) is unique, non-monotonicity of the bid function implies non-existence of Differentiating equilibrium. Since the bid function in Equation (A2) is only function of $\frac{\theta_2}{\theta_1}$ and r, we can verify its monotonicity just based on these two parameters. Figure A1 shows the region in which separating equilibrium exists. As we can see, separating equilibrium exists only if $\frac{\theta_2}{\theta_1} > 0.16$. Furthermore, low value of $\frac{\theta_2}{\theta_1}$ requires the reserve price to be low for the equilibrium to exist. This constraint could force the search engine to set the reserve price below the *optimal* that it would set if Differentiating equilibrium always existed. In fact, as depicted in Figure 8(a), when $\frac{\theta_2}{\theta_1}$ is small, since the search engine cannot set the reserve price high enough, GSP leads to higher revenue than Differentiating equilibrium of GSP_{2D}.

There are two more conditions necessary for existence of separating equilibrium. First, an S-bidder should have no incentive to bid for exclusivity. Second, a D-bidder should have no incentive to bid for non-exclusivity. Intuitively, the first condition is satisfied when θ_1 is large enough, θ_2 is large enough or α is large enough. If θ_1 is small, an S-bidder benefits from bidding for exclusivity even though it involves competing with D-bidders. As θ_2 becomes larger, it becomes easier for S-bidders to win the non-exclusive outcome. On the other hand, if θ_2 is very small (almost zero) non-exclusive outcome is almost the same as exclusive outcome but only with lower CTR. Finally, if α is large, D-bidder is stronger and therefore, S-bidders have less incentive to compete with them for exclusivity. Figure A2 displays the region in which Differentiating equilibrium exists as a function of $\frac{\theta_2}{\theta_1}$ and α for different values of θ_1.

A4 Reserve Price

In this section, we discuss how different parameters affect the optimum reserve price of GSP_{2D}. Before discussing GSP_{2D} we note that Edelman and Schwarz (2010) show that optimum reserve price in GSP can be obtained using Myerson's optimal auction. In particular, assuming regularity conditions, they show that optimum reserve price r^* of a regular second price auction, which is the solution to $r^* - (1 - F(r^*))/(f(r^*)) = 0$, is also the optimum reserve price for GSP. In this equation, $F(.)$ and $f(.)$, respectively represent the CDF and PDF of bidders' valuation. Since we assume that bidders' valuation come from a uniform distribution $U[0, 1]$, the optimum reserve price of GSP in our model is 1/2. The fact that optimum reserve price is independent of number of

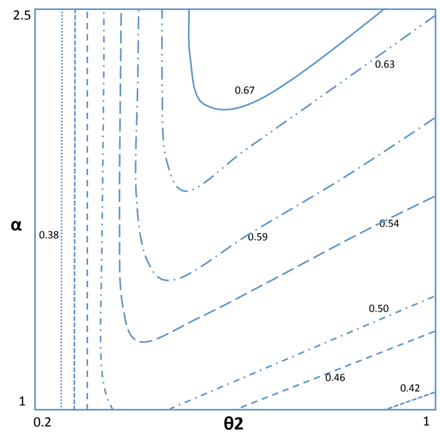

Figure A3: Contours represent optimum reserve price r^* as for different values of α and θ_2, assuming $\theta_1 = 1$.

bidders, number of slots, and corresponding click-through rates is particularly interesting.

For the All-Exclusive equilibrium of GSP_{2D}, Proposition 4 gives us the optimum reserve price. If α is large enough, the search engine ignores S-bidders and tries to maximize its revenue by setting the reserve price so that it would maximize the D-bidder's payment. In this case, reserve price r^* is set to $\alpha/2$. This is the same reserve price that an auctioneer sets if a single item is to be sold to a single bidder. On the other hand, if α is not large enough, the optimum reserve price is $\frac{\alpha+2}{6}$. In this case, when $\alpha = 1$, meaning that every bidders' valuation for exclusivity is drawn from $U[0, 1]$, optimum reserve price is exactly 1/2. As α increases, the search engine should increase the reserve price to increase the expected payment of the D-bidder with valuation $U[0, \alpha]$. The reserve price is not increasing as fast as α because higher reserve price decreases the expected payment of the other two S-bidders. In fact, search engine's strategy can be interpreted as calculating the reserve price for each bidder individually (1/2 for the S-bidders, and $\alpha/2$ for the D-bidder) and taking their average for the optimum reserve price: $(\frac{1}{2} + \frac{1}{2} + \frac{\alpha}{2})/3 = \frac{\alpha+2}{6}$.

Figure A3 shows the optimum reserve price for the Differentiating equilibrium of GSP_{2D}. As we can see, for the same value of θ_2, optimum reserve price increases with α. This is because larger α implies higher expected value for the D-bidder. Consequently, search engine wants to increase the reserve price to increase the expected payment of the D-bidder. For the same level of α, we see that optimum reserve price first increases and then decreases with θ_2. As discussed in Section A3, when θ_2 is small, reserve price has to be set sub-optimally low for Differentiating equilibrium to exist. In other words, when θ_2 is small, reserve price is set to the maximum value at which Differentiating equilibrium exists. Therefore, when θ_2 is small, optimum reserve price increases with θ_2. However, as θ_2 becomes larger, this condition is relaxed and reserve price is not forced down by existence of equilibrium conditions. When θ_2 is large enough, for the same level of α, as θ_2 increases, expected payment of the D-bidder also increase. The search engine, therefore, has to lower the reserve price to increase the probability of selling the slots, either exclusively or non-exclusively.