Errata for Chapter 11 of Subatomic Physics (3rd edition).

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1) Figure 11.8 on page 348: The caption should read "*Positron* spectrum…" instead of "*Electron* spectrum…". The paragraph on the right of the figure has the same mistake several times: all the instances of *electron* should be changed to *positron*. The spectra are measured from the decay of μ^+ rather than from μ^- because the latter can be captured into muonic atoms which introduces unwanted distortions.
2) Eqs. 11.48, and 11.49 (and the unnumbered Eq. following Eq. 11.42): What is stated here, that there is a difference between the decay rate w_μ and its parity mirror, w_μ^P, needs some remarks. This is true for example in the scattering of longitudinally polarized electrons (or hadrons) off nuclei. The rates w_μ and w_μ^P should be interpreted as the reaction rates corresponding to the two polarization states. In the case of observations of decays of polarized parents, the assertion that there is a difference between w_μ and w_μ^P is only true about the differential decay rates $dw_\mu(E_e)$ and $dw_\mu^P(E_e)$ that give the transition rates for emission of electrons (or positrons) in a particular direction . Once the spectrum is integrated over the electron (positron) emission directions, the interference term cancels and the parity violation is not observable. Thus, as is obvious in Eq. 11.52, for example, there is no way of checking for parity violation without looking at the electron (positron)'s direction with respect to the spin of the parent muon.
3) Eq. 11.52: missing here is a statement indicating that the equation is valid only for $0 < E_e < \frac{m_\mu c^2}{2}$. For $E_e > \frac{m_\mu c^2}{2}$ there is no decay probability. This restriction originates in the conservation of momentum and energy: in order for the electron to get the maximum possible energy its momentum should be equal and opposite to the total momenta of the two neutrinos and consequently, because all the particles are practically massless, will have approximately half of the total available energy (mass of the muon times c^2).
4) At the end of the phrase following Eq. 11.52 there is a reference to "Fig. 11.7" which should read "Fig. 11.8".
5) On page 364 we address neutrino oscillations. We assume that the time evolution is given by the factors $e^{-iEt/\hbar}$ so there is a phase difference between the two mass eigenstates given by $\delta\phi_{12} \approx \frac{(E_1 - E_2)t}{\hbar}$. Then, assuming $t \approx L/c$, using $p_1 = p_2$ and $E_i \approx pc + \frac{(m_i c^2)^2}{2pc}$ the phase difference becomes $\delta\phi_{12} \approx \frac{(m_1{}^2 - m_2{}^2)c^2 L}{2p\hbar}$, which, taking into account that the oscillation probability goes like $\sin^2(\frac{\delta\phi_{12}}{2})$, yields Eq. 11.83.

Now, arguably, rather than $t \approx L/c$, one should use $t_1 = L/v_1 = \frac{L}{c}\frac{E_1}{pc}$ and likewise for t_2. This would imply that the phase difference between the two neutrino states is $\delta\phi_{12} \approx \frac{\left(E_1{}^2 - E_2{}^2\right)}{pc^2} L/\hbar$ .So, using $E_i{}^2 = \left(pc\right)^2 + (m_i c^2)^2$ we would obtain for the appearance probability of a muon neutrino from an electron one $P_{\nu_\mu}(t) = \sin^2 2\theta_{12}\ \sin^2\left[\frac{1}{2}\frac{\left(m_1{}^2 - m_2{}^2\right)c^2 L}{p\ \hbar}\right]$. In brief, we get a factor of 2 difference in the oscillation frequency from Eq. 11.83! *This is incorrect and Eq. 11.83 is the correct formula.* Why is this? In our demonstration we started by assuming that the time evolution is given by the factors $e^{-iEt/\hbar}$ but this is correct only in the limit in which the oscillating system is at rest or when relativistic effects can be neglected. Here we are far from being able to meet these conditions since the neutrinos have a mass that is much smaller than their energies. The correct evolution is given by $e^{i(xp-Et)/\hbar}$. Then, at a distance L from the creation of the electron neutrino the *correct* phase difference between the two components is: $\delta\phi_{12} \approx \left[\left(p_1 - p_2\right)L - \left(E_1 - E_2\right)t\right]/\hbar = \left[\frac{\left(p_1{}^2 - p_2{}^2\right)}{p_1 + p_2} - \frac{\left(E_1{}^2 - E_2{}^2\right)}{\left(p_1 + p_2\right)c}\right]L/\hbar$, where we have assumed that the two neutrino species are moving with an average velocity $\bar{v} = \left(\frac{p_1 + p_2}{E_1 + E_2}\right)c$. The justification for this has been discussed, for instance, by Lipkin (Phys. Lett. **B642**, 366 (2006) and references therein). Now calculating the appearance probability we get agreement with Eq. 11.83 if we assume $p_1 = p_2$, as before. Note that in this limit the factor $\left(p_1 - p_2\right)L$ does not appear, so whether or not we take into account the momentum-space part of the phase shift is not so crucial. The crucial point is that one needs to use the average common speed. In order to do the calculations in a rigorous way, wave packets should be used and the same conclusions are reached.

In brief, the final formula for the oscillations, Eq. 11.83, is correct, but there are subtleties in the procedure we follow that may lead to significantly different results if not interpreted correctly.