Comparing frequency-difference beamforming and delay-and-sum beamforming as an estimator of direction-of-arrival

ELECTRICAL ENGINEERING
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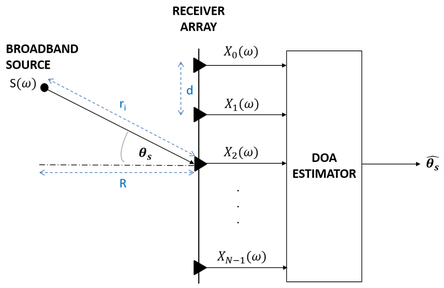
I. BACKGROUND

- **Problem Area**: direction-of-arrival (DOA) estimation



MAIN VARIABLES	
Broadband Source	$S(\omega)$
True DOA	θ_s
Num Elements	N
Element Spacing	d
Sound Speed	c
Received Signals	$X_0(\omega), \ldots, X_{N-1}(\omega)$
Estimated DOA	$\hat{\theta}_s$
Tx-Rx Hor. Distance	R
Path to ith Receiver	r_i

- **General Approach to DOA Estimation**: Process received signals at different search angles θ to form spatial spectrum $P(\theta)$, and then find the maximum

$$\hat{\theta}_s = \underset{\theta}{\operatorname{argmax}}\, P(\theta) \qquad (Eq.1)$$

- **Conventional Method for $P(\theta)$**: delay-and-sum beamforming (DASB)

Process received signals at different search angles θ (Eq.2a) → *Form spatial spectrum (Eq. 3a)*

$$b(\theta,\omega) = \sum_{i=0}^{N-1} X_i(\omega) e^{j\omega\left(i-\frac{N-1}{2}\right)\frac{d\sin(\theta)}{c}} \qquad P(\theta) = \int_{\omega_1}^{\omega_2} |b(\theta,\omega)|^2 d\omega$$

- **Unconventional Method for $P(\theta)$**: frequency-difference beamforming (FDB)

Process received signals at different search angles θ (Eq.2b) → *Form spatial spectrum (Eq. 3b)*

$$b(\theta,\omega) = \sum_{i=0}^{N-1} X_i^*(\omega) X_i(\omega+\Delta\omega) e^{j\Delta\omega\left(i-\frac{N-1}{2}\right)\frac{d\sin(\theta)}{c}} \qquad P(\theta) = \int_{\omega_1}^{\omega_2} |b(\theta,\omega)|^2 d\omega$$

  - Introduced by Abadi, Song, and Dowling in [1]
  - Reformulates DASB to use a field product $X_i^*(\omega)X_i(\omega+\Delta\omega)$ and an additional parameter called the frequency-difference $\Delta\omega$
  - Empirically showed in [1] that FDB can more accurately estimate DOA than DASB when processing a high-frequency broadband source and when linear receiver array is sparse

II. OBJECTIVE

- To study the relationship between FDB and DASB further, and investigate the following questions:
  - **How well** does FDB compare to DASB as an estimator of DOA?
  - **Are there conditions** on the source center frequency f_o or source bandwidth W in which FDB will outperform DASB?

III. APPROACH

- **DOA Estimator Equations**: Explicitly formulate the DOA estimator equations for DASB and FDB
- **One Propagation Path**: Assuming one path from source to receiver, analytically compare the estimator equations and simulate the effect of source center frequency f_o and bandwidth W on the DOA estimator error
- **Two Propagation Paths**: Assuming two paths from source to receiver, analytically compare the estimator equations and simulate the effect of source center frequency f_o and bandwidth W on the DOA estimator error

REFERENCES

[1] Abadi, Shima H., Hee Chun Song, and David R. Dowling. "Broadband sparse-array blind deconvolution using frequency-difference beamforming." *The Journal of the Acoustical Society of America* 132, no. 5 (2012): 3018-3029.

IV. DOA ESTIMATOR EQUATIONS

Eq. 1, 2a/2b, 3a/3b → Combine and simplify $\hat{\theta}_s$ in to matrix form

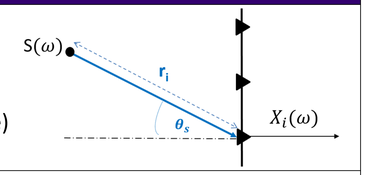
DASB	FDB
$\hat{\theta}_s = \underset{\theta}{\operatorname{argmax}} \int_{\omega_1}^{\omega_2} \boldsymbol{a}^H(\theta,\omega)\boldsymbol{G}\boldsymbol{a}(\theta,\omega)d\omega$	$\hat{\theta}_s = \underset{\theta}{\operatorname{argmax}}\ \boldsymbol{a}^H(\theta)\boldsymbol{G}\boldsymbol{a}(\theta)$
$\boldsymbol{G} = [\boldsymbol{G}]_{ik} = X_i(\omega)X_k^*(\omega)$	$\boldsymbol{G} = [\boldsymbol{G}]_{ik} = \int_{\omega_1}^{\omega_2} X_i^*(\omega)X_i(\omega+\Delta\omega)X_k(\omega)X_k^*(\omega+\Delta\omega)d\omega$
$\boldsymbol{a}(\theta,\omega) = [\boldsymbol{a}(\theta,\omega)]_i = e^{-j(\omega)(i)\left(\frac{d\sin(\theta)}{c}\right)}$	$\boldsymbol{a}(\theta) = [\boldsymbol{a}(\theta)]_i = e^{-j(\Delta\omega)(i)\left(\frac{d\sin(\theta)}{c}\right)}$

V. ONE PROPAGATION PATH

- **Model for Received Signal:**

$$X_i(\omega) = S(\omega)\frac{e^{-j\omega\frac{r_i}{c}}}{r_i} + N_i(\omega)$$

where $N_i(\omega) = \mathcal{F}\{\, n_w(t)\,\}$ (white Gaussian noise)



- **Comparing DOA Estimator Equations:**
  - DASB at one frequency $\omega = \Delta\omega$:

$$\hat{\theta}_s = \underset{\theta}{\operatorname{argmax}}\ \boldsymbol{a}^H(\theta)\boldsymbol{G}\boldsymbol{a}(\theta) \text{ where } \boldsymbol{G} = [\boldsymbol{G}]_{ik} = X_i(\Delta\omega)X_k^*(\Delta\omega) \text{ and } \boldsymbol{a}(\theta) = [\boldsymbol{a}(\theta)]_i = e^{-j(\Delta\omega)(i)\left(\frac{d\sin(\theta)}{c}\right)}$$

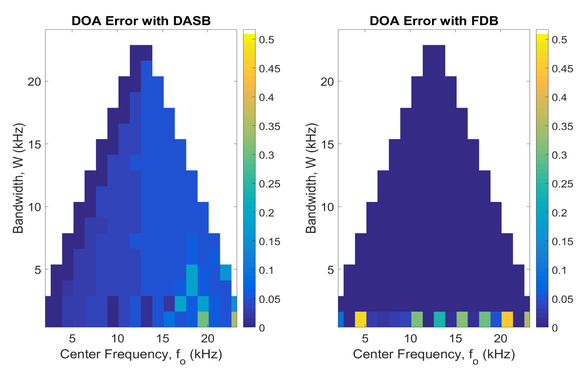
  - Matrix $\boldsymbol{G}$ with received signal model (assuming no noise):

Narrowband DASB	FDB
$\boldsymbol{G} = [\boldsymbol{G}]_{ik} = \frac{e^{-j\Delta\omega\left(\frac{r_i-r_k}{c}\right)}}{r_i r_k}\lvert S(\Delta\omega)\rvert^2$	$\boldsymbol{G} = [\boldsymbol{G}]_{ik} = \frac{e^{-j\Delta\omega\left(\frac{r_i-r_k}{c}\right)}}{r_i r_k}\frac{1}{r_i r_k}\int_{\omega_1}^{\omega_2}\lvert S(\omega)\rvert^2\lvert S(\omega+\Delta\omega)\rvert^2 d\omega$

- ***Observation***:

FDB on a high-frequency broadband $S(\omega)$ using lower-frequency $\Delta\omega$ ← Equivalent to within a scaling factor → **DASB** on a lower-frequency narrowband $S(\omega)$ centered at $\Delta\omega$

- **Effect of Source Center Frequency and Bandwidth:**



SIMULATION PARAMETERS	
$S(\omega)$	Chirp over frequency $\left[f_o - \frac{W}{2}, f_o - \frac{W}{2}\right]$
θ_s	10°
N	16
d	$3.75\ m$
c	$1500\ m/sec$
$\Delta\omega$	$2\pi \cdot 1523\ rad/sec$
R	$2000\ m$
DOA Search	Range: $5^\circ - 10^\circ$ Resolution: 0.1°
SNR	60 dB

DOA Error Definition
error = $\frac{\lvert\theta_s - \hat{\theta}_s\rvert}{\theta_s}$

- ***Observation***: At high SNR, the effect of changes in source f_o and W on DOA error is noticeably less severe in FDB than DASB
- ***Observation***: There exists a lower bound on the source W in which FDB will produce no significant error in DOA

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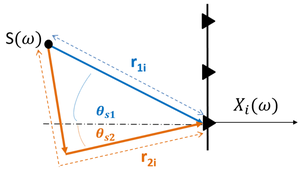
VI. TWO PROPAGATION PATHS

- **Model for Received Signal:**

$$X_i(\omega) = S(\omega)\left[\frac{e^{-j\omega\frac{r_{1i}}{c}}}{r_{1i}} + \frac{e^{-j\omega\frac{r_{2i}}{c}}}{r_{2i}}\right] + N_i(\omega)$$

where $N_i(\omega) = \mathcal{F}\{\, n_w(t)\,\}$ (white Gaussian noise)



- **Comparing DOA Estimator Equations:**
  - DASB at one frequency $\omega = \Delta\omega$: same as one propagation path

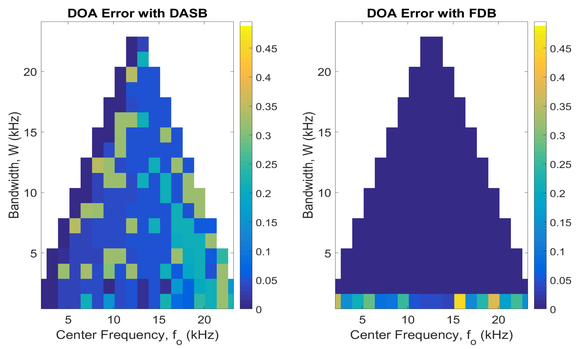
$$\hat{\theta}_s = \underset{\theta}{\operatorname{argmax}}\ \boldsymbol{a}^H(\theta)\boldsymbol{G}\boldsymbol{a}(\theta) \text{ where } \boldsymbol{G} = [\boldsymbol{G}]_{ik} = X_i(\Delta\omega)X_k^*(\Delta\omega) \text{ and } \boldsymbol{a}(\theta) = [\boldsymbol{a}(\theta)]_i = e^{-j(\Delta\omega)(i)\left(\frac{d\sin(\theta)}{c}\right)}$$

  - Matrix $\boldsymbol{G}$ with received signal model (assuming no noise):

Narrowband DASB	FDB
$\boldsymbol{G} = [\boldsymbol{G}]_{ik} =$ $\frac{e^{-j\Delta\omega\left(\frac{r_{1i}-r_{1k}}{c}\right)}}{r_{1i}r_{1k}}\lvert S(\Delta\omega)\rvert^2 +$ $\frac{e^{-j\Delta\omega\left(\frac{r_{1i}-r_{2k}}{c}\right)}}{r_{1i}r_{2k}}\lvert S(\Delta\omega)\rvert^2 +$ $\frac{e^{-j\Delta\omega\left(\frac{r_{2i}-r_{1k}}{c}\right)}}{r_{2i}r_{1k}}\lvert S(\Delta\omega)\rvert^2 +$ $\frac{e^{-j\Delta\omega\left(\frac{r_{2i}-r_{2k}}{c}\right)}}{r_{2i}r_{2k}}\lvert S(\Delta\omega)\rvert^2 +$	$\boldsymbol{G} = [\boldsymbol{G}]_{ik} =$ $\frac{e^{-j\Delta\omega\left(\frac{r_{1i}-r_{1k}}{c}\right)}}{r_{1i}r_{1k}}\frac{1}{r_{1i}r_{1k}}\int_{\omega_1}^{\omega_2}\lvert S(\omega)\rvert^2\lvert S(\omega+\Delta\omega)\rvert^2 d\omega +$ $\frac{e^{-j\Delta\omega\left(\frac{r_{1i}-r_{2k}}{c}\right)}}{r_{1i}r_{2k}}\frac{1}{r_{1i}r_{2k}}\int_{\omega_1}^{\omega_2}\lvert S(\omega)\rvert^2\lvert S(\omega+\Delta\omega)\rvert^2 d\omega +$ $\frac{e^{-j\Delta\omega\left(\frac{r_{2i}-r_{1k}}{c}\right)}}{r_{2i}r_{1k}}\frac{1}{r_{2i}r_{1k}}\int_{\omega_1}^{\omega_2}\lvert S(\omega)\rvert^2\lvert S(\omega+\Delta\omega)\rvert^2 d\omega +$ $\frac{e^{-j\Delta\omega\left(\frac{r_{2i}-r_{2k}}{c}\right)}}{r_{2i}r_{2k}}\frac{1}{r_{2i}r_{2k}}\int_{\omega_1}^{\omega_2}\lvert S(\omega)\rvert^2\lvert S(\omega+\Delta\omega)\rvert^2 d\omega +$ $\int_{\omega_1}^{\omega_2}\lvert S(\omega)\rvert^2\lvert S(\omega+\Delta\omega)\rvert^2[\text{sum of cross terms}]d\omega$

- ***Observation***: FDB is again similar in form to low-frequency narrowband DASB but with the addition of extra cross terms

- **Effect of Source Center Frequency and Bandwidth:**



SIMULATION PARAMETERS	
$S(\omega)$	Chirp over frequency $\left[f_o - \frac{W}{2}, f_o - \frac{W}{2}\right]$
θ_s	7.5° *and* 12.5°
N	16
d	$3.75\ m$
c	$1500\ m/sec$
$\Delta\omega$	$2\pi \cdot 1523\ rad/sec$
R	$2000\ m$
DOA Search	Range: $5^\circ - 10^\circ$ Resolution: 0.1°
SNR	60 dB

DOA Error Definition
error = $0.5\left(\frac{\lvert\theta_{s1} - \hat{\theta}_{s1}\rvert}{\theta_{s1}} + \frac{\lvert\theta_{s2} - \hat{\theta}_{s2}\rvert}{\theta_{s2}}\right)$

- ***Observation***: Under two propagation paths, the results are similar to the case of one path. FDB is still less affected by changes in source f_o and W, and a lower bound on W still exists

VII. CONCLUSIONS AND FUTURE WORK

- In terms of estimator equations, FDB compares very well in form to low-frequency narrowband DASB under one propagation path, but it includes an additional set of cross terms in the case of two propagation paths
- In terms of DOA error, conditions in f_o and W appear to exist under which FDB outperforms DASB. A lower bound for W in FDB is especially noticeable in the case of two propagation paths likely due to cross terms
- **Question: what are the conditions on W that will minimize the effect of cross terms under two propagation paths?**