Frequency Difference Beamforming with Sparse Arrays

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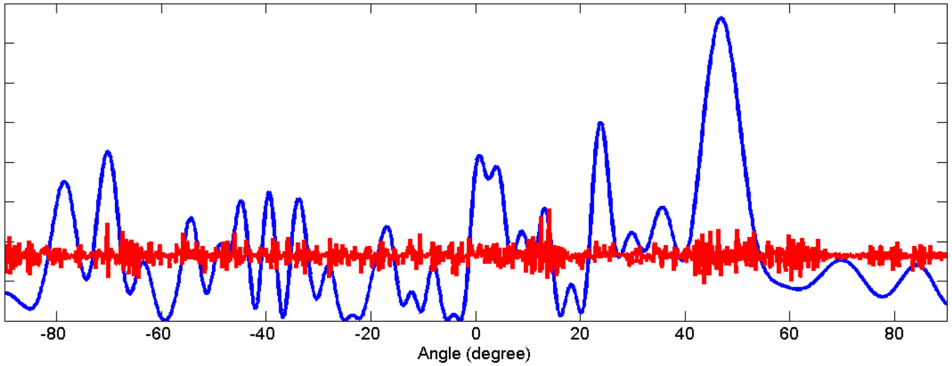

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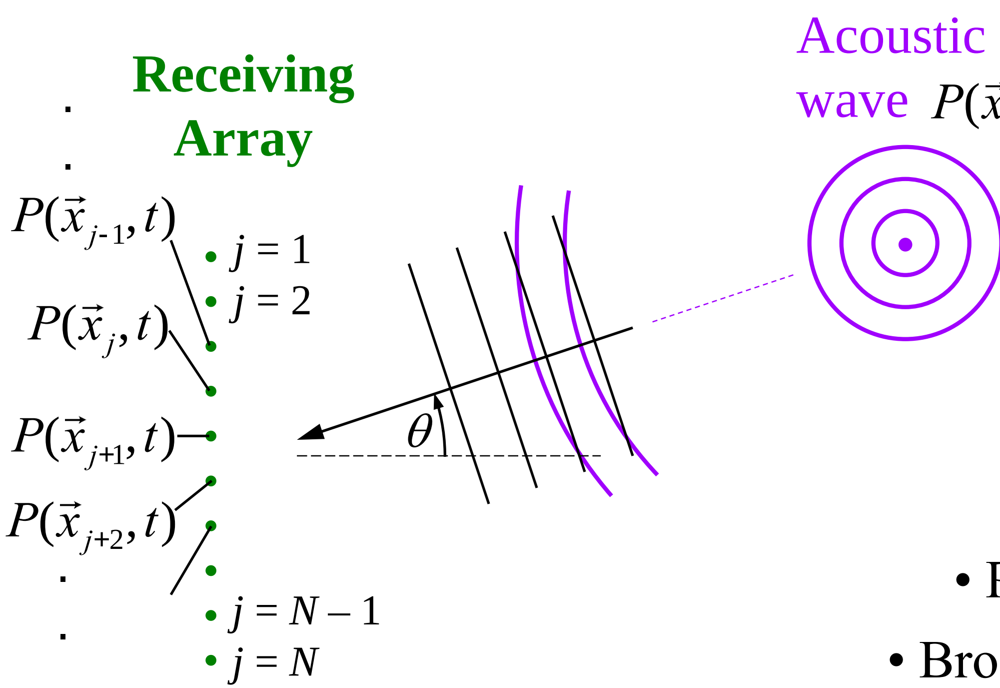
Motivation

1. Identifying source-waveform characteristics: 2pUW7
2. Conventional plane-wave beamforming deteriorates at high frequencies because of:
   - side lobes, and
     - actual vs. model wave-front mismatch

Can lower frequency information be *manufactured* from a *difference* of higher frequency information?

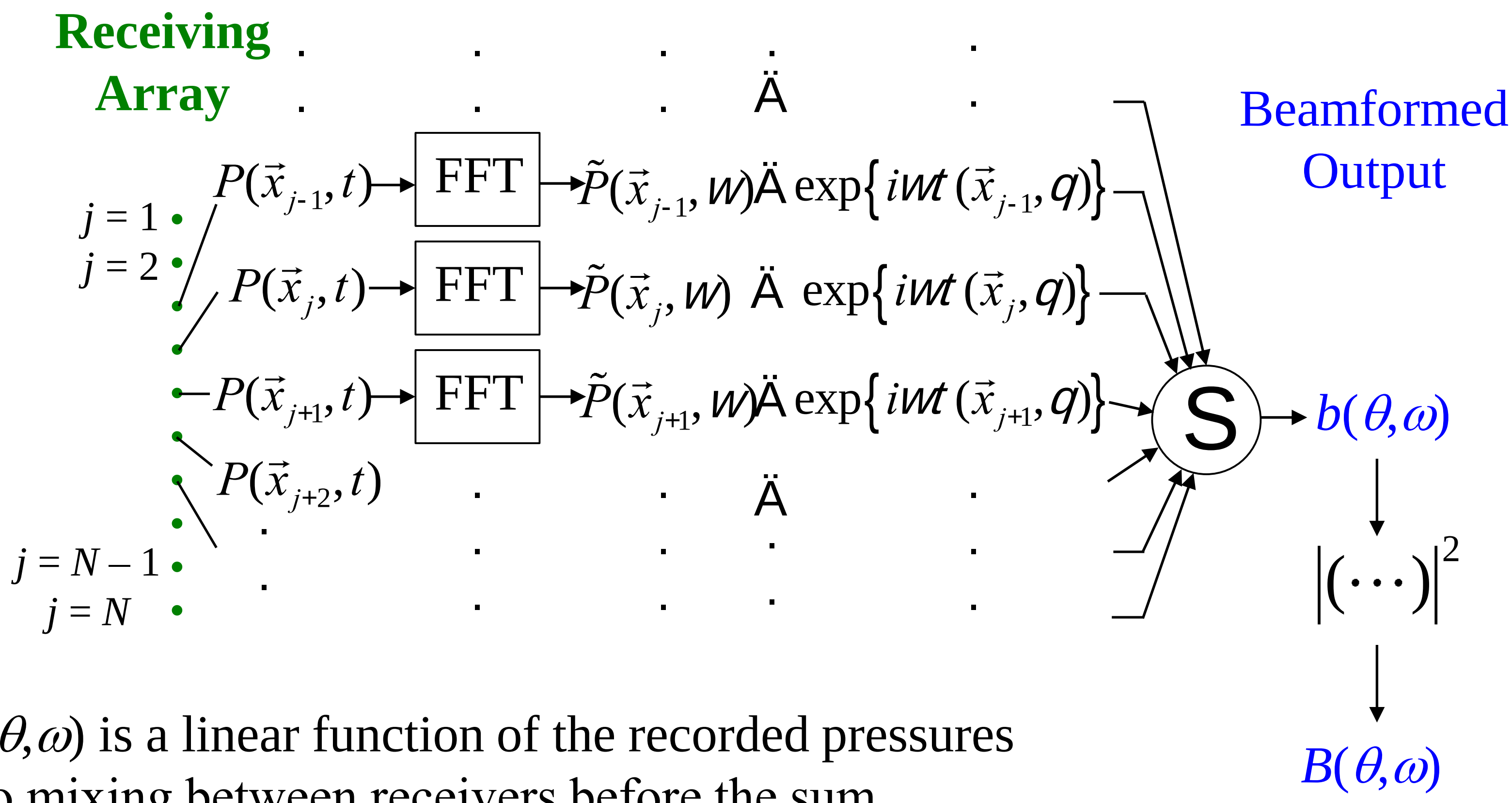
Three scenarios are considered.

First Beamforming Scenario



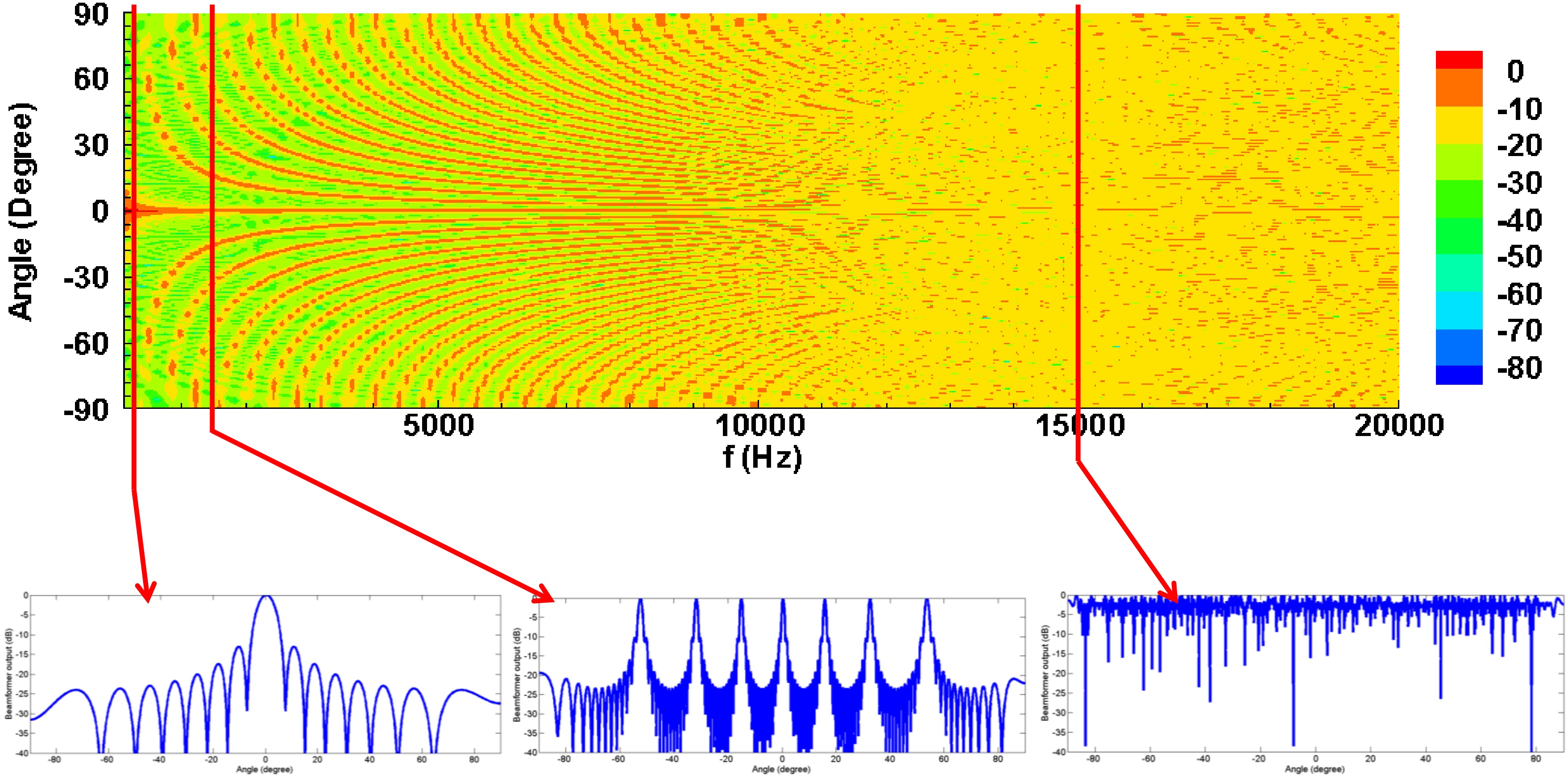
- **Free space**
- c = 1500 m/s
- Linear vertical array
- 16 elements
- Element spacing d = 3.75 m
- Point source; curved wave fronts
- Broadband: 100 Hz $\leq f \leq$ 20 kHz
- Source-array range R = 2.2 km
- Beam steering angle to the source $\theta = 0^\circ$

Conventional Plane-Wave Beamforming



- $b(\theta,\omega)$ is a linear function of the recorded pressures
- No mixing between receivers before the sum
- $\tau = (j-1)(d/c)\sin\theta$ for a vertical array with element spacing = d

Conventional Beamforming Output



200 Hz: results are good

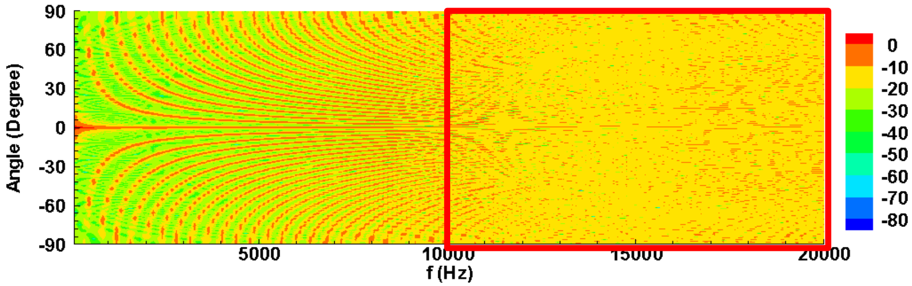
1.50 kHz: side lobes

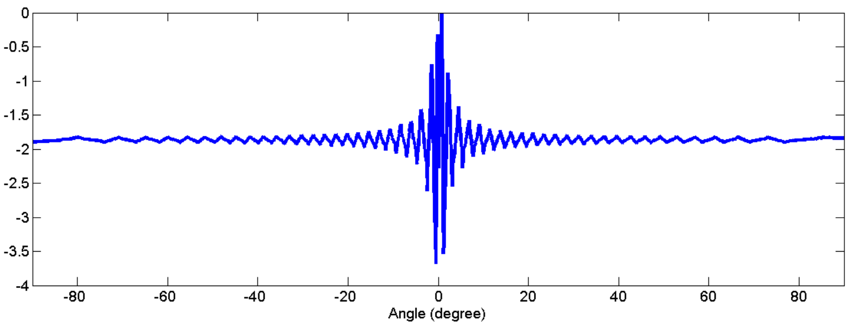
15.0 kHz: array is too sparse

Conventional Beamforming Limitations

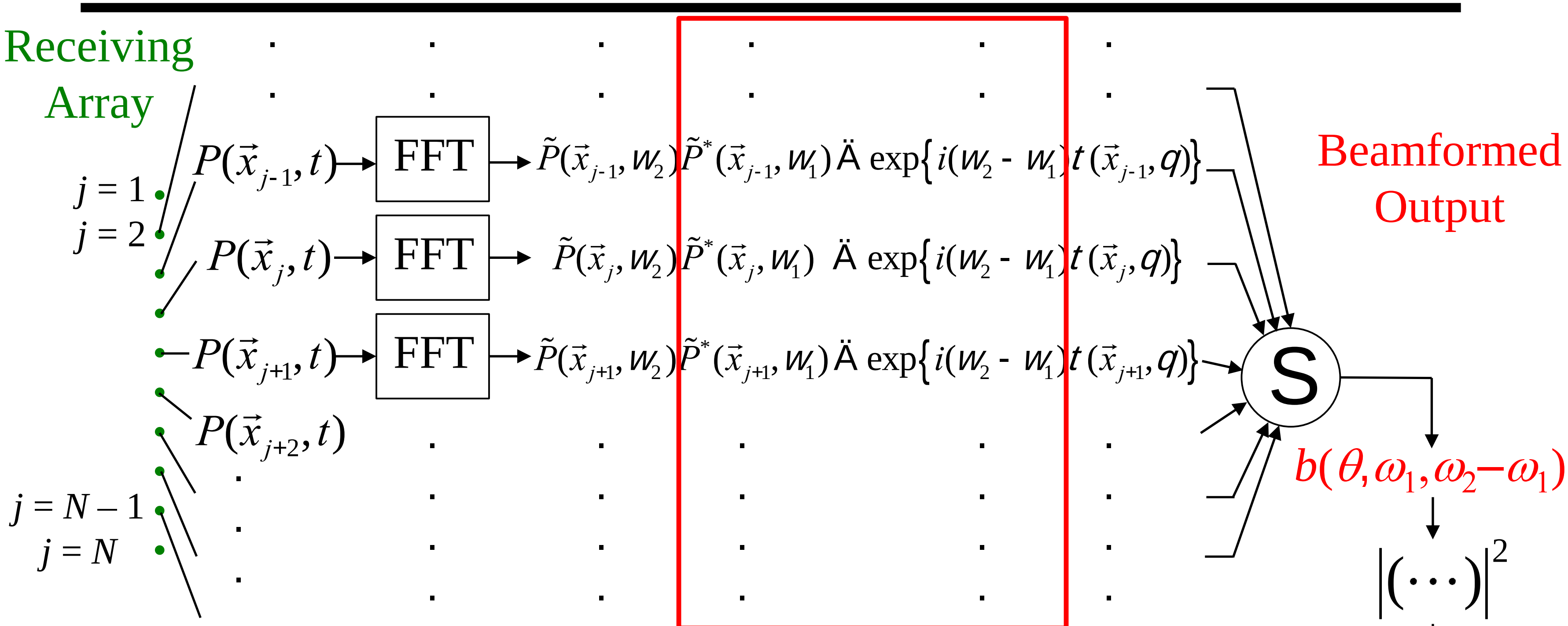
- Side lobes may be present when $d/\lambda > ½$
- Plane-wave/spherical-wave mismatch exists when: $\frac{L_A^2}{4\lambda R} > 1$
- What if the source-signal bandwidth is 10 kHz to 20 kHz?

Frequency-Averaged Results





Frequency-Difference Beamforming

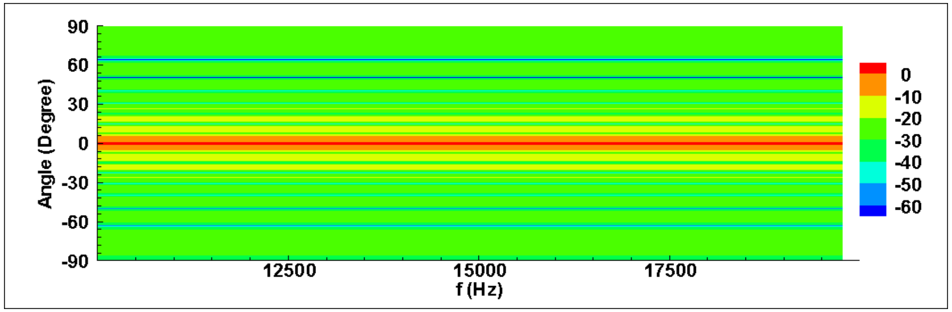


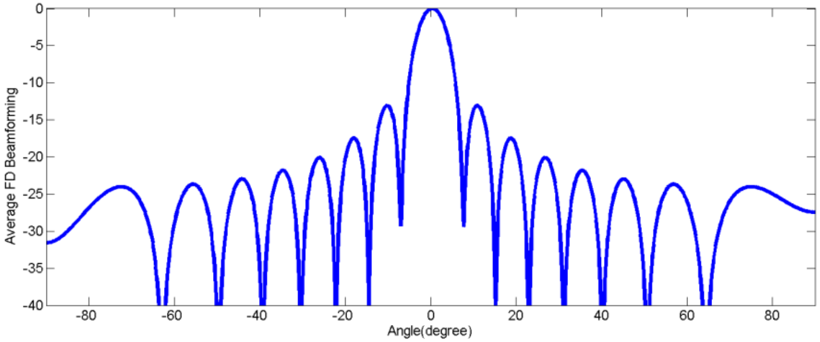
- $b(\theta,\omega_1,\omega_2-\omega_1)$ is a quadratic function of the recorded pressures
- No mixing between receivers before the sum
- $\tau = (j-1)(d/c)\sin\theta$ for a vertical array with element spacing = d
- The quadratic nonlinearity is reminiscent of the parametric array [Westervelt, 1963]

Δ*f* Beamforming, 10 kHz to 20 kHz

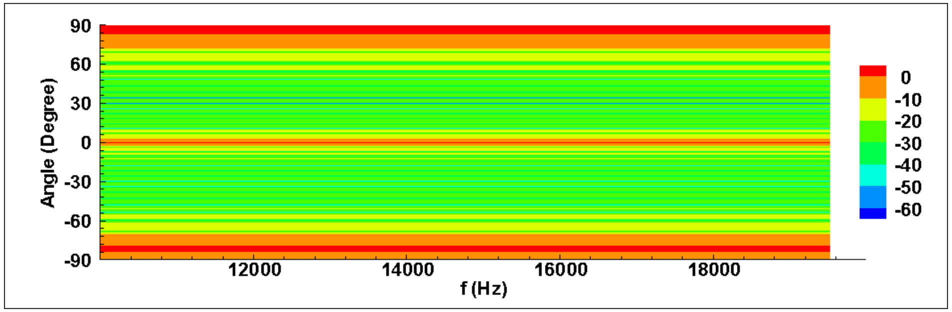
- Δ*f* = 200 Hz

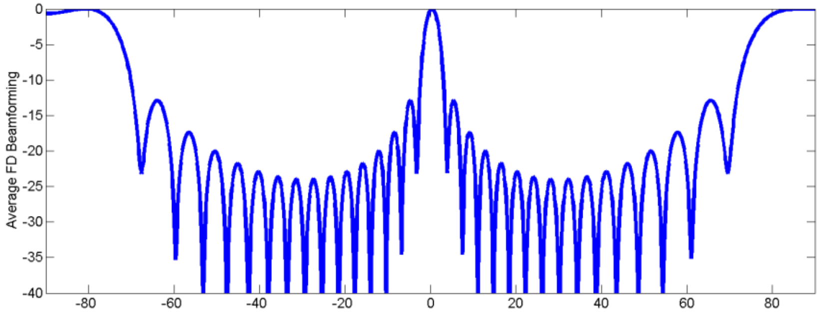
Frequency-Averaged Results





- Δ*f* = 400 Hz



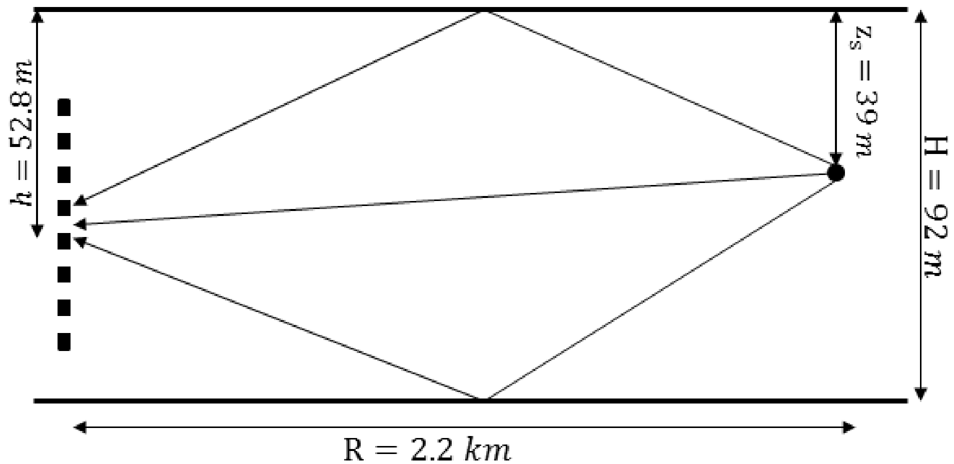


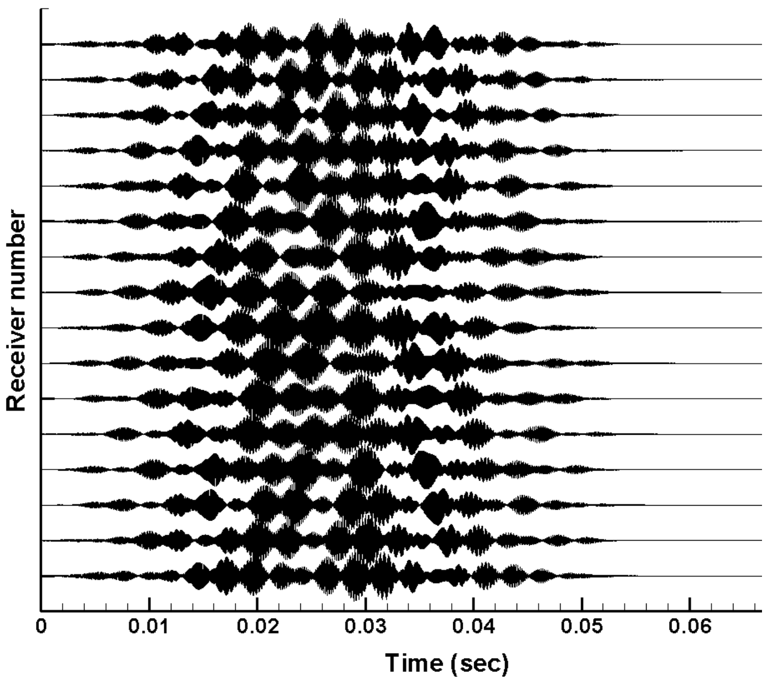
- What about a 10 kHz to 20 kHz signal with multipath?

Second Beamforming Scenario

Simple Simulations of FAF06 Experiment

- Three paths @ angles: – 2.4°, 0.3°, +2.6°
- Tapered chirp, 60 ms, 10 kHz to 20 kHz
- Same receiving array: linear, vertical, 16 phones, 3.75 m spacing, etc.

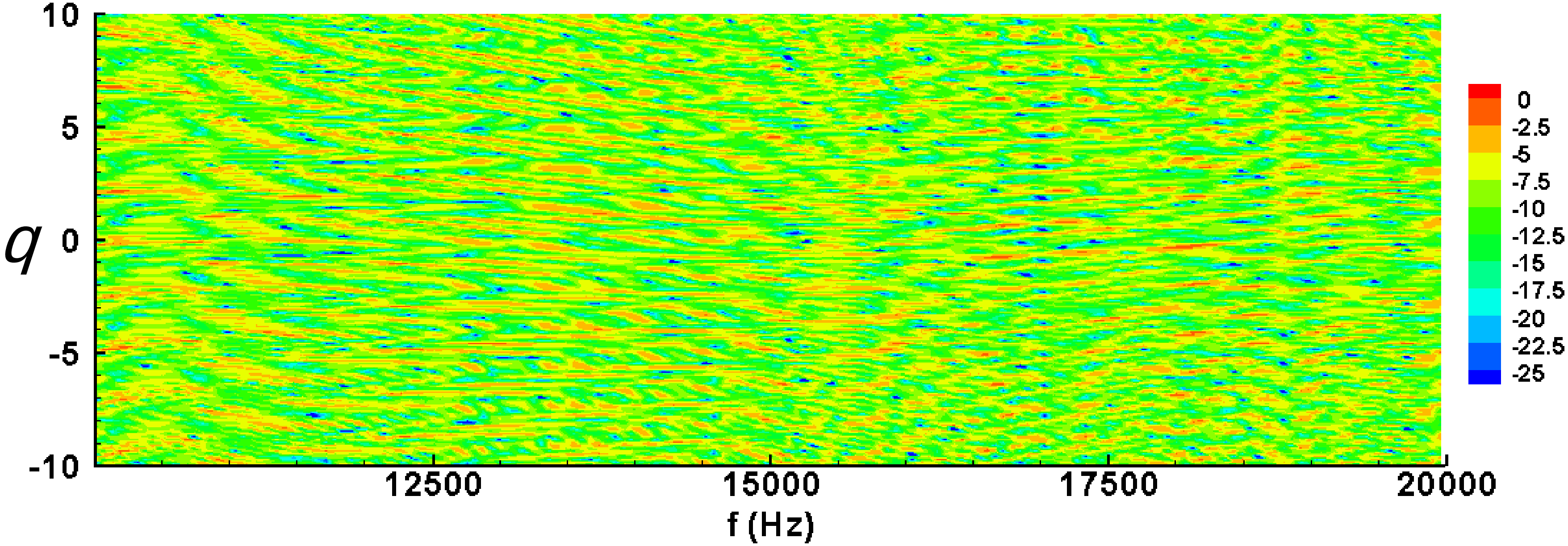




d/λ_c = 37.5; the array is sparse!

For these conditions, a *manufactured* frequency of $\Delta f \sim 1.5$ kHz is needed to resolve the ray paths

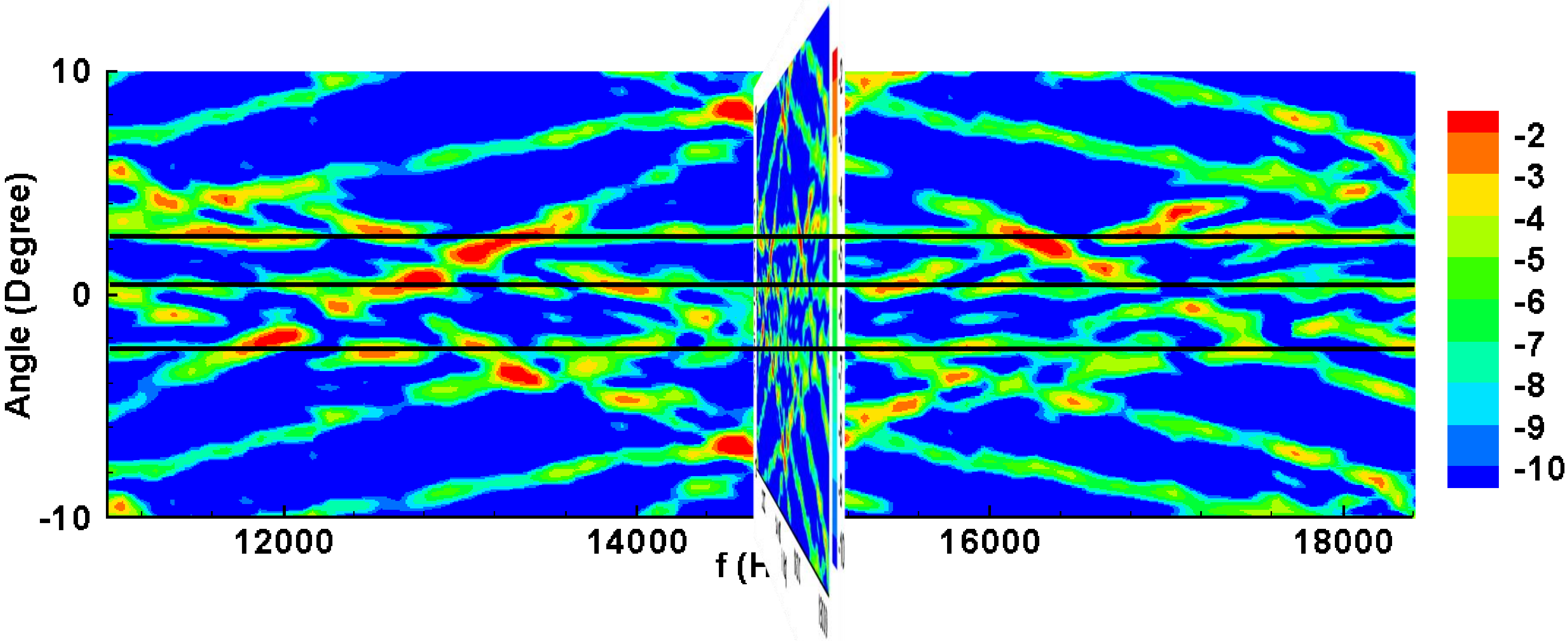
Conventional Beamforming: 10 kHz to 20 kHz



$$kL_A = \frac{2pf_c}{\bar{c}} L_A \sim 3500, \quad \frac{d}{l} = 37.5$$

Δf Beamforming: 10 kHz to 20 kHz

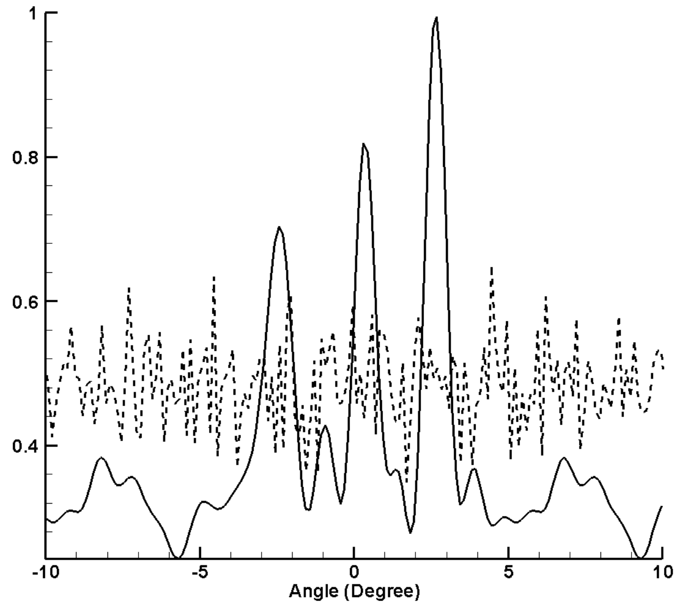
Δf = 1562 Hz



$$kL_A = \frac{2\pi\Delta f}{\bar{c}} L_A \sim 368, \quad \frac{d}{"\lambda"} = 3.9$$

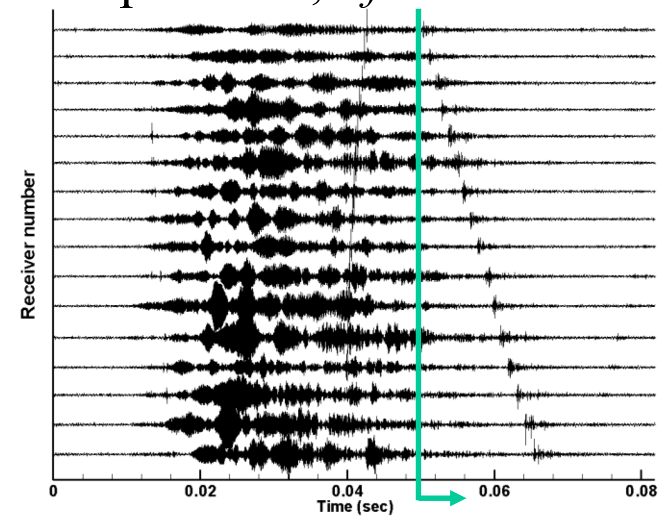
Integrated Δ*f* Beamforming

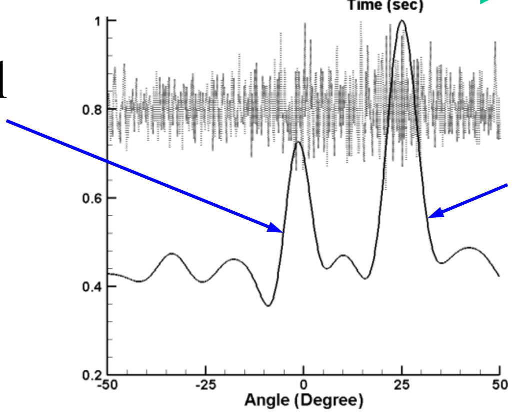
Simulations
Δ*f* = 1562 Hz



- - - - Conventional Beamforming,
—— Δ*f* Beamforming

FAF06 Experiment, Δ*f* = 195 Hz





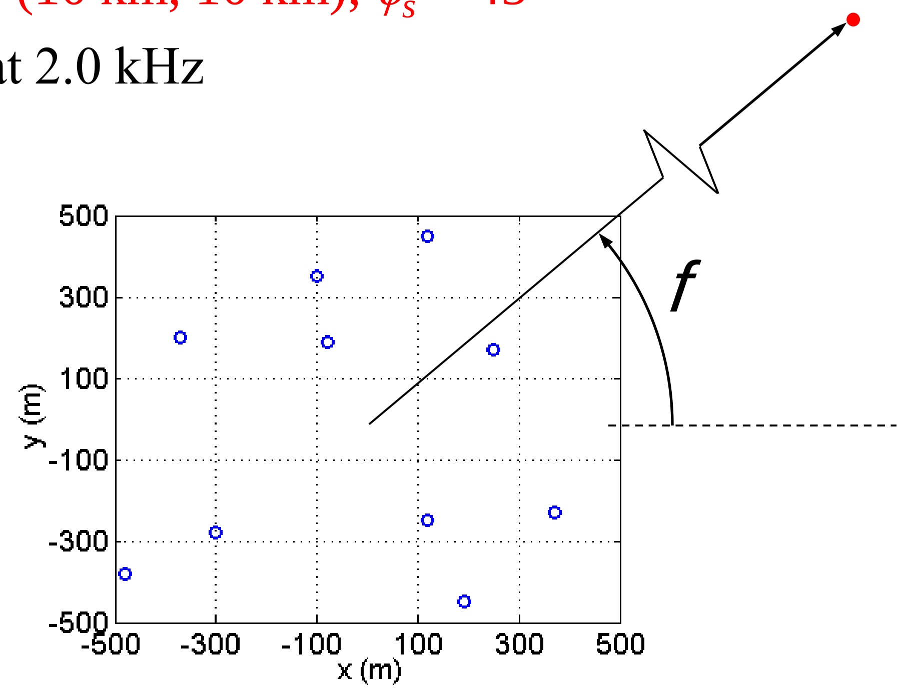
Third Beamforming Scenario

Simple Three-Dimensional Range-Independent Environment

- Same Sound Channel: 92 m depth, c = 1500 m/s, 3 paths, etc.
- Source: mid-water column, (x_s, y_s) = (10 km, 10 km); $\phi_s = 45^\circ$
- Signal: Frequency sweep, centered at 2.0 kHz
  500 Hz bandwidth
- Receiving Array:
  10 elements, 30 m depth,
  ~300 m average spacing

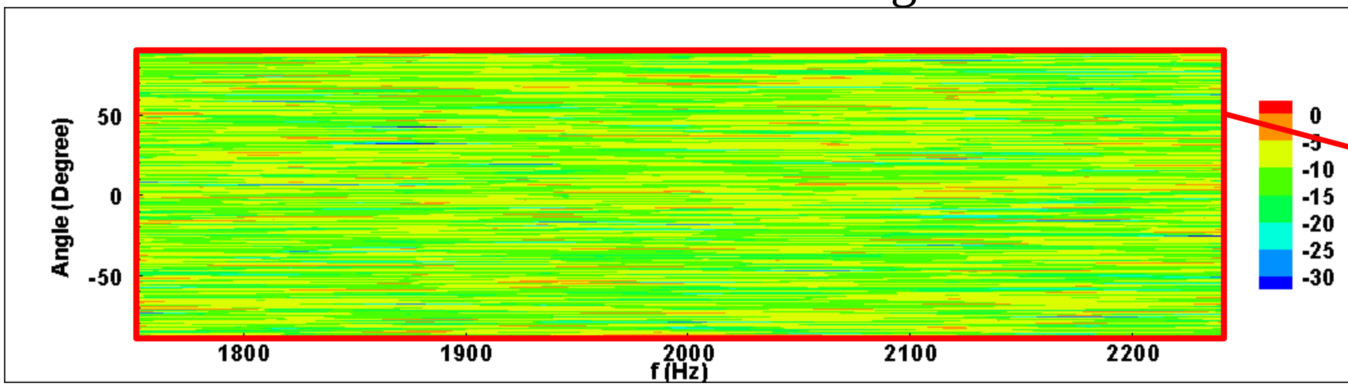
Top view of the horizontal
***x-y* plane**

d/λ_c ~ 100's; the array is **sparse!**

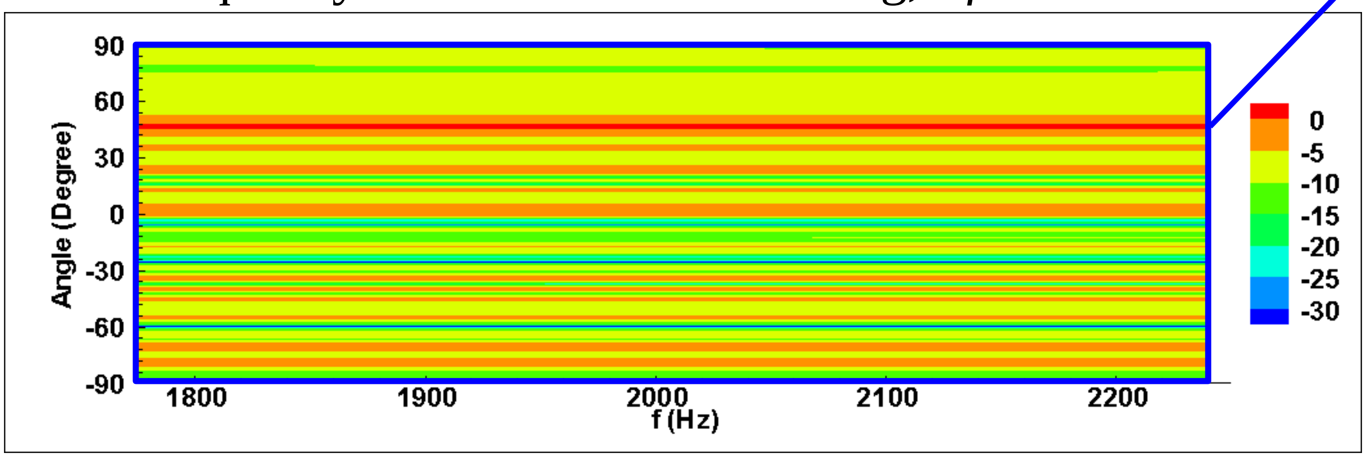


3D Env. Random Horizontal Array Results

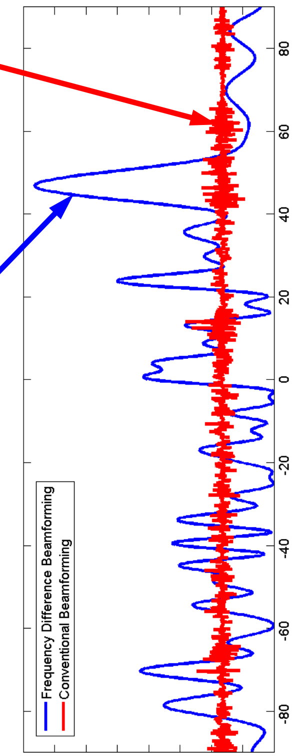
Conventional Beamforming



Frequency Difference Beamforming, Δf = 12.2 Hz



Frequency-Averaged Results



Conclusions

- Frequency difference beamforming allows sparse arrays to be used for acoustic ray-path direction finding
- Frequency difference beamforming is robust because of frequency downshifting.
  - Reduces side lobes
  - Reduces sensitivity to wave-front modeling errors
  - Successful in simulations of sparse random arrays with multipath
- Frequency difference beamforming may have applications beyond acoustic ray-path direction finding.