## Low Frequency Beamforming in Shallow Water Environments

Shima Abadi (abadi@uw.edu), Collette Amaratunge, Gavin Boyd, Mathew Daniels and Derek Thurmer University of Washington Bothell, Dept. of Engineering & Math., Bothell, WA 98011

## Abstract

Beamforming is a spatial filtering method that uses the ray theory to estimate the arrival angle from the array recordings that fails when water depth is smaller compared to the acoustic wavelength (shallow-water low-frequency propagation).

## Theory

In shallow water, the underwater sound propagation is modeled by normal-modes theory in which each mode propagates at a and range on beamforming. different incident angle and the total field is composed of a discrete sum of the  $0^\circ$  bearing angle propagation modes excited at the broadcast 30° bearing angle  $60^{\circ}$  bearing angle frequency. Μ

$$P(\omega, r, z) = -i\pi e^{i\omega t} \sum_{m=1}^{m} \psi_m(z_s) \psi_m(z) H_0^{(2)}$$

$$\frac{ie^{-\frac{i\pi}{4}}}{\rho\sqrt{8\pi r}}\sum_{m=1}^{M}\psi_m(z_s)\psi_m(z)\frac{e^{ik_{rm}r}}{\sqrt{k_{rm}}}$$

Beamforming techniques use recordings to determine wave propagation directions by relying on time delays to determine weights from a plane-wave signal model and is given by:

$$b(\theta,\omega) = \sum_{n=1}^{N} P(\omega,r_n,z_n)e^{-ik\left(\frac{N+1}{2}-n\right)dst}$$

## Simulation

The normal mode propagation algorithm KRAKEN (Porter and Reiss, 1984) is used to propagate a single frequency signal through a two-layer waveguide.





bearing angle.