Addendum to the paper on helicity and nuclear β decay correlations with calculations of neutron beta decay correlations

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This is an addendum to the paper of Ref.[2] addressing neutron beta decay.

I. NEUTRON β DECAY

The decay of the neutron ($n \to p + e^- + \bar{\nu}$) presents some additional calculation challenges, but there is also much to be learned for students interested in deeper understanding. The complete expressions for the correlations are available[1] and can be used as a 'solutions manual' to check that the simple arguments presented here lead to the correct expressions for the correlation coefficients. For brevity we work within the standard model: no scalar or tensor currents ($C_S = C'_S = C_T = C'_T = 0$) and no right-handed currents ($C_V = C'_V$ and $C_A = C'_A$).

Because both the neutron and proton are spin-1/2 particles both kinds of transitions, Fermi and GT, are allowed. Moreover, the GT part of the transition has a non-spin-flip component that interferes with the Fermi part and affects the lepton correlations. We start by considering the matrix elements of the nuclear current of the transition. The operator for the Fermi part is just unity with a coupling constant C_V, while the operator for the GT part is the Pauli matrices, $\boldsymbol{\sigma}$, with a coupling constant C_A. Thus, the total decay rate is simply proportional to (we show in the Appendix that the interferences cancel when integrating over the leptons directions):

$$C_V^2\left(1+3|\lambda|^2\right) \tag{1}$$

where we use the property of the Pauli matrices $\boldsymbol{\sigma}^2 = 3I$ (with I the 2×2 identity matrix) and we define the quantity $\lambda = C_A/C_V$ for the ratio of the axial to vector coupling constants.

To calculate the β asymmetry we consider a neutron with spin aligned in the $+z$ direction ($\boldsymbol{P} = \hat{\boldsymbol{z}}$). The GT operator can be considered separately for the non-spin-flip component, σ_z, and for the spin-flip component σ_- with ratio

$$\frac{\langle p\downarrow|\sigma_- t^+|n\uparrow\rangle}{\langle p\uparrow|\sigma_z t^+|n\uparrow\rangle} = \sqrt{2}, \tag{2}$$

where t^+ is the isospin raising operator that turns a neutron into a proton. If we ignore the interference between the GT and Fermi operators, the decay rate is proportional to a non-spin-flip component with probability proportional to $1+|\lambda|^2$ and a spin-flip component proportional to $2|\lambda|^2$. Following the arguments of Sect. II of Ref.[2] and integrating over all antineutrino directions, the differential decay rate is proportional to:

$$\left(1+|\lambda|^2\right) + 2|\lambda|^2\left(1-\boldsymbol{P}\cdot\frac{\boldsymbol{p}_e}{E_e}\right). \tag{3}$$

However, the non-spin-flip component of the axial current *can* interfere with the vector current because they both connect identical initial and final states. One may naively think that this interference does not depend on the neutron polarization or on the direction of the spin of the leptons, just as happened for the non-flip contributions considered above. However, the nuclear matrix element for the axial current depends on the spin of the neutron while the matrix element for the vector does not depend on it. As a consequence this interference depends linearly on the polarization and the z-projection of the spin of the electron, leading to an additional term $-2\lambda\boldsymbol{P}\cdot(\boldsymbol{p}_e/E_e)$, and Eq. 3 becomes

$$\left(1+|\lambda|^2\right) - 2\lambda\boldsymbol{P}\cdot\frac{\boldsymbol{p}_e}{E_e} + 2|\lambda|^2\left(1-\boldsymbol{P}\cdot\frac{\boldsymbol{p}_e}{E_e}\right). \tag{4}$$

More details, including the reason for the sign, are shown in Appendix 2.

In summary, the decay rate from polarized neutron β decay after integrating over antineutrino momentum is

$$\left(1+3|\lambda|^2\right)\left(1+A\boldsymbol{P}\cdot\frac{\boldsymbol{p}_e}{E_e}\right), \tag{5}$$

where the β asymmetry correlation coefficient A is

$$A = -2\frac{\lambda+|\lambda|^2}{1+3|\lambda|^2}. \tag{6}$$

Similarly, if the integration is performed over all electron directions, the differential decay rate is proportional to:

$$\left(1+3|\lambda|^2\right)\left(1+B\boldsymbol{P}\cdot\frac{\boldsymbol{p}_{\bar{\nu}}}{E_{\bar{\nu}}}\right), \tag{7}$$

where the antineutrino asymmetry correlation coefficient B is:

$$B = -2\frac{\lambda-|\lambda|^2}{1+3|\lambda|^2}. \tag{8}$$

Note that the sign of the non-interfering parts of A and B are opposite, while those of the interfering parts are identical.

The expression for the $e-\bar{\nu}$ correlation can be obtained by adding the non-spin-flip and spin-flip expressions of Eqs 26 and 27 multiplied by the respective probabilities:

$$\frac{1+|\lambda|^2}{1+3|\lambda|^2}\left(1+\frac{\boldsymbol{p}_e}{E_e}\cdot\frac{\boldsymbol{p}_{\bar{\nu}}}{E_{\bar{\nu}}}\right) + \frac{2|\lambda|^2}{1+3|\lambda|^2}\left(1-\frac{\boldsymbol{p}_e}{E_e}\cdot\frac{\boldsymbol{p}_{\bar{\nu}}}{E_{\bar{\nu}}}\right). \tag{9}$$

In summary, the differential decay rate for polarized neutrons is proportional to

$$1 + a\frac{\boldsymbol{p}_e}{E_e} \cdot \frac{\boldsymbol{p}_\nu}{E_\nu} + \boldsymbol{P} \cdot \left(A\,\frac{\boldsymbol{p}_e}{E_e} + B\,\frac{\boldsymbol{p}_{\bar{\nu}}}{E_{\bar{\nu}}} \right) \tag{10}$$

with A and B given by Eq. 6 and Eq. 8 and

$$a = \frac{1 - |\lambda|^2}{1 + 3|\lambda|^2}. \tag{11}$$

Because the ratio of the axial to vector coupling constants has the value $\lambda \sim -1.27$ the β asymmetry and $e - \bar{\nu}$ correlation coefficients end up being negative and small, while the antineutrino asymmetry is positive and large.

The careful reader may note that while the $e - \bar{\nu}$ correlation coefficient was derived assuming non-oriented neutrons we ended up quoting it for the decay from polarized neutrons. Arguably, if both the electron and antineutrino momenta are measured from β decay of nuclei oriented along the $+z$ direction, the differential decay rate should include a term proportional to $\frac{p_e^z}{E_e}\frac{p_{\bar{\nu}}^z}{E_{\bar{\nu}}}$:

$$1 + C_1\frac{p_e^z}{E_e} + C_2\frac{p_{\bar{\nu}}^z}{E_{\bar{\nu}}} + C_3\frac{\boldsymbol{p}_e}{E_e} \cdot \frac{\boldsymbol{p}_{\bar{\nu}}}{E_{\bar{\nu}}} + C_4\frac{p_e^z}{E_e}\frac{p_{\bar{\nu}}^z}{E_{\bar{\nu}}} \tag{12}$$

Both terms with p_e^z and $p_{\bar{\nu}}^z$ flip signs under the inversion of the polarization direction, but the $p_e^z p_{\bar{\nu}}^z$ term does not. If we average the rates with two opposite polarizations, the term with C_4 does not vanish. However, for spin-1/2 particles like neutrons, this average should be the same as the rate for non-polarized neutron decay, so it should not depend on any specific direction. Therefore C_4 *must* be zero for neutron decay. However, in other cases, such as the decay of ^{60}Co this term has to be included.

APPENDIX

The interference term in the neutron decay is the product of the non-spin-flip component of the axial current and the vector current, which yields

$$\begin{aligned} &4C_V^2\;\lambda\;(i)^2\bar{\psi}_e^L\gamma^3\gamma^5\psi_\nu^L\bar{\psi}_\nu^L\gamma^0\psi_e^L \times \\ &\langle p\uparrow|\sigma_z t^+|n\uparrow\rangle\langle n\uparrow|t^-|p\uparrow\rangle + h.c., \end{aligned} \tag{13}$$

where the product of the nuclear matrix elements $\langle p\uparrow|\sigma_z t^+|n\uparrow\rangle\langle n\uparrow|t^-|p\uparrow\rangle$ yields 1. Using the explicit expressions in Appendix 1 one gets

$$(i)^2\bar{\psi}_e^L\gamma^3\gamma^5\psi_\nu^L\bar{\psi}_\nu^L\gamma^0\psi_e^L = \psi_e^{L\dagger}\begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}\psi_\nu^L\psi_\nu^{L\dagger}\psi_e^L. \tag{14}$$

This indicates that this interference term has a linear dependence on the z component of the electron and antineutrino spins. One can obtain the dependence on lepton spins by working with the eigenstates of the σ_z operator. Because every spinor in Eq. 14 is left-handed, we can just work with the upper 2 components of the Dirac spinors, where $\chi^{\uparrow(\downarrow)}$ indicates spin-up(down) electrons, or spin-down(up) anti-neutrinos. The definition of $\chi^{\uparrow(\downarrow)}$ and the reason for the opposite signs for particle and antiparticle is explained in Appendix 1 of Ref.[2]. If the electron spin and antineutrino spin are identical, Eq. 14 is zero because $\chi^\uparrow$ is normal to $\chi^\downarrow$. This is consistent with the fact that the electron and antineutrino are emitted with total angular momentum zero along the z axis in non-spin-flip transitions and they should be opposite to each other. For spin-up electron and spin-down anti-neutrino, the value of Eq. 14 is positive (and negative for the opposite case). Therefore, the right-hand side of Eq. 14 should be proportional to $S_z^e - S_z^{\bar{\nu}}$, where S_z^e and $S_z^{\bar{\nu}}$ are the values of the z component of electron and antineutrino spins. We have seen before that the spin of the electron has a negative correlation with its direction, while the spin of the antineutrino has a positive correlation with its direction. Thus the interference term is $4C_V^2\,(-2\lambda\boldsymbol{P}\cdot(\boldsymbol{p}_e/E_e) - 2\lambda\boldsymbol{P}\cdot(\boldsymbol{p}_{\bar{\nu}}/E_{\bar{\nu}}))$, where the factor 2 comes from adding the hermitian conjugate term. After integrating over all electron directions and all antineutrino directions, the interference term disappears so it does not show up in the total decay rate.

[1] J. D. Jackson, S. B. Treiman, and H. W. Wyld, Phys. Rev. **106**, 517 (1957).

[2] R. Hong, M. Sternberg, and A. Garcia, Submitted to Am. Jour. of Phys. (2016).